Notation
https://github.com/Nazgand/nazgandMathBook

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Abstract

The goal of this paper is to clarify notation.

1 Substitution

This works similarly to Mathematica's Replace function. Example:

$$\left(\frac{\partial}{\partial x}x^2 : x \to a\right) = (2x : x \to a) = 2a \tag{1.1}$$

https://reference.wolfram.com/language/ref/Replace.html

2 Logic

And: Given statements A and B. The statement $A \wedge B$ means that both statements A and B are true.

Or: Given statements A and B. The statement $A \vee B$ means that at least 1 of the statements A or B is not false.

Implies: The statement $A \Rightarrow B$ is a conditional statement stating that if A is true, then B is true.

$A \Leftrightarrow B$ means $[A \Rightarrow B \wedge B \Rightarrow A]$.

Brackets: Normally, logic is read from left to right, yet sometimes brackets are used to change the order or add clarification. Example: $A \vee [B \wedge C]$.

Exists: $\exists a$ means that some a exists.

$\exists a \wedge P(a)$ can be thought as $\{\} \neq \{a \mid P(a)\}$.

https://en.wikipedia.org/wiki/List_of_logic_symbols

Kronecker delta function:

$$\delta(0) = 1 \wedge [x \neq 0 \Rightarrow \delta(x) = 0] \tag{2.1}$$

https://en.wikipedia.org/wiki/Kronecker_delta

3 Order and Equality

Equals: $a = b$ means that a is equal to b.

https://en.wikipedia.org/wiki/Equality_(mathematics)

Greater than: $a > b$ means that a is greater than b. $a \geq b$ means that a is greater than or equal to b.

Less than: $a < b$ means that a is less than b. $a \leq b$ means that a is less than or equal to b.

https://en.wikipedia.org/wiki/Inequality_(mathematics)

4 Sets

Sets: Sets either have an element or they do not have the element; no element is listed more than 1 time. Example sets are: $\{1, 2, 3\}, \{1, \{1, a, b\}\}$.

Element of: $\in$ means "is an element of". Examples: $1 \in \{1, 2, 3\}$ and $\{1, a, b\} \in \{1, \{1, a, b\}\}$.

Subset: $\subseteq$ means "is a subset of". Examples: $\{1, 2, 3\} \subseteq \{1, 2, 3\}$ and $\{3\} \subseteq \{1, 2, 3\}$ and $\{\} \subseteq \{1, 2, 3\}$.

Union: $A \cup B$ is the minimal set which contains all elements either A or B contain.

Intersection: $A \cap B$ is the set which contains all elements both A and B contain.

https://en.wikipedia.org/wiki/Set_(mathematics)

Set builder notation:

The set of all things a which satisfy the constraining statement $P(a)$:

$$\{a \mid P(a)\} \tag{4.1}$$

The set of all things a in the set A which satisfy the statement $P(a)$:

$$\{a \in A \mid P(a)\} \tag{4.2}$$

https://en.wikipedia.org/wiki/Set-builder_notation

Integers:

$$0 \in \mathbb{Z} \land [a \in \mathbb{Z} \Leftrightarrow (a+1) \in \mathbb{Z}] \tag{4.3}$$

$$\mathbb{Z}^{+} = \{n \in \mathbb{Z} \mid n > 0\} \tag{4.4}$$

$$\mathbb{Z}^{\geq 0} = \{n \in \mathbb{Z} \mid n \geq 0\} \tag{4.5}$$

https://en.wikipedia.org/wiki/Integer
https://en.wikipedia.org/wiki/Natural_number#Notation
Rational numbers:

$$\mathbb{Q} = \left\{\frac{a}{b} \mid a \in \mathbb{Z} \land b \in \mathbb{Z}^{+}\right\} \tag{4.6}$$

$$\mathbb{Q}^{+} = \{q \in \mathbb{Q} \mid q > 0\} \tag{4.7}$$

$$\mathbb{Q}^{\geq 0} = \{q \in \mathbb{Q} \mid q \geq 0\} \tag{4.8}$$

https://en.wikipedia.org/wiki/Rational_number
Real numbers:

$$\mathbb{R}^{\geq 0} = \left\{\inf A \mid A \subseteq \mathbb{Q}^{+} \land A \neq \{\}\right\} \tag{4.9}$$

$$\mathbb{R} = \left\{a - b \mid \{a, b\} \subseteq \mathbb{R}^{\geq 0}\right\} \tag{4.10}$$

$$\mathbb{R}^{+} = \left\{a \in \mathbb{R}^{\geq 0} \mid a > 0\right\} \tag{4.11}$$

https://en.wikipedia.org/wiki/Real_number

5 Sigma summation

$$\sum_{k \in \{\}} f(k) = 0 \tag{5.1}$$

$$\sum_{k \in \{a\}} f(k) = f(a) \tag{5.2}$$

$$\sum_{k \in \{S_1 \cup S_2\}} f(k) = \left(\sum_{k \in \{S_1\}} f(k)\right) + \left(\sum_{k \in \{S_2\}} f(k)\right) - \left(\sum_{k \in \{S_1 \cap S_2\}} f(k)\right) \tag{5.3}$$

Shorthand

$$\sum_{k=a}^{a-1} f(k) = 0 \tag{5.4}$$

$$\sum_{k=a}^{b+1} f(k) = \left(\sum_{k=a}^{b} f(k)\right) + f(b+1) = f(a) + \sum_{k=a+1}^{b} f(k) \tag{5.5}$$

6 Calculus

A limit $\lim_{a \to b} f(a)$ is the value usually approaching $f(b)$

nth derivative, not to be confused with $f^{n}(x)$:

$$f^{(n)}(x) = \frac{\partial^n}{\partial x^n} f(x) \tag{6.1}$$

https://en.wikipedia.org/wiki/Derivative#Leibniz's_notation

Integrals can be thought as the area under a curve. My notation differs slightly by using ∂ instead of d. The integral from a to b of function $f(x)$ with respect to x is:

$$\int_a^b f(x) \partial x \tag{6.2}$$

Laplace transform:

$$\mathcal{L}\{f(t)\}(s) = \int_{t=0}^{\infty} f(t) e^{-st} \partial t \tag{6.3}$$

Laplace transform inverse:

$$\mathcal{L}^{-1}\{\mathcal{L}\{f(t)\}(s)\}(t) = f(t) \tag{6.4}$$

https://en.wikipedia.org/wiki/Laplace_transform#Formal_definition

Modular arithmetic function, $\operatorname{mod} : (\mathbb{C}, \mathbb{C}) \to \mathbb{C}$

$$0 \leq \Re\left(\frac{\operatorname{mod}(a, b)}{b}\right) < 1 \land \operatorname{mod}(a, b) = \operatorname{mod}(a + b, b) \tag{6.5}$$