

# Argument Sum Rules From Homogeneous Linear Differential Equations Of Constant Coefficients Conjecture

<https://github.com/Nazgand/nazgandMathBook>

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October 1, 2022

## Abstract

The goal of this paper is to conjecture a seemingly fundamental calculus fact.

A [homogeneous linear differential equation of constant coefficients] has the form

$$0 = \sum_{k=0}^n a_k \frac{\partial^k}{\partial z^k} f(z) = \sum_{k=0}^n a_k f^{(k)}(z) \quad (0.1)$$

where  $a_k \in \mathbb{C}$ ,  $f : \mathbb{C} \rightarrow \mathbb{C}$ . Let  $f_1(z), \dots, f_n(z)$  be solutions that span the vector space of solutions of the [homogeneous linear differential equation of constant coefficients]. To be clear:

$$\left\{ f(z) \mid 0 = \sum_{k=0}^n a_k f^{(k)}(z) \right\} = \left\{ \sum_{k=1}^n b_k f_k(z) \mid b_k \in \mathbb{C} \right\} \quad (0.2)$$

Let us define a column vector and clarify its transpose (a row vector):

$$v(z_0) = \begin{pmatrix} f_1(z_0) \\ \vdots \\ f_n(z_0) \end{pmatrix}, v(z_1)^T = (f_1(z_1) \quad \dots \quad f_n(z_1)) \quad (0.3)$$

I conjecture there exists a complex-valued constant  $n$  by  $n$  symmetric matrix  $A$  ( $A = A^T$ ) such that

$$f(z_0 + z_1) = v(z_1)^T A v(z_0) = v(z_0)^T A v(z_1) \quad (0.4)$$