# CALCULUS AND PROBABILITY BASED STATISTICS

MARK ANDREW GERADS

## 1. Finite Statistics

The purpose of this text is to have statistics built on a solid foundation of probability and calculus.

**Definition 1.1.** $A$ is a repeatable event.

**Definition 1.2.** $p$ is the probability that $A$ will happen when $A$ is given a chance to happen. For simplicity, we assume that $p$ is the same every time $A$ is given a chance to happen.

**Definition 1.3.** $n \in \mathbb{W}$ is the number of times $A$ had a chance to happen.

**Definition 1.4.** $k \in \mathbb{W}$ is the number of times $A$ actually happened.

*Remark* 1.5. $k \leq n$.

**Definition 1.6.** $f_p$ is the probability is the probability distribution for $p$.

*Remark* 1.7. $1 = \int_0^1 f_p \mathrm{d}p$

**Definition 1.8.** $(a, b)$ is the prediction interval of $p$ such that $b - a$ is minimized.

**Definition 1.9.** $c$ is the desired certainty for the prediction interval of $p$.

*Remark* 1.10. $0 \leq c = \int_a^b f_p \mathrm{d}p \leq 1$

*Remark* 1.11. $0 \leq a \leq b \leq 1$

Now that we have the definitions, here is the problem: $n, k, c$ are known, and we must solve for $a, b$.

*Remark* 1.12. The probability of $k$ events given $n$ chances is $\binom{n}{k} p^k (1-p)^{n-k}$, but we must scale it to satisfy 1.7:

$$f_p = \frac{\binom{n}{k} p^k (1-p)^{n-k}}{\int_0^1 \binom{n}{k} p^k (1-p)^{n-k} \mathrm{d}p} = \frac{p^k (1-p)^{n-k}}{\int_0^1 p^k (1-p)^{n-k} \mathrm{d}p} = (n+1)\binom{n}{k} p^k (1-p)^{n-k} \tag{1.1}$$

318 Thomas Drive, Marshalltown, Iowa, 50158
*E-mail address*: MGerads11@winona.edu, nazgand@gmail.com

*Date*: February 18, 2016.