UNIVERSITÉ Concordia UNIVERSITY

Department of Mechanical Engineering

Design of Industrial Control Systems
(MECH 6021)

"Design of Flight Controller of a UAV- Quadrotor"

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"We certify that this submission is the original work of members of the group and meets the Faculty's Expectations of Originality"

Final Project Report Submitted to:
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5th December 2021

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1. INTRODUCTION

1.1 Rationale and the Problem Statement

Nowadays new technology works towards controlling aerial vehicles without the onboard presence of a pilot, this is also known in the academic community as autonomous control systems, and while the drones (quadrotors) are known as unmanned aerial vehicles (UAV).

A huge amount of investment is made in the research and development of these unmanned aircraft systems (UAVS), due to several of their basic advantages over manned systems: lower risk to crews, increased maneuverability, lower cost, lower radar signatures, and longer endurance. Additionally, the vertical lift allows the system further maneuverability. Thus, requiring less human interaction from take-off to landing. Unmanned aircraft system is presented differently. The UAV can be controlled by either a human operator using ground-based controllers or can be provided semi-autonomy or autonomy, as per the requirement.

Additionally, due to drones advanced movement systems, any imbalance in their system generates angular and linear accelerations; thus, prompting them to collide easily. As the quadrotor system is nonlinear and the movements are linked with each other. System feedback illuminates what is required to control all aspects of the system. In this part, PID control comes to roll its part to treat with every variable to present quadrotor as a linear system.

Thus, our team worked on the development of the PID system; however, inclined towards a more commercially used application of it. We tried to determine a new model design method for the flight control of an autonomous quadrotor, known as mini-drones in layman terms. In the following mathematic model, we study the behavior of a four-rotor vertical take-off and landing (VTOL) unmanned air vehicle known as quadrotor aircraft. Designing the mathematical model of the quadrotor allows us to determine the response of the quadrotor when various inputs are provided to it. The objective is to develop a model of the vehicle as realistic as possible.

1.2 Quadrotor

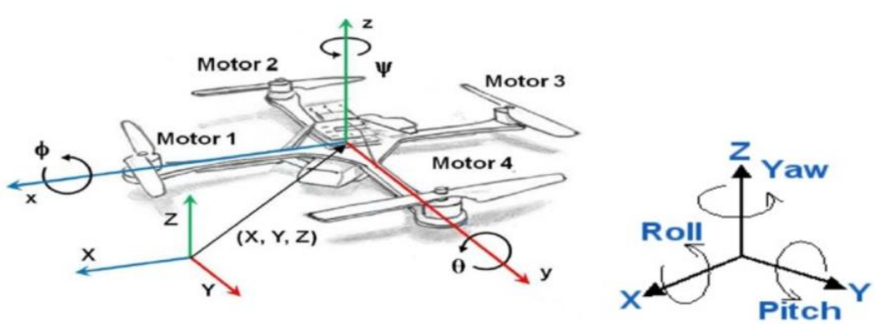


Fig 1.

A quadrotor encompasses a power source and the four-motor used in controlling it. As shown in Figure 1, the rotational speeds of the four rotors are independent of each other. Also, as a quadrotor is a six-degree of freedom system (DoF) system, it can move along the three space axes X, Y, and Z, and able to turn along its body axes describing roll (ϕ), pitch (θ), and yaw (ψ) angles. Thus, we can control the pitch, roll, and yaw nature of the vehicle. While the displacement is produced by the total thrust of the rotors whose direction varies according to the nature of the quadrotor. Hence, we can control the quadrotors motion. Additionally, roll and pitch movements also generate motion along the Y and X axes of space, respectively.

The model has four input forces which are the thrust provided by each propeller connected to each rotor with a fixed angle. Rotors 1 and 3 move counterclockwise, while rotors 4 and 2 move counterclockwise. Hence, by decreasing and increasing the speed at rotors 1 and 3 and rotors 1 and 4 simultaneously we can produce a forward (backward) motion in the quadrotor by changing the pitch angle. While similarly, we can produce left and right motion by changing the roll angle. By increasing (decreasing) counterclockwise motors speed while decreasing (increasing) clockwise motor speeds we can derive the yaw angle.

Since the quadrotor uses four rotors instead of one rotor to provide thrust, it has four input forces and six output coordinates. Thus, the payload capacity for it is larger than conventional aerial robots. Allowing it to carry more weight. In addition, as there are four input forces and six output states, the quadrotor is an under-actuated system.

Also, the design of the quadrotor is much simpler compared to conventional UAVs as its direction is changed by manipulating the individual rotor's speed and does not require cyclic and collective pitch control. Thus, the production cost of the flying robot is much smaller. However, the space required for the four rotors and the power consumption of them is much larger compared to the conventional single motor UAV's.

Due to the use of four motors as actuators, there is a high-power consumption. Although the minimal cross-coupling simplifies the quad-rotor dynamics, the dynamics of the quadrotor and specifically its low-rate damping can make the vehicle difficult to control. The challenge of controlling the vehicle can be even more difficult for a small, low-cost flying vehicle (Stone, 2002). Existing quadrotor dynamic models are developed on the basis of one unique rigid body; however, it does not account for the fact the system is composed of five rigid bodies: Four rotors and a crossing body frame. Thus, making the clarifications of several features, like gyroscopic effects, complex. Additionally, simplification hypotheses are generally introduced early in the model development and lead in general to misleading interpretations.

Hence, this work presents the development of a quadrotor's model considering its dynamic behavior. A simplification is made based on the analysis of its involved; therefore, making the implementation of then PID controllers easier.

1.3 PID Controllers

PID (Proportional-Integral-Derivative) controllers process every variable independently within a limited range in which the behavior of the system needs to be stabilized. Generally, PID presents a control loop system to correct the system behavior continuously. This system has feedback with respect to system target and it interprets as a PID controller input to adjust the overshoot, time to peak, and settling time. PID controller uses some constant value as gain to compensate system response to get a better result.

The cruise control on a car and thermostat at home is a simple example of a PID controller. Cruise control adjusts engine rotation to supply enough power for keeping the car's speed on desired value in different situations like a road with slop or flat road, or thermostat try to keep the temperature in constant degree with minimum overshoot or fluctuation.

PID were used from early 1920 onwards in the field of automatic steering systems for ships. Then it was used for automation in the manufacturing industry, where it was widely implemented at pneumatic and electronic controllers. These days any applications requiring precise and optimized automatic control use PID concepts.

PID controllers have the ability to use three controlling features of proportional, integral, and derivative which provides accurate and optimal control at the output. PID constantly evaluates the error difference between the desired and measured values which are feedback from the system as e(t). This difference is applied as a correction based on PID gain values. Therefore, the error is reduced.

"P" is proportional to the current value of the setpoint- process error $e(t)$. Taking into account the gain factor "K", the error can be large and positive, while the control output will be proportional to it.

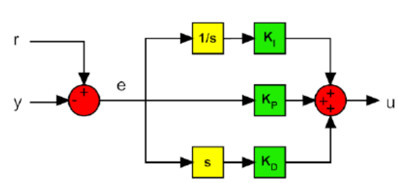
"I" is integral to the error $e(t)$ processed due to the proportional control which reduced the residual error, concerning the historical error data. When the error is removed, the integral term will start to grow. On the other hand, the growing integral effect will result in the proportional effect decreases and the error too, but this is compensated for the integral term.

"D" is a derivative, and it reduces the effect of the setpoint- process error by applying a control influence generated by the rate of error change. It is a damping effect.

The balance of these effects is achieved by loop control tuning to rich the optimal control function. The tuning constants are shown below as "K" and must be derived for each control application, as they depend on the response characteristics of the complete loop external to the controller.

The overall function: $u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d_\tau + K_d \frac{de(t)}{dt}$

Where K_p, K_i and K_d all are positive, denote the coefficients for the proportional, integral, and derivative terms respectively.



Traditional PID structure

2. LITERATURE REVIEW

2.1 Flight PID controller design for a UAV quadrotor

Salih, Moghavvemi, Mohamed, and Gaeid (2010) studied the development of a PID (proportional-integral-derivative) for a quad-rotor flying object to obtain the control method for its stability. The quad-rotor used had four inputs (u1, u2, u3, u4) and six outputs (x, y, z, phi, theta, psi); hence, making it an underactuated UAV (Unmanned Aerial Vehicle), as shown in the schematic of figure 2.

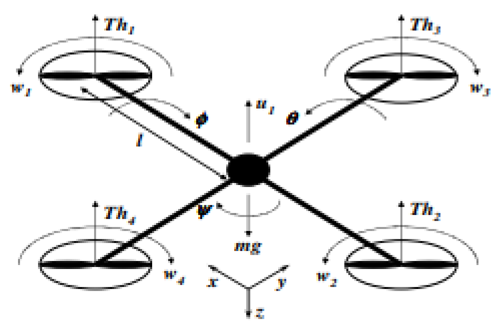


Figure 1. The quadrotor schematic.

Figure 2.

The roll and pitch were stabilized when developing the mathematical and simulation model, as all states are unable to be controlled at the same time, using the output combination of x, y, z, and yaw angle. In addition, the following assumptions and cancellations were made when doing the mathematical modeling:

1. The quadrotor structure was symmetrical and rigid.
2. The Inertia matrix (I) of the vehicle was very small and be neglected.
3. The center of mass and o’ coincides.
4. The propellers are rigid.
5. Thrust and drag are proportional to the square of the propeller's speed.

In addition, the PID controller was developed based on fast and accurate responses. Hence, the following simulation model was developed.

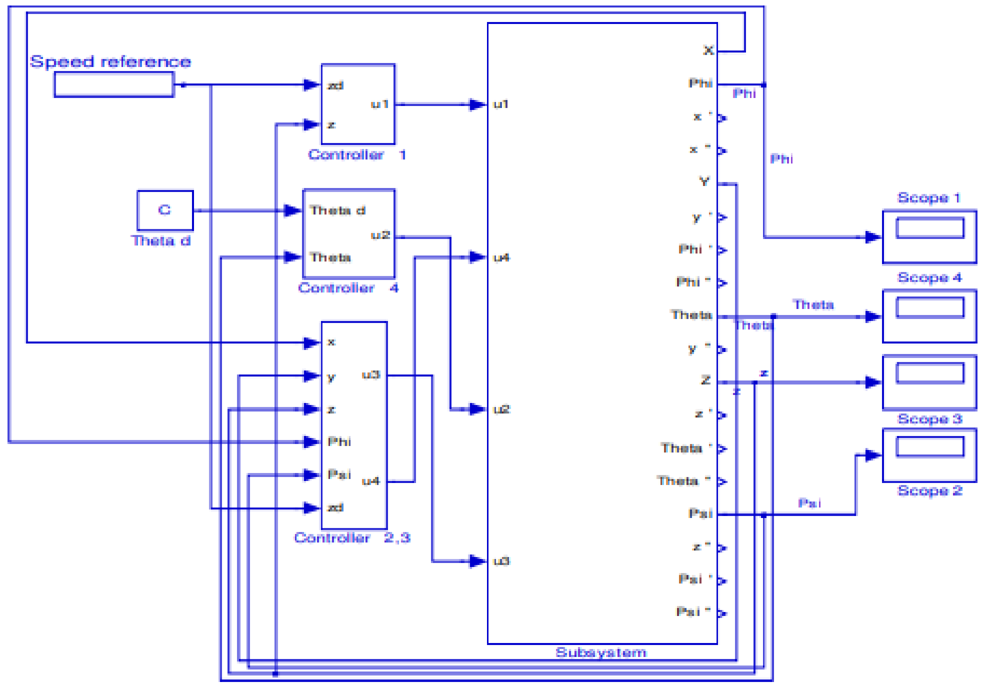


Figure 3

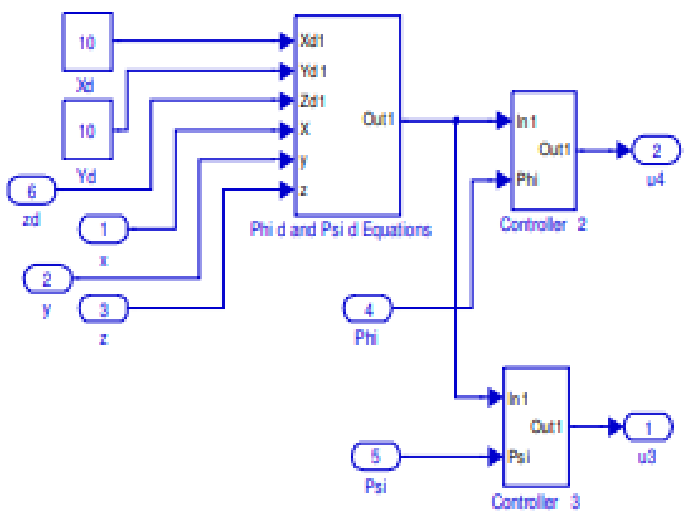


Figure 4

Therefore, PID controllers were used to control the system-whose model was simplified and linearized for ease of analysis. State-space representations were used and finally, the models were converted to MATLAB Simulink and the system responses were studied.

The PID controllers were tuned and optimized initially using the Ziegler-Nichols method. Following which a trial-and-error method was utilized to obtain a fast and robust responsive system.

When the system was supplied three-step inputs (3, 10, 20) for the change in the altitude in the z-axis. The roll, pitch, and yaw angle responded at different time intervals. There were some transient overshoots in the z-axis as shown in Figure 5. While the roll angle started responding after 3s and started moving to the desired outcome, as shown in Figure 6. Whereas the yaw angle started moving to the desired outcome after 5s, as shown in Figure 7. The pitch angle had a 5% overshoot when subjected to the step input, as shown in Figure 8. These transient perturbations were due to many reasons such as simplification of controller design and certain of some mechanical parameters in the design.

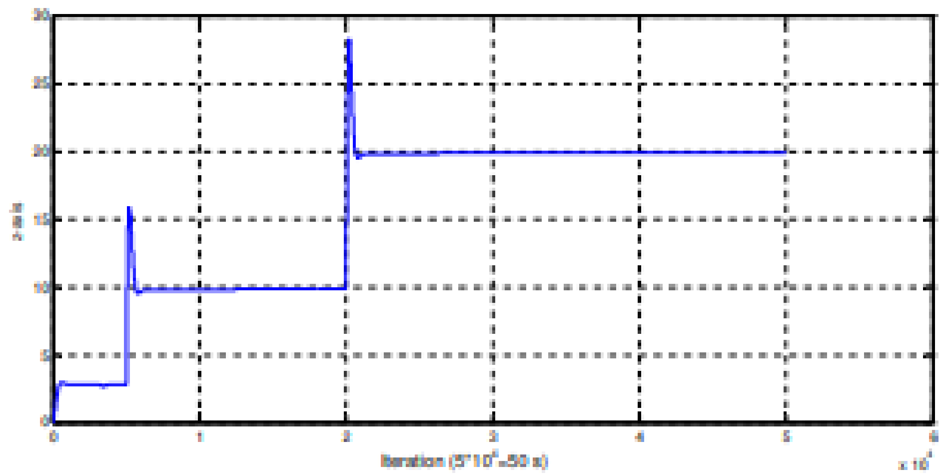


Figure 5. Plot drawing represent the z-axis moving to the desired z-point.

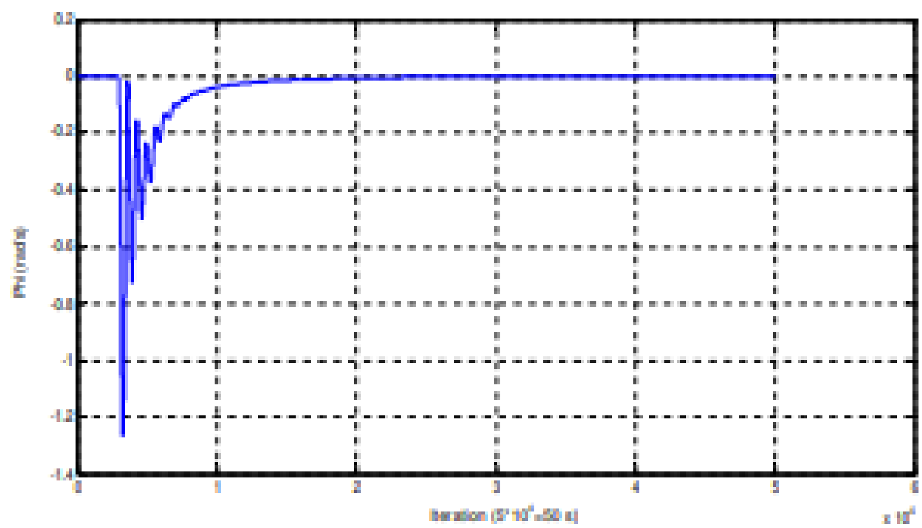


Figure 6. Plot drawing represent the Phi (Roll) angle after 3 seconds to start moving to the desired point.

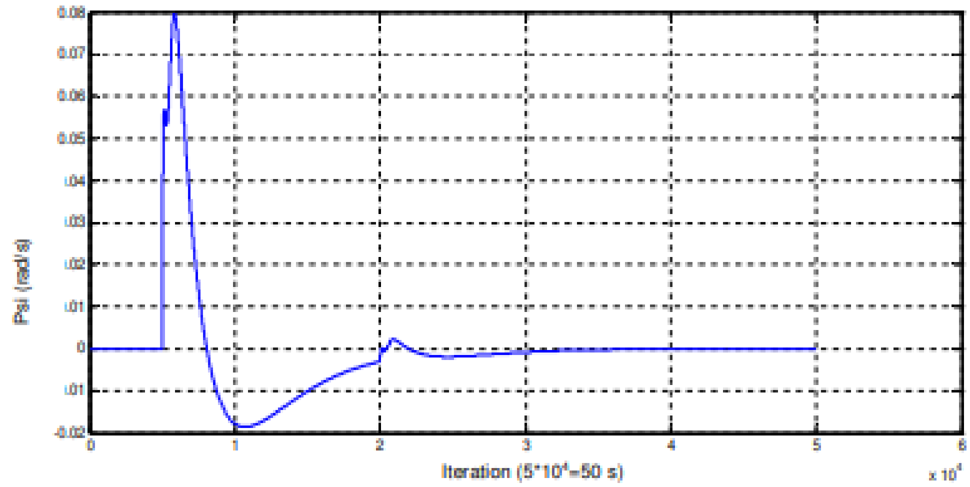


Figure 7. Plot drawing represent the Psi (Yaw) angle after 5 seconds to start moving to the desired point.

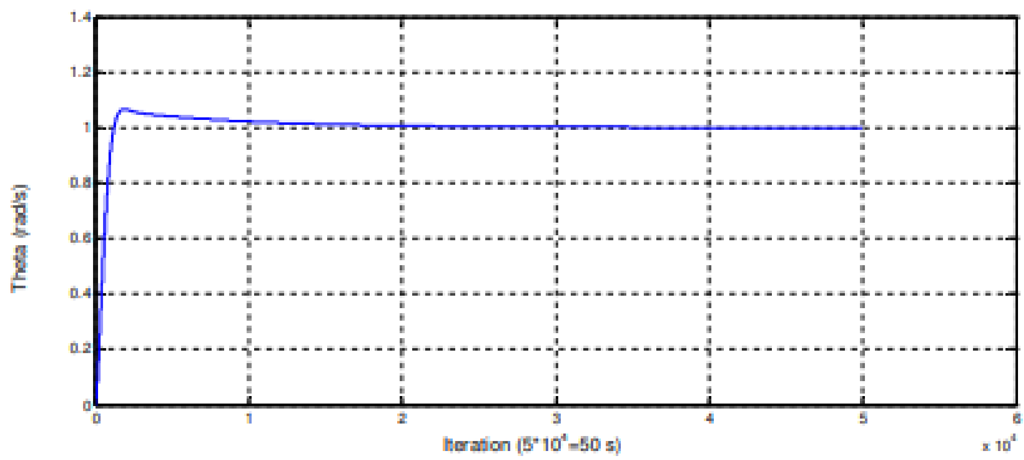


Figure 8. Plot drawing represent the Theta (Pitch) angle start moving to the desired point.

Finally. the quadrotor controller successfully stabilized the roll and pitch angles to zero to reach the desired altitude z at a constant fixed yaw angle provided. Hence the overall system showcases the desired performance and its simple design.

3. MATHEMATICAL MODELLING

A mathematical model is developed for the quadrotor to understand its behavior for a PID controller to be designed. Firstly, we need to understand the motion of the quadrotor, by studying its kinematics and dynamics to obtain the equations of motion.

Two reference frames are defined. The first is the EF system (Earth-Fixed Frame), represented by the X, Y, and Z-axis, which has a fixed origin to the earth. Second is the BF system (Body-Fixed Frame), whose origin is fixed to the center of the quadrotor (Bresciani, 2008). This allows us to analyze the dynamics of the motion of the quadrotor; hence, allowing us to develop the required PID controller to stabilize the system.

3.1 Kinematics:

The linear kinematics of the quadrotor is summarized as shown below, (Bresciani, 2008):

$$\Gamma_E = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \textbf{Eq.1}$$

$$\dot{\Gamma}_E = R_\Theta V_B \quad \textbf{Eq.2}$$

$$\boldsymbol{R_\Theta} = \begin{bmatrix} c\psi c\theta - s\phi s\psi s\theta & -c\phi s\psi & c\psi s\theta + c\theta s\phi s\psi \\ c\theta s\psi + c\psi s\theta s\phi & c\phi c\psi & s\psi s\theta - c\psi c\theta s\phi \\ -c\phi s\theta & s\phi & c\phi c\theta \end{bmatrix} \quad \textbf{Eq.3}$$

Wherewith respect to EF, $\dot{\boldsymbol{r}}_{\boldsymbol{E}}$ is the linear velocity vector. Whereas, concerning BF, $\boldsymbol{V_B}$ is the linear velocity vector. $\boldsymbol{R_\theta}$ is the rotation matrice obtained using the three basic rotation matrices (Z-Y-X Euler angles). Additionally in the equations, the expressions have been abbreviated as s for sine, c for cosine.

The Angular Kinematics is given by:

$$\omega_B = T_\theta \dot{\theta}_E \quad \textbf{Eq.4}$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} c\theta & 0 & -c\phi s\theta \\ 0 & 1 & s\phi \\ s\theta & 0 & c\phi c\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \quad \textbf{Eq.5}$$

$$p = \dot{\phi} c\theta - \dot{\psi} c\phi s\theta$$
$$q = \dot{\theta} + \dot{\psi} s\phi$$
$$r = \dot{\phi} s\theta + \dot{\psi} c\phi c\theta \quad \textbf{Eq.6}$$

Where $\dot{\boldsymbol{\theta}}_{\boldsymbol{E}}$ is the vector of the angular velocity vector with respect to EF ($\dot{\emptyset}, \dot{\theta}, \dot{\varphi}$, are the angular velocity- roll rate, pitch rate, and yaw rate in EF) and $\boldsymbol{\omega}_{\boldsymbol{B}}$ is the vector of angular velocities with respect to BF (p, q, r –angular velocities in BF). $\boldsymbol{T}_{\boldsymbol{\theta}}$ is the transfer matrix used to map angular velocities from BF to EF.

3.2 Dynamics:

The quadrotor dynamics are obtained using the EF and BF, linear and angular components respectively. The force contribution in the system generates linear movements, and so they are defined in EF. The general equation of forces according to Newton's laws is:

$$\boldsymbol{F}_{\boldsymbol{E}} = m\, \ddot{\boldsymbol{r}}_{\boldsymbol{E}} \qquad \textbf{Eq.7}$$

$\boldsymbol{F}_{\boldsymbol{E}}$ is the vector force on EF, $\ddot{\boldsymbol{r}}_{\boldsymbol{E}}$ is the linear vector acceleration of the quadrotor on EF, and m, is the mass of the quadrotor. For force modeling, every force acting on the aircraft needs to be considered. The gravitational force $\boldsymbol{G}_{\boldsymbol{E}}$ on EF is a constant vector pointing downward along the Z-axis of EF, represented with.

$$\boldsymbol{G}_{\boldsymbol{E}} = \begin{bmatrix} 0 \\ 0 \\ -m\, g \end{bmatrix} \qquad \textbf{Eq.8}$$

The thrust (F1+F2+F3+F4) generated by the propellers is a constant direction vector on the Z-axis of BF. Therefore, the thrust on EF is:

$$\mathrm{m}\ddot{\boldsymbol{r}}_{\boldsymbol{E}} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ F_1 + F_2 + F_3 + F_4 \end{bmatrix} \qquad \textbf{Eq.9}$$

$$\mathrm{m}\ddot{\boldsymbol{r}}_{\boldsymbol{E}} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + \begin{bmatrix} c\psi c\theta - s\phi s\psi s\theta & -c\phi s\psi & c\psi s\theta + c\theta s\phi s\psi \\ c\theta s\psi + c\psi s\theta s\phi & c\phi c\psi & s\psi s\theta - c\psi c\theta s\phi \\ -c\phi s\theta & s\phi & c\phi c\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ u_1 \end{bmatrix} \qquad \textbf{Eq.10}$$

$$U_1 = b\left({\Omega_1}^2 + {\Omega_2}^2 + {\Omega_3}^2 + {\Omega_4}^2\right) \qquad \textbf{Eq.11}$$

In the above, U_1 is the value of the thrust generated by the quadrotor where, b is the aerodynamic lift coefficient (assumed as b=1), and Ω is the speed of each rotor (Bresciani, 2008). While, the total forces acting on the quadrotor results in the following system of equations, describing the linear dynamics of the quadrotor on EF.

$$
\begin{aligned}
m\ddot{x} &= (c\psi s\theta + c\theta s\phi s\psi)\, u_1 \\
m\ddot{y} &= (s\psi s\theta - c\psi c\theta s\phi)\, u_1 \\
m\ddot{z} &= -mg + (c\phi c\theta)\, u_1
\end{aligned}
\quad \textbf{Eq.12}
$$

The angular dynamics use the BF and compute the torque which generates the angular acceleration on the system. The equation for the torque on a body is given by Euler's equation of rotation:

$$
\tau_B = I\, \dot{\omega}_B + \omega_B \times (I\, \omega_B) \quad \textbf{Eq.13}
$$

Where, $\boldsymbol{\tau_B}$ is the torque vector acting on a body on BF, **I** is the inertia, known as the matrix of inertia, and $\boldsymbol{\omega_B}$ is the angular acceleration vector on BF. Certain assumptions are taken for the mathematical modeling for the quadrotor.

- The quadrotor structure is symmetrical and rigid.
- The Inertia (I) of the vehicle is very small and to be neglected.
- The center of mass and the frame origin coincide.
- The rotor propellers are rigid.
- Thrust is proportional to the square of the propeller speed, also the drag is negligible.
- The structure is rigid and the gyroscopic forces from the propeller's rotation had been neglected.

Due to the assumptions made regarding the origin of BF the quadrotor's center of gravity, the axes of the quadrotor match the axes of BF, which are defined as:

$$
\boldsymbol{I} = \begin{bmatrix} I_{XX} & 0 & 0 \\ 0 & I_{YY} & 0 \\ 0 & 0 & I_{ZZ} \end{bmatrix} \quad \textbf{Eq.14}
$$

$$
\begin{aligned}
\tau_x &= I_{XX}\dot{p} + q\, r\, (I_{ZZ} - I_{YY}) \\
\tau_y &= I_{YY}\dot{q} + p\, r\, (I_{XX} - I_{ZZ}) \\
\tau_z &= I_{ZZ}\dot{r} + p\, q\, (I_{YY} - I_{XX})
\end{aligned}
\quad \textbf{Eq.15}
$$

In the aforementioned, p, q, r is the angular velocities on BF. While, the torque produced by the propeller, obtained by the basic working principle of the quadrotor is given by:

$$\tau_{\varphi} = U_2 = b\, l \left(\Omega_4{}^2 - \Omega_2{}^2\right)$$
$$\tau_{\theta} = U_3 = b\, l \left(\Omega_3{}^2 - \Omega_1{}^2\right)$$
$$\tau_{\psi} = U_4 = d \left(\Omega_2{}^2 + \Omega_4{}^2 - \Omega_1{}^2 - \Omega_3{}^2\right)$$

Eq.16

Additionally, the torques produced by the propellers are directly linked with the system inputs. The torque contribution gives the angular dynamics of the system.

$$\dot{p} = \frac{(I_{YY} - I_{ZZ})}{I_{XX}}\, q\, r + \frac{\tau_{\varphi}}{I_{XX}}$$
$$\dot{q} = \frac{(I_{ZZ} - I_{XX})}{I_{YY}}\, p\, r + \frac{\tau_{\theta}}{I_{YY}}$$
$$\dot{r} = \frac{(I_{XX} - I_{YY})}{I_{ZZ}}\, p\, q + \frac{\tau_{\psi}}{I_{ZZ}}$$

Eq.17

In the model shown above, the dynamics of the DC motors have been disregarded because it is much faster than the quadrotor dynamics. PID controllers are only suitable for linear systems, so the model must be linearized around an operating point, which is hovering flight. This quadrotor helicopter model has six outputs (x, y, z, phi, theta, psi) while it only has four independent inputs, therefore the quadrotor is an under-actuated system. We are not able to control all the states at the same time.

Therefore, a possible combination of controlled parameters can be x, y, z, and yaw in order to track the desired trajectory and stabilize the other two angles- roll and pitch (phi & theta), which introduces stable zero dynamics into the system (Altug et al., 2002; Pounds et al., 2002). Using small-angle approximations and the system of equations is linearized.

Hence, the below system of linear equations (of the quadrotor dynamic model), and the original model and the linear model have similar behavior for small-angle inputs because the coupling between variables becomes insignificant.

$$m\ddot{x} = (\theta c\psi + \phi s\psi)\, u_1$$
$$m\ddot{y} = (\theta s\psi - \phi c\psi)\, u_1$$
$$m\ddot{z} = -mg + u_1$$

Eq.18

$$\ddot{\phi} = \frac{u_{2x}}{I_{xx}}$$
$$\ddot{\theta} = \frac{u_{2y}}{I_{yy}}$$
$$\ddot{\psi} = \frac{u_{2z}}{I_{zz}}$$

Eq.19

The above-mentioned equations represent the linear and angular behavior of the linearized quadrotor model at the hover state. While below are the assumptions made to linearize the equations at hover state.

$$p = \dot{\phi} \qquad q = \dot{\theta} \qquad r = \dot{\psi}$$

$$qr \approx pr \approx pq \approx \dot{\theta}\dot{\psi} \approx \dot{\phi}\dot{\psi} \approx \dot{\phi}\dot{\theta} \approx 0$$

$$\sin(\theta) \approx \theta, \sin(\phi) \approx \phi, \cos(\theta) \approx \cos(\phi) \approx 1$$

The following parameters of the quadrotor were considered for modeling purposes.

Parameters	Value	Unit
Mass (m)	2	Kg
Gravity (G)	9.81	m/s^2
Moment of Inertia (Ixx)	1.25	Ns^2/rad
Moment of Inertia (Iyy)	1.25	Ns^2/rad
Moment of Inertia (Izz)	2.5	Ns^2/rad
Span of Quadrotor (l)	0.2	M

Table 1

4. MODEL SIMULATION

The model Simulation process is used to analyze a physical model (quadrotor) to study its performance in real environments. It is a tool used by engineers to predict the system's behavior under different scenarios without developing an actual physical prototype. In the following study, the MATLAB Simulink package is used.

4.1 Quad Rotor Plant Simulink Model

The state-space ordinary differential equations of motion (eq.18 and eq.19) which represent the linearized mathematical model of the quadrotor plant, is converted to its respective Simulink model as shown in Fig. 9 and Fig. 10. The plant model relates the input forces and moments (u1, u2, u3, and u4) to the output x, y, z, psi, phi, theta. Therefore, representing an underactuated system.

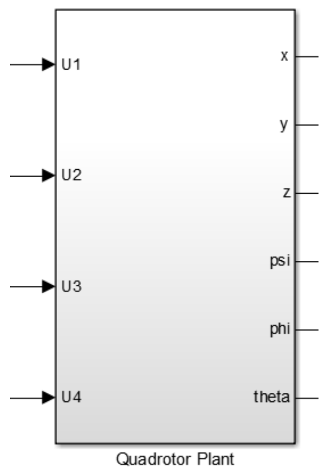


Figure. 9

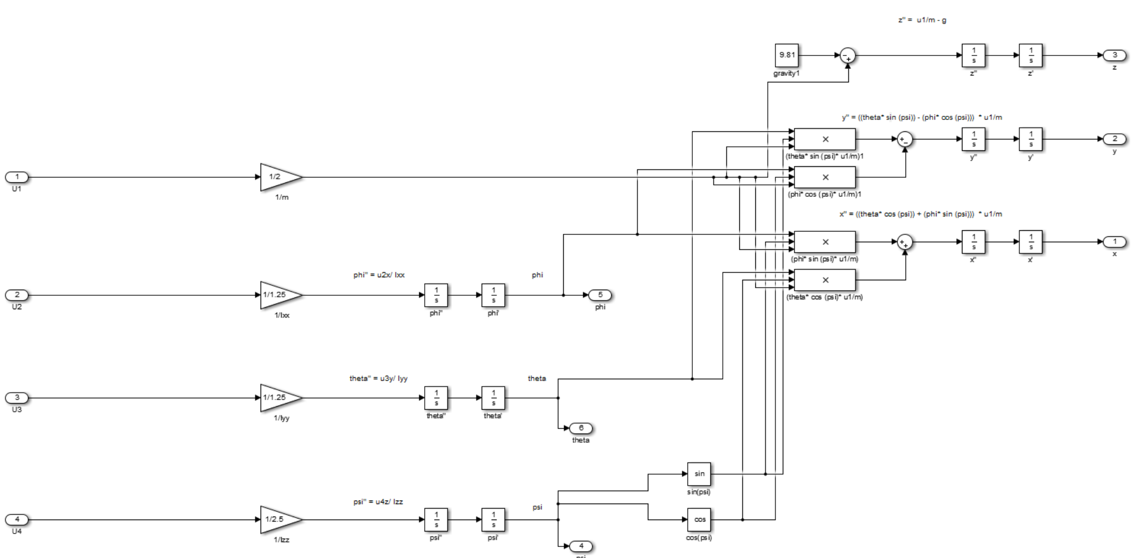


Figure. 10

4.2 PID Controller

While trying to determine the design for the controller it was identified that as the system is underactuated it is unable to control all 6 output states at the same time. Thus, to design a suitable controller for the system, we combine the x, y, z, and psi (yaw) outputs of the quadrotor to track the desired position and thereby stabilize the phi (roll) and theta (pitch) angles. (Altug et al., 2002; Pounds et al., 2002). Therefore, the controller must be designed to make the quadrotor reach the desired position and yaw angle keeping the pitch and roll angles fixed.

Hence, we need to feed in the desired trajectory given by a vector r_{des}

$$r_d = \begin{bmatrix} x_d \\ y_d \\ z_d \\ \varphi_d \end{bmatrix} \quad \textbf{Eq. 20}$$

Thereby, we assume a simple trajectory where the quadrotor increases in altitude (z) with time using a step response, at a constant yaw angle and keeping a fixed x and y value. The error between the actual trajectory and the desired trajectory (position— e_{pos} and velocity error— e_{vel}) must be eliminated.

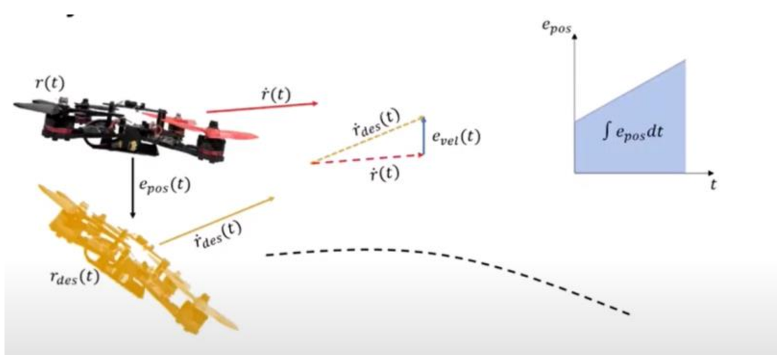


Figure 11

Following this, by using the actual position obtained from the close loop state-feedback of the quadrotor and the desired trajectory position. A Pythagoras relationship is determined as shown in Fig. 12. Using which desired roll and pitch angles (Eq. 21 and Eq.22) are obtained.

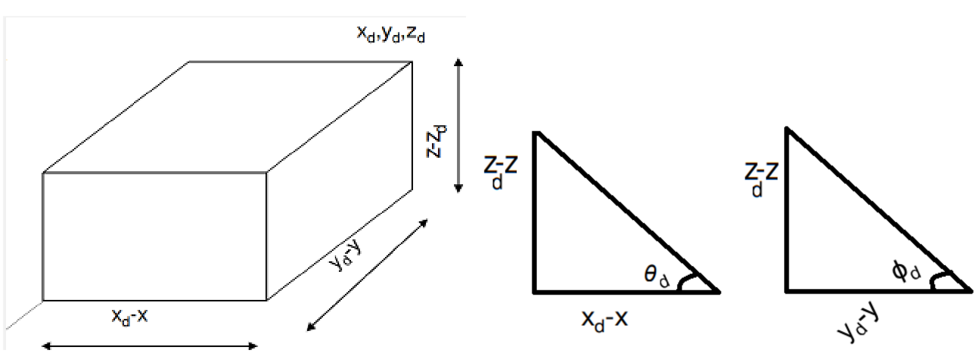


Figure 12

$$\emptyset_d = \tan^{-1}\frac{(z_d - z)}{(y_d - y)} \quad \textbf{Eq. 21}$$

$$\theta_d = \tan^{-1}\frac{(z_d - z)}{(x_d - x)} \quad \textbf{Eq. 22}$$

The actual state variables obtained through the close loop feedback from the quadrotor plant (x, y, z, phi, theta, and psi) and desired input values (z_d, φ_d, θ_d, $\emptyset_d$) are inputted in the PID controllers as shown in Fig. 13 and 14.

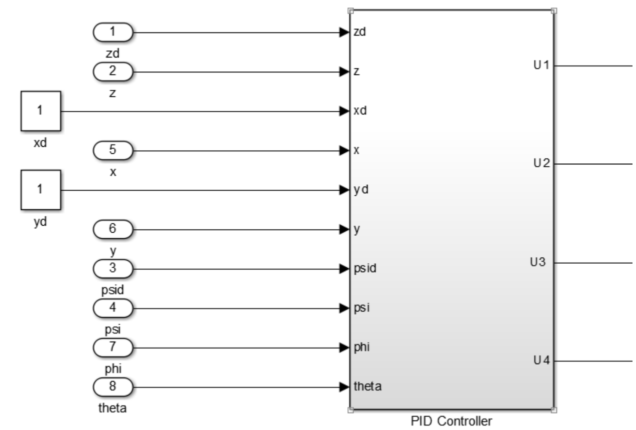


Figure 13

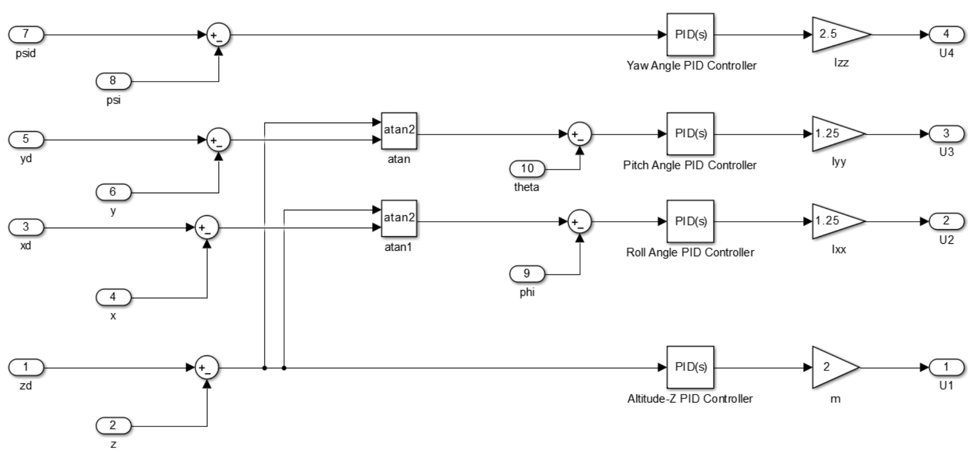


Figure 14

Using the input values the errors are computed as shown in Fig. 14. These errors are then forwarded to the four PID controllers – Yaw Angle PID Controller, Pitch Angle PID Controller, Roll Angle PID Controller, and Altitude-Z PID Controller.

These four PID controllers were individually tuned to obtain the desired system response and performance. Initially, the PID gain values for the individual controllers are determined using Ziegler-Nichol's method, as shown in Table 2.

PID parameter	KP	Kp/Ki	Kd/Kp
P	Time constant/delay time	∞	0
PI	0.9*TC/delay time	Delay time/0.3	0
PID	1.2*TC/delay time	2*delay time	0.5*delay time

Table 2

Further optimization of the PID controller was done via the trial-and-error method, and using the inbuilt PID tuner in MATLAB Simulink, as shown in Fig. 15. Different PID values were evaluated to obtain a faster response and a robust system.

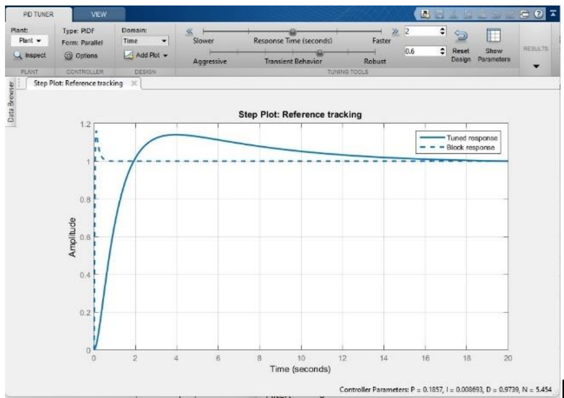


Figure 15

Thereby, we obtain the output signal required to stabilize the overall system from the PID controllers.

4.3 Motor Mixing Model

The output signal obtained from the PID controllers is fed into a motor mixing model, which represents a relationship between the output signal and the actuating signal fed into the motors, to obtain the actuating thrust and moments fed into the quadrotor plant.

The following equations (Eq.23 and Eq.24) were used in developing the motor mixing model (as represented in the below Figure 16 and 17)

$$
\begin{aligned}
U_1 &= b\left(\Omega_1{}^2 + \Omega_2{}^2 + \Omega_3{}^2 + \Omega_4{}^2\right) \\
\tau_\varphi &= U_2 = b\,l\left(\Omega_4{}^2 - \Omega_2{}^2\right) \\
\tau_\theta &= U_3 = b\,l\left(\Omega_3{}^2 - \Omega_1{}^2\right) \\
\tau_\psi &= U_4 = d\left(\Omega_2{}^2 + \Omega_4{}^2 - \Omega_1{}^2 - \Omega_3{}^2\right)
\end{aligned}
\qquad \textbf{Eq.23}
$$

$$
\begin{aligned}
\Omega_1{}^2 &= -(2 * U_3 - l * U_1 + l * U_4)/(4 * l) \\
\Omega_2{}^2 &= (l * U_1 - 2 * U_2 + l * U_4)/(4 * l) \\
\Omega_3{}^2 &= (2 * U_3 + l * U_1 - l * U_4)/(4 * l) \\
\Omega_4{}^2 &= (2 * U_2 + l * U_1 + l * U_4)/(4 * l)
\end{aligned}
\qquad \textbf{Eq.24}
$$

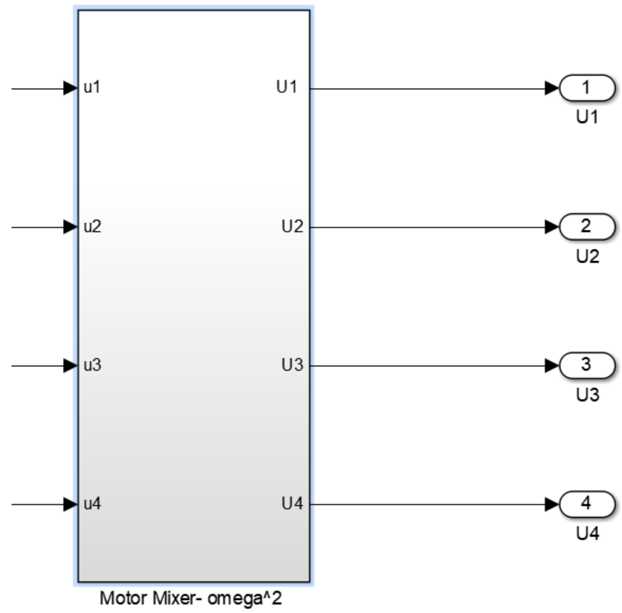


Figure 16

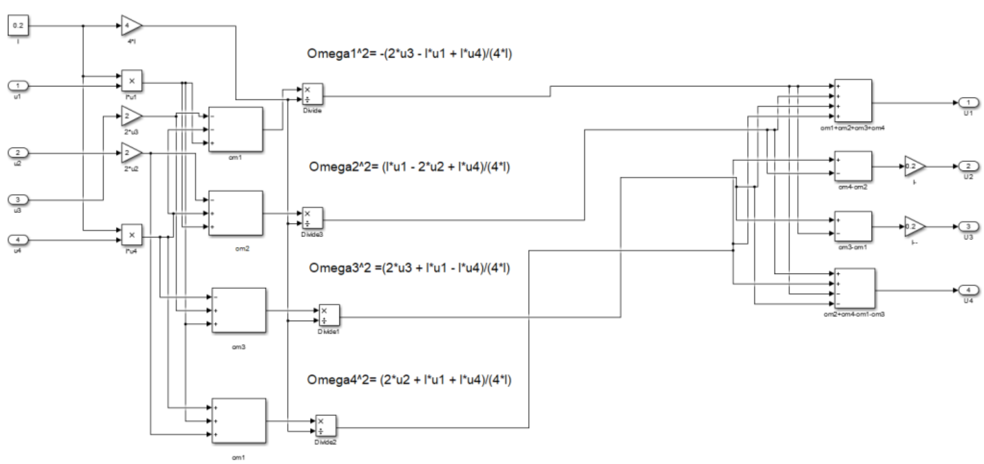


Figure 17

Hence the flight controller for the quadrotor is designed using a combination of the PID controller and Motor Mixing Model, as shown in Figure 18 and 19.

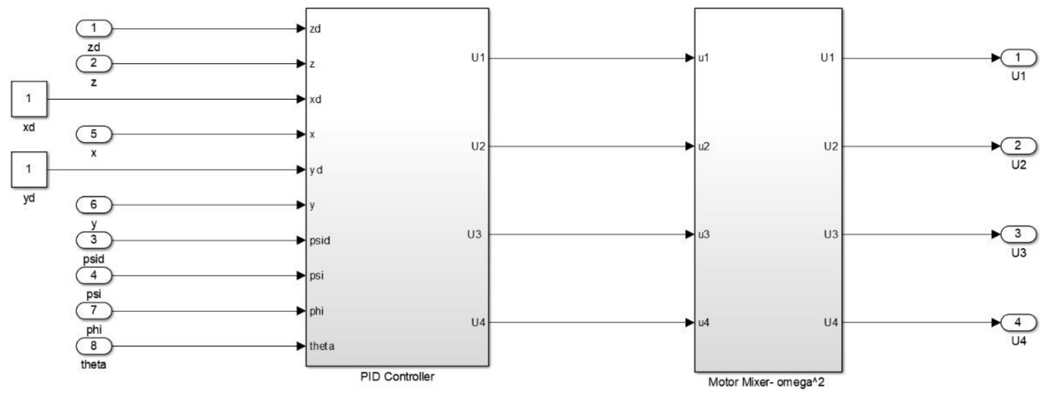


Figure 18

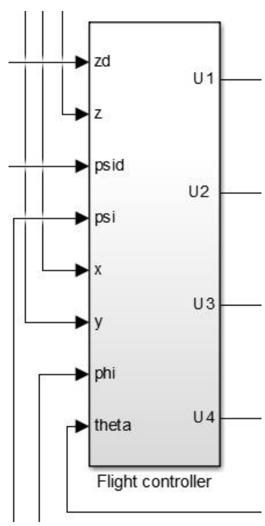


Figure 19

4.4 Quadrotor Plant with Flight Controller Model

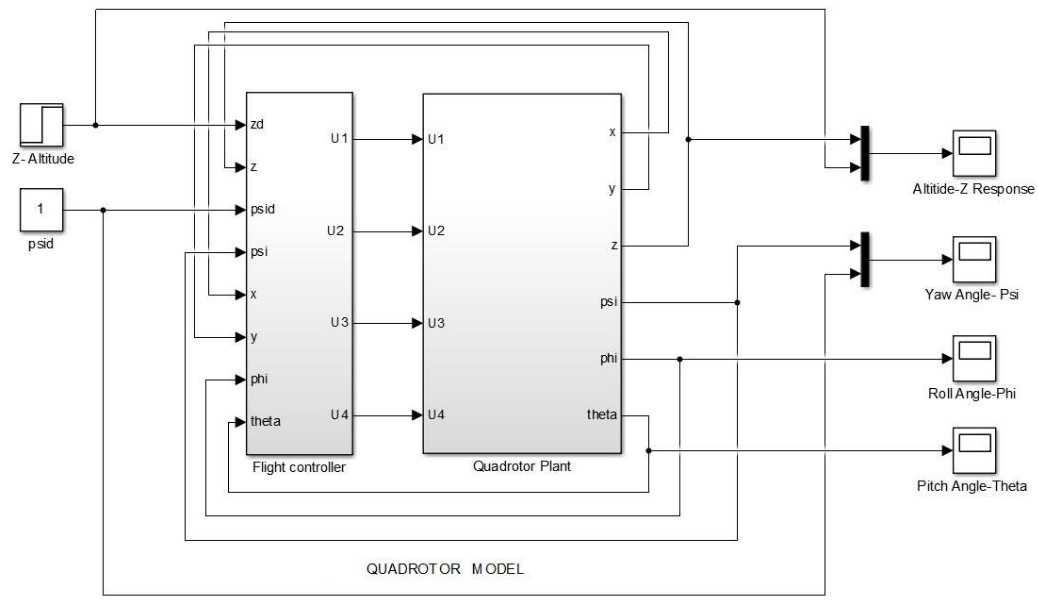


Figure 20

Using the Flight controller (combination of PID controller and Motor Mixing Model) and the quadrotor plant the overall system response is observed, as shown in Figure 20.

The following parameters were considered to observe the system's response.

Parameter	Value
x_d	10 m
y_d	10 m
z_d	Step 0 m to 20 m in 1 s
φ_d	1 rad

Table 3

5. RESULTS, COMPARISON, AND CONCLUSION

By providing the input parameters as stated in Table 3 into the quadrotor model, and optimizing the PID controllers, the following system responses were observed. In Figure 21 we obtained an overshoot within the change in the altitude (z-axis). Thereby obtaining a faster, stable, and robust system as compared to Figure 5.

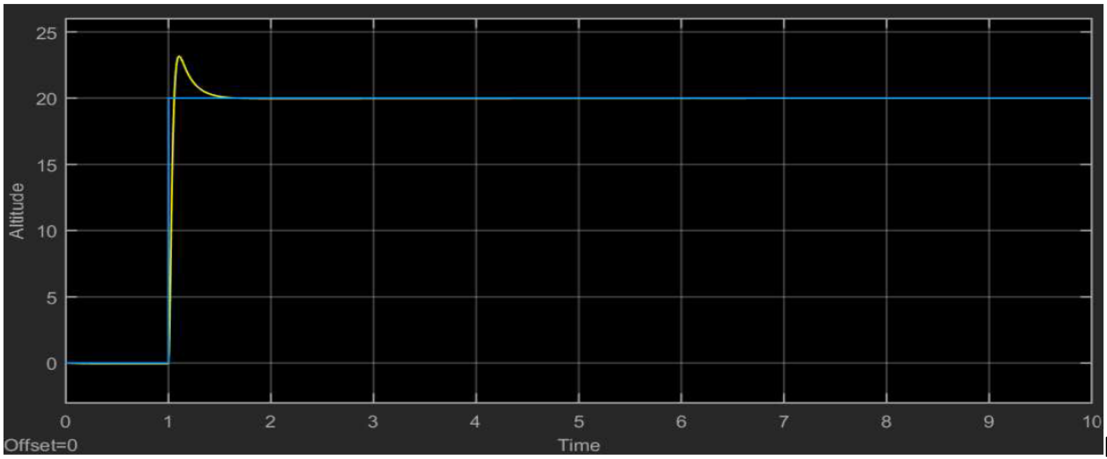


Figure 21

Additionally, it was noted the roll and pitch (Figure 22 and 23) started stabilizing and coming to the desired outcome after 2 seconds with a small overshoot. Hence, achieving a faster-stabilized response compared to Figures. 6 and 7 (ref. to literature review).

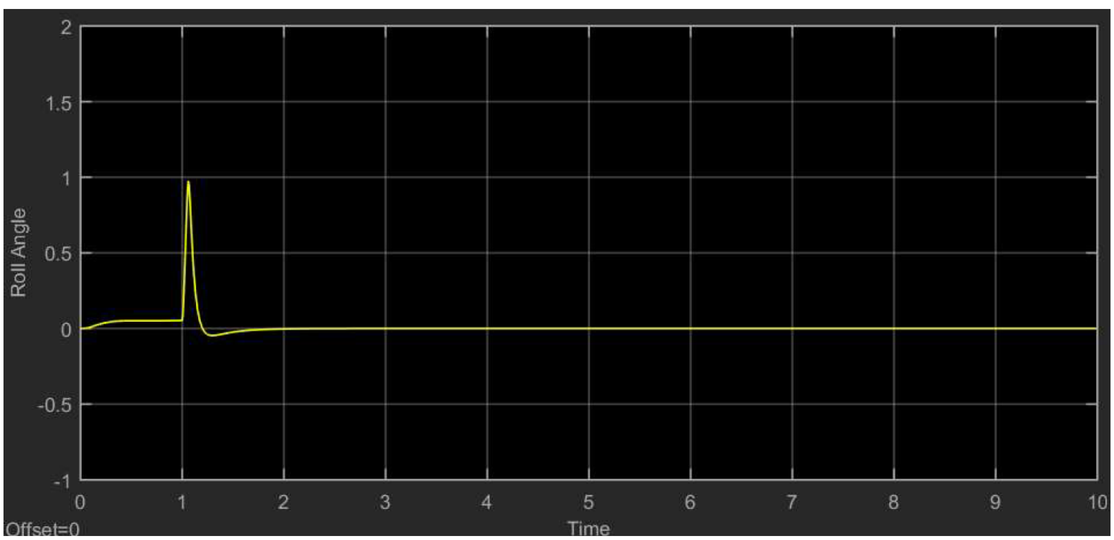


Figure 22

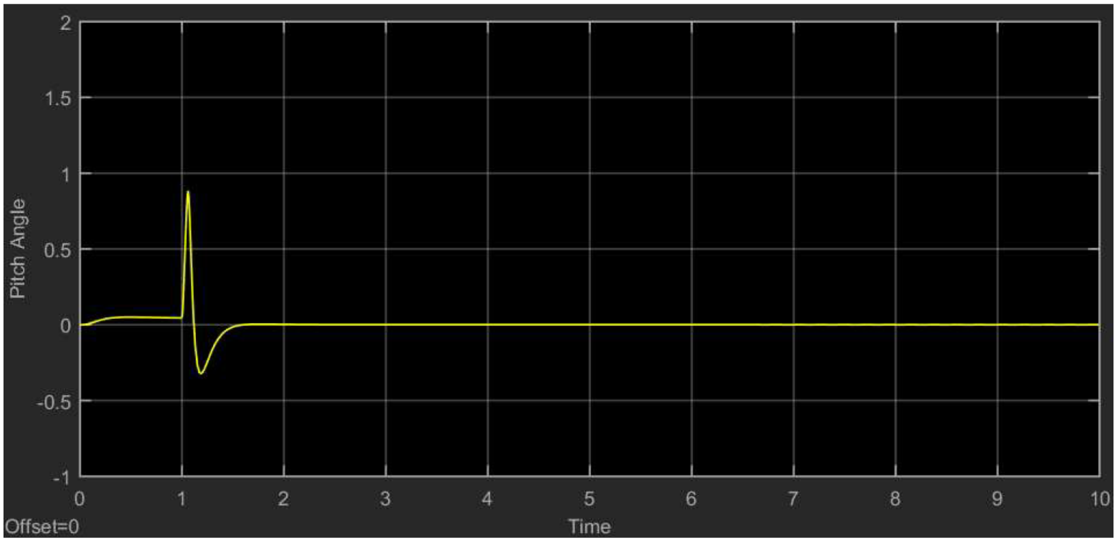


Figure 24

The Yaw angle which was set to the desired value (1 rad) is observed to stabilize itself within 1.5 seconds. Producing a faster response compared to Figure 8 (ref. literature review) but with more overshoot.

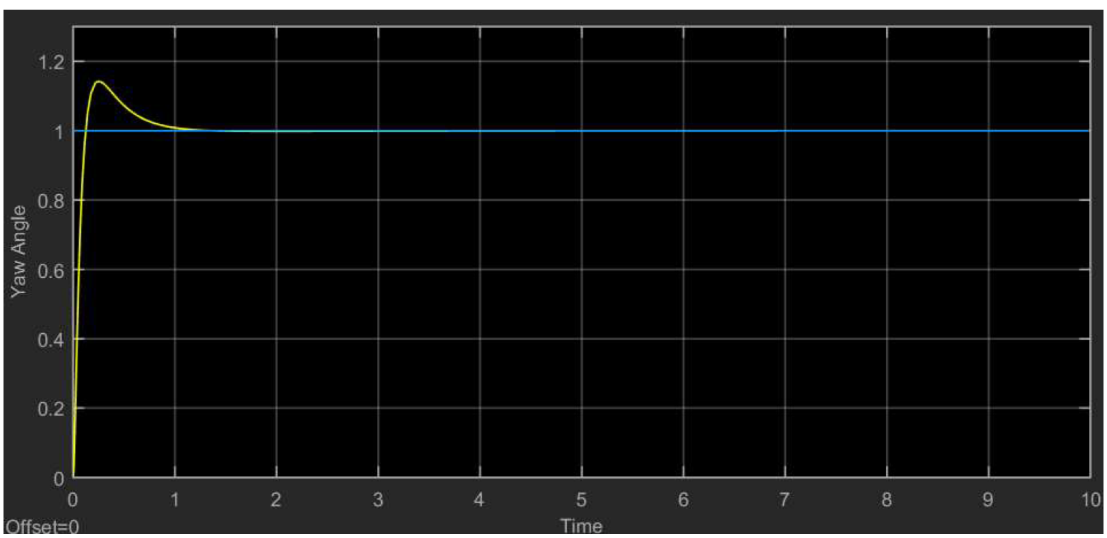


Figure 23

The report showcases the design of the PID flight controller for the quadrotor wherein the desired system performance is obtained. Initially, a simple mathematical model was developed for the quadrotor plant, representing and analyzing it in the MATLAB Simulink environment. Finally, a PID controller and motor mixing algorithms were designed and developed, using which the system was optimized, and thereby the desired system was controlled, and the motive was achieved.

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6 https://www.youtube.com/channel/UCOBPDRR5uTqh7EZR27kH6Bg