Gamma Function

https://github.com/Nazgand/nazgandMathBook

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Abstract

The goal of this paper is to review the gamma function.

1 Definition

Definition 1.1.

$$\Gamma(a+1)=\int_{t=0}^{\infty}t^a e^{-t}\partial t \tag{1.1}$$

2 Convergence of Integral

For $t^a e^{-t}$ shrinks exponentially quickly as $t\to\infty$, thus the integral $\int_{t=1}^{\infty}t^a e^{-t}\partial t$ converges. The integral $\int_{t=0}^{1}t^a e^{-t}\partial t$ only converges for $a>-1$.

3 Recursive Property

From (1.1), integrate by parts with $u=t^a, v=-e^{-t}$

$$\Gamma(a+1)=\left[-t^a e^{-t}\right]_{t=0}^{\infty}-\int_{t=0}^{\infty}-e^{-t}at^{a-1}\partial t \tag{3.1}$$

Simplify

$$\Gamma(a+1)=a\int_{t=0}^{\infty}e^{-t}t^{a-1}\partial t=a\Gamma(a) \tag{3.2}$$