
Tools

```
import numpy as np
import pandas as pd
import warnings
from keras import backend as K
from scipy.stats import norm
import matplotlib.pyplot as plt
import seaborn as sb
import numpy as geek
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import MinMaxScaler
import tensorflow as tf
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import Dense, Dropout
from tensorflow.keras.optimizers import Adam
from tensorflow.keras.initializers import GlorotUniform
from sklearn.metrics import mean_squared_error, mean_absolute_error, r2_score
```

```
# Check if GPU is available

if tf.test.gpu_device_name():
    print('GPU found')
else:
    print("No GPU found")

# Create a TensorFlow session and set it to use the GPU
config = tf.compat.v1.ConfigProto()
config.gpu_options.allow_growth = True
session = tf.compat.v1.Session(config=config)
```

```
No GPU found
```

Step 1: Data Generation with the Black-Scholes Model

BS-ANN Parameters	Range	Unit
Input		
Stock price S0/K (SK)	[0.5, 1.5]	-
Time to Maturity τ (T)	[0.3, 0.95]	year
Risk free rate (r)	[0.03, 0.08]	-
Volatility σ (sigma)	[0.02, 0.9]	-
Output		
Call price V/K (Price)	(0.0, 0.73)	-

```
# Generate random samples for input parameters

np.random.seed(42)
num_samples = 1000000
```

```
# Define the ranges for input parameters

param_range = {
    'Sk': [0.5, 1.5],
    'r': [0.03, 0.08],
    'T': [0.3, 0.95],
    'sigma': [0.02, 0.9]
}

# Generate random samples within the specified ranges
Sk = np.random.uniform(*param_range['Sk'], num_samples)
r = np.random.uniform(*param_range['r'], num_samples)
T = np.random.uniform(*param_range['T'], num_samples)
sigma = np.random.uniform(*param_range['sigma'], num_samples)
```

```
warnings.filterwarnings('ignore')

# Black-Scholes formula for generating data

def black_scholes(Sk, r, T, sigma):
    d1 = (np.log(Sk) + (r + sigma**2 / 2) * T) / (sigma * np.sqrt(T))
    d2 = d1 - sigma * np.sqrt(T)
    call_price = Sk * norm.cdf(d1) - np.exp(-r * T) * norm.cdf(d2)
    return call_price

option_prices = black_scholes(Sk, r, T, sigma)

# Create a DataFrame with input parameters and option prices
```

```
df = pd.DataFrame({'Sk': Sk, 'r': r, 'T': T, 'sigma': sigma, 'Price': option_prices})

# Split the dataset into train and test sets
# This article takes 90% of the data as the training data and 10% as test data
train_df, test_df = train_test_split(df, test_size=0.1, random_state=42)

# Scale the output option prices
scaler = MinMaxScaler()
train_df['Price'] = scaler.fit_transform(train_df['Price'].values.reshape(-1, 1))
test_df['Price'] = scaler.transform(test_df['Price'].values.reshape(-1, 1))
```

```
train_df.head()
```

	Sk	r	T	sigma	Price
378046	1.119234	0.053364	0.756577	0.180502	0.234627
510062	0.895161	0.043464	0.467723	0.627440	0.163255
534815	0.958026	0.074538	0.339123	0.246096	0.063904
342590	1.220338	0.036695	0.815436	0.594285	0.511054
559414	0.898186	0.067342	0.338859	0.324806	0.051439

- Parameters of the ANN

Parameters	Options
Hidden layers	4
Neurons(each layer)	400
Activation	ReLU
Dropout rate	0.0
Batch-normalization	No
Initialization	Glorot_uniform
Optimizer	Adam
Batch size	1024

Step 3: Black-Scholes Option Pricing ANN

```
# Model definition

model = Sequential()
model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform(), input_shape=(4,)))
model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
```

```
model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
model.add(Dense(1))
model.compile(optimizer=Adam(), loss='mean_squared_error')
batch_size = 1024
epochs = 10
with tf.device('/GPU:0'):
    history = model.fit(
        train_df[['Sk', 'r', 'T', 'sigma']],
        train_df['Price'],
        batch_size=batch_size,
        epochs=epochs,
        verbose=1
    )
```

```
Epoch 1/10
879/879 [==============================] - 69s 76ms/step - loss: 0.0013
Epoch 2/10
879/879 [==============================] - 58s 66ms/step - loss: 4.2035e-05
Epoch 3/10
879/879 [==============================] - 55s 63ms/step - loss: 3.6857e-05
Epoch 4/10
879/879 [==============================] - 60s 68ms/step - loss: 3.1670e-05
Epoch 5/10
879/879 [==============================] - 58s 66ms/step - loss: 2.7447e-05
Epoch 6/10
879/879 [==============================] - 58s 66ms/step - loss: 2.2108e-05
Epoch 7/10
879/879 [==============================] - 58s 66ms/step - loss: 2.1693e-05
Epoch 8/10
879/879 [==============================] - 57s 65ms/step - loss: 2.1971e-05
Epoch 9/10
879/879 [==============================] - 56s 63ms/step - loss: 2.2927e-05
Epoch 10/10
879/879 [==============================] - 56s 63ms/step - loss: 1.1461e-05
```

Evaluation

```
def calc_metrics(y_true, y_pred):
    mse = mean_squared_error(y_true, y_pred)
    mae = mean_absolute_error(y_true, y_pred)
    r2 = r2_score(y_true, y_pred)
    return {'mse': mse, 'mae': mae, 'r2': r2}
```

```
# Evaluate the trained model on the test sets on the GPU

with tf.device('/GPU:0'):
    wide_test_loss = model.evaluate(
```

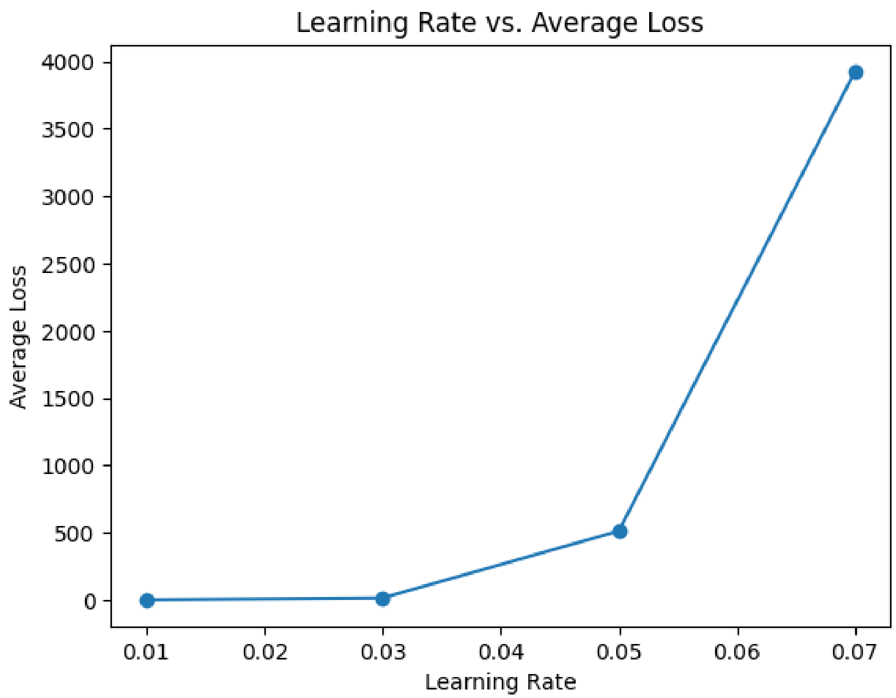
```
        test_df[['Sk', 'r', 'T', 'sigma']],
        test_df['Price'],
        verbose=0
    )

wide_test_predictions = model.predict(test_df[['Sk', 'r', 'T', 'sigma']])
wide_test_predictions = scaler.inverse_transform(wide_test_predictions)  # Inverse scaling

# Reshape wide_test_predictions to match the shape of test_df['Price']
wide_test_predictions = wide_test_predictions.flatten()

# Calculate performance metrics for wide test set
res= calc_metrics(test_df['Price'], wide_test_predictions)
print(res)
```

```
3125/3125 [==============================] - 8s 3ms/step
{'mse': 0.00994309406799212, 'mae': 0.07533613762760052, 'r2': 0.8373844851410981}
```

```
# Train with different lrs
batch_size = 1024
epochs = 2
average_losses = []
learning_rate= []
for i in range(4):
  model = Sequential()
  model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform(), input_shape=(4,)))
  model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
  model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
  model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
  model.add(Dense(1))
  model.compile(optimizer=Adam(), loss='mean_squared_error')
  K.set_value(model.optimizer.learning_rate, 0.01+i*0.02)
  learning_rate.append(0.01+i*0.02)
  with tf.device('/GPU:0'):
      history = model.fit(
          train_df[['Sk', 'r', 'T', 'sigma']],
          train_df['Price'],
          batch_size=batch_size,
          epochs=epochs,
          verbose=1
      )
  average_losses.append(
      np.average(history.history['loss'])
  )

import matplotlib.pyplot as plt
import numpy as np
plt.plot(learning_rate, average_losses, marker='o')
plt.xlabel('Learning Rate')
```

```
plt.ylabel('Average Loss')
plt.title('Learning Rate vs. Average Loss')
plt.show()
```

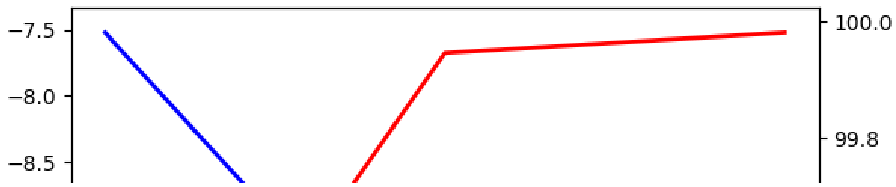
```
Epoch 1/2
879/879 [==============================] - 70s 76ms/step - loss: 0.0739
Epoch 2/2
879/879 [==============================] - 56s 64ms/step - loss: 1.0200e-04
Epoch 1/2
879/879 [==============================] - 58s 64ms/step - loss: 27.1939
Epoch 2/2
879/879 [==============================] - 56s 64ms/step - loss: 2.8678e-04
Epoch 1/2
879/879 [==============================] - 57s 64ms/step - loss: 1021.6788
Epoch 2/2
879/879 [==============================] - 56s 64ms/step - loss: 0.0012
Epoch 1/2
879/879 [==============================] - 56s 62ms/step - loss: 7853.6475
Epoch 2/2
879/879 [==============================] - 56s 64ms/step - loss: 0.0045
```




```
# Train with differnt dataset size
batch_size = 1024
epochs = 2
R2s = []
```

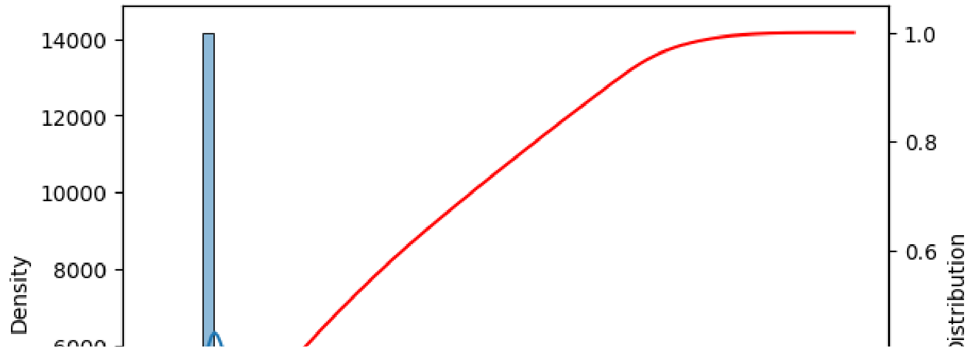
```
mses = []
x=[]
for i in range(3):
  model = Sequential()
  model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform(), input_shape=(4,)))
  model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
  model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
  model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
  model.add(Dense(1))
  model.compile(optimizer=Adam(), loss='mean_squared_error')
  K.set_value(model.optimizer.learning_rate, 0.0001)
  with tf.device('/GPU:0'):
      history = model.fit(
          train_df[['Sk', 'r', 'T', 'sigma']][:int((i+1)/8*len(train_df))],
          train_df['Price'][:int((i+1)/8*len(train_df))],
          batch_size=batch_size,
          epochs=epochs,
          verbose=1
      )
  train_predicteds = model.predict(train_df[['Sk', 'r', 'T', 'sigma']][:int((i+1)/8*len(train_df))])
  _metrics = calc_metrics(train_df['Price'][:int((i+1)/8*len(train_df))], train_predicteds)
  R2s.append(_metrics['r2']*100)
  mses.append(np.log(_metrics['mse']))
  x.append(i)
fig, ax1 = plt.subplots()

ax1.plot(x, mses, lw=2, color="blue")
ax1.set_ylabel(r"Log(MSE)", fontsize=10)

ax2 = ax1.twinx()
ax2.plot(x, R2s, lw=2, color="red")
ax2.set_ylabel(r"R2(%)", fontsize=10 )
```

```
Epoch 1/2
110/110 [==============================] - 10s 78ms/step - loss: 0.0243
Epoch 2/2
110/110 [==============================] - 8s 69ms/step - loss: 0.0023
3516/3516 [==============================] - 11s 3ms/step
Epoch 1/2
220/220 [==============================] - 15s 65ms/step - loss: 0.0161
Epoch 2/2
220/220 [==============================] - 14s 64ms/step - loss: 2.0012e-04
7032/7032 [==============================] - 18s 2ms/step
Epoch 1/2
330/330 [==============================] - 22s 65ms/step - loss: 0.0090
Epoch 2/2
330/330 [==============================] - 21s 64ms/step - loss: 4.7098e-05
10547/10547 [==============================] - 28s 3ms/step
Text(0, 0.5, 'R2(%)')
```




```
print(len(test_df['sigma']))
print(len(wide_test_predictions))
error=geek.subtract(test_df['Price'], wide_test_predictions)

# Plotting histogram with KDE
sb.histplot(error, kde=True, edgecolor='black')
plt.ylabel("Density")


# Plotting cumulative KDE
ax2 = plt.twinx()
sb.kdeplot(error, cumulative=True, color="red", ax=ax2)
ax2.set_ylabel("Distribution", fontsize=10)

plt.show()
```

```
100000
100000
```



Step 4: Black-Scholes Implicit Volatility ANN

Part 1 –The resulting input is then given by {V/K, S0/K,r, τ}

```
# Train the model on the GPU
model = Sequential()
model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform(), input_shape=(4,)))
model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
model.add(Dense(1))
model.compile(optimizer=Adam(), loss='mean_squared_error')
batch_size = 1024
epochs = 10
with tf.device('/GPU:0'):
    history = model.fit(
        train_df[['Sk', 'r', 'T', 'Price']],
        train_df['sigma'],
        batch_size=batch_size,
        epochs=epochs,
        verbose=1
    )
```

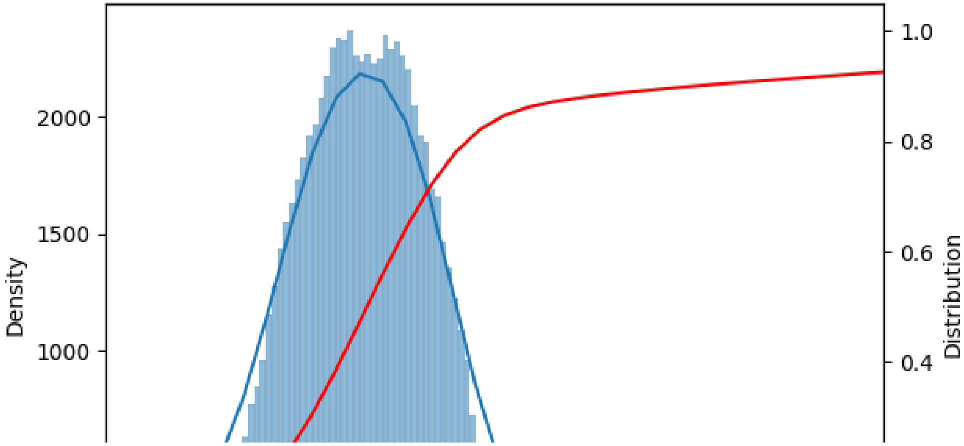
```
Epoch 1/10
879/879 [==============================] - 58s 64ms/step - loss: 0.0091
Epoch 2/10
879/879 [==============================] - 57s 65ms/step - loss: 0.0018
Epoch 3/10
879/879 [==============================] - 56s 64ms/step - loss: 0.0015
Epoch 4/10
879/879 [==============================] - 56s 63ms/step - loss: 0.0014
Epoch 5/10
879/879 [==============================] - 55s 63ms/step - loss: 0.0013
```

```
Epoch 6/10
879/879 [==============================] - 57s 65ms/step - loss: 0.0012
Epoch 7/10
879/879 [==============================] - 56s 64ms/step - loss: 0.0012
Epoch 8/10
879/879 [==============================] - 55s 63ms/step - loss: 0.0011
Epoch 9/10
879/879 [==============================] - 57s 65ms/step - loss: 0.0011
Epoch 10/10
879/879 [==============================] - 58s 66ms/step - loss: 0.0011
```

```
# Evaluate the trained model on the test sets on the GPU

with tf.device('/GPU:0'):
    wide_test_loss = model.evaluate(
        test_df[['Sk', 'r', 'T', 'Price']],
        test_df['sigma'],
        verbose=0
    )

wide_test_predictions = model.predict(test_df[['Sk', 'r', 'T', 'Price']])
res= calc_metrics(test_df['sigma'], wide_test_predictions)
print(res)
```

```
3125/3125 [==============================] - 7s 2ms/step
{'mse': 0.0012293477194626039, 'mae': 0.02291394119952112, 'r2': 0.9809931881803292}
```

```
error = []
for myindex, sigma in enumerate(test_df['sigma']):
  error.append(sigma-wide_test_predictions[myindex][0])
sb.histplot(error, kde=True, edgecolor='black')
plt.ylabel("Density")


# Plotting cumulative KDE
ax2 = plt.twinx()
sb.kdeplot(error, cumulative=True, color="red", ax=ax2)
ax2.set_ylabel("Distribution", fontsize=10)

plt.xlim(-0.01, 0.06)
plt.show()
```




```
plt.plot([min(test_df['sigma']), max(test_df['sigma'])], [min(wide_test_predictions), max(wide_test_predictions)], 'k-',  label='R2')
# Calculate the midpoint coordinates of the line
x_mid = (min(test_df['sigma']) + max(test_df['sigma'])) / 2
y_mid = (min(wide_test_predictions) + max(wide_test_predictions)) / 2

# Add the 'R2' label at the midpoint
plt.text(x_mid, y_mid, 'R2  ', ha='right', va='center')
plt.xlabel("Actual Value")
plt.ylabel("Predicted Value")
plt.show()
```

Part 2 –

$$Vhat = V - MAX(S - Ke^{-r\tau}, 0)$$

where Vhat is the option time value.

The proposed approach to overcome approximation issues is to reduce the gradient's steepness by furthermore working under a log-transformation of the option value. The resulting input is then given by {log (Vhat/K), S0/K,r, τ}. The adapted gradient approach increases the prediction accuracy significantly

```
warnings.filterwarnings('ignore')

Vhat=np.log(option_prices-np.maximum(Sk -  np.exp(-r * T), 0))

df2= pd.DataFrame({'Sk': Sk, 'r': r, 'T': T, 'sigma': sigma, 'Price': Vhat})
df2.replace([-np.inf, -np.inf], np.nan, inplace=True)
df2.dropna(inplace=True)

# Split the dataset into train and testnan sets
train_df2, test_df2 = train_test_split(df2, test_size=0.1, random_state=42)

```

```
# Train the model on the GPU
model = Sequential()
model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform(), input_shape=(4,)))
model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
model.add(Dense(400, activation='relu', kernel_initializer=GlorotUniform()))
model.add(Dense(1))
model.compile(optimizer=Adam(), loss='mean_squared_error')
batch_size = 1024
epochs = 10
with tf.device('/GPU:0'):
    history = model.fit(
        train_df2[['Sk', 'r', 'T', 'Price']],
        train_df2['sigma'],
        batch_size=batch_size,
        epochs=epochs,
        verbose=1
    )
```

```
Epoch 1/10
868/868 [==============================] - 57s 65ms/step - loss: 0.0254
Epoch 2/10
868/868 [==============================] - 54s 62ms/step - loss: 0.0011
Epoch 3/10
868/868 [==============================] - 56s 64ms/step - loss: 7.7130e-08
Epoch 4/10
```

```
868/868 [==============================] - 55s 63ms/step - loss: 6.4656e-08
Epoch 5/10
868/868 [==============================] - 54s 63ms/step - loss: 3.6444e-05
Epoch 6/10
868/868 [==============================] - 58s 67ms/step - loss: 3.6026e-06
Epoch 7/10
868/868 [==============================] - 57s 65ms/step - loss: 4.5707e-06
Epoch 8/10
868/868 [==============================] - 55s 63ms/step - loss: 6.8074e-06
Epoch 9/10
868/868 [==============================] - 55s 63ms/step - loss: 2.3071e-05
Epoch 10/10
868/868 [==============================] - 57s 66ms/step - loss: 6.3326e-09
```

```
# Evaluate the trained model on the test sets on the GPU

with tf.device('/GPU:0'):
    wide_test_loss = model.evaluate(
        test_df2[['Sk', 'r', 'T', 'Price']],
        test_df2['sigma'],
        verbose=0
    )

wide_test_predictions = model.predict(test_df2[['Sk', 'r', 'T', 'Price']])

res= calc_metrics(test_df2['sigma'], wide_test_predictions)
print(res)
```

```
3084/3084 [==============================] - 8s 3ms/step
{'mse': 4.210007605292294e-09, 'mae': 4.757111173965783e-05, 'r2': -2.428845473707435e+24}
```