

# Cauchy Sequence Permutations

<https://github.com/Nazgand/nazgandMathBook>

Mark Andrew Gerads: [Nazgand@Gmail.Com](mailto:Nazgand@Gmail.Com)

June 12, 2023

## Abstract

The goal of this paper is to notice that [every reordering of a Cauchy Sequence] is a Cauchy Sequence.

Let a bijection exist on  $\mathbb{N}_0$  (The set of natural numbers including 0),  $perm : \mathbb{N}_0 \rightarrow \mathbb{N}_0$ .  $perm$  is short for 'permutation', because every bijection from a set to itself is a permutation.

Let a Cauchy Sequence exist,  $cs : \mathbb{N}_0 \rightarrow \mathbb{C}$ , with the limit

$$l = \lim_{k \rightarrow \infty} cs(k) \in \mathbb{C} \quad (0.1)$$

Then

$$l = \lim_{k \rightarrow \infty} cs(perm(k)) \in \mathbb{C} \quad (0.2)$$

Let  $\epsilon \in \mathbb{R}^+$ . Then consider the set

$$OutsideCs(\epsilon) = \{k \in \mathbb{N}_0 \mid |cs(k) - l| > \epsilon\} \quad (0.3)$$

Notice  $OutsideCs(\epsilon)$  is finite [because if  $OutsideCs(\epsilon)$  were infinite in cardinality, then the limit could not be  $l$ ]. Maximums of finite non-empty sets are well defined, so...

$$csSkip(\epsilon) = \max(\{-1\} \cup OutsideCs(\epsilon)) + 1 \quad (0.4)$$

This function tells [how much of the Cauchy Series to skip] such that [the new Cauchy Sequence created by the skip] in a precision circle of size  $\epsilon$ .

$$cs_2(k, \epsilon) = cs(k + csSkip(\epsilon)) \quad (0.5)$$

$$[k \in \mathbb{N}_0, \epsilon \in \mathbb{R}^+] \Rightarrow |cs_2(k, \epsilon) - l| \leq \epsilon \quad (0.6)$$

We need to create an analogous  $csPermSkip(\epsilon)$  function for the second series, requiring

$$cs_3(k, \epsilon) = cs(perm(k + csPermSkip(\epsilon))) \quad (0.7)$$

$$[k \in \mathbb{N}_0, \epsilon \in \mathbb{R}^+] \Rightarrow |cs_3(k, \epsilon) - l| \leq \epsilon \quad (0.8)$$

$$OutsideCsPerm(\epsilon) = \{k \in \mathbb{N}_0 \mid |cs(perm(k)) - l| > \epsilon\} \quad (0.9)$$

It should be noticed that

$$OutsideCsPerm(\epsilon) = \{k \in \mathbb{N}_0 \mid perm(k) \in OutsideCs(\epsilon)\} \quad (0.10)$$

The finiteness of  $OutsideCsPerm(\epsilon)$  is enough to show the limit is  $l$ , yet we explicitly define the skip function:

$$csPermSkip(\epsilon) = \max(\{-1\} \cup OutsideCsPerm(\epsilon)) + 1 \quad (0.11)$$

Equivalently,

$$csPermSkip(\epsilon) = \max(\{-1\} \cup \{perm^{-1}(k) \mid k \in OutsideCs(\epsilon)\}) + 1 \quad (0.12)$$

□