Flight Control Systems
(MECH 6091)

F-16 Autopilot - Final Report

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Submitted to *Dr. Youmin Zhang*

"I certify that this submission is our original work and meets the Faculty's Expectations of Originality"

December 19th, 2022

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Introduction

As aircraft design has progressed from the first powered flight of the Wright brothers to longer durations and heights, so has the need of alleviating the pilot of unneeded stress. The inception of the first primitive autopilot systems were as simple as blocking the control stick or pedals to compensate for wind direction during flight. As technology progressed, feedback systems were developed to ensure stable, level flight from a set of navigation coordinates or heading. Furthermore, as militaries saw the importance of airspace dominance during armed conflict, armament manufacturers sought maneuverability advantages over their opponent. The result was the inception of unstable airframes, which need an active control system to maintain level flight.

The F-16 Fighting Falcon is an example of such control requirements. With over 4600 airframes produced since 1974 and exported to over 27 countries, it remains one of the most ubiquitous and successful military aircraft ever produced. It also uses cutting-edge aerodynamics and avionics, including the first application of a relaxed static stability/fly-by- flight control system for improved maneuverability.

The study of this airframe and its flight control system for an autopilot in the longitudinal and lateral planes are the subject of this paper.

Figure 1 An F-16 model [1]

F-16 Properties

In this section properties of F-16 are defined based on empirical values and some assumed parameters. These properties are defined as shown in the following tables:

Parameter	**Symbol**	**Value**
Vehicle Weight (lbs)	W	20500
Wing span (ft)	B	30
Wing area (ft^2)	S	300
Mean aerodynamic chord (ft)	$\bar{c}$	11.32
Roll moment of inertia (slug-ft^2)	I_X	9496
Pitch moment of inertia (slug-ft^2)	Iy	55814
Yaw moment of inertia (slug-ft^2)	Iz	63100
Product moment of inertia (slug-ft^2)	I_{Xx}	982
Product moment of inertia (slug-ft^2)	I_{Xy}	0
Product moment of inertia (slug-ft^2)	Iyz	0

Table 1 Mass and Geometry Parameter [2]

Parameter	Symbol	Value
Reference CG Location (ft)	X_{cgR}	0.35 $\bar{c}$
Gravitational constant (ft/sec^2)	g	32.174
Engine Angular Momentum (slug-ft^2 /s) (assumed fixed !)	h_E	160.0
Radian-to-degree constant	d_r	57.29578

Table 2 Other parameters used in the model [2]

Model and Assumptions

Reference Frame

A reference frame is required to explain the position and behavior of an aircraft (RF). Depending on the situation, a reference frame is chosen based on practicality, usually to simplify mathematical calculations. The Earth-fixed reference denoted as FE, and the body-fixed reference denoted as FB, are the two most widely used frames.

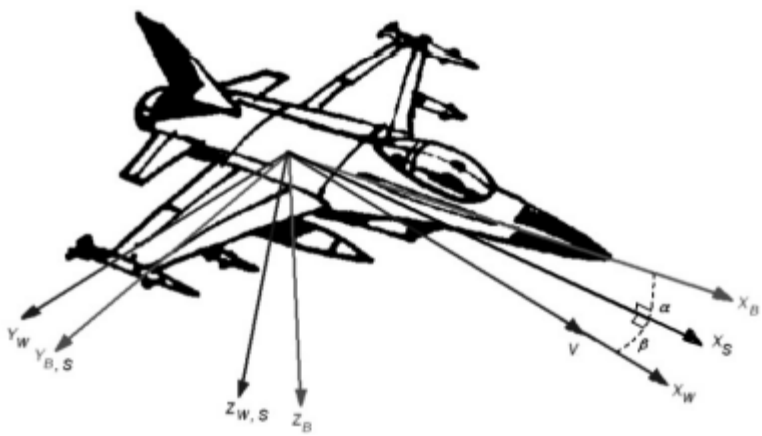

Figure 2 Aircraft Reference frames [3]

Assumptions

The following assumptions were made for the derivations of the equations required for the mathematical model.

- The plane is considered to be a rigid body. Of course, this is an idealization as all solid materials change shape to some extent when forces are applied to them.
- Flat, non-revolving Earth when used as an inertial reference. This is not true when examining inertial guiding systems.
- The mass remains constant and fuel consumption is ignored during this time frame.
- Compared to the X_BOZ_B-plane, the aircraft's mass distribution is symmetric.
- The aircraft is symmetric.
- There is constant wind.

Aircraft Control Surfaces

Control surfaces are typically utilized to control an aircraft. Elevators, flaps, and spoilers are a few examples. We can distinguish between primary and secondary flight control surfaces. The entire airplane becomes uncontrollable when the major control surfaces malfunction. (Elevators, ailerons, and rudders are examples.) However, the aircraft becomes slightly more difficult to operate when secondary control surfaces malfunction. (Flaps and trim tabs serve as examples.) The control system refers to the entire system required to control the aircraft and is referred to as reversible when it gives the pilot immediate feedback.

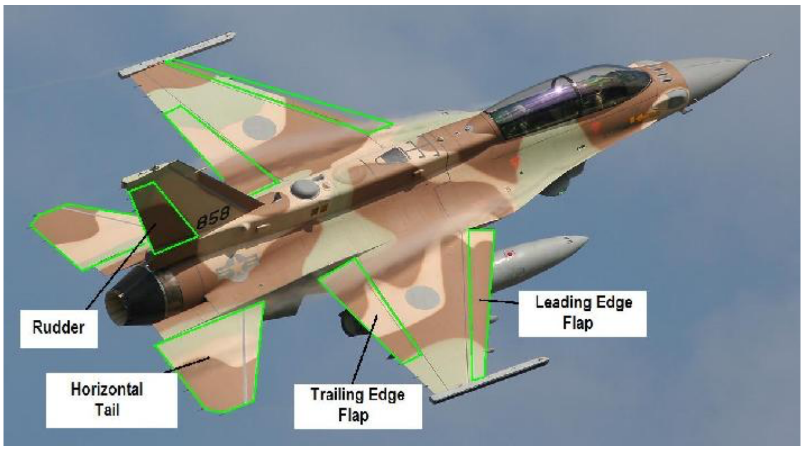

Figure 3 Control Surfaces of an F-16 Model [4]

The F-16 model has controls for the elevator, aileron, rudder, and thrust. The thrust works positively along the positive body x-axis and is measured in pounds. Surface deflection results in a change in body rates.

The following table defines the maximum positive orientations for each control surface:

Symbol	**Command name**	**Deflection limit**	**Rate limit**	**Time constant**	**Positive sign convention**	**Effect**
δ_E	Elevator	± 25.0˚	60˚/s	0.0495sec lag	Trailing edge down	Negative pitching moment
δ_A	Ailerons	± 21.5˚	80˚/s	0.0495sec lag	Right-wing trailing edge down	Negative prolling moment
δ_R	Rudder	± 30.0˚	120˚/s	0.0495sec lag	Trailing edge left	Negative yawing moment, positive rolling moment

Table 3 Control Surface Actuator Models [2]

Mathematical Model

Non-Linear Equations

Since Newton's Second Law of Motion is only applicable in the inertial reference frame, it is assumed that the earth is flat and not spinning when defining the force and moment equations for the earth fixed reference frame. The equations of motion and force may then be obtained by switching from the earth fixed frame to the body fixed frame using Newton's Second Law of Motion.

Forces expressed in earth fixed frame are shown in equation 1. Where m is the mass of the aircraft and V is the total velocity of the aircraft.

$$F = [\frac{d(mV)}{dt}]_E$$

While the forces expressed in the body frame are expressed as:

$$F = [\frac{d(mV)}{dt}]_B + \omega + mV$$

Roll, pitch and yaw rate can be reflected in the vector components of the angular velocity which in return can be represented in p, q and r which will be in the x, y and z direction. While the velocity vector V has components in u, v and w in the x, y and z directions.

$$F = \left(\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} + \begin{bmatrix} p \\ q \\ r \end{bmatrix} x \begin{bmatrix} u \\ v \\ w \end{bmatrix} \right)$$

This can be further simplified to represent forces in x, y and z directions

$$F_x = m\ (\dot{u} + qw - rv)$$

$$F_y = m\ (\dot{v} + ru - pw)$$

$$F_z = m\ (\dot{w} + pv - qv)$$

Kinetic Equations

Bank angle (Φ) for roll, Elevation angle (Θ) for pitch, and heading angle(Ψ) for yaw with respect to the body and earth frame are represented as follows:

$$\dot{\varphi} = p + tan\theta\ (q\ sin\varphi + r\ cos\varphi)$$

$$\dot{\theta} = q\ cos\varphi - rsin\varphi$$

$$\dot{\Psi} = \frac{q\ sin\,\Phi + r\ cos\,\Phi}{cos\,\Theta}$$

Forces acting on an Aircraft

Weight

Weight (W) of the aircraft is defined based on the gravity acting on it. Weight is further represented in x, y, and z directions.

$$W_x = mg\ sin\,\Theta$$

$$W_y = mg\ sin\,\Phi\ cos\,\Theta$$

$$W_z = mg\ cos\,\Phi\ cos\,\Theta$$

Aerodynamic Forces

The angle of attack(α), side slip angle(β), roll(p), pitch(q), and yaw (r) rates, as well as the density of the air entering the aircraft and its Mach number, all have a significant impact on the aerodynamic forces acting on it. Other important factors include the geometry of the wings, which includes their span (b), mean aerodynamic chord, and wing area S.

$$\hat{X} = \bar{q}SCx_T(\alpha, \beta, p, q, r, \delta)$$

$$\hat{Y} = \bar{q}SCy_T(\alpha, \beta, p, q, r, \delta)$$

$$\hat{Z} = \bar{q}SCZ_T(\alpha, \beta, p, q, r, \delta)$$

Where the $\bar{q}$ is a function of the density and total velocity and represents the aerodynamic pressure.

$$\bar{q} = \frac{1}{2}\rho V_t^2$$

Thrust

The thrust vector component will only have an x component because it is assumed to be acting parallel to the aircraft's nose in that direction.

$$E_x = F_T$$

The total force equation can be represented as shown, after assembling the weight, thrust, and aerodynamic forces.

$$m(\dot{u} + qw - rv) = \bar{X} + F_T - mg \sin \Theta$$

$$m(\dot{v} + ru - pw) = \bar{Y} + mg \cos \Theta \sin \Phi$$

$$m(\dot{w} + pv - qu) = \bar{Z} + mg \cos \Theta \cos \Phi$$

Moments on an Aircraft

The total sum of all moments acting on the aircraft is equal to the rate of change of angular momentum according to Newton's Second Law of Motion.

$$M = \begin{bmatrix} \dot{p}I_x - \dot{r}I_{xz} \\ \dot{q}I_r \\ -\dot{p}I_{xz} + \dot{r}I_z \end{bmatrix} + \begin{bmatrix} rq(I_z - I_y) - pqI_{xz} \\ pq(I_x - I_z) + (p^2 - r^2)I_{xz} \\ pq(I_y - I_z) + qrI_{xz} \end{bmatrix}$$

Now using the equations roll, pitch, and yaw moment equations in the x, y, and z directions can be represented as follows:

$$M_x = \dot{p}I_x - \dot{r}I_{xz} + rq(\dot{I}_z - I_y) - pqI_{xz}$$

$$M_y = \dot{q}I_y + pq(I_x - I_z) + (p^2 - r^2)I_{xz}$$

$$M_z = -\dot{p}I_{xz} + \dot{r}I_z + pq(I_y - I_z) + qrI_{xz}$$

Aerodynamic Moments

The roll, pitch, and yaw moments for aerodynamic moments can be represented as:

$$\bar{L} = \bar{q}SC_{LT}\,(\alpha, \beta, p, q, r, \delta)$$

$$\bar{M} = \bar{q}SC_{MT}\,(\alpha, \beta, p, q, r, \delta)$$

$$\bar{N} = \bar{q}SC_{NT}\,(\alpha, \beta, p, q, r, \delta)$$

Thrust Moments

The thrust moments for roll, pitch, and yaw are L_T, M_T, and N_T, respectively. The body fixed frame's total moment equation can be written as the sum of the aerodynamic and thrust moments.

$$\bar{L} + L_T = \dot{p}I_x - \dot{r}I_{xz} + rq(I_z - I_y) - pqI_{xz}$$

$$\bar{M} + M_T = \dot{q}I_y + pq(I_x - I_z) + (p^2 - r^2)I_{xz}$$

$$\bar{N} + N_T = -\dot{p}I_{xz} + \dot{r}I_z + pq(I_y - I_z) + qrI_{xz}$$

Navigation

The aircraft navigation was derived with respect to north, east and Altitude frame, represented by N, E, and h.

$$\dot{N}_{dis} = V\ \cos\alpha \cos\beta \cos\Theta \cos\Psi + V\ \sin\beta(\sin\Phi \cos\Psi \sin\Theta - \cos\Phi \sin\Psi\, 7 + V\ \sin\alpha \cos\beta(\cos\Phi \cos\Psi \sin\Theta - \sin\Phi \sin\Psi)$$

$$\dot{E}_{dis} = V\ \cos\alpha \cos\beta \cos\Theta \sin\Psi + V\ \sin\beta(\sin\Phi \sin\Psi \sin\Theta - \cos\Phi \cos\Psi + V\ \sin\alpha \cos\beta(\cos\Phi \sin\Psi \sin\Theta - \sin\Phi \cos\Psi)$$

$$\dot{h} = = V\cos\alpha\ \cos\beta\ \sin\theta - V\ \sin\beta\ \sin\Phi\ \cos\theta - V\ \sin\alpha\ \cos\Phi\cos\theta$$

MATLAB Implementation

The following equations & aerodynamic coefficient values were referred when implementing the non-linear model in MATLAB.

The density of the air is taken to be the following, with h being the altitude.

$$\rho = 2.377\ x\ 10^{-3}(1.0 - 0.703\ x\ 10^{-5}h)^{4.14}$$

While the temperature is taken to be the following with respect to altitude.

$$t = \begin{cases} 519(1.0 - 0.703x10^{-5}h) & h < 35000.00 \\ 390.0 & h > 35000.00 \end{cases}$$

The Mach Number is derived with respect to the velocity and temperature.

$$M = \frac{V}{\sqrt{1.4x1716.3xt}}$$

While the coefficients of force in the direction of X, Y and Z are represented as follows.

$$C_{x,t} = \frac{\bar{c}}{2V} C_{xq}(\alpha_d)q + C_x(\alpha_d, \delta_E)$$

$$C_{Y,t} = C_y(\beta_d, \delta_A, \delta_R) + \frac{b}{2V}\left[C_{Y,r}(\alpha_d)r + C_{Yp}(\alpha_d)p\right]$$

$$C_{Y,t} = -0.02\beta_d + \frac{b}{2V}\left[rC_{Yr}(\alpha_d) + C_{Yp}(\alpha_d)p\right] + 0.021\frac{\delta_A}{20} + 0.086\frac{\delta_R}{30}$$

$$C_{Z,t} = C_Z(\alpha_d, \beta_d, \delta_E) + \frac{\bar{c}}{2V} C_{Zq}(\alpha_d)q$$

$$C_{Z,t} = C_{Z,1}(\alpha_d, \beta_d) + \frac{\bar{c}}{2V} C_{Zq}(\alpha_d)q - 0.19\frac{\delta_E}{25}$$

Furthermore, the Roll, pitch and yaw coefficients are represented as shown:

$$C_{L,t} = C_L(\alpha_d, \beta_d, \delta_A, \delta_R) + \frac{b}{2V}[rC_{L,r}(\alpha_d) + C_{L,p}(\alpha_d)p]$$

$$C_{L,t} = C_{l,1}(\alpha_d, \beta_d) + \frac{b}{2V}[C_{L,r}(\alpha_d)r + C_{L,p}(\alpha_d)p] + C_{l,2}(\alpha_d, \beta_d)\frac{\delta_A}{20} + C_{l,3}(\alpha_d, \beta_d)\frac{\delta_R}{30}$$

$$C_{M,t} = \frac{\bar{c}}{2V} C_{Mq}(\alpha_d)q + C_{Z,t}(X_{cgR} - X_{cg}) + C_m(\alpha_d, \delta_E)$$

$$C_{N,t} = C_n(\alpha_d, \beta_d, \delta_A, \delta_R) + \frac{b}{2V}\left[C_{Nr}(\alpha_d)r + C_{Np}(\alpha_d)p\right] - \frac{\bar{c}}{b} C_{Y,t}\left(X_{cgr} - X_{cg}\right)$$

$$C_{N,t} = C_{n,1}(\alpha_d, \beta_d) + \frac{b}{2V}\left[C_{Nr}(\alpha_d)r + C_{Np}(\alpha_d)p\right] - \frac{\bar{c}}{b} C_{Y,t}\left(X_{cgr} - X_{cg}\right) + C_{n,2}(\alpha_d, \beta_d)\frac{\delta_A}{20} + C_{n,3}(\alpha_d, \beta_d)\frac{\delta_R}{30}$$

The state and control variables implemented in MATLAB are shown in the following matrix.

$$x = [v_T \quad \alpha \quad \beta \quad \Phi \quad \Theta \quad \Psi \quad P \quad Q \quad r \quad N_{dis} \quad E_{dis} \quad h \quad power]$$

$$u = [throttle \;\; \delta_E \;\; \delta_a \;\; \delta_r]$$

Linearization about a Trim Point

System of the equations can be represented as:

$$\dot{x} = f(x, u)$$

$$y = c(x, u)$$

At trim point

$$0 = f(x_e, u_e)$$

Using Taylor expansion and the following perturbated variables:

$$\delta_x = x - x_e$$

$$\delta_y = x - x_y$$

$$\delta_z = x - x_z$$

Hence, trimming the f-16 aircraft will give us a set of Eigen values which are as follow

$$-0.90832 + 1.4472\mathrm{i}$$

$$-0.90832 - 1.4472\mathrm{i}$$

$$-0.00439 + 0.072\mathrm{i}$$

$$-0.00439 - 0.072\mathrm{i}$$

$$8.1796 + 10^{-14}$$

Linear Equations

The equations should be represented in the Wind-Axes Reference Frame for linearization and for improved control design. The body fixed reference frame is converted to the wind axis frame by rotating over the side slip angle and then rotating over the angle of attack. The velocity vector V is made up of three components: the normal velocity (w), the lateral velocity (u), and the longitudinal velocity (v).

$$v_T = \sqrt{u^2 + v^2 + w^2}$$

The angle of attack and slide slip angle can be represented in the form of u, v and w.

$$\alpha = tan^{-1}\frac{w}{u}$$

$$\beta = sin^{-1}\frac{v}{v_T}$$

The velocities u, v and w are as follow:

$$u = v_T \cos\alpha \cos\beta$$

$$v = v_T \sin\beta$$

$$w = v_T \sin\alpha \cos\beta$$

Taking the derivatives of these total velocity and aerodynamic angles equations will produce the following:

$$\dot{v_T} = \frac{u\dot{u} + v\dot{v} + w\dot{w}}{v_T}$$

$$\dot{\alpha} = \frac{u\dot{w} - wu}{u^2 + w^2}$$

$$\dot{\beta} = \frac{\dot{v}v_T - \dot{v_T}v}{v_T^2 \cos\beta}$$

Substituting the equations above, along with the force equations derived previously:

$$\dot{v_T} = \frac{1}{m}(-D - F_T \cos\alpha \cos\beta + mg_1)$$

$$\dot{\alpha} = q - (p\cos\alpha + r\sin\alpha)\tan\beta + \frac{-L - F_T \sin\alpha + mg_3}{mv_T cos\beta}$$

$$\dot{\beta} = psin\alpha - rcos\alpha + \frac{1}{mv_T}(Y - F_T\, cos\alpha sin\beta + mg_2)$$

Moreover, the drag force (D), side force (Y) and lift force (L) can be represented as follows:

$$D = -\bar{X}cos\alpha cos\beta - \bar{Y}\, sin\beta - \bar{Z}sin\alpha cos\beta$$

$$Y = -\bar{X}cos\alpha sin\beta - \bar{Y}\, cos\beta - \bar{Z}sin\alpha sin\beta$$

$$L = \bar{X}sin\alpha - \bar{Z}cos\alpha$$

It is necessary to linearize the above equations of the aircraft's non-linear model to examine characteristic modes, handling quality, stability, and develop control systems using straightforward linear techniques. This can be done analytically or numerically. The system is linearized numerically by first determining an initial condition, and then the system is linearized around that condition. MATLAB is used to implement these conditions.

$$y = f(x_0) + fx_1(X_0)\Delta x_1 + fx_2X_0)\Delta x_2 + \ldots. + fx_n(X_0)\Delta x_n$$

The function is linearized around the n-dimensional component X_0. It is assumed that the aircraft is flying steadily, straight, and symmetrically:

$$u_0 = V$$

$$w_0 = 0$$

$$\alpha_0 = 0$$

$$\Theta_0 = \gamma_0$$

$$\theta = \gamma + \alpha$$

The linearized system will take the following form when using MATLAB's linearization function and the trim condition for both longitudinal and lateral motion.

$$\dot{x} = A\dot{x} + Bu$$

$$y = Cx + Du$$

Different trim conditions are input to introduce small perturbations in the commands because the linear model should be able to express the non-linear one correctly.

MATLAB Implementation

Longitudinal Dynamics Model

Expanding the state-space equations we achieve the following matrix forms for the longitudinal dynamics.

$$x = [\theta\ v\ \alpha\ q]^T \quad u = [\delta_e] \quad y = [\theta\ v\ \alpha\ q]^T$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ -32.1 & -0.013 & -2.66 & -1.18 \\ 0 & -0.00 & -0.67 & 0.93 \\ 0 & 0 & -0.57 & -0.87 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0.0387 \\ -0.0014 \\ -0.1188 \end{bmatrix}$$

$$C = \begin{bmatrix} 57.2958 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 57.2958 & 0 \\ 0 & 0 & 0 & 57.2958 \end{bmatrix} \quad D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The above Longitudinal state space equations obtained were further studied and analyzed using transfer functions by computing the root locus, PZ map & Bode plot of the Longitudinal dynamic modes- Phugoid (long period) and short period are represented in figure 4.

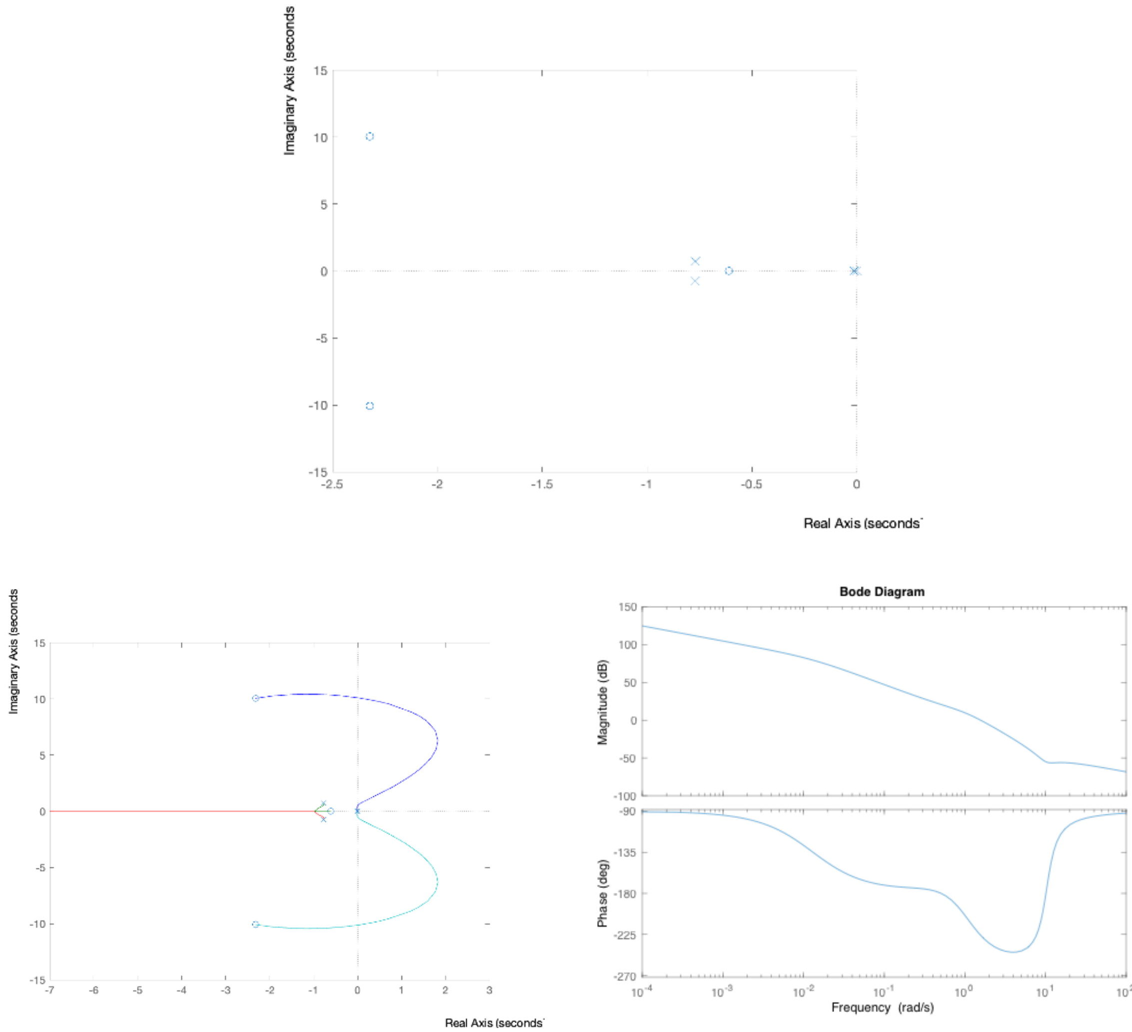

Figure 4 Longitudinal Equation: Pole-Zero map, Root Locus and Bode Plot

Lateral Dynamics Model

While the lateral dynamics matrix is obtained from the state-space equations:

$$x^T(t) = [\Phi\ \beta\ p\ r]^T \quad u^T(t) = [\delta_r\ \delta_a]$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0.078 \\ 0.064 & -0.202 & 0.078 & -0.99 \\ 0 & -22.92 & -2.25 & 0.54 \\ 0 & 6.00 & -0.04 & -0.31 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 \\ 0.0002 & 0.0005 \\ -0.4623 & 0.0569 \\ -0.0244 & -0.0469 \end{bmatrix}$$

$$C = \begin{bmatrix} 57.29 & 0 & 0 & 0 \\ 0 & 57.29 & 0 & 0 \\ 0 & 0 & 57.29 & 0 \\ 0 & 0 & 0 & 57.29 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The above Lateral state space equations obtained were further studied and analyzed using transfer functions by computing the root locus, PZ map & Bode plot of the Lateral dynamic modes - Roll mode, Spiral and Dutch roll Modes are represented in figure 5.

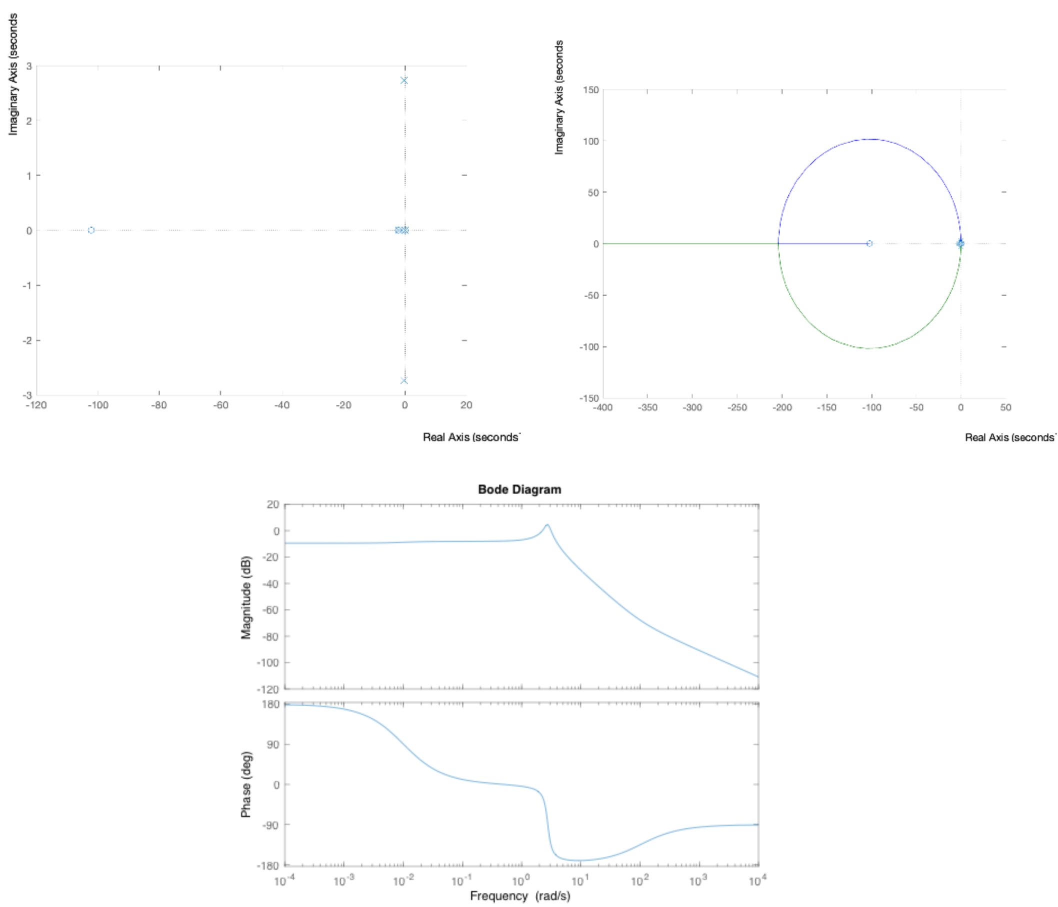

Figure 5 Lateral Equation: Pole-Zero map, Root Locus and Bode Plot

Autopilot Design

Closed-loop control systems are used in design applications to stabilize a system, follow a reference more precisely, and add stability to uncertainties. The simplicity, and excellent performance of PID controller design make them the most alluring and often utilized approach. A cascade PID control system structure is recommended in the work's autopilot design.

A cascade control system has two loops: the primary loop, also known as the outer loop, and the secondary loop, often known as the inner loop. The input or set point of the secondary controller is the control signal of the primary controller. The system's operation is made possible by properly adjusting the gain values (Kp, Ki and Kd), which is represented in the following equation:

$$G(s) = K_p + K_i \frac{1}{s} + K_d s E$$

Non-Linear System

Altitude PID Control

The Altitude PID controller was implemented in the non-linear model. The inner loop will use a PI controller, while the outer loop will use a PD controller. Cascade control is suitable in this design and the Ziegler-Method was used to tune the gains. The altitude autopilot's resulting Simulink structure is depicted in figure 6.

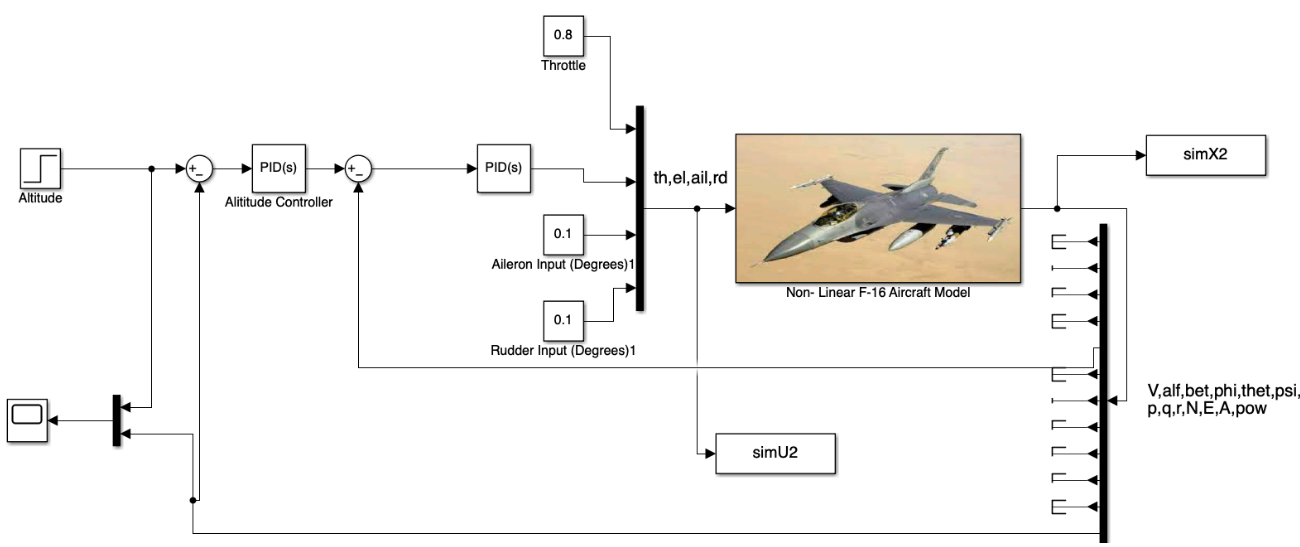

Figure 6 Simulink model of Altitude PID

Figure 7 shows the step response of the designed PID altitude controller.

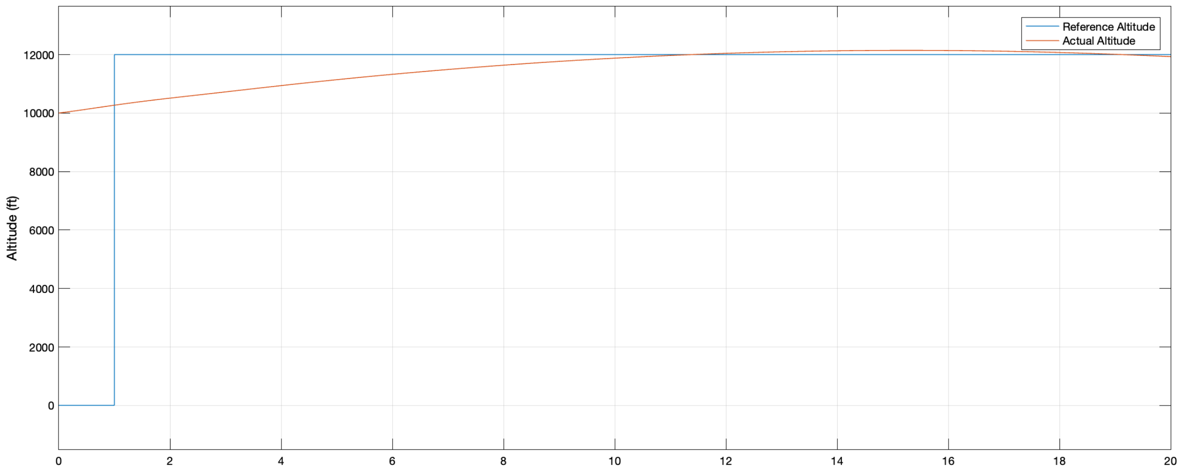

Figure 7 Step Response of the Altitude Controlled System

Velocity PID Control

The scalar value of the aircraft's velocity is used as the PI controller's input in velocity autopilot. The output of this structure controls the overall thrust generated by the engine.

The velocity autopilot's resulting Simulink structure is depicted in figure 8.

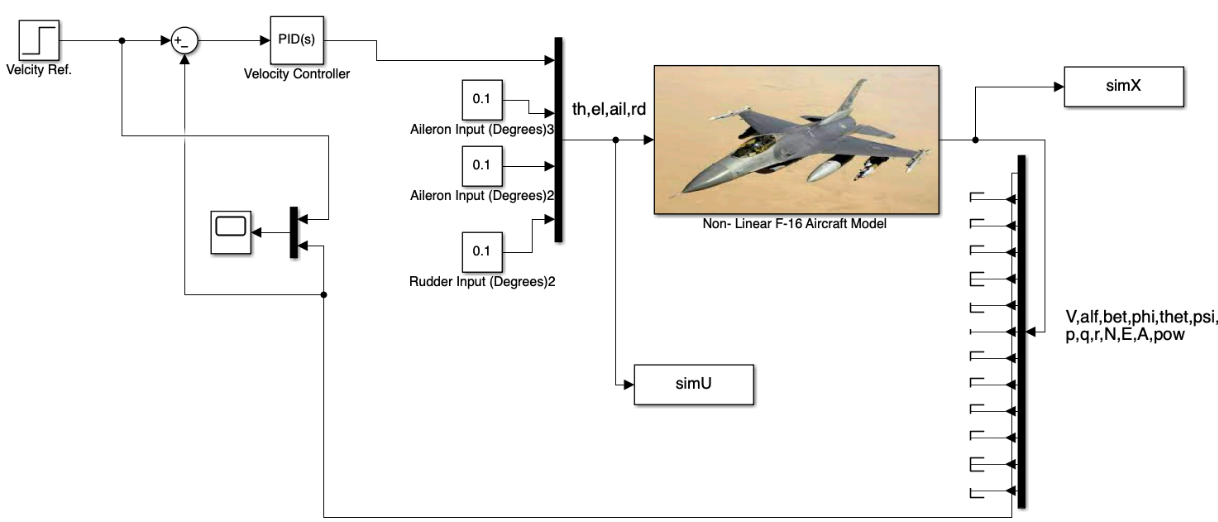

Figure 8 Simulink model of Velocity PID

Figure 9 shows the step response of the designed PID velocity controller.

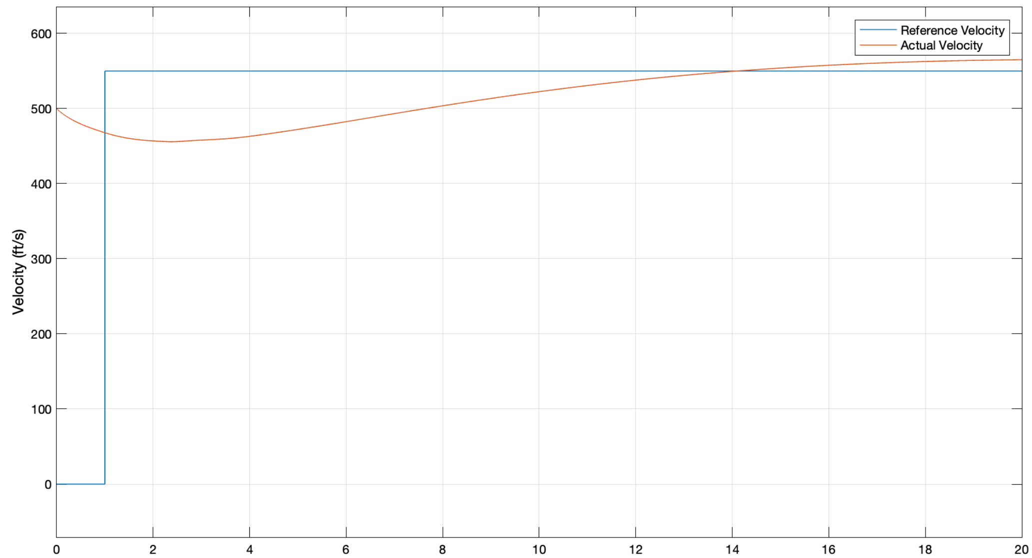

Figure 9 Step Response of the Altitude Controlled System

Linear System

Altitude Longitudinal PID Control

The alititude PID controller was similarly designed for the F-16 aircraft's linearized model using the similar implementation method as shown above. The altitude longitudinal resulting Simulink structure is depicted in figure 10.

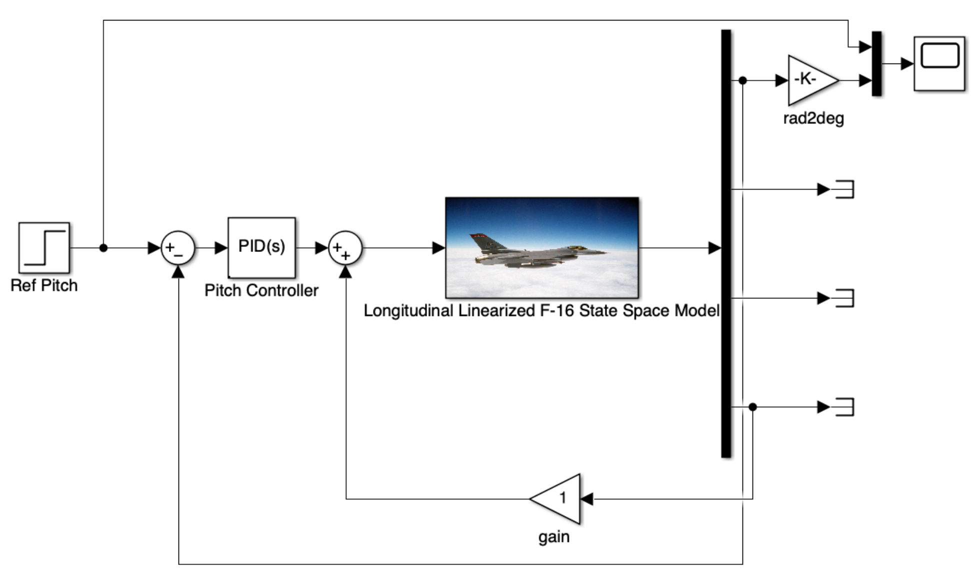

Figure 10 Simulink model of Altitude Longitudinal PID

In figures 11 shows the step response of the designed PID longitudinal altitude controller.

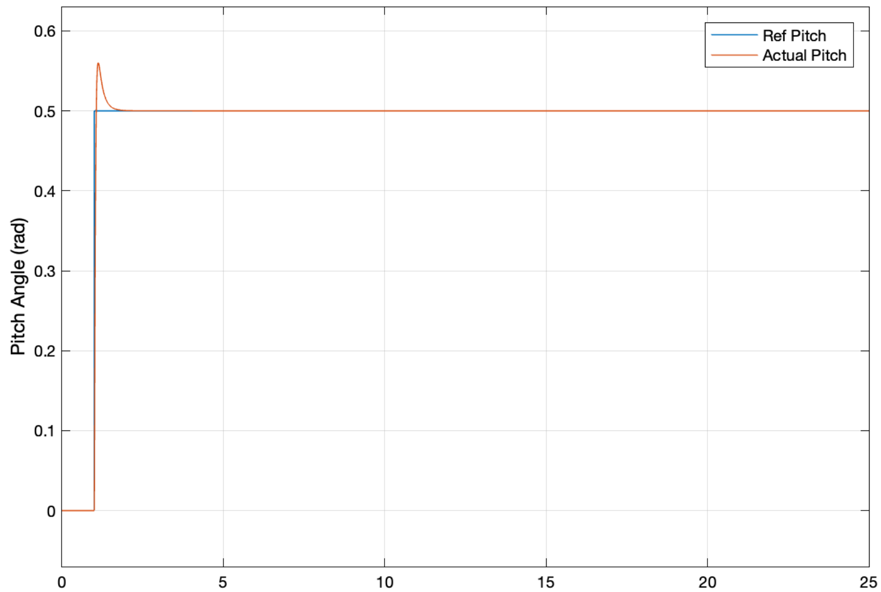

Figure 11 Step Response of the Altitude Longitudinal Controlled System

Roll Lateral PID Control

The roll PID controller was similarly designed for the F-16 aircraft's linearized model using the similar implementation method as shown above. The roll lateral resulting Simulink structure is depicted in figure 12.

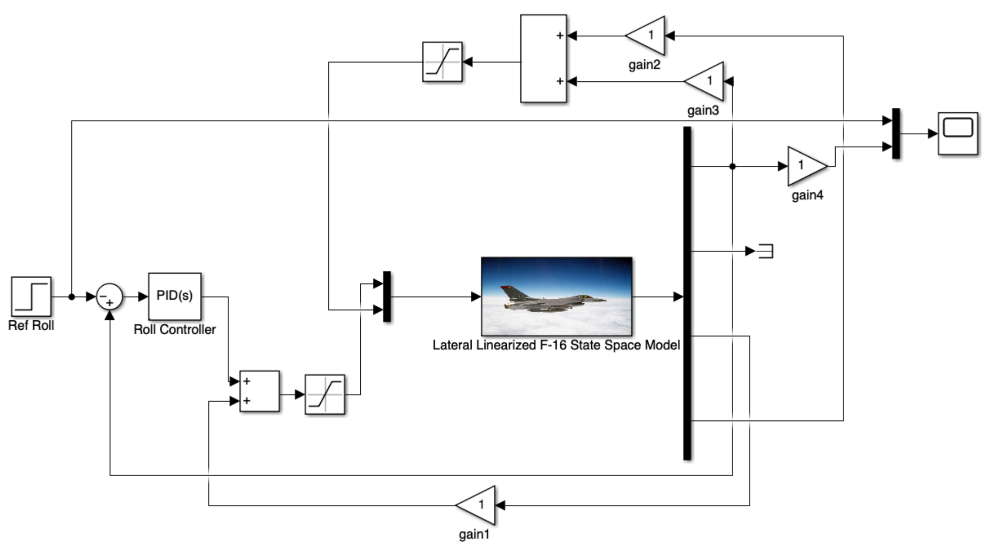

Figure 12 Simulink model of Altitude Lateral PID

In figures 13 shows the step response of the designed PID roll lateral controller.

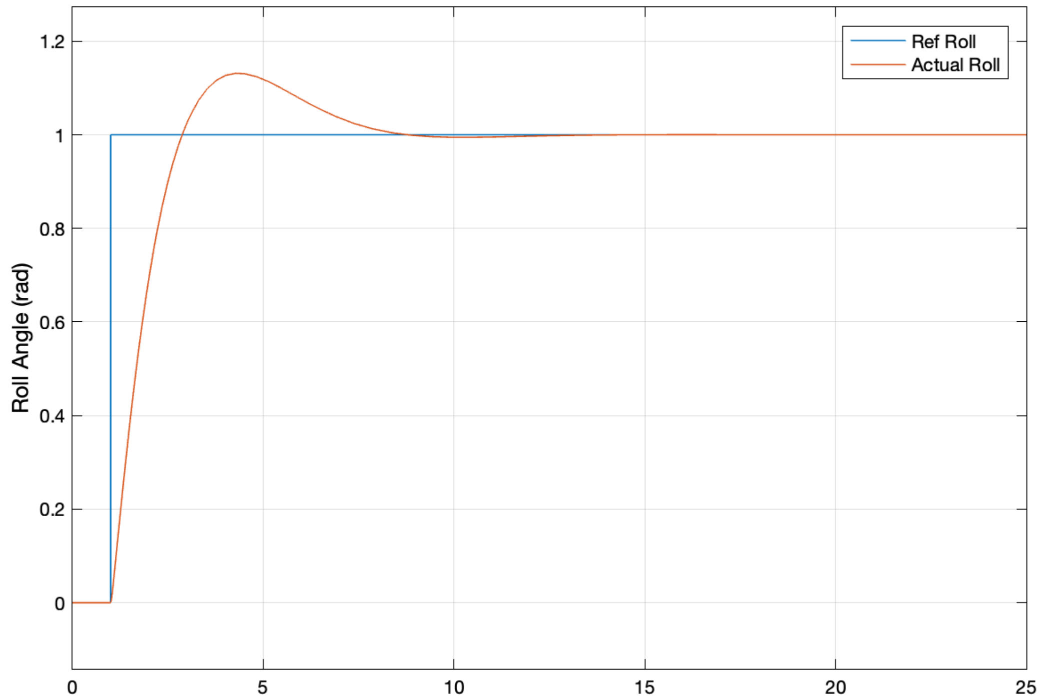

Figure 13 Step Response of the Altitude Lateral Controlled System

3D Visualization of Aircraft Navigation

As per the requirement of this project, a 3D visualization of the F-16 Aircraft was developed as shown in figure 14. This involves the use of the non-linear aircraft model with basic linear PID controllers, along with the navigation equations which were mentioned previously.

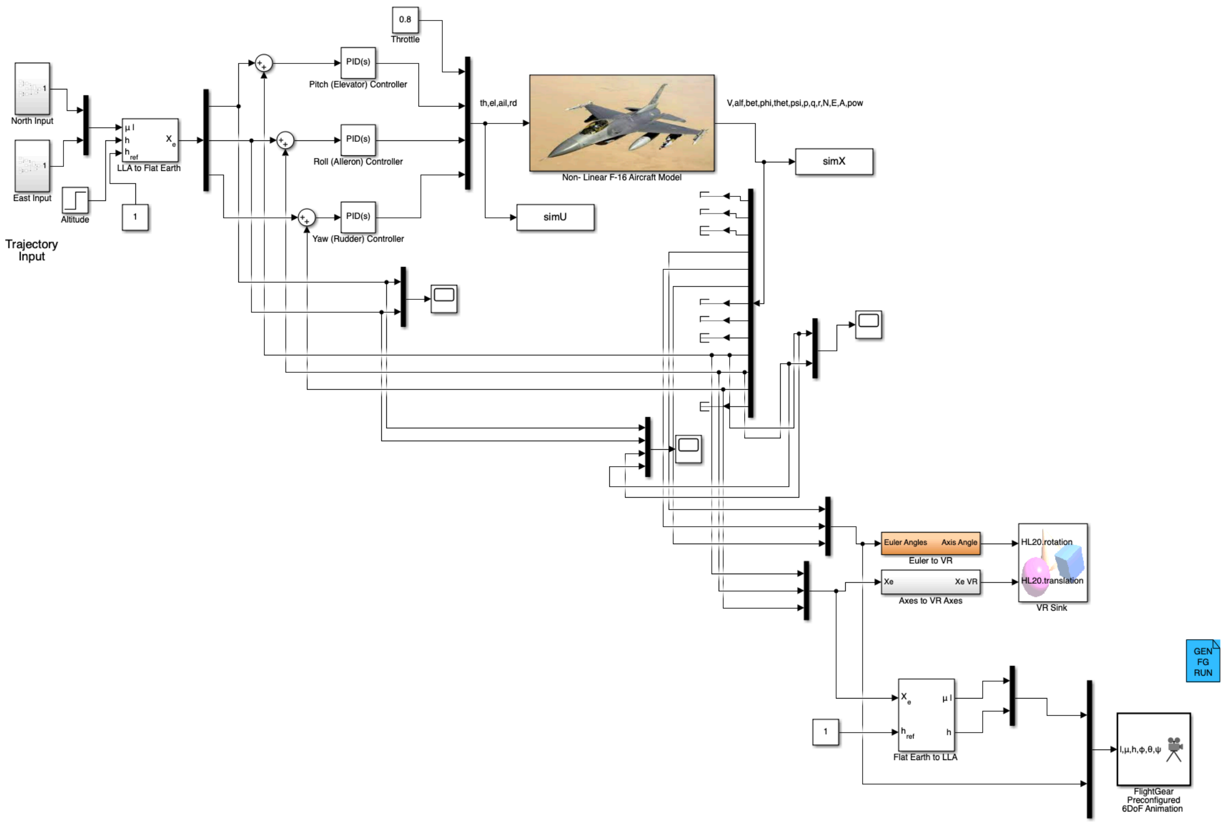

Figure 14 Non-Linear Model with a 3D Visualization for a Trajectory

North, East, and Altitude navigation parameters were converted to Latitude, Longitude and Altitude (LLA) form to feed the desired circular trajectory into the system for which the Aerospace Block from Simulink was used.

The Simulink Aerospace Block and its tools were used for obtaining the 3D visualization of the aircraft. The block was used to develop a 3D visualization in MATLAB and flight gear, a third-party flight simulator software. An F-16 Model was selected in Flight gear for visualization. Additionally, the visualization was validated using the standard aircraft of the built-in Simulink 3D flight simulation tool (aeroblk_HL20), as shown in Figure 15.

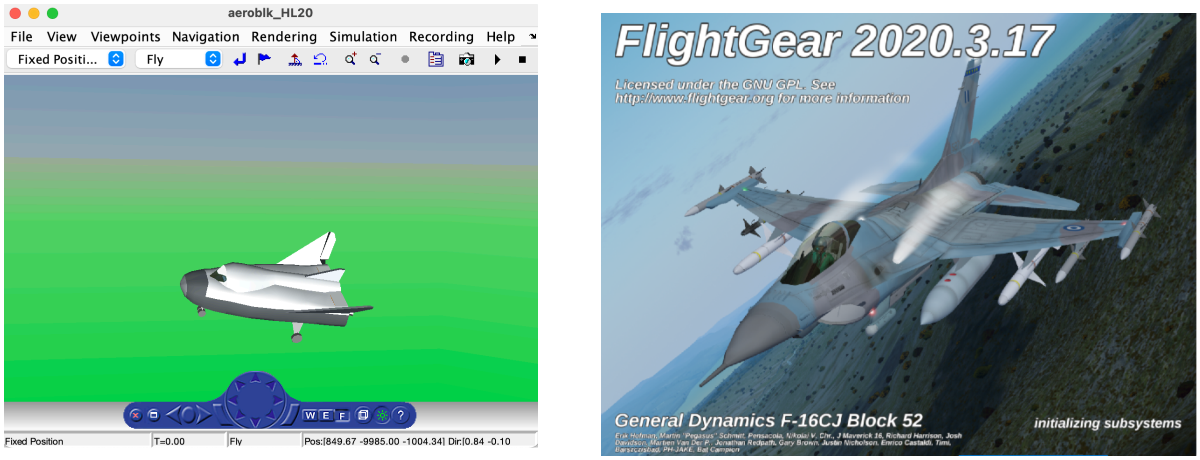

Figure 15 aeroblk_HL20

Following inputs shown in figure 16 were fed to cater to a circular trajectory:

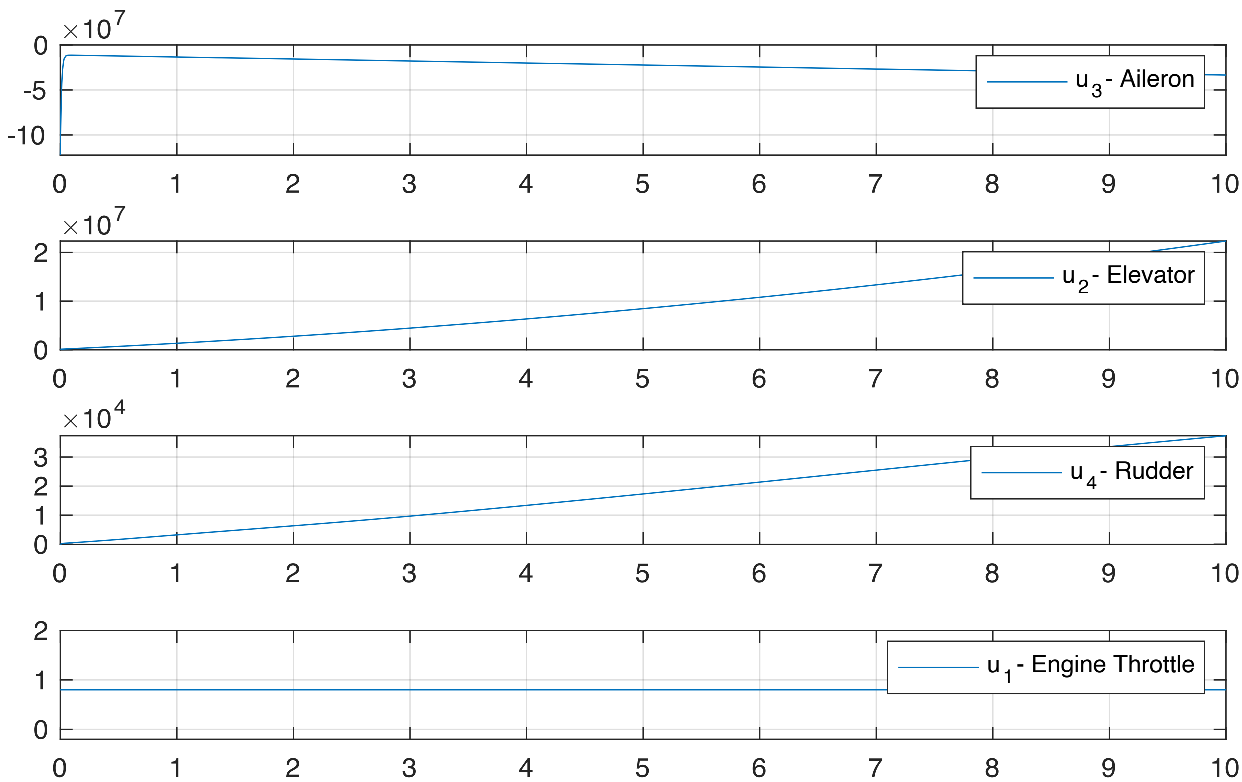

Figure 16 input signals for a circular trajectory

Responses shown in figure 17 for the state variables (u, v, w, p, q, r) were obtained:

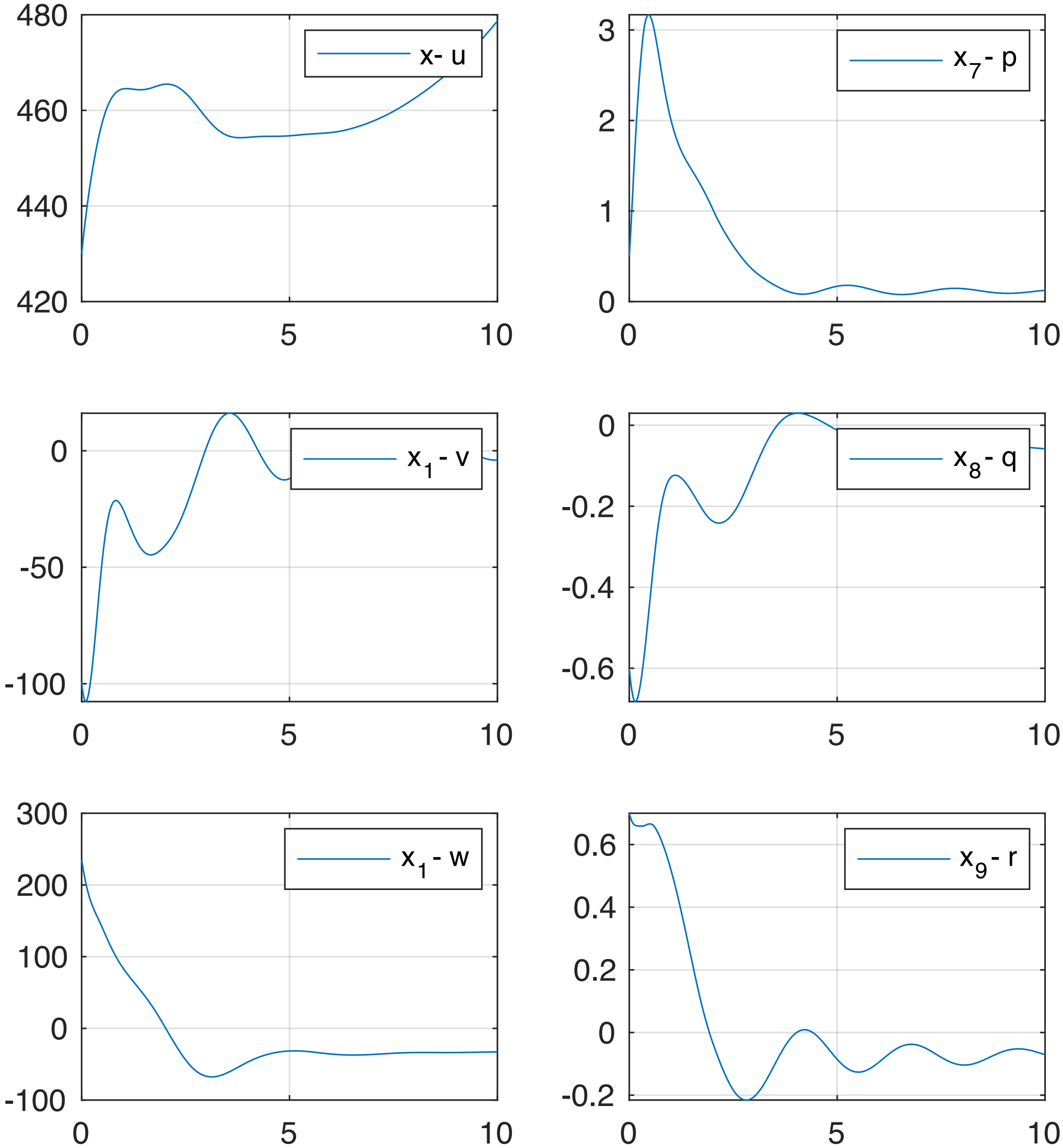

Figure 17 output state variable responses for a circular trajectory

Conclusion

Due to the limitations of PIDs, linearized models must be used for state feedback control. While linear approximations can be valid for steady flight, extreme aerodynamic maneuvers such as those performed in air combat or acrobatic performances push linearization assumptions to their limit. While this might not be an issue for leisure aircraft, gliders or even passenger airplanes, the ability to maintain control in these extreme conditions is certainly a requirement for the F-16.

While linear control methods might not be suitable for extreme aerial acrobatics, like all vehicles the F-16 spends a majority of its time in steady conditions where it is simply navigating to a destination. For this, our control strategy has succeeded in bringing stable, controllable, and safe flight behavior.

Other linear controllers such as LQRs would have also been very suitable for this objective. Where weight matrices are defined for the input and output, and a cost function is solved to obtain the optimal gains. While peak performance is similar to PIDs, the ability to instantly find the optimal controller gains based on your design criteria is a very attractive solution. This approach is most useful when controlling multiple state variables such as ours.

The sources of error in our analysis include the linearization assumptions, and the difficulty of obtaining complete aerodynamic data for the F-16 airframe. Many variants were produced for various export customers and military branch requirements. While mostly similar, significant wingspan, weight, or payload differences are possible between variants. Most of the aerodynamic data was obtained from NASA for the F-16XL variant but was supplemented from data from other variants.

Furthermore, our control system does not consider many real-world effects. These include signal noise present in all digital sensors used to assess the aircraft's state through the fly-by-wire system. The airframe was also considered as a rigid body, but significant deflections are present during high rates of turn or even turbulence. Additionally, the aircraft weight was kept constant. However, an aircraft's fuel weight is very significant and in the case of the F-16, can change rapidly. For example, an F-16's fuel capacity is approximately 3200 kg, and at full afterburner thrust the F-16 can burn 8 kg of fuel per second. That means it's weight can change significantly within the same maneuver.

Overall, adequate performance was achieved with linear control techniques given the complexity of the system model and the limited resources available.

Bibliography

[1] [Online]. Available: https://en.wikipedia.org/wiki/General_Dynamics_F-16_Fighting_Falcon.

[2] Y. Huo, "Model of F-16 Fighter Aircraft Equation of Motions".

[3] M. E. O. W. P. G. K. S. K. P. W. B. a. P. L. D. Luat T. Nguyen, "Simulator Study of Stall/PostStall Characteristics of a Fighter Airplane With Relaxed Longitudal Static Stability".

[4] [Online]. Available: https://www.nasa.gov/sites/default/files/atoms/files/principles-of-flight-in-action-9-12.pdf.

[5] [Online]. Available: https://www.f-16.net/forum/viewtopic.php?f=23&t=11863&start=60.

[6] W. Ahmed, "System Modeling and Controller Design for Lateral and Longitudinal Motion of F-16," 2019.

[7] H. C. Atak, "MATHEMATICAL MODEL AND AUTOPILOT DESIGN OF A TWIN," p. 109, 2020.