Argument Sum Rules From Homogeneous Linear Differential Equations Of Constant Coefficients Conjecture https://github.com/Nazgand/nazgandMathBook

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Abstract

The goal of this paper is to conjecture a seemingly fundamental calculus fact.

A [homogeneous linear differential equation of constant coefficients] has the form

$$0=\sum_{k=0}^{n}a_k\frac{\partial^k}{\partial z^k}f(z)=\sum_{k=0}^{n}a_k f^{(k)}(z) \tag{0.1}$$

where $a_k\in\mathbb{C}, f:\mathbb{C}\to\mathbb{C}$. Let $f_1(z),\ldots,f_n(z)$ be solutions that span the vector space of solutions of the [homogeneous linear differential equation of constant coefficients]. To be clear:

$$\left\{f(z)\mid 0=\sum_{k=0}^{n}a_k f^{(k)}(z)\right\}=\left\{\sum_{k=1}^{n}b_k f_k(z)\mid b_k\in\mathbb{C}\right\} \tag{0.2}$$

Let us define a column vector and clarify its transpose (a row vector):

$$v(z_0)=\begin{pmatrix}f_1(z_0)\\ \vdots\\ f_n(z_0)\end{pmatrix}, v(z_1)^T=\begin{pmatrix}f_1(z_1) & \ldots & f_n(z_1)\end{pmatrix} \tag{0.3}$$

I conjecture there exists a complex-valued constant n by n symmetric matrix A ($A=A^T$) such that

$$f(z_0+z_1)=v(z_1)^T A v(z_0)=v(z_0)^T A v(z_1) \tag{0.4}$$

1 A reason A is symmetric

Suppose instead of a symmetric matrix A, we find a matrix B such that

$$f(z_0 + z_1) = v(z_1)^T B v(z_0) \tag{1.1}$$

Then take the transpose of the equation and substitute $z_0 \to z_1, z_1 \to z_0$, resulting in the following equation:

$$f(z_0 + z_1) = v(z_1)^T B^T v(z_0) \tag{1.2}$$

Average both equations:

$$f(z_0 + z_1) = v(z_1)^T \frac{B + B^T}{2} v(z_0) \tag{1.3}$$

Note that we can set $A = \frac{B+B^T}{2}$ because it is symmetric. $\square$