#### 1. Heston Model

Step 1: Data Generation with the Heston Model

Step 2: Heston Option Pricing ANN

Step 3: Heston Implicit Volatility ANN

### **Tools**

```
1 %%capture
 2 !pip install pyDOE tensorflow addons
 3 import pickle
 4 import numpy as np
 5 np.random.seed(42)
 6 import pandas as pd
 7 from scipy import integrate
 8 from pyDOE import lhs
 9 import matplotlib.pyplot as plt
10 import seaborn as sb
11 import warnings
12 warnings.filterwarnings('ignore')
13 from sklearn.model_selection import train_test_split
14 from sklearn.preprocessing import MinMaxScaler
15 import tensorflow as tf
16 import tensorflow_addons as tfa
17 from tensorflow.keras.models import Sequential
18 from tensorflow.keras.layers import Dense, Dropout
19 from tensorflow.keras.optimizers import Adam
20 from tensorflow.keras.initializers import GlorotUniform
21 from sklearn.metrics import mean squared error, mean absolute error, r2 score
22 from tensorflow.keras.layers import BatchNormalization
23 from tensorflow.keras.layers import Activation
24 from tensorflow.keras import regularizers
25 from tensorflow.keras.models import Sequential, save model, load model
26 from google.colab import drive
27
```

# Step 1: Data Generation with the Heston Model

ANN Parameters	Range	Method
Input		
Moneyness, m = S0/K	(0.6, 1.4)	LHS
Time to maturity, $\boldsymbol{\tau}$	(0.1, 1.4)(year)	LHS
Risk free rate, r	(0.0%, 10%)	LHS
Correlation, ρ	(-0.95, 0.0)	LHS
Reversion speed, κ	(0.0, 2.0)	LHS
Long average variance, $\theta$	(0.0, 0.5)	LHS
Volatility of volatility, $\sigma$	(0.0, 0.5)	LHS
Initial variance, v0	(0.05, 0.5)	LHS
Output		
European call price, V	(0, 0.67)	

The LHS method, also known as Latin Hypercube Sampling, is a statistical sampling technique used in various fields, including engineering, computer experiments, and simulation studies. It is designed to create a representative sample of parameter values from a multivariate distribution.

In Latin Hypercube Sampling, the parameter space is divided into equal intervals along each dimension, and then one random sample is selected from each interval. This ensures that the resulting sample covers the entire parameter space uniformly and efficiently. The Latin Hypercube structure helps to reduce sampling variability and ensures a more representative sample compared to simple random sampling. 
image.png

```
1 def heston price(m, v0, kappa, theta, sigma, rho, tau, r, a, b, N cos):
 3
       sum=0
       sum1=0
 5
 6
      moneyness = m - r* tau
 7
      for k in range(0, N_cos):
         args = (m, v0, kappa, theta, sigma, rho, tau, r, a, b)
 9
         res= heston charfunc(k* np.pi/(b-a), *args)* U k(k, a, b)* np.exp(-1j* k* np.pi* ((moneyness+ a)/(b-a)))
         if k==0:
10
           sum1=res
11
12
         else:
13
           sum+=res
14
15
       result= np.exp(-r* tau)* np.real(0.5* sum1+ sum)
16
17
       return result
18
19 def heston price(m, v0, kappa, theta, sigma, rho, tau, r, a, b, N cos):
```

```
20
21
      sum=0
22
      sum1=0
23
24
      moneyness = m - r* tau
25
      for k in range(0, N_cos):
26
        args = (m, v0, kappa, theta, sigma, rho, tau, r, a, b)
27
         res= heston_charfunc(k* np.pi/(b-a), *args)* U_k_calc(k, a, b)* np.exp(-1j* k* np.pi* ((moneyness+ a)/(b-a)))
28
        if k==0:
29
          sum1=res
30
         else:
31
          sum+=res
32
33
34
      result= np.exp(-r* tau)* np.real(0.5* sum1+ sum)
35
36
37
      return result
38
39 def heston_charfunc(w, m, v0, kappa, theta, sigma, rho, tau, r, a, b):
40
41
      args = (w, m, v0, kappa, theta, sigma, rho, tau, r, a, b)
42
      D= D_calc(*args)
43
      G= G_calc(*args)
      result= np.exp(1j* w* r* tau+ (v0/(sigma**2))* ((1- np.exp(-1* D* tau))/(1-G* np.exp(-1* D* tau)))
44
45
      * (kappa- (1j* rho* sigma* w)- D))* np.exp((kappa* theta/ (sigma**2))* (tau* (kappa- 1j* rho* sigma* w- D)-
46
                                                                                       2* np.log((1- G* np.exp(-1* D* tau))/(1-G))))
47
48
      return result
49
50 def D_calc(w, m, v0, kappa, theta, sigma, rho, tau, r, a, b):
51
52
      result= np.sqrt((kappa- 1j* rho* sigma* w)**2 + (w**2+ 1j* w)* (sigma**2))
53
54
      return result
55
56 def G_calc(w, m, v0, kappa, theta, sigma, rho, tau, r, a, b):
57
58
      args = (w, m, v0, kappa, theta, sigma, rho, tau, r, a, b)
59
      D= D calc(*args)
60
      result= (kappa- 1j* rho* sigma* w- D )/ (kappa- 1j* rho* sigma* w+ D)
61
62
      return result
63
64 def U_k_calc(k, a, b):
65
66
      result= (2/(b-a))^* (Chi_k(k, 0, b, a, b)- Psi_k(k, 0, b, a, b))
67
68
      return result
```

```
69
70 def Chi_k(k, c, d, a, b):
71
72
       result= (1/((1+(k*np.pi/(b-a))**2)))*((np.cos(k*np.pi*((d-a)/(b-a)))*np.exp(d))-(np.cos(k*np.pi*((c-a)/(b-a)))*np.exp(c))
73
       +((k* np.pi/(b-a))*np.sin(k* np.pi*((d-a)/(b-a)))* np.exp(d)) - ((k* np.pi/(b-a))*np.sin(k* np.pi*((c-a)/(b-a)))* np.exp(c)))
74
75
       return result
76
77 def Psi_k(k, c, d, a, b):
78
79
       if k==0:
 80
         result= d-c
81
       else:
82
         result=(np.sin(k* np.pi* ((d-a)/(b-a))) - np.sin(k* np.pi* ((c-a)/(b-a))))* (b-a)/ (k* np.pi)
83
 84
       # print('say_x', result)
85
       return result
 86
87 def calc_a_b(m, v0, kappa, theta, sigma, rho, tau, r, L_cos):
 88
89
       c1= r* tau+ (1- np.exp(-1* kappa* tau))* ((theta- v0)/ (2* kappa))- 0.5* theta* tau
90
91
       c2= (1/(8* (kappa**3)))* (sigma* tau* kappa* np.exp(-1* kappa* tau)*(v0-theta)* (8* kappa* rho- 4* sigma)+
92
                               kappa* rho* sigma* (1- np.exp(-1* kappa* tau))* (16* theta- 8* v0)+
93
                               2* theta* kappa* tau* (-4* kappa* rho* sigma+ sigma**2+ 4* (kappa**2))+
 94
                               sigma**2 * ((theta- 2*v0)* np.exp(-2* kappa* tau)+ theta* (6* np.exp(-1* kappa* tau)-7)+ 2* v0)+
95
                               8* (kappa**2) * (v0- theta)* (1-np.exp(-1* kappa * tau)))
96
97
       a= c1- L cos* (np.sqrt(np.abs(c2)))
98
       b= c1+ L_cos* (np.sqrt(np.abs(c2)))
99
100
       return a, b
101
102 # Define the LHS method with lhs function in Python library pyDOE
103
104 def lhs sample(range values):
105
       lower bound, upper bound = range values
106
       samples = lhs(1, samples=1) # Generating a single sample
107
       scaled sample = samples * (upper bound - lower bound) + lower bound
108
       return scaled sample.item()
109
110 # Create an empty data frame to store the values
111 df = pd.DataFrame(columns=['m', 'tau', 'r', 'rho', 'kappa', 'theta', 'sigma', 'v0', 'option_price'])
112
113
114 num samples = 1000000
116 # Define the ranges for input parameters
117 dataset = []
```

```
118 for i in range(num_samples):
       if i % 3000 == 0:
119
120
            print(i)
121
       m = lhs_sample([0.6, 1.4])
122
       tau = lhs_sample([0.1, 1.4])
123
       r = lhs_sample([0.0, 0.1])
124
       rho = lhs_sample([-0.95, 0.0])
       kappa = lhs_sample([0.0, 2.0])
125
       theta = lhs_sample([0.0, 0.5])
126
127
       sigma = lhs_sample([0.0, 0.5])
       v0 = lhs_sample([0.05, 0.5])
128
129
130
       a, b = calc_a_b(np.log(m), v0, kappa, theta, sigma, rho, tau, r, L_cos=50)
131
            #print(a, b)
132
       option_price = heston_price(np.log(m), v0, kappa, theta, sigma, rho, tau, r, a, b, N_cos=1500)
133
            # dataset.append([m, tau, r, rho, kappa, theta, sigma, v0, ])
134
            # Append the values to the data frame
135
       df = df.append({
136
            'm': m,
137
            'tau': tau,
138
            'r': r,
139
            'rho': rho,
140
            'kappa': kappa,
141
            'theta': theta,
142
            'sigma': sigma,
143
            'v0': v0,
            'option price': option price
144
145
       }, ignore index=True)
146
147 with open('df_{}.pickle'.format(len(df)), 'wb') as f:
     pickle.dump(df, f)
  1 # load df by pickle
  2 with open('df_1000000.pickle', 'rb') as f:
 3
       df = pickle.load(f)
  4
  1 df.dropna(inplace=True)
  2
  3 df.reset_index(drop=True, inplace=True)
  5 df_filtered = df[df['option_price'] < 1]</pre>
  6 df_filtered = df_filtered[df_filtered['option_price'] >= 0]
  8 df filtered.head(10)
```

```
sigma
                                                                             v0 o
                      tau
                                         rho
                                               kappa
                                                        theta
     1 1.080892 1.020494 0.002058 -0.028586 1.664885 0.106170 0.090912 0.132532
        0.843394 0.782183 0.043195 -0.673332 1.223706 0.069747 0.146072 0.214863
        0.964856 1.120729 0.019967 -0.461477 1.184829 0.023225 0.303772 0.126736
        0.652041 1.333551 0.096563
                                  -0.182023 0.609228 0.048836 0.342117 0.248069
     5 0.697631 0.743730 0.003439
                                  -0.086146  0.517560  0.331261  0.155856  0.284031
     6 1.037368 0.340311 0.096958
                                  -0.213624 1.878998 0.447414 0.298950 0.464843
        0.670794 0.354778 0.004523
                                  -0.640936 0.777355 0.135675 0.414369 0.210539
        0.604418 1.160100 0.070686 -0.257443 1.542541 0.037022 0.179233 0.102141
     10 1.290483 0.910288 0.033090 -0.889620 0.621965 0.162592 0.364803 0.336901
1 # Split the dataset into train and test sets
3 train_df, test_df = train_test_split(df_filtered, test_size=0.1, random_state=55)
4 train_df, eval_df = train_test_split(train_df, test_size=1/9, random_state=55)
1 # Check if GPU is available
2
3 if tf.test.gpu_device_name():
      print('GPU found')
5 else:
      print("No GPU found")
6
8 # Create a TensorFlow session and set it to use the GPU
9 config = tf.compat.v1.ConfigProto()
10 config.gpu_options.allow_growth = True
11 session = tf.compat.v1.Session(config=config)
    GPU found
1 def calc_metrics(y_true, y_pred):
2
3
      mse = mean squared error(y true, y pred)
      mae = mean_absolute_error(y_true, y_pred)
4
      r2 = r2_score(y_true, y_pred)
      return {'mse': mse, 'mae': mae, 'r2': r2}
```

## Step 2: Heston Option Pricing ANN

#### Parameters of the ANN

Parameters	Options	
Hidden layers	4	
Neurons(each layer)	400	
Activation	ReLU	
Dropout rate	0.0	
Batch-normalization	No	
Initialization	Glorot_uniform	
Optimizer	Adam	
Batch size	1024	

```
1 model = Sequential()
2 model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform(), input_shape=(8,)))
3 model.add(Dense(400, activation='relu', kernel initializer=GlorotUniform()))
4 model.add(Dense(400, activation='relu', kernel initializer=GlorotUniform()))
5 model.add(Dense(400, activation='relu', kernel initializer=GlorotUniform()))
6 model.add(Dense(1))
7 model.compile(optimizer=Adam(), loss='mean_squared_error')
9 # Train the model on the GPU
10
11 \text{ batch\_size} = 1024
12 \text{ epochs} = 100
13 with tf.device('/GPU:0'):
     history = model.fit(
15
        x = train_df[['m', 'tau', 'r', 'rho', 'kappa', 'theta', 'sigma', 'v0']],
16
        y = train df['option price'],
17
        batch size=batch size,
18
        epochs=epochs,
19
         verbose=1
20
     )
    Epoch 1/100
    719/719 [=========== ] - 10s 4ms/step - loss: 0.0033
    Epoch 2/100
    Epoch 3/100
    719/719 [=========== ] - 3s 4ms/step - loss: 0.0014
    Epoch 4/100
    Epoch 5/100
    Epoch 6/100
```

2

4

5

```
719/719 [============ - - 3s 4ms/step - loss: 0.0012
  Epoch 7/100
  719/719 [============ - 3s 4ms/step - loss: 0.0011
  Epoch 8/100
  719/719 [=========== ] - 3s 4ms/step - loss: 0.0011
  Epoch 9/100
  Epoch 10/100
  719/719 [============= ] - 3s 4ms/step - loss: 0.0011
  Epoch 11/100
  719/719 [============== ] - 3s 4ms/step - loss: 0.0011
  Epoch 12/100
  719/719 [============== ] - 3s 4ms/step - loss: 0.0011
  Epoch 13/100
  719/719 [============= - - 4s 5ms/step - loss: 0.0010
  Epoch 14/100
  719/719 [============== ] - 3s 4ms/step - loss: 0.0010
  Epoch 15/100
  719/719 [============ - - 3s 5ms/step - loss: 0.0010
  Epoch 16/100
  Epoch 17/100
  719/719 [=========== ] - 3s 4ms/step - loss: 0.0010
  Epoch 18/100
  719/719 [============== ] - 3s 4ms/step - loss: 0.0010
  Epoch 19/100
  Epoch 20/100
  Epoch 21/100
  719/719 [========== ] - 3s 4ms/step - loss: 9.9153e-04
  Epoch 22/100
  Epoch 23/100
  719/719 [============ ] - 3s 4ms/step - loss: 9.8305e-04
  Epoch 24/100
  719/719 [=============== ] - 3s 4ms/step - loss: 9.6651e-04
  Epoch 25/100
  719/719 [========= ] - 3s 4ms/step - loss: 9.5987e-04
  Epoch 26/100
  719/719 [============ ] - 3s 4ms/step - loss: 9.6322e-04
  Epoch 27/100
  719/719 [========== ] - 3s 4ms/step - loss: 9.5242e-04
  Epoch 28/100
  719/719 [========== ] - 3s 4ms/step - loss: 9.4937e-04
  Epoch 29/100
  1 # Evaluate the trained model on the test sets on the GPU
3 with tf.device('/GPU:0'):
   wide_test_loss = model.evaluate(
      test_df[['m', 'tau', 'r', 'rho', 'kappa', 'theta', 'sigma', 'v0']],
```

```
6
          test_df['option_price'],
7
          verbose=0
 8
 9
10 wide_test_predictions = model.predict(test_df[['m', 'tau', 'r', 'rho', 'kappa', 'theta', 'sigma', 'v0']])
12
13 # Reshape wide test predictions to match the shape of test df['option price']
14 wide test predictions = wide test predictions.flatten()
15
16 # Calculate performance metrics for wide test set
17 res = calc_metrics(test_df['option_price'], wide_test_predictions)
18 print(res)
     2874/2874 [============] - 5s 2ms/step
     {'mse': 0.001004394715258821, 'mae': 0.009025426892304482, 'r2': 0.9800865105589234}
```

## Step 3: Heston Implicit Volatility ANN

```
1 model = Sequential()
 2 model.add(Dense(600, ))
 3 model.add(BatchNormalization())
 4 model.add(Activation('relu'))
 5 model.add(Dropout(0.1))
 6 model.add(Dense(600, ))
 7 model.add(BatchNormalization())
 8 model.add(Activation('relu'))
 9 model.add(Dropout(0.1))
10 model.add(Dense(600, ))
11 model.add(BatchNormalization())
12 model.add(Activation('relu'))
13 model.add(Dropout(0.1))
14 model.add(Dense(600, ))
15 model.add(BatchNormalization())
16 model.add(Activation('relu'))
17 model.add(Dropout(0.3))
18 model.add(Dense(1, ))
19 \text{ epochs} = 200
20 \text{ batch\_size} = 1024
21
22 initial_learning_rate = 0.01
23 lr_schedule = tf.keras.optimizers.schedules.ExponentialDecay(
       initial_learning_rate,
25
       decay steps=100000,
26
      decay_rate=0.99,
27
       staircase=True)
```

```
28
29 steps_per_epoch = batch_size
30 INIT LR = 1e-5
31 MAX_LR = 1e-1
32
33 clr = tfa.optimizers.CyclicalLearningRate(initial_learning_rate=INIT_LR,
      maximal learning rate=MAX LR,
34
      scale fn=lambda x: 1/(2.**(x-1)),
35
      step size=2 * steps per epoch
36
37)
38
39 filepath = 'best model'
40 checkpoint = tf.keras.callbacks.ModelCheckpoint(filepath=filepath, mode='min', monitor='val loss', verbose=1, save best only=True)
41 callbacks list = [checkpoint]
42 callbacks=callbacks_list
43 model.compile(optimizer=tf.keras.optimizers.Adam(learning_rate=clr), loss='mean_squared_error', )
44
45 # Train the model on the GPU
46
47 with tf.device('/GPU:0'):
48
      history = model.fit(
49
          x = train_df[['m', 'tau', 'r', 'rho', 'kappa', 'theta', 'v0', 'option_price' ]],
50
          y = train df['sigma'],
51
          validation_data = (eval_df[['m', 'tau', 'r', 'rho', 'kappa', 'theta', 'v0', 'option_price' ]], eval_df['sigma']),
52
          batch size=batch size,
53
          epochs=epochs,
          verbose=1,
54
55
           callbacks=callbacks list
56
      )
57
58 # plot lines
59 plt.plot([i for i in range(1, len(history.history['loss'])+1)], history.history['loss'], label = "training loss")
60 plt.plot([i for i in range(1, len(history.history['val loss'])+1)], history.history['val loss'], label = "validation loss")
61 plt.legend()
62 plt.show()
63
```

```
Epoch 1/333
Epoch 1: val loss improved from inf to 0.02358, saving model to best model
WARNING:absl:Found untraced functions such as _update_step_xla while saving (showing 1 of 1). These functions will not
Epoch 2/333
Epoch 2: val loss improved from 0.02358 to 0.02115, saving model to best model
WARNING:absl:Found untraced functions such as _update_step_xla while saving (showing 1 of 1). These functions will not
Epoch 3/333
662/662 [============ ] - ETA: 0s - loss: 0.0406
Epoch 3: val loss improved from 0.02115 to 0.02100, saving model to best model
WARNING:absl:Found untraced functions such as update step xla while saving (showing 1 of 1). These functions will not
662/662 [============== ] - 8s 12ms/step - loss: 0.0406 - val loss: 0.0210
Epoch 4/333
Epoch 4: val loss improved from 0.02100 to 0.02091, saving model to best model
WARNING:absl:Found untraced functions such as _update_step_xla while saving (showing 1 of 1). These functions will not
662/662 [============= ] - 8s 13ms/step - loss: 0.0280 - val loss: 0.0209
Epoch 5/333
Epoch 5: val loss did not improve from 0.02091
662/662 [================ ] - 5s 7ms/step - loss: 0.0234 - val loss: 0.0210
Epoch 6/333
Epoch 6: val loss improved from 0.02091 to 0.02091, saving model to best model
WARNING:absl:Found untraced functions such as _update_step_xla while saving (showing 1 of 1). These functions will not
Epoch 7/333
659/662 [============>.] - ETA: 0s - loss: 0.0210
Epoch 7: val loss improved from 0.02091 to 0.02074, saving model to best model
WARNING:absl:Found untraced functions such as _update step_xla while saving (showing 1 of 1). These functions will not
662/662 [============= ] - 7s 11ms/step - loss: 0.0210 - val loss: 0.0207
Epoch 8/333
660/662 [=============>.] - ETA: 0s - loss: 0.0208
Epoch 8: val loss did not improve from 0.02074
662/662 [=============== ] - 6s 9ms/step - loss: 0.0208 - val loss: 0.0208
Epoch 9/333
656/662 [============>.] - ETA: 0s - loss: 0.0208
Epoch 9: val loss improved from 0.02074 to 0.02072, saving model to best model
WARNING:absl:Found untraced functions such as update step xla while saving (showing 1 of 1). These functions will not
662/662 [================= ] - 7s 11ms/step - loss: 0.0208 - val_loss: 0.0207
Epoch 10/333
657/662 [=============>.] - ETA: 0s - loss: 0.0207
Epoch 10: val_loss improved from 0.02072 to 0.02070, saving model to best_model
WARNING:absl:Found untraced functions such as update step_xla while saving (showing 1 of 1). These functions will not
662/662 [================ ] - 8s 12ms/step - loss: 0.0207 - val_loss: 0.0207
Epoch 11/333
656/662 [=============>.] - ETA: 0s - loss: 0.0207
Epoch 11: val loss did not improve from 0.02070
662/662 [================= ] - 5s 7ms/step - loss: 0.0207 - val loss: 0.0207
Epoch 12/333
```

```
Epoch 12: val loss did not improve from 0.02070
Epoch 13/333
659/662 [=============>.] - ETA: 0s - loss: 0.0207
Epoch 13: val_loss did not improve from 0.02070
662/662 [======================== ] - 6s 9ms/step - loss: 0.0207 - val loss: 0.0207
Epoch 14/333
Epoch 14: val loss improved from 0.02070 to 0.02070, saving model to best model
WARNING:absl:Found untraced functions such as _update step_xla while saving (showing 1 of 1). These functions will not
662/662 [============= ] - 7s 11ms/step - loss: 0.0207 - val loss: 0.0207
Epoch 15/333
Epoch 15: val loss improved from 0.02070 to 0.02068, saving model to best model
WARNING:absl:Found untraced functions such as update step xla while saving (showing 1 of 1). These functions will not
662/662 [============= ] - 8s 12ms/step - loss: 0.0207 - val loss: 0.0207
Epoch 16/333
Epoch 16: val loss did not improve from 0.02068
Epoch 17/333
Epoch 17: val loss improved from 0.02068 to 0.02065, saving model to best model
WARNING:absl:Found untraced functions such as _update step_xla while saving (showing 1 of 1). These functions will not
662/662 [============= ] - 8s 12ms/step - loss: 0.0206 - val loss: 0.0207
Epoch 18/333
659/662 [============>.] - ETA: 0s - loss: 0.0206
Epoch 18: val loss did not improve from 0.02065
Epoch 19/333
Epoch 19: val loss did not improve from 0.02065
Epoch 20/333
Epoch 20: val loss did not improve from 0.02065
Epoch 21/333
660/662 [=============>.] - ETA: 0s - loss: 0.0206
Epoch 21: val loss did not improve from 0.02065
662/662 [=============== ] - 5s 7ms/step - loss: 0.0206 - val loss: 0.0207
Epoch 22/333
662/662 [============ ] - ETA: 0s - loss: 0.0206
Epoch 22: val loss did not improve from 0.02065
Epoch 23/333
Epoch 23: val loss did not improve from 0.02065
Epoch 24/333
659/662 [============>.] - ETA: 0s - loss: 0.0206
Epoch 24: val loss improved from 0.02065 to 0.02065, saving model to best model
WARNING:absl:Found untraced functions such as update step xla while saving (showing 1 of 1). These functions will not
662/662 [============= ] - 8s 12ms/step - loss: 0.0206 - val loss: 0.0206
```

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Epoch 25/333
Epoch 25: val loss did not improve from 0.02065
662/662 [======================== ] - 5s 7ms/step - loss: 0.0206 - val loss: 0.0207
Epoch 26/333
Epoch 26: val loss did not improve from 0.02065
Epoch 27/333
Epoch 27: val loss did not improve from 0.02065
Epoch 28/333
659/662 [============>.] - ETA: 0s - loss: 0.0206
Epoch 28: val loss did not improve from 0.02065
662/662 [================ ] - 5s 7ms/step - loss: 0.0206 - val loss: 0.0207
Epoch 29/333
Epoch 29: val loss did not improve from 0.02065
662/662 [=============== ] - 6s 9ms/step - loss: 0.0206 - val loss: 0.0207
Epoch 30/333
Epoch 30: val loss did not improve from 0.02065
662/662 [============= ] - 5s 8ms/step - loss: 0.0206 - val loss: 0.0207
Epoch 31/333
Epoch 31: val_loss did not improve from 0.02065
Epoch 32/333
658/662 [=============>.] - ETA: 0s - loss: 0.0206
Epoch 32: val loss did not improve from 0.02065
Epoch 33/333
657/662 [============>.] - ETA: 0s - loss: 0.0206
Epoch 33: val_loss did not improve from 0.02065
Epoch 34/333
Epoch 34: val loss did not improve from 0.02065
662/662 [==================] - 7s 10ms/step - loss: 0.0206 - val_loss: 0.0207
Epoch 35/333
662/662 [============== ] - ETA: 0s - loss: 0.0206
Epoch 35: val loss improved from 0.02065 to 0.02064, saving model to best model
WARNING:absl:Found untraced functions such as _update_step_xla while saving (showing 1 of 1). These functions will not
662/662 [============= ] - 8s 12ms/step - loss: 0.0206 - val loss: 0.0206
Epoch 36/333
660/662 [=============>.] - ETA: 0s - loss: 0.0206
Epoch 36: val loss did not improve from 0.02064
662/662 [=============== ] - 5s 8ms/step - loss: 0.0206 - val loss: 0.0207
Epoch 37/333
660/662 [=============>.] - ETA: 0s - loss: 0.0206
Epoch 37: val loss improved from 0.02064 to 0.02064, saving model to best model
WARNING:absl:Found untraced functions such as _update_step_xla while saving (showing 1 of 1). These functions will not
662/662 [============ ] - 8s 12ms/step - loss: 0.0206 - val loss: 0.0206
```

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Epoch 38/333
Epoch 38: val loss did not improve from 0.02064
662/662 [=============== ] - 5s 8ms/step - loss: 0.0206 - val loss: 0.0207
Epoch 39/333
Epoch 39: val_loss did not improve from 0.02064
Epoch 40/333
Epoch 40: val loss did not improve from 0.02064
Epoch 41/333
Epoch 41: val loss did not improve from 0.02064
662/662 [============================ ] - 6s 9ms/step - loss: 0.0206 - val loss: 0.0206
Epoch 42/333
662/662 [============== ] - ETA: 0s - loss: 0.0206
Epoch 42: val_loss did not improve from 0.02064
Epoch 43/333
Epoch 43: val loss did not improve from 0.02064
662/662 [============= ] - 5s 7ms/step - loss: 0.0206 - val loss: 0.0206
Epoch 44/333
Epoch 44: val loss did not improve from 0.02064
Epoch 45/333
659/662 [=============>.] - ETA: 0s - loss: 0.0206
Epoch 45: val loss improved from 0.02064 to 0.02063, saving model to best model
WARNING:absl:Found untraced functions such as _update step_xla while saving (showing 1 of 1). These functions will not
662/662 [================ ] - 7s 11ms/step - loss: 0.0206 - val loss: 0.0206
Epoch 46/333
657/662 [============>.] - ETA: 0s - loss: 0.0206
Epoch 46: val loss did not improve from 0.02063
Epoch 47/333
658/662 [=============>.] - ETA: 0s - loss: 0.0206
Epoch 47: val loss did not improve from 0.02063
Epoch 48/333
657/662 [============>.] - ETA: 0s - loss: 0.0206
Epoch 48: val loss did not improve from 0.02063
662/662 [================= ] - 5s 7ms/step - loss: 0.0206 - val_loss: 0.0206
Epoch 49/333
660/662 [============>.] - ETA: 0s - loss: 0.0206
Epoch 49: val loss did not improve from 0.02063
662/662 [============== ] - 6s 9ms/step - loss: 0.0206 - val loss: 0.0207
Epoch 50/333
Epoch 50: val loss did not improve from 0.02063
Epoch 51/333
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Epoch 51: val loss did not improve from 0.02063
662/662 [================ ] - 5s 8ms/step - loss: 0.0206 - val loss: 0.0207
Epoch 52/333
656/662 [============>.] - ETA: 0s - loss: 0.0206
Epoch 52: val loss did not improve from 0.02063
Epoch 53/333
Epoch 53: val loss did not improve from 0.02063
Epoch 54/333
Epoch 54: val loss did not improve from 0.02063
662/662 [============= ] - 6s 9ms/step - loss: 0.0206 - val loss: 0.0206
Epoch 55/333
Epoch 55: val loss did not improve from 0.02063
Epoch 56/333
Epoch 56: val_loss did not improve from 0.02063
662/662 [============= ] - 5s 8ms/step - loss: 0.0206 - val loss: 0.0207
Epoch 57/333
Epoch 57: val_loss did not improve from 0.02063
Epoch 58/333
662/662 [============= ] - ETA: 0s - loss: 0.0206
Epoch 58: val_loss did not improve from 0.02063
662/662 [================ ] - 5s 7ms/step - loss: 0.0206 - val_loss: 0.0206
Epoch 59/333
Epoch 59: val loss did not improve from 0.02063
Epoch 60/333
Epoch 60: val loss did not improve from 0.02063
Epoch 61/333
Epoch 61: val loss did not improve from 0.02063
662/662 [============= ] - 5s 7ms/step - loss: 0.0205 - val loss: 0.0208
Epoch 62/333
Epoch 62: val loss did not improve from 0.02063
662/662 [============= ] - 6s 9ms/step - loss: 0.0206 - val loss: 0.0206
Epoch 63/333
662/662 [=============] - ETA: 0s - loss: 0.0206
Epoch 63: val_loss did not improve from 0.02063
Epoch 64/333
Epoch 64: val loss did not improve from 0.02063
```

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                     1000. 0.0200
Epoch 65/333
Epoch 65: val loss did not improve from 0.02063
Epoch 66/333
Epoch 66: val_loss did not improve from 0.02063
Epoch 67/333
662/662 [=============== ] - ETA: 0s - loss: 0.0205
Epoch 67: val loss did not improve from 0.02063
662/662 [======================= ] - 6s 9ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 68/333
Epoch 68: val_loss did not improve from 0.02063
Epoch 69/333
Epoch 69: val loss did not improve from 0.02063
Epoch 70/333
Epoch 70: val loss did not improve from 0.02063
Epoch 71/333
Epoch 71: val loss did not improve from 0.02063
Epoch 72/333
Epoch 72: val loss did not improve from 0.02063
Epoch 73/333
Epoch 73: val loss did not improve from 0.02063
Epoch 74/333
Epoch 74: val_loss did not improve from 0.02063
Epoch 75/333
Epoch 75: val_loss did not improve from 0.02063
Epoch 76/333
Epoch 76: val loss did not improve from 0.02063
662/662 [================== ] - 5s 7ms/step - loss: 0.0205 - val_loss: 0.0206
Epoch 77/333
Epoch 77: val_loss did not improve from 0.02063
662/662 [================= ] - 5s 8ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 78/333
```

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Epoch 78: val loss improved from 0.02063 to 0.02063, saving model to best model
WARNING:absl:Found untraced functions such as update step xla while saving (showing 1 of 1). These functions will not
662/662 [================ ] - 8s 12ms/step - loss: 0.0205 - val loss: 0.0206
Epoch 79/333
661/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 79: val_loss did not improve from 0.02063
Epoch 80/333
Epoch 80: val loss did not improve from 0.02063
Epoch 81/333
661/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 81: val loss did not improve from 0.02063
662/662 [================ ] - 5s 7ms/step - loss: 0.0205 - val loss: 0.0206
Epoch 82/333
Epoch 82: val loss did not improve from 0.02063
Epoch 83/333
Epoch 83: val loss did not improve from 0.02063
Epoch 84/333
661/662 [=============>.] - ETA: 0s - loss: 0.0205
Epoch 84: val_loss did not improve from 0.02063
Epoch 85/333
Epoch 85: val_loss did not improve from 0.02063
Epoch 86/333
Epoch 86: val_loss did not improve from 0.02063
Epoch 87/333
662/662 [============ ] - ETA: 0s - loss: 0.0205
Epoch 87: val loss did not improve from 0.02063
662/662 [================ ] - 5s 8ms/step - loss: 0.0205 - val_loss: 0.0206
Epoch 88/333
661/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 88: val loss did not improve from 0.02063
Epoch 89/333
Epoch 89: val loss did not improve from 0.02063
Epoch 90/333
Epoch 90: val loss did not improve from 0.02063
662/662 [============= ] - 6s 9ms/step - loss: 0.0205 - val loss: 0.0206
Epoch 91/333
Epoch 91: val_loss did not improve from 0.02063
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Epoch 92: val loss did not improve from 0.02063
662/662 [================= ] - 5s 7ms/step - loss: 0.0205 - val_loss: 0.0206
Epoch 93/333
Epoch 93: val loss did not improve from 0.02063
662/662 [================ ] - 6s 10ms/step - loss: 0.0205 - val loss: 0.0206
Epoch 94/333
657/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 94: val loss did not improve from 0.02063
Epoch 95/333
Epoch 95: val loss did not improve from 0.02063
Epoch 96/333
662/662 [=============== ] - ETA: 0s - loss: 0.0205
Epoch 96: val loss did not improve from 0.02063
Epoch 97/333
Epoch 97: val_loss did not improve from 0.02063
662/662 [============================ ] - 5s 7ms/step - loss: 0.0205 - val loss: 0.0206
Epoch 98/333
Epoch 98: val_loss did not improve from 0.02063
Epoch 99/333
Epoch 99: val loss did not improve from 0.02063
662/662 [============================ ] - 5s 7ms/step - loss: 0.0205 - val loss: 0.0206
Epoch 100/333
Epoch 100: val loss did not improve from 0.02063
Epoch 101/333
Epoch 101: val loss did not improve from 0.02063
Epoch 102/333
Epoch 102: val loss did not improve from 0.02063
Epoch 103/333
Epoch 103: val loss did not improve from 0.02063
Epoch 104/333
Epoch 104: val loss did not improve from 0.02063
Epoch 105/333
```

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662/662 |============== | - ETA: 0s - loss: 0.0205
Epoch 105: val loss did not improve from 0.02063
Epoch 106/333
660/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 106: val loss did not improve from 0.02063
Epoch 107/333
662/662 [============= ] - ETA: 0s - loss: 0.0205
Epoch 107: val_loss did not improve from 0.02063
Epoch 108/333
662/662 [============ ] - ETA: 0s - loss: 0.0205
Epoch 108: val loss did not improve from 0.02063
662/662 [======================== ] - 5s 7ms/step - loss: 0.0205 - val loss: 0.0206
Epoch 109/333
661/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 109: val loss did not improve from 0.02063
662/662 [================ ] - 6s 9ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 110/333
Epoch 110: val loss did not improve from 0.02063
Epoch 111/333
662/662 [============ ] - ETA: 0s - loss: 0.0205
Epoch 111: val loss did not improve from 0.02063
Epoch 112/333
Epoch 112: val loss did not improve from 0.02063
Epoch 113/333
Epoch 113: val_loss did not improve from 0.02063
Epoch 114/333
Epoch 114: val loss did not improve from 0.02063
Epoch 115/333
Epoch 115: val loss did not improve from 0.02063
Epoch 116/333
662/662 [============= ] - ETA: 0s - loss: 0.0205
Epoch 116: val loss did not improve from 0.02063
Epoch 117/333
Epoch 117: val_loss did not improve from 0.02063
Epoch 118/333
658/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 118: val loss did not improve from 0.02063
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Epoch 119/333
662/662 [============= ] - ETA: 0s - loss: 0.0205
Epoch 119: val_loss did not improve from 0.02063
Epoch 120/333
Epoch 120: val_loss did not improve from 0.02063
Epoch 121/333
Epoch 121: val loss did not improve from 0.02063
Epoch 122/333
Epoch 122: val loss did not improve from 0.02063
662/662 [=============== ] - 6s 9ms/step - loss: 0.0205 - val loss: 0.0206
Epoch 123/333
Epoch 123: val loss did not improve from 0.02063
Epoch 124/333
Epoch 124: val loss did not improve from 0.02063
662/662 [============= ] - 6s 8ms/step - loss: 0.0205 - val loss: 0.0206
Epoch 125/333
Epoch 125: val loss did not improve from 0.02063
662/662 [============= ] - 7s 10ms/step - loss: 0.0205 - val loss: 0.0206
Epoch 126/333
Epoch 126: val_loss did not improve from 0.02063
Epoch 127/333
Epoch 127: val_loss did not improve from 0.02063
Epoch 128/333
659/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 128: val loss did not improve from 0.02063
Epoch 129/333
Epoch 129: val loss did not improve from 0.02063
Epoch 130/333
662/662 [============= ] - ETA: 0s - loss: 0.0205
Epoch 130: val loss did not improve from 0.02063
Epoch 131/333
658/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 131: val loss did not improve from 0.02063
Epoch 132/333
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Epoch 132: val\_loss did not improve from 0.02063 Epoch 133/333 Epoch 133: val loss did not improve from 0.02063 Epoch 134/333 660/662 [============>.] - ETA: 0s - loss: 0.0205 Epoch 134: val loss did not improve from 0.02063 Epoch 135/333 660/662 [============>.] - ETA: 0s - loss: 0.0205 Epoch 135: val loss did not improve from 0.02063 662/662 [============= ] - 5s 8ms/step - loss: 0.0205 - val loss: 0.0206 Epoch 136/333 Epoch 136: val loss did not improve from 0.02063 Epoch 137/333 662/662 [=============== ] - ETA: 0s - loss: 0.0205 Epoch 137: val loss did not improve from 0.02063 Epoch 138/333 Epoch 138: val\_loss did not improve from 0.02063 Epoch 139/333 Epoch 139: val loss did not improve from 0.02063 Epoch 140/333 Epoch 140: val loss did not improve from 0.02063 Epoch 141/333 661/662 [============>.] - ETA: 0s - loss: 0.0205 Epoch 141: val loss did not improve from 0.02063 Epoch 142/333 Epoch 142: val loss did not improve from 0.02063 Epoch 143/333 Epoch 143: val loss did not improve from 0.02063 662/662 [============================ ] - 6s 9ms/step - loss: 0.0205 - val loss: 0.0207 Epoch 144/333 657/662 [============>.] - ETA: 0s - loss: 0.0205 Epoch 144: val loss did not improve from 0.02063 Epoch 145/333 662/662 [============= ] - ETA: 0s - loss: 0.0205 Epoch 145: val\_loss did not improve from 0.02063 

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Epoch 146/333
Epoch 146: val loss did not improve from 0.02063
662/662 [======================== ] - 5s 8ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 147/333
662/662 [============ ] - ETA: 0s - loss: 0.0205
Epoch 147: val loss did not improve from 0.02063
662/662 [=============== ] - 5s 7ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 148/333
Epoch 148: val loss did not improve from 0.02063
Epoch 149/333
Epoch 149: val_loss did not improve from 0.02063
662/662 [======================== ] - 5s 7ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 150/333
Epoch 150: val loss did not improve from 0.02063
662/662 [================ ] - 5s 8ms/step - loss: 0.0205 - val loss: 0.0206
Epoch 151/333
Epoch 151: val_loss did not improve from 0.02063
662/662 [=================== ] - 5s 8ms/step - loss: 0.0205 - val_loss: 0.0207
Epoch 152/333
662/662 [================== ] - ETA: 0s - loss: 0.0205
Epoch 152: val loss did not improve from 0.02063
Epoch 153/333
662/662 [============= ] - ETA: 0s - loss: 0.0205
Epoch 153: val loss did not improve from 0.02063
Epoch 154/333
656/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 154: val loss did not improve from 0.02063
Epoch 155/333
Epoch 155: val loss did not improve from 0.02063
662/662 [============================ ] - 6s 9ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 156/333
Epoch 156: val loss did not improve from 0.02063
Epoch 157/333
Epoch 157: val_loss did not improve from 0.02063
Epoch 158/333
662/662 [============= ] - ETA: 0s - loss: 0.0205
Epoch 158: val loss did not improve from 0.02063
Epoch 159/333
659/662 [============>.] - ETA: 0s - loss: 0.0205
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Epoch 159: val loss did not improve from 0.02063
662/662 [=============== ] - 5s 7ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 160/333
660/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 160: val_loss did not improve from 0.02063
Epoch 161/333
Epoch 161: val loss did not improve from 0.02063
662/662 [============================ ] - 6s 9ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 162/333
658/662 [=============>.] - ETA: 0s - loss: 0.0205
Epoch 162: val loss did not improve from 0.02063
662/662 [================ ] - 5s 7ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 163/333
656/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 163: val loss did not improve from 0.02063
Epoch 164/333
659/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 164: val_loss did not improve from 0.02063
662/662 [================ ] - 5s 7ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 165/333
662/662 [============ ] - ETA: 0s - loss: 0.0205
Epoch 165: val loss did not improve from 0.02063
662/662 [================ ] - 5s 7ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 166/333
656/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 166: val loss did not improve from 0.02063
Epoch 167/333
Epoch 167: val loss did not improve from 0.02063
662/662 [======================== ] - 5s 7ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 168/333
Epoch 168: val loss did not improve from 0.02063
662/662 [============================ ] - 6s 9ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 169/333
Epoch 169: val loss did not improve from 0.02063
Epoch 170/333
Epoch 170: val loss did not improve from 0.02063
Epoch 171/333
656/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 171: val_loss did not improve from 0.02063
Epoch 172/333
Epoch 172: val loss did not improve from 0.02063
662/662 [================ ] - 5s 7ms/step - loss: 0.0205 - val loss: 0.0207
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Epoch 173/333
Epoch 173: val_loss did not improve from 0.02063
Epoch 174/333
Epoch 174: val_loss did not improve from 0.02063
662/662 [=================== ] - 5s 7ms/step - loss: 0.0205 - val_loss: 0.0207
Epoch 175/333
662/662 [============ ] - ETA: 0s - loss: 0.0205
Epoch 175: val loss did not improve from 0.02063
Epoch 176/333
662/662 [============ ] - ETA: 0s - loss: 0.0205
Epoch 176: val loss did not improve from 0.02063
Epoch 177/333
Epoch 177: val loss did not improve from 0.02063
662/662 [======================== ] - 5s 7ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 178/333
Epoch 178: val loss did not improve from 0.02063
662/662 [=============== ] - 5s 8ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 179/333
Epoch 179: val_loss did not improve from 0.02063
Epoch 180/333
662/662 [================== ] - ETA: 0s - loss: 0.0205
Epoch 180: val loss did not improve from 0.02063
Epoch 181/333
Epoch 181: val_loss did not improve from 0.02063
Epoch 182/333
Epoch 182: val_loss did not improve from 0.02063
Epoch 183/333
Epoch 183: val loss did not improve from 0.02063
Epoch 184/333
Epoch 184: val loss did not improve from 0.02063
Epoch 185/333
Epoch 185: val loss did not improve from 0.02063
Epoch 186/333
Frach 186. val loss did not improve from a araka
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Epoch 187/333
659/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 187: val loss did not improve from 0.02063
Epoch 188/333
660/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 188: val loss did not improve from 0.02063
662/662 [================ ] - 6s 9ms/step - loss: 0.0205 - val_loss: 0.0207
Epoch 189/333
Epoch 189: val loss did not improve from 0.02063
Epoch 190/333
Epoch 190: val loss did not improve from 0.02063
Epoch 191/333
Epoch 191: val loss did not improve from 0.02063
Epoch 192/333
Epoch 192: val_loss did not improve from 0.02063
662/662 [================= ] - 5s 7ms/step - loss: 0.0205 - val_loss: 0.0207
Epoch 193/333
Epoch 193: val loss did not improve from 0.02063
662/662 [=============== ] - 6s 8ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 194/333
656/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 194: val loss did not improve from 0.02063
Epoch 195/333
Epoch 195: val loss did not improve from 0.02063
Epoch 196/333
Epoch 196: val loss did not improve from 0.02063
662/662 [=========================== ] - 6s 9ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 197/333
Epoch 197: val loss did not improve from 0.02063
Epoch 198/333
662/662 [=============== ] - ETA: 0s - loss: 0.0205
Epoch 198: val loss did not improve from 0.02063
Epoch 199/333
Epoch 199: val loss did not improve from 0.02063
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Epoch 200: val loss did not improve from 0.02063
662/662 [=============== ] - 5s 7ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 201/333
659/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 201: val loss did not improve from 0.02063
662/662 [======================== ] - 6s 9ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 202/333
Epoch 202: val_loss did not improve from 0.02063
662/662 [=============== ] - 5s 7ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 203/333
661/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 203: val loss did not improve from 0.02063
662/662 [================ ] - 5s 8ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 204/333
Epoch 204: val_loss did not improve from 0.02063
Epoch 205/333
657/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 205: val_loss did not improve from 0.02063
662/662 [================ ] - 5s 7ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 206/333
Epoch 206: val loss did not improve from 0.02063
662/662 [=============== ] - 6s 8ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 207/333
662/662 [============= ] - ETA: 0s - loss: 0.0205
Epoch 207: val loss did not improve from 0.02063
Epoch 208/333
Epoch 208: val loss did not improve from 0.02063
662/662 [======================== ] - 5s 8ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 209/333
662/662 [============= ] - ETA: 0s - loss: 0.0205
Epoch 209: val loss did not improve from 0.02063
Epoch 210/333
658/662 [=============>.] - ETA: 0s - loss: 0.0205
Epoch 210: val_loss did not improve from 0.02063
Epoch 211/333
Epoch 211: val loss did not improve from 0.02063
Epoch 212/333
Epoch 212: val loss did not improve from 0.02063
Epoch 213/333
Epoch 213: val loss did not improve from 0.02063
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Epoch 214/333
Epoch 214: val_loss did not improve from 0.02063
Epoch 215/333
Epoch 215: val loss did not improve from 0.02063
Epoch 216/333
660/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 216: val loss did not improve from 0.02063
Epoch 217/333
Epoch 217: val loss did not improve from 0.02063
662/662 [================ ] - 5s 7ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 218/333
660/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 218: val loss did not improve from 0.02063
662/662 [================ ] - 5s 8ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 219/333
Epoch 219: val loss did not improve from 0.02063
662/662 [============= ] - 7s 10ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 220/333
Epoch 220: val loss did not improve from 0.02063
Epoch 221/333
Epoch 221: val loss did not improve from 0.02063
662/662 [======================== ] - 5s 8ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 222/333
Epoch 222: val_loss did not improve from 0.02063
Epoch 223/333
Epoch 223: val loss did not improve from 0.02063
Epoch 224/333
662/662 [============== ] - ETA: 0s - loss: 0.0205
Epoch 224: val loss did not improve from 0.02063
662/662 [=============== ] - 6s 9ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 225/333
Epoch 225: val loss did not improve from 0.02063
Epoch 226/333
Epoch 226: val loss did not improve from 0.02063
Epoch 227/333
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Epoch 227: val loss did not improve from 0.02063
Epoch 228/333
Epoch 228: val_loss did not improve from 0.02063
Epoch 229/333
Epoch 229: val loss did not improve from 0.02063
Epoch 230/333
662/662 [=============== ] - ETA: 0s - loss: 0.0205
Epoch 230: val loss did not improve from 0.02063
662/662 [======================== ] - 5s 8ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 231/333
662/662 [=============== ] - ETA: 0s - loss: 0.0205
Epoch 231: val loss did not improve from 0.02063
662/662 [============= ] - 5s 8ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 232/333
Epoch 232: val loss did not improve from 0.02063
Epoch 233/333
Epoch 233: val loss did not improve from 0.02063
Epoch 234/333
Epoch 234: val_loss did not improve from 0.02063
662/662 [=============== ] - 6s 9ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 235/333
Epoch 235: val_loss did not improve from 0.02063
Epoch 236/333
Epoch 236: val loss did not improve from 0.02063
Epoch 237/333
Epoch 237: val loss did not improve from 0.02063
Epoch 238/333
Epoch 238: val loss did not improve from 0.02063
662/662 [================ ] - 5s 8ms/step - loss: 0.0205 - val_loss: 0.0207
Epoch 239/333
Epoch 239: val loss did not improve from 0.02063
662/662 [============= ] - 6s 9ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 240/333
Epoch 240: val_loss did not improve from 0.02063
```

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Epoch 241/333
Epoch 241: val loss did not improve from 0.02063
Epoch 242/333
Epoch 242: val loss did not improve from 0.02063
Epoch 243/333
Epoch 243: val loss did not improve from 0.02063
662/662 [============= ] - 5s 7ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 244/333
Epoch 244: val loss did not improve from 0.02063
Epoch 245/333
662/662 [=============== ] - ETA: 0s - loss: 0.0205
Epoch 245: val_loss did not improve from 0.02063
Epoch 246/333
662/662 [============ ] - ETA: 0s - loss: 0.0205
Epoch 246: val loss did not improve from 0.02063
662/662 [================ ] - 5s 8ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 247/333
658/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 247: val_loss did not improve from 0.02063
Epoch 248/333
Epoch 248: val loss did not improve from 0.02063
Epoch 249/333
662/662 [============== ] - ETA: 0s - loss: 0.0205
Epoch 249: val loss did not improve from 0.02063
662/662 [================ ] - 6s 10ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 250/333
Epoch 250: val_loss did not improve from 0.02063
662/662 [============= ] - 5s 8ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 251/333
662/662 [=============== ] - ETA: 0s - loss: 0.0205
Epoch 251: val loss did not improve from 0.02063
Epoch 252/333
Epoch 252: val loss did not improve from 0.02063
Epoch 253/333
660/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 253: val loss did not improve from 0.02063
Epoch 254/333
     ----- 1 _ ETA+ Qc _ 10cc+ Q Q2QC
```

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OUT/OUZ |----- UD55. W.WAD5
Epoch 254: val loss did not improve from 0.02063
Epoch 255/333
Epoch 255: val loss did not improve from 0.02063
Epoch 256/333
Epoch 256: val loss did not improve from 0.02063
Epoch 257/333
Epoch 257: val_loss did not improve from 0.02063
Epoch 258/333
Epoch 258: val_loss did not improve from 0.02063
662/662 [================ ] - 5s 7ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 259/333
660/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 259: val loss did not improve from 0.02063
Epoch 260/333
Epoch 260: val loss did not improve from 0.02063
Epoch 261/333
Epoch 261: val loss did not improve from 0.02063
662/662 [============= ] - 6s 9ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 262/333
Epoch 262: val loss did not improve from 0.02063
Epoch 263/333
661/662 [=============>.] - ETA: 0s - loss: 0.0205
Epoch 263: val_loss did not improve from 0.02063
Epoch 264/333
Epoch 264: val_loss did not improve from 0.02063
662/662 [======================= ] - 6s 9ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 265/333
Epoch 265: val loss did not improve from 0.02063
Epoch 266/333
Epoch 266: val loss did not improve from 0.02063
662/662 [============================ ] - 6s 9ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 267/333
660/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 267: val loss did not improve from 0.02063
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Epoch 268/333
662/662 [============= ] - ETA: 0s - loss: 0.0205
Epoch 268: val loss did not improve from 0.02063
662/662 [======================== ] - 5s 8ms/step - loss: 0.0205 - val loss: 0.0207
Epoch 269/333
Epoch 269: val_loss did not improve from 0.02063
Epoch 270/333
Epoch 270: val loss did not improve from 0.02063
Epoch 271/333
659/662 [============>.] - ETA: 0s - loss: 0.0205
Epoch 271: val loss did not improve from 0.02063
662/662 [=============== ] - 6s 9ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 272/333
Epoch 272: val loss did not improve from 0.02063
Epoch 273/333
Epoch 273: val loss did not improve from 0.02063
Epoch 274/333
Epoch 274: val loss did not improve from 0.02063
662/662 [======================= ] - 6s 9ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 275/333
661/662 [=============>.] - ETA: 0s - loss: 0.0204
Epoch 275: val_loss did not improve from 0.02063
Epoch 276/333
Epoch 276: val_loss did not improve from 0.02063
Epoch 277/333
662/662 [============== ] - ETA: 0s - loss: 0.0204
Epoch 277: val loss did not improve from 0.02063
662/662 [================ ] - 5s 8ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 278/333
662/662 [============= ] - ETA: 0s - loss: 0.0204
Epoch 278: val loss did not improve from 0.02063
Epoch 279/333
Epoch 279: val loss did not improve from 0.02063
Epoch 280/333
Epoch 280: val loss did not improve from 0.02063
Epoch 281/333
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Epoch 281: val_loss did not improve from 0.02063
Epoch 282/333
Epoch 282: val loss did not improve from 0.02063
Epoch 283/333
Epoch 283: val loss did not improve from 0.02063
Epoch 284/333
Epoch 284: val loss did not improve from 0.02063
662/662 [============= ] - 6s 9ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 285/333
Epoch 285: val loss did not improve from 0.02063
Epoch 286/333
Epoch 286: val loss did not improve from 0.02063
Epoch 287/333
658/662 [=============>.] - ETA: 0s - loss: 0.0204
Epoch 287: val loss did not improve from 0.02063
662/662 [======================== ] - 5s 8ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 288/333
Epoch 288: val_loss did not improve from 0.02063
662/662 [================ ] - 5s 7ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 289/333
Epoch 289: val loss did not improve from 0.02063
662/662 [=============== ] - 6s 9ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 290/333
Epoch 290: val loss did not improve from 0.02063
Epoch 291/333
Epoch 291: val loss did not improve from 0.02063
662/662 [=============== ] - 6s 9ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 292/333
656/662 [============>.] - ETA: 0s - loss: 0.0204
Epoch 292: val loss did not improve from 0.02063
662/662 [============= ] - 5s 8ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 293/333
Epoch 293: val loss did not improve from 0.02063
662/662 [=============] - 5s 8ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 294/333
Epoch 294: val loss did not improve from 0.02063
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Epoch 295/333
Epoch 295: val loss did not improve from 0.02063
662/662 [=============== ] - 5s 8ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 296/333
662/662 [============= ] - ETA: 0s - loss: 0.0204
Epoch 296: val loss did not improve from 0.02063
Epoch 297/333
656/662 [=============>.] - ETA: 0s - loss: 0.0204
Epoch 297: val_loss did not improve from 0.02063
Epoch 298/333
Epoch 298: val_loss did not improve from 0.02063
Epoch 299/333
658/662 [=============>.] - ETA: 0s - loss: 0.0204
Epoch 299: val loss did not improve from 0.02063
662/662 [=============== ] - 6s 9ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 300/333
661/662 [=============>.] - ETA: 0s - loss: 0.0204
Epoch 300: val_loss did not improve from 0.02063
662/662 [================ ] - 5s 8ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 301/333
Epoch 301: val_loss did not improve from 0.02063
Epoch 302/333
662/662 [============ ] - ETA: 0s - loss: 0.0204
Epoch 302: val loss did not improve from 0.02063
662/662 [================ ] - 5s 8ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 303/333
Epoch 303: val loss did not improve from 0.02063
662/662 [=============== ] - 5s 7ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 304/333
Epoch 304: val loss did not improve from 0.02063
Epoch 305/333
662/662 [=============== ] - ETA: 0s - loss: 0.0204
Epoch 305: val loss did not improve from 0.02063
662/662 [======================== ] - 5s 7ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 306/333
Epoch 306: val loss did not improve from 0.02063
Epoch 307/333
Epoch 307: val_loss did not improve from 0.02063
Epoch 308/333
1 1 111 11
```

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EDOCH 308: VAL LOSS GLG NOT IMPROVE TROM 0.02063
662/662 [======================== ] - 5s 7ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 309/333
Epoch 309: val loss did not improve from 0.02063
662/662 [======================= ] - 6s 9ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 310/333
Epoch 310: val_loss did not improve from 0.02063
Epoch 311/333
662/662 [============ ] - ETA: 0s - loss: 0.0204
Epoch 311: val loss did not improve from 0.02063
662/662 [================= ] - 7s 11ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 312/333
660/662 [============>.] - ETA: 0s - loss: 0.0204
Epoch 312: val loss did not improve from 0.02063
Epoch 313/333
Epoch 313: val loss did not improve from 0.02063
Epoch 314/333
659/662 [============>.] - ETA: 0s - loss: 0.0204
Epoch 314: val loss did not improve from 0.02063
662/662 [=========================== ] - 6s 9ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 315/333
662/662 [============= ] - ETA: 0s - loss: 0.0204
Epoch 315: val loss did not improve from 0.02063
662/662 [======================== ] - 5s 7ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 316/333
Epoch 316: val_loss did not improve from 0.02063
Epoch 317/333
Epoch 317: val_loss did not improve from 0.02063
662/662 [======================== ] - 5s 8ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 318/333
659/662 [============>.] - ETA: 0s - loss: 0.0204
Epoch 318: val loss did not improve from 0.02063
Epoch 319/333
Epoch 319: val_loss did not improve from 0.02063
Epoch 320/333
Epoch 320: val loss did not improve from 0.02063
Epoch 321/333
Epoch 321: val_loss did not improve from 0.02063
662/662 [================ ] - 6s 8ms/step - loss: 0.0204 - val loss: 0.0207
```

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LPUCII JEE/JJJ
Epoch 322: val_loss did not improve from 0.02063
Epoch 323/333
Epoch 323: val loss did not improve from 0.02063
Epoch 324/333
Epoch 324: val loss did not improve from 0.02063
662/662 [=============== ] - 6s 9ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 325/333
Epoch 325: val loss did not improve from 0.02063
Epoch 326/333
Epoch 326: val loss did not improve from 0.02063
Epoch 327/333
662/662 [=============] - ETA: 0s - loss: 0.0204
Epoch 327: val loss did not improve from 0.02063
Epoch 328/333
Epoch 328: val loss did not improve from 0.02063
Epoch 329/333
Epoch 329: val_loss did not improve from 0.02063
Epoch 330/333
Epoch 330: val loss did not improve from 0.02063
662/662 [================ ] - 5s 7ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 331/333
Epoch 331: val_loss did not improve from 0.02063
Epoch 332/333
Epoch 332: val loss did not improve from 0.02063
662/662 [================ ] - 5s 8ms/step - loss: 0.0204 - val loss: 0.0207
Epoch 333/333
Epoch 333: val_loss did not improve from 0.02063
662/662 [======================== ] - 5s 8ms/step - loss: 0.0204 - val loss: 0.0208
                        training loss
                        validation loss
0.30
0.25
```

```
0.20 -
```

```
1 # Evaluate the trained model on the test sets on the GPU
 2 # model = tf.keras.models.load model('best model')
 4 with tf.device('/GPU:0'):
      wide_test_loss = model.evaluate(
 6
          test_df[['m', 'tau', 'r', 'rho', 'kappa', 'theta', 'v0', 'option_price']],
 7
          test_df['sigma'],
 8
          verbose=0
 9
      )
10
11 wide_test_predictions = model.predict(test_df[['m', 'tau', 'r', 'rho', 'kappa', 'theta', 'v0', 'option_price']])
12
13 # Calculate performance metrics for wide test set
14
15 res = calc_metrics(test_df['sigma'], wide_test_predictions)
16 print(res)
     2874/2874 [=========== ] - 6s 2ms/step
     {'mse': 0.0064118987324165225, 'mae': 0.053238941802368274, 'r2': 0.6892325231064504}
```

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