MODERN CONTROL THEORY
PROJECT-2

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NOTE: We have included all the Six cases in the MPC_Main, Since I took N_Sim = 1000 the run time will be a little longer i.e. around 15 secs. We can see whether the given case is controllable, Observable or not in the command window.

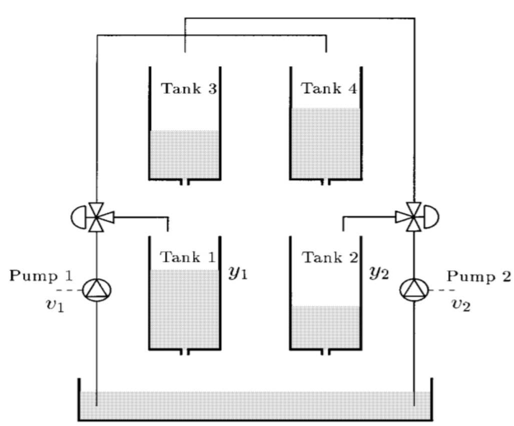
Model Predictive Control

Model Predictive Control is a method that is used to control a process while also making sure that certain constraints are met. This method has been in use in plants and industries since the 1980s. MPC involves the effects of past and the future control steps. In MPC, the controller predicts the states till the control horizon and then implements the first control move. After the move, it again predicts the next set of control moves and then implements the first move. The process is repeated till the set point is reached while making sure that the constraints are not violated. MPC contains a controlled variable and a manipulated variable. A cost function is set up between the predicted response and set point. This function is minimized to reach the set point faster.
$J = (R_s - Y)^T (R_s - Y) + \Delta U^T R \Delta U$ is the cost function.

In real life there is noise in the measurements and the process so to make sure these are handled properly we use Kalman Filtering to get a prediction of the states from the measurements and the state space model which is then fed to the MPC to reach the desired set point.

Problem

We have been given the same 4 tank problem from Project 1 which involved the implementation of Kalman Filtering. However in this Project we are given multiple cases in which the levels of two tanks are measured and the other two have to be controlled.



The initial states given are $[12.4 \quad 12.7 \quad 1.8 \quad 1.4]^T$ and the sampling time is 0.1s. Given constraints are

$DUmin = 5 * [-1\ \ -1]^T$
$DUmax = 5 * [1\ \ 1]^T$
$Umin = 0 * [-1\ \ -1]^T$
$Umax = 20 * [1\ \ 1]^T$

EXPLANATION FOR CODE:

There are 5 files **MPC_main.m, Kalman_init_.m, mpcgain_MIMO, Constraint_Matrices, Kalman_Filter.**

Run File: MPC_main.m

The first section of the file includes all the given cases, then in the second section Structure array of Constraints is added, then we have prediction horizon (Np), control horizon (Nc) and Number of iteration (N_sim) initialised in the next section. After that we called for Kalman_init_ function which gives the linearized State-space model as the output arguments for each case,then we called Mpcgain_MIMO function which gives required matrices for the cost function as output arguments. Next Setpoints for the controlled tank heights w.r.t initial heights (h0) are found. Then we found the coefficients of DeltaU in the cost function and called for the Constraint_Matrices function which gives the Constraint matrices as the output arguments. Next using quadprog function we optimized the cost function and found input change for Nc time samples (Deltau), from this we have used input change at the present instance (deltau) to find Present Input(u) and then sensor measurement of the measured tanks are found in the next steps. Next we called Kalman_Filter function to find the estimated states i.e. (X_k+) with which I found the controlled tank heights.Then we stored the required parameters and also stored the current states as previous states and continued the iterations. At the end of each case we plotted Measured tank heights,Controlled tank heights and Inputs.

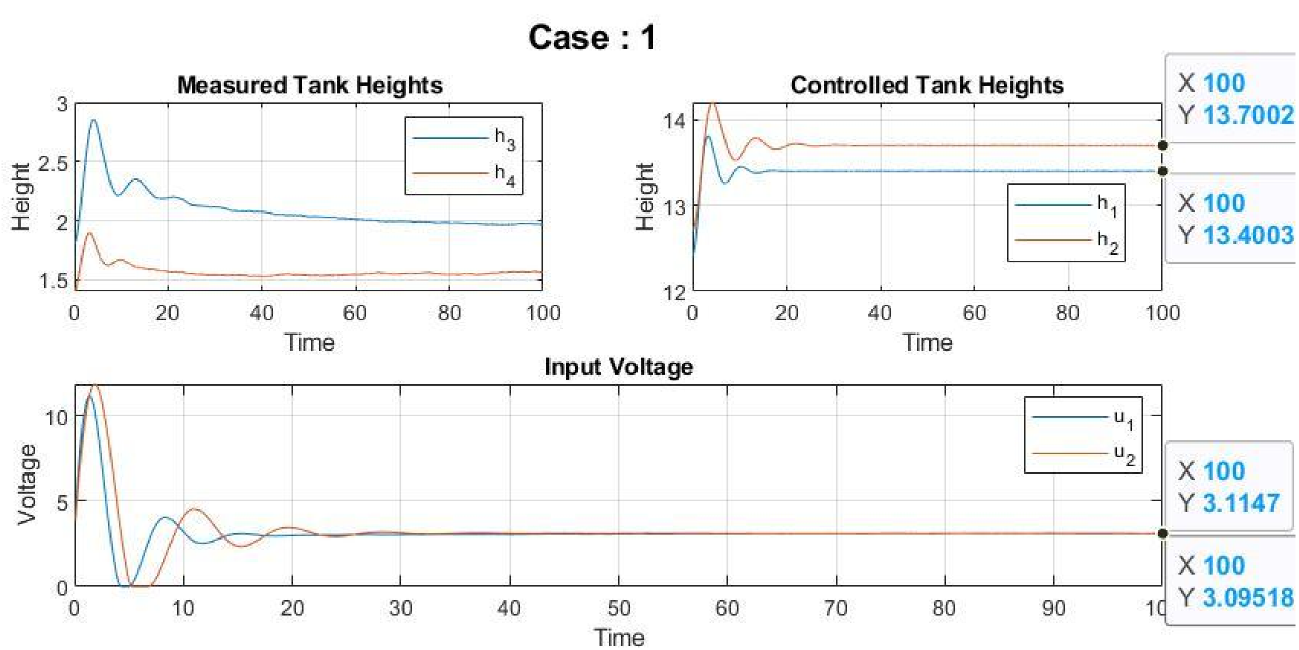
Assumptions:

Np = 8; Nc = 3; N_sim = 1000; input_weights = [0.5,0.5]; Q = 0.02*eye(4); R_m = 0.001*eye(2);

a)

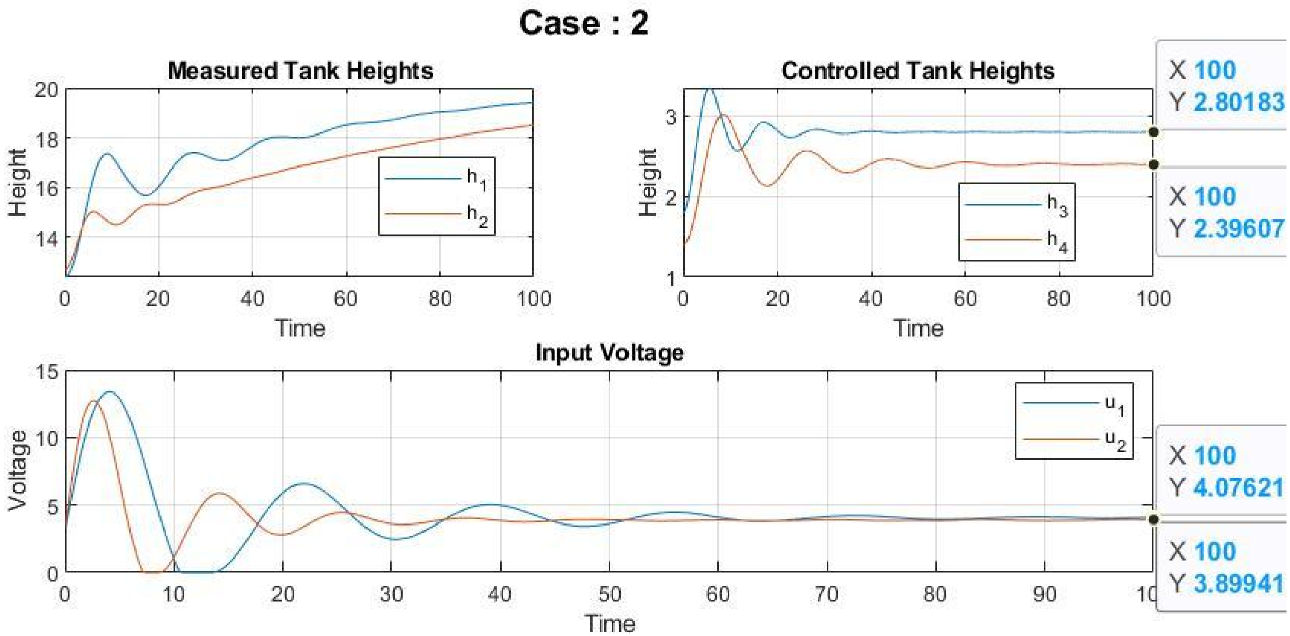
i)

Case 1: Measured heights = h3, h4 ;Controlled heights = h1, h2 ;Set_points : h1 = 13.4,h2 = 13.7 :



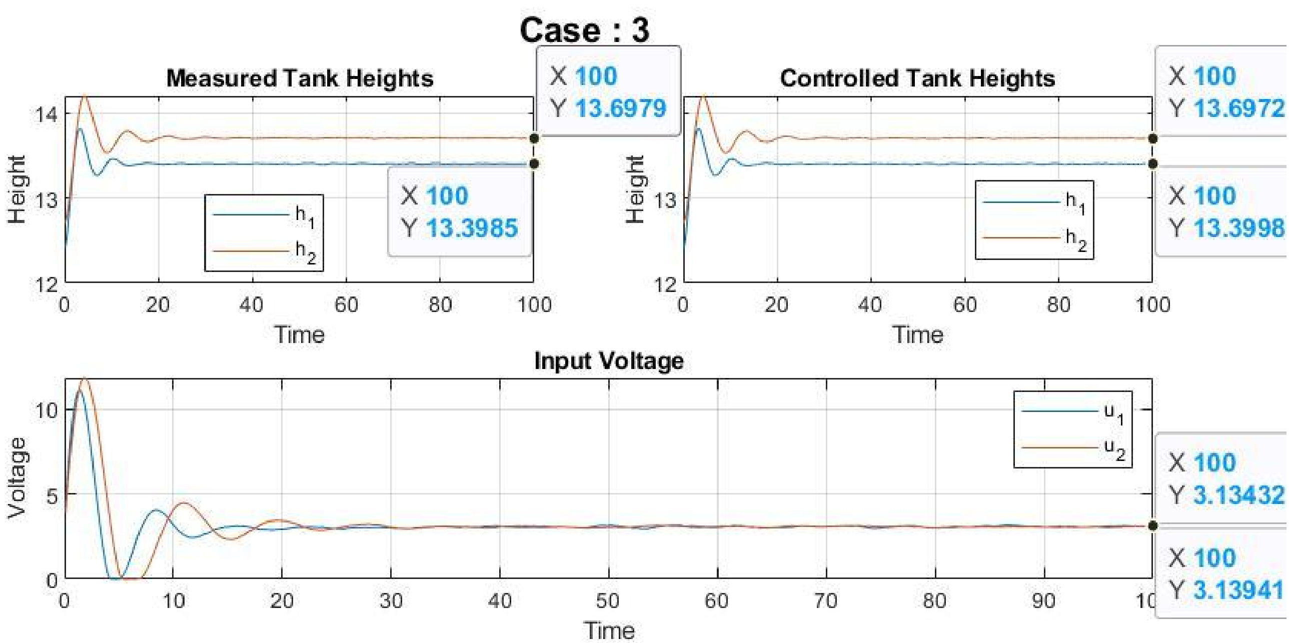
From the graph shown above we can see that h1,h2 attained steady state after t=25 secs. And also the heights [h1,h2] = [13.4003,13.7002] attained after t = 100secs are very close to the setpoints [13.4,13.7]. We can also notice that the Input voltages are at their minimum(Umin = 0) in between 4 secs to 8 secs and then reached steady values at t = 35 secs.

Case 2: Measured heights = h1, h2 ;Controlled heights = h3, h4 ;Set_points : h1 = 2.8 ,h2 = 2.4 :



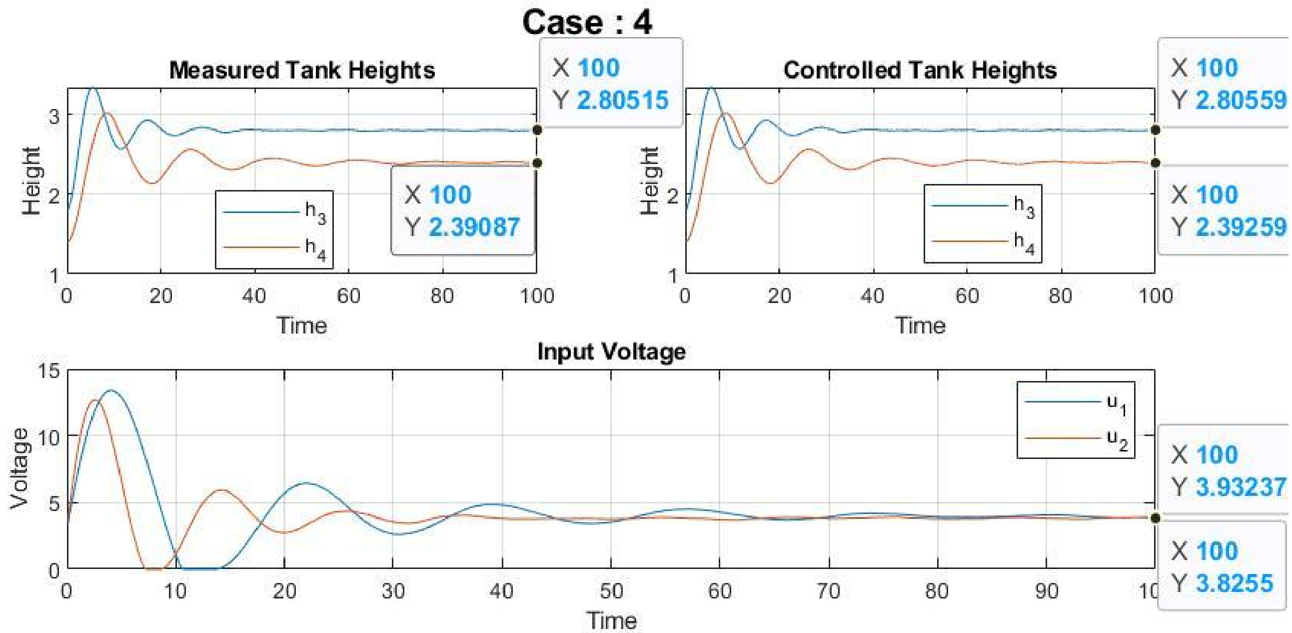
From the graph shown above we can see that h3,h4 nearly attained steady state after t=80 secs. And also the heights [h3,h4] = [2.80183,2.39607] attained after t = 100secs are very close to the setpoints [2.8,2.4]. We can notice that the Input voltages are at their minimum(Umin = 0) in between 8 secs to 15 secs and then reached nearly steady values at t = 80 secs. We can also see that h1,h2 are increasing even though Input voltage and Controlled tank heights attained steady states.

Case 3: Measured heights = h1, h2 ;Controlled heights = h1, h2 ;Set_points : h1 = 13.4 ,h2 = 13.7 :



From the graph shown above we can see that controlled h1,h2 attained steady state after t=35 secs. And also the heights [h1,h2] = [13.3998,13.6972] attained after t = 100secs are very close to the setpoints [13.4,13.7]. We can notice that the Input voltages are at their minimum(Umin = 0) in between 4 secs to 8 secs and then reached steady values at t = 30 secs. We can also see that measured h1,h2 and controlled h1,h2 are nearly the same, and the slight variation is because of measurement,process noises.

Case 4: Measured heights = h3, h4 ;Controlled heights = h3, h4 ;Set_points : h3 = 2.8 ,h4 = 2.4 :



From the graph shown above we can see that controlled h3,h4 nearly attained steady state after t=80 secs. And also the heights [h3,h4] = [2.80559,2.39259] attained after t = 100secs are very close to the setpoints [2.8,2.4]. We can notice that the Input voltages are at their minimum(Umin = 0) in between 7 secs to 15 secs and then reached nearly steady values at t = 80 secs. We can also see that measured h3,h4 and controlled h1,h2 are nearly the same, and the slight variation is because of measurement,process noises.

ii)

Yes, we can see good control performance in the above four cases. Even though in all the four cases the controlled tanks are reaching their set-points the settling time varies from case to case. We can see the settling time is longer for cases 2 and 4 when compared to cases 1 and 3.

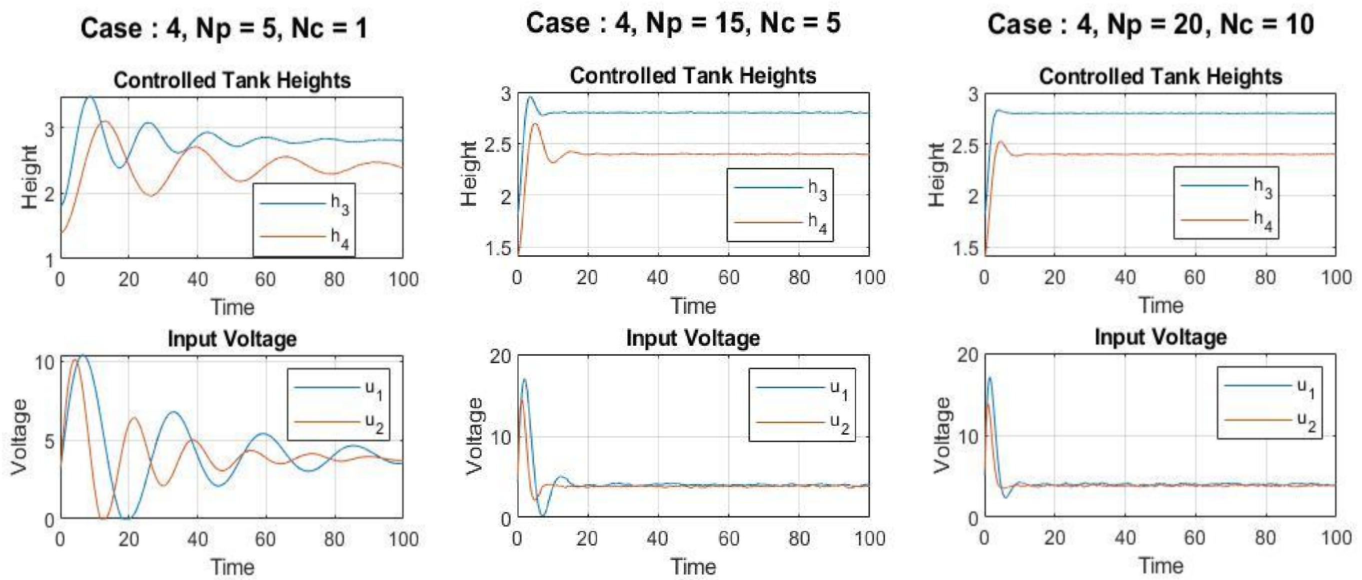
iii)

Here the Performance deterioration between the cases is due to MPC performance,because we have used the same Kalman filter for estimating all the four states and with the same initial conditions in every case. The only difference between the cases is measured heights,controlled heights, which brings the changes in MPC controller and this affects MPC performance.

iv) Ways to improve MPC Performance:

- **Tuning Prediction Horizon and Control Horizon**

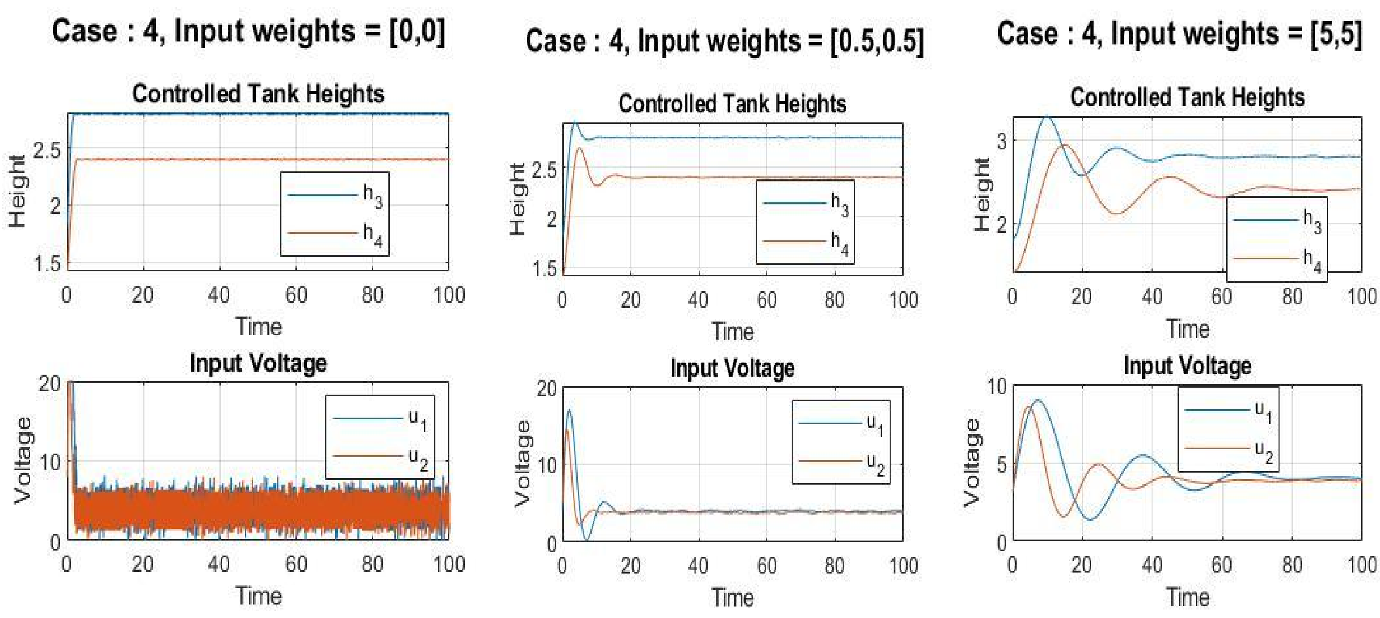
  When the control horizon and the prediction horizon were increased the MPC performance improved. The system reached the set point much faster,there was less overshoot and there were lesser oscillations once the system reached the set point. However the drawback of increasing the control horizon and the prediction horizon is the increased computational time required. So there exists a trade off between MPC performance and computational time.



After numerous iterations considering performance and computational time we found the optimum value for N_P and N_C to be 15 and 5 respectively.

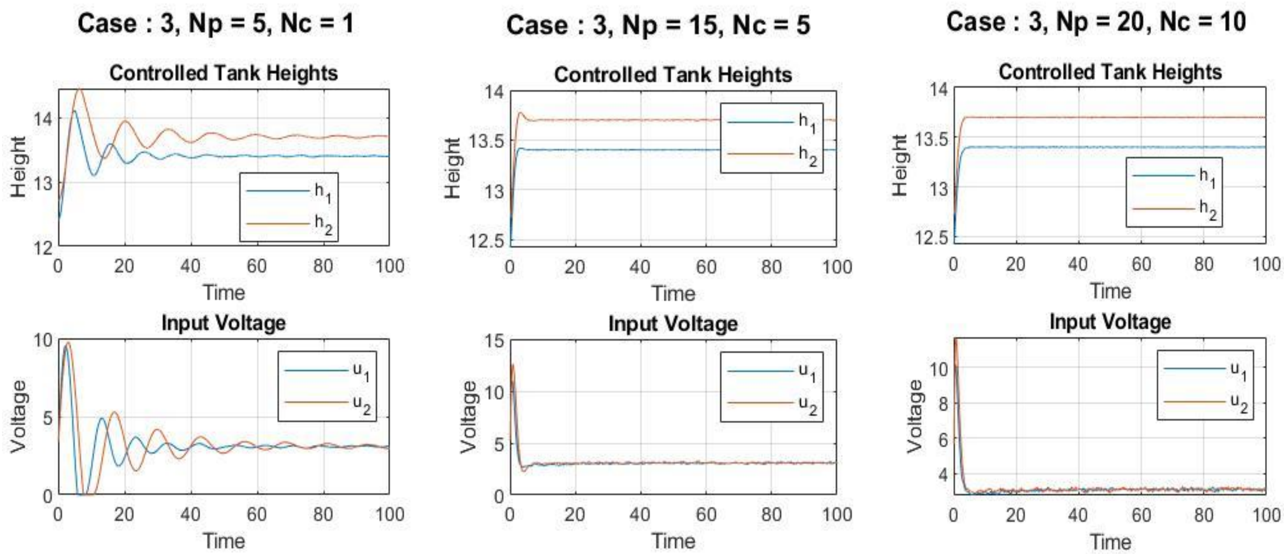
- **Tuning Input Weights:**

When the input weights are increased then the oscillations in the input voltage decreases but when the weights are increased the set points are reached later. So we have to select the optimum input weights to balance the time to reach the set point and the oscillations of input voltage.

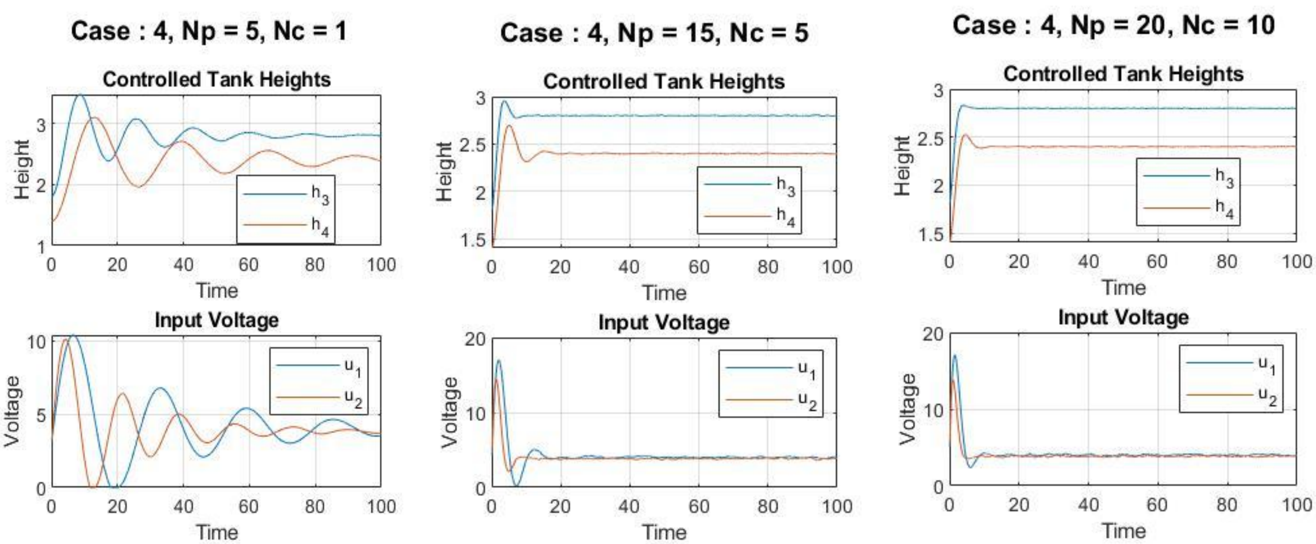


After numerous iterations considering performance and oscillations we found the optimum input weights to be [0.5, 0.5].

b) When the control horizon and the prediction horizon were increased the MPC performance improved. The system reached the set point much faster,there was less overshoot and there were lesser oscillations once the system reached the set point. This trend can be seen in both Case 3 and Case 4 as shown below. But the increase in the control horizon and the prediction horizon increased computational time required.

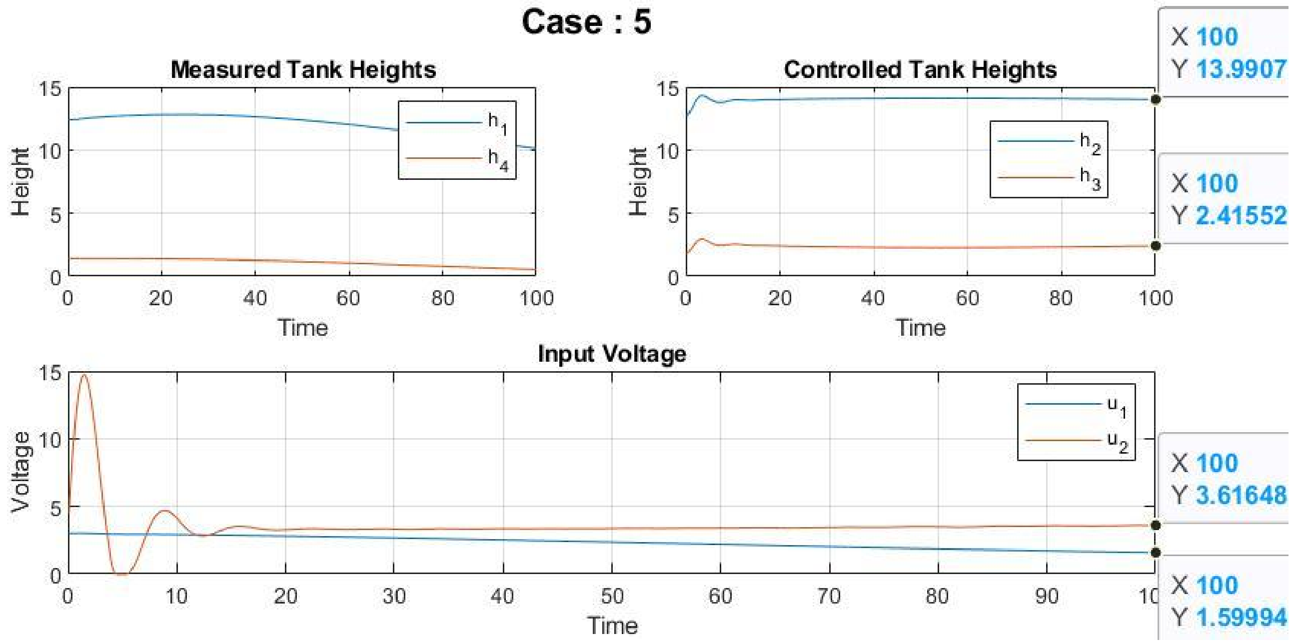


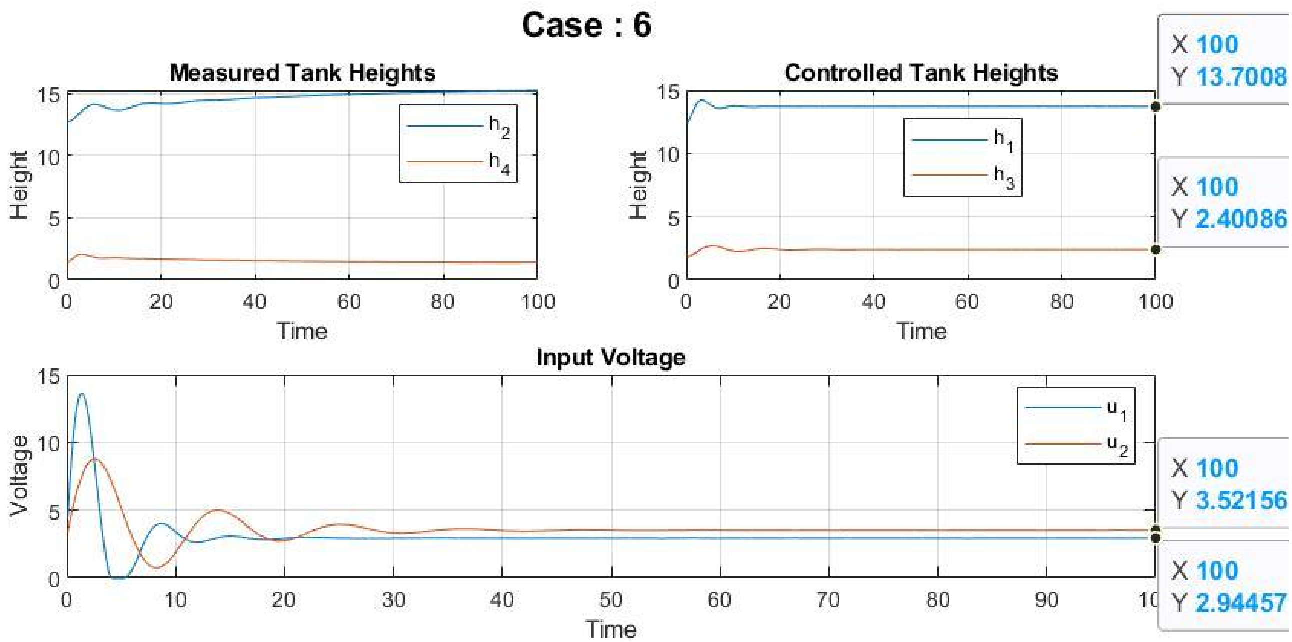
Effect of changes in Nc and Np on the MPC performance for **Case 3**



Effect of changes in Nc and Np on the MPC performance for **Case 4**

c)





In the figures shown above we can see that MPC is able to achieve the set point in the case of b) i.e. Case 6 but not in the case of a) i.e. Case 5 there is a steady state error . This is because in the problem tank 1 and tank 4 are connected to each other via a single pump and we are measuring the heights of tank 1 and tank 4 but we don't have measurement data about tanks 2 and 3 leading to improper estimates. Whereas in Case b we are measuring one tank each connected to each pump and we are able to estimate the states properly resulting in better MPC performance.