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Introduction to Artificial Intelligence, Winter Term 2021
Problem Set 7

Discussion: December 4 - December 9

Exercise 7-1

If there are three stacked blocks. The top one is green and the bottom one is not green. Is there a green block directly on top of a non-green block? Answer the question using resolution.

Solution:

1. $\{green(top)\}$ (Premises)
2. $\{\neg green(bottom)\}$ (Premises)
3. $\{on(top, middle)\}$ (Premises)
4. $\{on(middle, bottom)\}$ (Premises)
5. $\{\neg green(x), green(y), \neg On(x, y)\}$ (negation of the consequant in clausal NF)
6. $\{\neg On(top, y), green(y)\}$ (from 1,5, $\mu = \{top/x\}$)
7. $\{green(middle)\}$ (3,6, $\mu = \{middle/y\}$)
8. $\{\neg On(middle, y), green(y)\}$ (5,7, $\mu = \{middle/x\}$)
9. $\{green(bottom)\}$ (4,8, $\mu = \{bottom/y\}$)
10. $\{\}$ (2,9,$\mu = \{\}$)

Hence, Contradiction.

Exercise 7-2

Using resolution, together with demodulation, prove that (c) follows from (a) and (b).

a) $\forall x Married(Father(x), Mother(x))$

b) $Father(John) = Bill$

c) $Married(Bill, Mother(John))$

Solution:

Statement (c) comes from directly applying the demodulation rule where

- $t1 = Father(John)$

- $t2 = Bill$
- $t3 = Father(x)$

Exercise 7-3 (From R&N)

From "Horses are animals," it follows that "The head of a horse is the head of an animal." Demonstrate that this inference is valid by carrying out the following steps:

a) Translate the premise and the conclusion into the language of first-order logic. Use three predicates: $HeadOf(h, x)$, $Horse(x)$, and $Animal(x)$.

b) Negate the conclusion, and convert the premise and negated conclusion into implicative normal form.

c) Use resolution to show that the conclusion follows from the premise.

Solution:

Premises: $\forall x[Horse(x) \Rightarrow Animal(x)]$

Conclusion (to be proved): $\forall x, y[(Horse(x) \wedge HeadOf(y, x)) \Rightarrow \exists w[Animal(w) \wedge HeadOf(y, w)]]$

Negating the conclusion: $\exists x, y[\neg[(Horse(x) \wedge HeadOf(y, x)) \Rightarrow \exists w[Animal(w) \wedge HeadOf(y, w)]]]$

- Represent the knowledge base in INF:

1. remove $\Rightarrow$:
   $\forall x[\neg Horse(x) \vee Animal(x)]$
   $\wedge \exists x, y[\neg[\neg(Horse(x) \wedge HeadOf(y, x)) \vee \exists w[Animal(w) \wedge HeadOf(y, w)]]]$

2. Push $\neg$ inwards:
   $\forall x[\neg Horse(x) \vee Animal(x)]$
   $\wedge \exists x, y[Horse(x) \wedge HeadOf(y, x) \wedge \forall w[\neg Animal(w) \vee \neg HeadOf(y, w)]]$

3. Standardize Apart:
   $\forall u[\neg Horse(u) \vee Animal(u)]$
   $\wedge \exists x, y[Horse(x) \wedge HeadOf(y, x) \wedge \forall w[\neg Animal(w) \vee \neg HeadOf(y, w)]]$

4. Skolemize:
   $\forall u[\neg Horse(u) \vee Animal(u)]$
   $\wedge [Horse(H) \wedge HeadOf(A, H) \wedge \forall w[\neg Animal(w) \vee \neg HeadOf(A, w)]]$

5. Discard $\forall$:
   $[\neg Horse(u) \vee Animal(u)]$
   $\wedge [Horse(H) \wedge HeadOf(A, H) \wedge [\neg Animal(w) \vee \neg HeadOf(A, w)]]$

6. Put into CNF:
   $[\neg Horse(u) \vee Animal(u)]$
   $\wedge [Horse(H)]$

$\wedge[HeadOf(A, H)]$
$\wedge[\neg Animal(w) \vee \neg HeadOf(A, w)]$

7. Put into INF:
   $Horse(u) \Rightarrow Animal(u)$
   $true \Rightarrow Horse(H)$
   $true \Rightarrow HeadOf(A, H)$
   $Animal(w) \wedge HeadOf(A, w) \Rightarrow false$

- Apply Resolution Refutation:

  1. $Horse(u) \Rightarrow Animal(u)$ (Premises)
  2. $true \Rightarrow Horse(H)$ (Premises)
  3. $true \Rightarrow HeadOf(A, H)$ (Premises)
  4. $Animal(w) \wedge HeadOf(A, w) \Rightarrow false$ (Premises)
  5. $true \Rightarrow Animal(H)$ (1,2, $\mu = \{H/u\}$)
  6. $HeadOf(A, H) \Rightarrow false$ (4,5, $\mu = \{H/w\}$)
  7. $true \Rightarrow false$ (3,6, $\mu = \{\}$)

  Hence, Contradiction.