

Gamma Function

Mark Andrew Gerads <Nazgand@Gmail.Com>

2026-01-21 04:46:20-08:00

1 Definition

Definition 1.1.

$$\forall z : \mathbb{C}, \operatorname{Re}(z) > -1 \Rightarrow \Pi(z) = \Gamma(z+1) = \int_0^\infty t^z e^{-t} \, dt \quad (1.1)$$

2 Convergence of Integral

$t^z e^{-t}$ shrinks exponentially quickly as $t \rightarrow \infty$, thus the integral $\int_1^\infty t^z e^{-t} \, dt$ converges. The integral $\int_0^1 t^z e^{-t} \, dt$ only converges for $\operatorname{Re}(z) > -1$.

3 Recursive Property

From (1.1), integrate by parts with $u = t^z, v = -e^{-t}$

$$\Gamma(z+1) = \left[-t^z e^{-t} \right]_{t=0}^\infty - \int_0^\infty -e^{-t} z t^{z-1} \, dt \quad (3.1)$$

Simplify

$$\Gamma(z+1) = z \int_0^\infty e^{-t} t^{z-1} \, dt = z \Gamma(z) \quad (3.2)$$