https://GitHub.Com/Nazgand/NazgandMathBook

Cauchy Sequence Permutations

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Abstract

The goal of this paper is to notice that [every reordering of a Cauchy Sequence] is a Cauchy Sequence.

Let a bijection exist on $\mathbb{Z}_{\geq 0}$ (The set of natural numbers including 0), $perm : \mathbb{Z}_{\geq 0} \to \mathbb{Z}_{\geq 0}$. $perm$ is short for 'permutation', because every bijection from a set to itself is a permutation.

Let a Cauchy Sequence exist, $cs : \mathbb{Z}_{\geq 0} \to \mathbb{C}$, with the limit

$$l = \lim_{k \to \infty} cs(k) \in \mathbb{C} \tag{0.1}$$

Then

$$l = \lim_{k \to \infty} cs(perm(k)) \in \mathbb{C} \tag{0.2}$$

Let $\epsilon \in \mathbb{R}^+$. Then consider the set

$$OutsideCs(\epsilon) = \{k \in \mathbb{Z}_{\geq 0} | |cs(k) - l| > \epsilon\} \tag{0.3}$$

Notice $OutsideCs(\epsilon)$ is finite [because if $OutsideCs(\epsilon)$ were infinite in cardinality, then the limit could not be l]. Maximums of finite non-empty sets are well defined, so...

$$csSkip(\epsilon) = \max\left(\{-1\} \cup OutsideCs(\epsilon)\right) + 1 \tag{0.4}$$

This function tells [how much of the Cauchy Series to skip] such that [the new Cauchy Sequence created by the skip] in a precision circle of size ϵ.

$$cs_2(k, \epsilon) = cs(k + csSkip(\epsilon)) \tag{0.5}$$

$$\left[k \in \mathbb{Z}_{\geq 0}, \epsilon \in \mathbb{R}^+\right] \Rightarrow |cs_2(k, \epsilon) - l| \leq \epsilon \tag{0.6}$$

We need to create an analogous $csPermSkip(\epsilon)$ function for the second series, requiring

$$cs_3(k, \epsilon) = cs(perm(k + csPermSkip(\epsilon))) \tag{0.7}$$

$$\left[k \in \mathbb{Z}_{\geq 0}, \epsilon \in \mathbb{R}^+\right] \Rightarrow |cs_3(k, \epsilon) - l| \leq \epsilon \tag{0.8}$$

$$OutsideCsPerm(\epsilon) = \{k \in \mathbb{Z}_{\geq 0} | |cs(perm(k)) - l| > \epsilon\} \tag{0.9}$$

It should be noticed that

$$OutsideCsPerm(\epsilon) = \{k \in \mathbb{Z}_{\geq 0} | perm(k) \in OutsideCs(\epsilon)\} \tag{0.10}$$

The finiteness of $OutsideCsPerm(\epsilon)$ is enough to show the limit is l, yet we explicitly define the skip function:

$$csPermSkip(\epsilon) = \max\left(\{-1\} \cup OutsideCsPerm(\epsilon)\right) + 1 \tag{0.11}$$

Equivalently,

$$csPermSkip(\epsilon) = \max\left(\{-1\} \cup \left\{perm^{-1}(k) | k \in OutsideCs(\epsilon)\right\}\right) + 1 \tag{0.12}$$

$\square$