Gamma Function

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Abstract

The goal of this paper is to review the gamma function.

1 Definition

Definition 1.1.

$$\Gamma(a+1) = \int_{t=0}^{\infty} t^a e^{-t}\,\partial t \tag{1.1}$$

2 Convergence of Integral

For $t^a e^{-t}$ shrinks exponentially quickly as $t \to \infty$, thus the integral $\int_{t=1}^{\infty} t^a e^{-t}\,\partial t$ converges. The integral $\int_{t=0}^{1} t^a e^{-t}\,\partial t$ only converges for $a > -1$.

3 Recursive Property

From (1.1), integrate by parts with $u = t^a, v = -e^{-t}$

$$\Gamma(a+1) = \left[-t^a e^{-t}\right]_{t=0}^{\infty} - \int_{t=0}^{\infty} -e^{-t}a t^{a-1}\,\partial t \tag{3.1}$$

Simplify

$$\Gamma(a+1) = a\int_{t=0}^{\infty} e^{-t}t^{a-1}\,\partial t = a\Gamma(a) \tag{3.2}$$