

1. [Black-Scholes Model](#)
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Tools

```
1 import numpy as np
2 import pandas as pd
3 import warnings
4 from keras import backend as K
5 from scipy.stats import norm
6 import matplotlib.pyplot as plt
7 import seaborn as sb
8 import numpy as geek
9 from sklearn.model_selection import train_test_split
10 from sklearn.preprocessing import MinMaxScaler
11 import tensorflow as tf
12 from tensorflow.keras.models import Sequential
13 from tensorflow.keras.layers import Dense, Dropout
14 from tensorflow.keras.optimizers import Adam
15 from tensorflow.keras.initializers import GlorotUniform
16 from sklearn.metrics import mean_squared_error, mean_absolute_error, r2_score
```

```
1 # Check if GPU is available
2
3 if tf.test.gpu_device_name():
4     print('GPU found')
5 else:
6     print("No GPU found")
7
8 # Create a TensorFlow session and set it to use the GPU
9 config = tf.compat.v1.ConfigProto()
10 config.gpu_options.allow_growth = True
11 session = tf.compat.v1.Session(config=config)
```

No GPU found

Step 1: Data Generation with the Black-Scholes Model

BS-ANN Parameters	Range	Unit
Input		
Stock price S_0/K (Sk)	[0.5, 1.5]	-
Time to Maturity τ (T)	[0.3, 0.95]	year
Risk free rate (r)	[0.03, 0.08]	-
Volatility σ (sigma)	[0.02, 0.9]	-
Output		
Call price V/K (Price)	(0.0, 0.73)	-

```

1 # Generate random samples for input parameters
2
3 np.random.seed(42)
4 num_samples = 1000000

1 # Define the ranges for input parameters
2
3 param_range = {
4     'Sk': [0.5, 1.5],
5     'r': [0.03, 0.08],
6     'T': [0.3, 0.95],
7     'sigma': [0.02, 0.9]
8 }
9
10 # Generate random samples within the specified ranges
11 Sk = np.random.uniform(*param_range['Sk'], num_samples)
12 r = np.random.uniform(*param_range['r'], num_samples)
13 T = np.random.uniform(*param_range['T'], num_samples)
14 sigma = np.random.uniform(*param_range['sigma'], num_samples)

1 warnings.filterwarnings('ignore')
2
3 # Black-Scholes formula for generating data
4
5 def black_scholes(Sk, r, T, sigma):
6     d1 = (np.log(Sk) + (r + sigma**2 / 2) * T) / (sigma * np.sqrt(T))
7     d2 = d1 - sigma * np.sqrt(T)
8     call_price = Sk * norm.cdf(d1) - np.exp(-r * T) * norm.cdf(d2)
9     return call_price
10
11 option_prices = black_scholes(Sk, r, T, sigma)
12
13 # Create a DataFrame with input parameters and option prices

```

```

14 df = pd.DataFrame({'Sk': Sk, 'r': r, 'T': T, 'sigma': sigma, 'Price': option_prices})
15
16 # Split the dataset into train and test sets
17 # This article takes 90% of the data as the training data and 10% as test data
18 train_df, test_df = train_test_split(df, test_size=0.1, random_state=42)
19
20 # Scale the output option prices
21 scaler = MinMaxScaler()
22 train_df['Price'] = scaler.fit_transform(train_df['Price'].values.reshape(-1, 1))
23 test_df['Price'] = scaler.transform(test_df['Price'].values.reshape(-1, 1))

```

```
1 train_df.head()
```

	Sk	r	T	sigma	Price
378046	1.119234	0.053364	0.756577	0.180502	0.234627
510062	0.895161	0.043464	0.467723	0.627440	0.163255
534815	0.958026	0.074538	0.339123	0.246096	0.063904
342590	1.220338	0.036695	0.815436	0.594285	0.511054
559414	0.898186	0.067342	0.338859	0.324806	0.051439

- Parameters of the ANN

Parameters	Options
Hidden layers	4
Neurons(each layer)	400
Activation	ReLU
Dropout rate	0.0
Batch-normalization	No
Initialization	Glorot_uniform
Optimizer	Adam
Batch size	1024

Step 3: Black-Scholes Option Pricing ANN

```

1 # Model definition
2
3 model = Sequential()
4 model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform(), input_shape=(4,)))
5 model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))

```

```

6 model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
7 model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
8 model.add(Dense(1))
9 model.compile(optimizer=Adam(), loss='mean_squared_error')
10 batch_size = 1024
11 epochs = 10
12 with tf.device('/GPU:0'):
13     history = model.fit(
14         train_df[['Sk', 'r', 'T', 'sigma']],
15         train_df['Price'],
16         batch_size=batch_size,
17         epochs=epochs,
18         verbose=1
19     )

Epoch 1/10
879/879 [=====] - 69s 76ms/step - loss: 0.0013
Epoch 2/10
879/879 [=====] - 58s 66ms/step - loss: 4.2035e-05
Epoch 3/10
879/879 [=====] - 55s 63ms/step - loss: 3.6857e-05
Epoch 4/10
879/879 [=====] - 60s 68ms/step - loss: 3.1670e-05
Epoch 5/10
879/879 [=====] - 58s 66ms/step - loss: 2.7447e-05
Epoch 6/10
879/879 [=====] - 58s 66ms/step - loss: 2.2108e-05
Epoch 7/10
879/879 [=====] - 58s 66ms/step - loss: 2.1693e-05
Epoch 8/10
879/879 [=====] - 57s 65ms/step - loss: 2.1971e-05
Epoch 9/10
879/879 [=====] - 56s 63ms/step - loss: 2.2927e-05
Epoch 10/10
879/879 [=====] - 56s 63ms/step - loss: 1.1461e-05

```

Evaluation

```

1 def calc_metrics(y_true, y_pred):
2     mse = mean_squared_error(y_true, y_pred)
3     mae = mean_absolute_error(y_true, y_pred)
4     r2 = r2_score(y_true, y_pred)
5     return {'mse': mse, 'mae': mae, 'r2': r2}

1 # Evaluate the trained model on the test sets on the GPU
2
3 with tf.device('/GPU:0'):
4     wide_test_loss = model.evaluate(

```

```

5     test_df[['Sk', 'r', 'T', 'sigma']],
6     test_df['Price']],
7     verbose=0
8 )
9
10 wide_test_predictions = model.predict(test_df[['Sk', 'r', 'T', 'sigma']])
11 wide_test_predictions = scaler.inverse_transform(wide_test_predictions) # Inverse scaling
12
13 # Reshape wide_test_predictions to match the shape of test_df['Price']
14 wide_test_predictions = wide_test_predictions.flatten()
15
16 # Calculate performance metrics for wide test set
17 res= calc_metrics(test_df['Price'], wide_test_predictions)
18 print(res)

```

```

3125/3125 [=====] - 8s 3ms/step
{'mse': 0.00994309406799212, 'mae': 0.07533613762760052, 'r2': 0.8373844851410981}

```

```

1 # Train with different lrs
2 batch_size = 1024
3 epochs = 2
4 average_losses = []
5 learning_rate= []
6 for i in range(4):
7     model = Sequential()
8     model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform(), input_shape=(4,)))
9     model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
10    model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
11    model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
12    model.add(Dense(1))
13    model.compile(optimizer=Adam(), loss='mean_squared_error')
14    K.set_value(model.optimizer.learning_rate, 0.01+i*0.02)
15    learning_rate.append(0.01+i*0.02)
16    with tf.device('/GPU:0'):
17        history = model.fit(
18            train_df[['Sk', 'r', 'T', 'sigma']],
19            train_df['Price'],
20            batch_size=batch_size,
21            epochs=epochs,
22            verbose=1
23        )
24    average_losses.append(
25        np.average(history.history['loss'])
26    )
27
28 import matplotlib.pyplot as plt
29 import numpy as np
30 plt.plot(learning_rate, average_losses, marker='o')
31 plt.xlabel('Learning Rate')

```

```

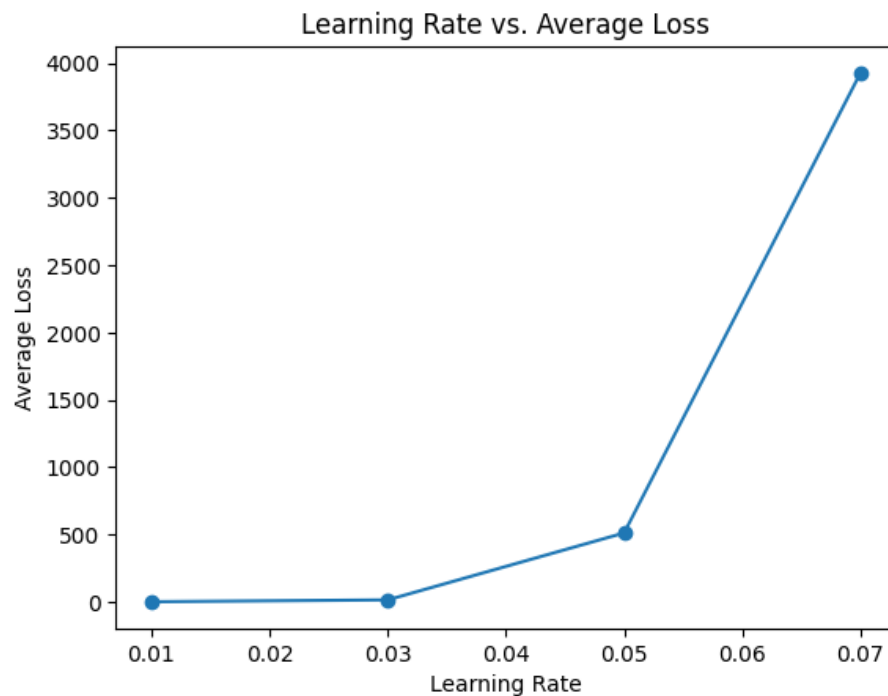
32 plt.ylabel('Average Loss')
33 plt.title('Learning Rate vs. Average Loss')
34 plt.show()

```

```

Epoch 1/2
879/879 [=====] - 70s 76ms/step - loss: 0.0739
Epoch 2/2
879/879 [=====] - 56s 64ms/step - loss: 1.0200e-04
Epoch 1/2
879/879 [=====] - 58s 64ms/step - loss: 27.1939
Epoch 2/2
879/879 [=====] - 56s 64ms/step - loss: 2.8678e-04
Epoch 1/2
879/879 [=====] - 57s 64ms/step - loss: 1021.6788
Epoch 2/2
879/879 [=====] - 56s 64ms/step - loss: 0.0012
Epoch 1/2
879/879 [=====] - 56s 62ms/step - loss: 7853.6475
Epoch 2/2
879/879 [=====] - 56s 64ms/step - loss: 0.0045

```



```

1 # Train with differnt dataset size
2 batch_size = 1024
3 epochs = 2
4 R2s = []

```

```

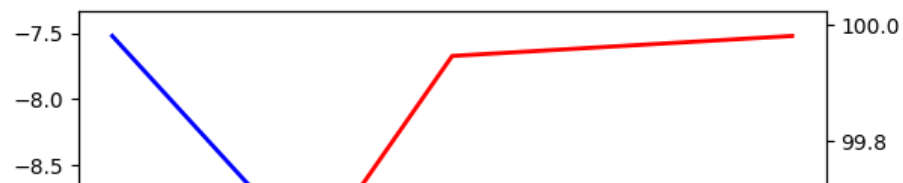
5 mses = []
6 x=[]
7 for i in range(3):
8     model = Sequential()
9     model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform(), input_shape=(4,)))
10    model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
11    model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
12    model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
13    model.add(Dense(1))
14    model.compile(optimizer=Adam(), loss='mean_squared_error')
15    K.set_value(model.optimizer.learning_rate, 0.0001)
16    with tf.device('/GPU:0'):
17        history = model.fit(
18            train_df[['Sk', 'r', 'T', 'sigma'][:int((i+1)/8*len(train_df))],
19            train_df['Price'][:int((i+1)/8*len(train_df))],
20            batch_size=batch_size,
21            epochs=epochs,
22            verbose=1
23        )
24    train_predicted = model.predict(train_df[['Sk', 'r', 'T', 'sigma'][:int((i+1)/8*len(train_df))]])
25    _metrics = calc_metrics(train_df['Price'][:int((i+1)/8*len(train_df))], train_predicted)
26    R2s.append(_metrics['r2']*100)
27    mses.append(np.log(_metrics['mse']))
28    x.append(i)
29 fig, ax1 = plt.subplots()
30
31 ax1.plot(x, mses, lw=2, color="blue")
32 ax1.set_ylabel(r"Log(MSE)", fontsize=10)
33
34 ax2 = ax1.twinx()
35 ax2.plot(x, R2s, lw=2, color="red")
36 ax2.set_ylabel(r"R2(%)", fontsize=10 )

```

```

Epoch 1/2
110/110 [=====] - 10s 78ms/step - loss: 0.0243
Epoch 2/2
110/110 [=====] - 8s 69ms/step - loss: 0.0023
3516/3516 [=====] - 11s 3ms/step
Epoch 1/2
220/220 [=====] - 15s 65ms/step - loss: 0.0161
Epoch 2/2
220/220 [=====] - 14s 64ms/step - loss: 2.0012e-04
7032/7032 [=====] - 18s 2ms/step
Epoch 1/2
330/330 [=====] - 22s 65ms/step - loss: 0.0090
Epoch 2/2
330/330 [=====] - 21s 64ms/step - loss: 4.7098e-05
10547/10547 [=====] - 28s 3ms/step
Text(0, 0.5, 'R2(%)')

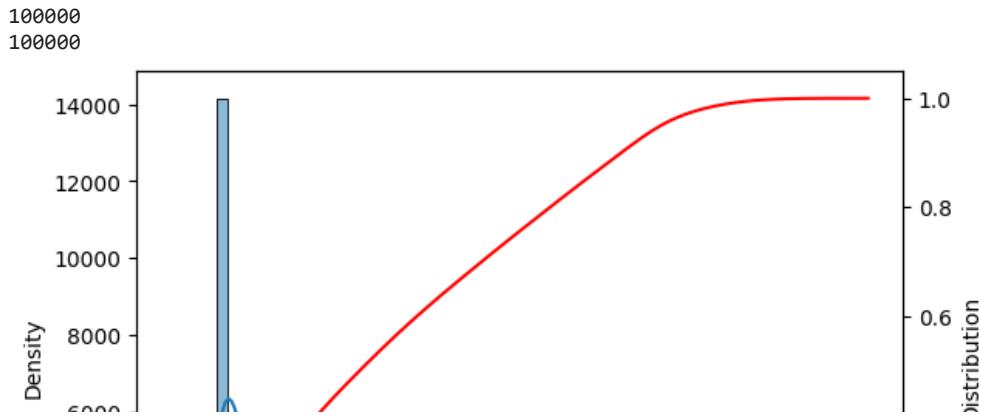
```



```

1 print(len(test_df['sigma']))
2 print(len(wide_test_predictions))
3 error=geek.subtract(test_df['Price'], wide_test_predictions)
4
5 # Plotting histogram with KDE
6 sb.histplot(error, kde=True, edgecolor='black')
7 plt.ylabel("Density")
8
9
10 # Plotting cumulative KDE
11 ax2 = plt.twinx()
12 sb.kdeplot(error, cumulative=True, color="red", ax=ax2)
13 ax2.set_ylabel("Distribution", fontsize=10)
14
15 plt.show()

```

Step 4: Black-Scholes Implicit Volatility ANN

Part 1 –The resulting input is then given by $\{V/K, S_0/K, r, \tau\}$

```

1 # Train the model on the GPU
2 model = Sequential()
3 model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform(), input_shape=(4,)))
4 model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
5 model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
6 model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
7 model.add(Dense(1))
8 model.compile(optimizer=Adam(), loss='mean_squared_error')
9 batch_size = 1024
10 epochs = 10
11 with tf.device('/GPU:0'):
12     history = model.fit(
13         train_df[['Sk', 'r', 'T', 'Price']],
14         train_df['sigma'],
15         batch_size=batch_size,
16         epochs=epochs,
17         verbose=1
18     )

```

```

Epoch 1/10
879/879 [=====] - 58s 64ms/step - loss: 0.0091
Epoch 2/10
879/879 [=====] - 57s 65ms/step - loss: 0.0018
Epoch 3/10
879/879 [=====] - 56s 64ms/step - loss: 0.0015
Epoch 4/10
879/879 [=====] - 56s 63ms/step - loss: 0.0014
Epoch 5/10
879/879 [=====] - 55s 63ms/step - loss: 0.0013

```

```

Epoch 6/10
879/879 [=====] - 57s 65ms/step - loss: 0.0012
Epoch 7/10
879/879 [=====] - 56s 64ms/step - loss: 0.0012
Epoch 8/10
879/879 [=====] - 55s 63ms/step - loss: 0.0011
Epoch 9/10
879/879 [=====] - 57s 65ms/step - loss: 0.0011
Epoch 10/10
879/879 [=====] - 58s 66ms/step - loss: 0.0011

```

```

1 # Evaluate the trained model on the test sets on the GPU
2
3 with tf.device('/GPU:0'):
4     wide_test_loss = model.evaluate(
5         test_df[['Sk', 'r', 'T', 'Price']],
6         test_df['sigma'],
7         verbose=0
8     )
9
10 wide_test_predictions = model.predict(test_df[['Sk', 'r', 'T', 'Price']])
11 res= calc_metrics(test_df['sigma'], wide_test_predictions)
12 print(res)

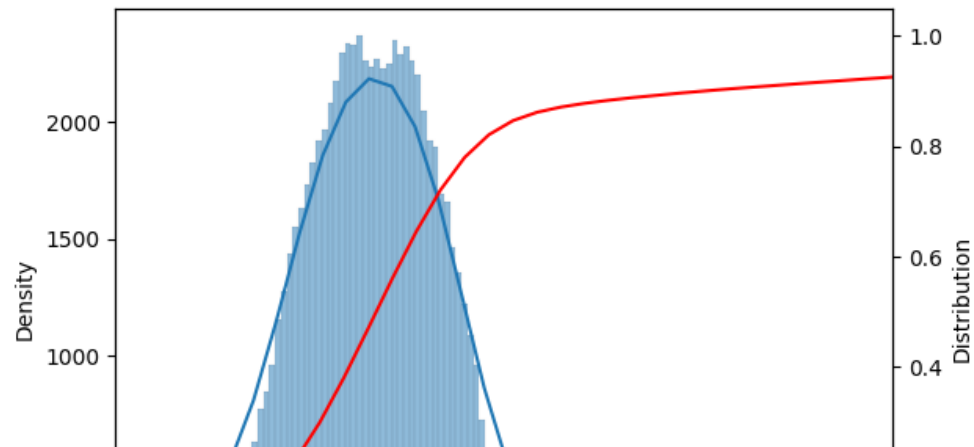
3125/3125 [=====] - 7s 2ms/step
{'mse': 0.0012293477194626039, 'mae': 0.02291394119952112, 'r2': 0.9809931881803292}

```

```

1 error = []
2 for myindex, sigma in enumerate(test_df['sigma']):
3     error.append(sigma-wide_test_predictions[myindex][0])
4 sb.histplot(error, kde=True, edgecolor='black')
5 plt.ylabel("Density")
6
7
8 # Plotting cumulative KDE
9 ax2 = plt.twinx()
10 sb.kdeplot(error, cumulative=True, color="red", ax=ax2)
11 ax2.set_ylabel("Distribution", fontsize=10)
12
13 plt.xlim(-0.01, 0.06)
14 plt.show()

```



```

1 plt.plot([min(test_df['sigma']), max(test_df['sigma'])], [min(wide_test_predictions), max(wide_test_predictions)], 'k-', label='R2')
2 # Calculate the midpoint coordinates of the line
3 x_mid = (min(test_df['sigma']) + max(test_df['sigma'])) / 2
4 y_mid = (min(wide_test_predictions) + max(wide_test_predictions)) / 2
5
6 # Add the 'R2' label at the midpoint
7 plt.text(x_mid, y_mid, 'R2 ', ha='right', va='center')
8 plt.xlabel("Actual Value")
9 plt.ylabel("Predicted Value")
10 plt.show()

```

Part 2 –

$$V_{\text{hat}} = V - \text{MAX}(S - Ke^{-r\tau}, 0)$$

where Vhat is the option time value.

The proposed approach to overcome approximation issues is to reduce the gradient's steepness by furthermore working under a log-transformation of the option value. The resulting input is then given by $\{\log(V_{\text{hat}}/K), S_0/K, r, \tau\}$. The adapted gradient approach increases the prediction accuracy significantly

```

1 warnings.filterwarnings('ignore')
2
3 Vhat=np.log(option_prices-np.maximum(Sk - np.exp(-r * T), 0))
4
5 df2= pd.DataFrame({'Sk': Sk, 'r': r, 'T': T, 'sigma': sigma, 'Price': Vhat})
6 df2.replace([-np.inf, -np.inf], np.nan, inplace=True)
7 df2.dropna(inplace=True)
8
9 # Split the dataset into train and testnan sets
10 train_df2, test_df2 = train_test_split(df2, test_size=0.1, random_state=42)
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868/868 [=====] - 55s 63ms/step - loss: 6.4656e-08
Epoch 5/10
868/868 [=====] - 54s 63ms/step - loss: 3.6444e-05
Epoch 6/10
868/868 [=====] - 58s 67ms/step - loss: 3.6026e-06
Epoch 7/10
868/868 [=====] - 57s 65ms/step - loss: 4.5707e-06
Epoch 8/10
868/868 [=====] - 55s 63ms/step - loss: 6.8074e-06
Epoch 9/10
868/868 [=====] - 55s 63ms/step - loss: 2.3071e-05
Epoch 10/10
868/868 [=====] - 57s 66ms/step - loss: 6.3326e-09

```

```

1 # Evaluate the trained model on the test sets on the GPU
2
3 with tf.device('/GPU:0'):
4     wide_test_loss = model.evaluate(
5         test_df2[['Sk', 'r', 'T', 'Price']],
6         test_df2['sigma'],
7         verbose=0
8     )
9
10 wide_test_predictions = model.predict(test_df2[['Sk', 'r', 'T', 'Price']])
11
12 res= calc_metrics(test_df2['sigma'], wide_test_predictions)
13 print(res)

```

```

3084/3084 [=====] - 8s 3ms/step
{'mse': 4.210007605292294e-09, 'mae': 4.757111173965783e-05, 'r2': -2.428845473707435e+24}

```

