Simulations

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1 Spectral separation.... foreground (Guillaume 2021)

Assumption: A and E channels have the same GW and same noise, T channel has just noise.

The data is simulated in the frequency domain. PSD for the three channels is given as:

$$PSD_A = S_A + N_A, \tag{1}$$
$$PSD_E = S_E + N_E, \tag{2}$$
$$PSD_T = N_T, \tag{3}$$

with

$$S_A = S_E = \frac{3H_0^2}{4\pi^2}\frac{\Omega_{GW}R(f)}{f^3}, \tag{4}$$
$$\Omega_{GW} = b\Big(\frac{f}{f_{ref}}\Big)^g, \tag{5}$$
$$R(f) = \frac{3}{10}\frac{(\frac{2\pi fL}{c})^2}{1+0.6(\frac{2\pi fL}{c})^2}. \tag{6}$$

Where $R(f)$ is the LISA response function and Ω_{GW} is the energy density for SGWB (power law model).

Noise is given as:

$$N_A = N_E = N_1 - N_2, \tag{7}$$
$$N_T = N_1 + 2N_2, \tag{8}$$

with

$$N_1 = \Big(4S_s(f) + 8\Big(1 + cos^2\Big(\frac{f}{f_{ref}}\Big)\Big)S_a(f)\Big)|W(f)|^2, \tag{9}$$
$$N_2 = -\Big(2S_s(f) + 8S_a(f)\Big)cos\Big(\frac{f}{f_{ref}}\Big)|W(f)|^2, \tag{10}$$

where $W(f) = 1 - e^{\frac{-2\pi i f}{f_{ref}}}$ and $S_a(f) = N_{pos}$, and $S_s(f) = \frac{N_{acc}}{(2\pi f)^2}(1+(\frac{4\times10^{-4}\mathrm{Hz}}{f})^2)$ with acceleration noise level $N_{acc} = 1.44 \times 10^{-48}\mathrm{s}^{-4}\mathrm{Hz}^{-1}$, and optical path length fluctuation noise level $N_{pos} = 3.6 \times 10^{-41}\mathrm{Hz}^{-1}$.

2 Prospects.....transitions (Guillaume 2023)

Assumption: A and E channels have the same GW and same noise, T channel has just noise.

Energy spectral density of noise for three channels is given as:

$$\Omega_A = \Omega_{NE} = S_A(f)\frac{4\pi^2 f^3}{3H_0} \tag{11}$$

$$\Omega_T = S_T(f)\frac{4\pi^2 f^3}{3H_0^2} \tag{12}$$

where the noise spectral densities S_A, S_E and S_T are given as:

$$S_E(f) = S_A(f) = \frac{N_A(f)}{R_A(f)} \tag{13}$$

$$S_T(f) = \frac{N_T(f)}{R_T(f)} \tag{14}$$

LISA response functions for three channels:

$$R_A(f) = R_E(f) = \frac{9}{10}|W(f)|^2\Big[1 + \Big(\frac{f}{4f_*/3}\Big)^2\Big]^{-1}, \tag{15}$$

$$R_T(f) = \frac{1}{4032}\Big(\frac{f}{f_*}\Big)^6|W(f)|^2\Big[1 + \frac{5}{16128}\Big(\frac{f}{f_*}\Big)^8\Big]^{-1}, \tag{16}$$

where $W(f) = 1 - e^{-2if/f_*}$ and $f_* = c/2\pi L$.

The noise model is:

$$N_A(f) = N_E(f) = N_X(f) - N_{XY}(f) \tag{17}$$

$$N_T(f) = N_X(f) + 2N_{XY}(f) \tag{18}$$

$$N_X(f) = (4P_s(f) + 8\Big[1 + \cos^2\Big(\frac{f}{f_*}\Big)\Big]P_a(f))|W(f)|^2 \tag{19}$$

$$N_{XY}(f) = -\Big[2P_s(f) + 8P_a(f)\Big]\cos\Big(\frac{f}{f_*}\Big)|W(f)|^2 \tag{20}$$

with optical path-length fluctuation noise $P_s(f)$, and single test mass acceleration noise $P_a(f)$:

$$P_s(f) = N_{pos}, \tag{21}$$

$$P_a(f) = \frac{N_{acc}}{(2\pi f)^4}\Big(1 + \Big(\frac{0.4\mathrm{mHz}}{f}\Big)^2\Big), \tag{22}$$

where $N_{pos} = 3.6 \times 10^{-41}\mathrm{Hz}^{-}1$ is optical pathlength fluctuation noise level, and $N_{acc} = 1.44 \times 10^{-48}\mathrm{s}^{-4}\mathrm{Hz}^{-1}$ is acceleration noise level.

The energy spectral density for SGWB is given as:

$$\Omega_{GW} = b\Big(\frac{f}{f_{ref}}\Big)^{g} \tag{23}$$