EE5175: Image Signal Processing

Lab-9

1. Assume a Markov-1 process with covariance matrix R of size 8 $\times$ 8 and $\rho = 0.91$. Compute the Energy Packing Efficiency and De-correlation Efficiency of the Walsh-Haddamard Transform and Discrete Cosine Transform for the above process. What is your observation about the eigenvectors of R in relation to the DCT basis?

2. Find $\beta^2 R^{-1}$, where $\beta^2 = \dfrac{1-\rho^2}{1+\rho^2}$. Does $\beta^2 R^{-1}$ have a tridiagonal structure?. Is it close to the tridiagonal matrix Q given by,

$$\mathbf{Q} = \begin{bmatrix} 1-\alpha & -\alpha & 0 & 0 & 0 & 0 & 0 & 0 \\ -\alpha & 1 & -\alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & -\alpha & 1 & -\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha & 1 & -\alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & -\alpha & 1 & -\alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & -\alpha & 1 & -\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & -\alpha & 1 & -\alpha \\ 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & 1-\alpha \end{bmatrix}$$

where $\alpha = \dfrac{\rho}{1+\rho^2}$

Try diagonalizing $\beta^2 R^{-1}$ and Q using the DCT matrix. What is your observation.?

3. Compute SVD for the given 8×8 image $\mathbf{g}$ (provided in `imageFile.mat` and also given below) using the following steps:

   (a) Perform eigen-value decomposition of $\mathbf{g}^T\mathbf{g}$ and $\mathbf{g}\mathbf{g}^T$.

   (b) Find the singular value matrix $\mathbf{\Sigma}$.

   (c) Reconstruct the image using $\mathbf{\Sigma}$ and the eigen-vector matrices.

4. Remove one singular value at a time from $\mathbf{\Sigma}$ and reconstruct the image ($\widehat{\mathbf{g}_k}$). Compute $\|\mathbf{g}-\widehat{\mathbf{g}_k}\|^2$ and compare it with the sum of the squares of the first k singular values.

$$
\text{Image } \mathbf{g} = \begin{bmatrix} 207 & 244 & 107 & 173 & 70 & 111 & 180 & 244 \\ 230 & 246 & 233 & 193 & 11 & 97 & 192 & 86 \\ 3 & 40 & 202 & 189 & 24 & 195 & 70 & 149 \\ 232 & 247 & 244 & 100 & 209 & 202 & 173 & 57 \\ 161 & 244 & 167 & 167 & 177 & 47 & 167 & 191 \\ 24 & 123 & 9 & 43 & 80 & 124 & 41 & 65 \\ 71 & 204 & 216 & 180 & 242 & 113 & 30 & 129 \\ 139 & 36 & 238 & 8 & 8 & 164 & 127 & 178 \end{bmatrix}
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