ECE 514 Random Process - Project II Report

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ECHO SONAR - SYSTEM MODEL

A modern approach to estimating the number of fishes in an area by transmitting sound pulses is called the echo sonar technique. In this approach the reflected waveforms from a school of fish are modeled as a Gaussian random process.

This technique is more accurate and inexpensive compared to the traditional method of fish capturing in the net and counting them manually. A transmitter on the boat transmits a sinusoidal pulse and the receiver measures the count of reflected waveform. The number of reflected waveform indicates the number of fishes present in the school of fish.

Considering the transmit pulse to be given by the equation below:

$$T(t) = A_T \, cos(2\pi f_0 t)$$

where A_T and f_0 are the amplitude and frequency of the transmitted pulse. The received signal can be written as:

$$X_i(t) = A_i \, cos(2\pi \, f_0 \, (t - \tau_i) + \theta_i)$$

$X_i(t)$ is the i^{th} reflected signal. A_i and θ_i are the amplitude and phase of the i^{th} reflected signal. Here τ_i is the time delay of the pulse reflected from the i^{th} fish and f_0 is still the transmit frequency in Hz (1Khz).

1. ***Equation of the received waveform over all fish*:**

The received signal is the superposition of all the reflected waveforms. Hence the received signal can be written as:

$$X(t) = \sum_{i=1}^{N} X_i(t) = \sum_{i=1}^{N} A_i \, cos(2\pi \, f_0 t + \Theta_i)$$

Using the trigonometry identity: $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

$$X(t) = cos(2\pi \, f_0 t) \sum_{i=1}^{N} A_i \, cos(\Theta_i) - sin(2\pi \, f_0 t) \sum_{i=1}^{N} A_i \, sin(\Theta_i)$$

$$X(t) = cos(2\pi \, f_0 t) \sum_{i=1}^{N} U_i - sin(2\pi \, f_0 t) \sum_{i=1}^{N} V_i$$

Here, $U_i = A_i \, cos(\Theta_i)$ and $V_i = A_i \, sin(\Theta_i)$

Without loss of generality, we assume that all fish are about the same size and hence the echo amplitudes are about the same i.e $Ai = A$; $for \, all \, i$. This means that all fishes are identical. The phase shift by reflection from surface of all fish can hence be assumed independent, $i.e. \; \Theta_i \; ; \; for \, all \, i$ are independent random variables.

The received waveform can be simplified as follows:

$$X(t) = A \, cos(2\pi \, f_0 t) \sum_{i=1}^{N} \cos(\Theta_i) - A \, sin(2\pi \, f_0 t) \sum_{i=1}^{N} sin(\Theta_i)$$

Θ_i is an independent random variable and so is the $\cos(\Theta_i)$ and $\sin(\Theta_i)$. Hence, in the above equation we can clearly see that the received waveform is sum of independent random variables. The different reflections of the fish do not interfere with each other.

2. *Received Waveform as Gaussian Random Process:*

It is stated that $U = \sum_{i=1}^{N} U_i$ where $U_i = A_i\, cos(\Theta_i)$. Since we assumed all fishes to be independent we know that Θ_i is an independent random variable which in turn indicates that $\cos(\Theta_i)$ are independent for all i.

U can therefore be called as a sum of N-independent random variables. Based on the Central Limit Theorem, we can conclude that random variable U converges to a Gaussian random variable in distribution for a large value of N. This can be though of as a convolution between multiple positive pulses that represent the independent uniform distributions that would result to form a Gaussian distribution.

Similarly, since $V = \sum_{i=1}^{N} V_i$ where $V_i = A_i\, sin(\Theta_i)$, for a large value of N we say that the random variable V converges to Gaussian random variable in distribution.

3. *Theoretical number of fish:*

Since U_i and V_i are independent random variables say with a finite variance of ${\sigma_i}^2$, then the sum of N such random variates, $U\ and\ V$ will be Gaussian Random variables with mean value of 0 and a finite variance of $N\sigma^2$. The amplitude or envelope of $X(t)$ can be written as

$$A = |X(t)| = \sqrt{(U^2 + V^2)}$$

A has a Rayleigh distribution with parameter $N\sigma^2$. The expected value of Rayleigh distribution is given by:

$$E[A] = \sqrt{\frac{\pi\ N\sigma^2}{2}}$$

Hence, if we know the receive amplitudes, we can estimate N, number of fishes. σ^2 can be obtained by measuring the reflection characteristics of a single fish. We assume that this parameter is known for our analysis (given).

4. *Empirical measurement of the number of fish:*

To measure the number of fishes empirically in a region, we transmit many pulses and measure the received echoes. Let P be the number of transmit signals. We measure the echoes corresponding to each of the transmitted signal. Consider $A'_1,\ A'_2\ \dots\ A'_P$ be the amplitudes of the set of received pulses $X'_1,\ X'_2\ \dots\ X'_P$. Using frequency interpretation, we can write the expected values of the received amplitudes as

$$E[A'] = \frac{A'_1 +\ A'_2 +\ \dots + A'_P}{P}$$

We can estimate the total number of fishes by using the analytical formula with the replacement of empirical mean. This value can be written as

$$N' = \frac{2}{\pi\sigma^2} E^2[A']$$

5. *Simulation of one Reflected Waveform:*

To simulate $X_i(t)$ is a continuous we perform its sampling. From the Fig. 1 we can see that the histogram resembles that of a cosine wave. $X_i(t)$ has larger number of samples present near the amplitude region than the mean.

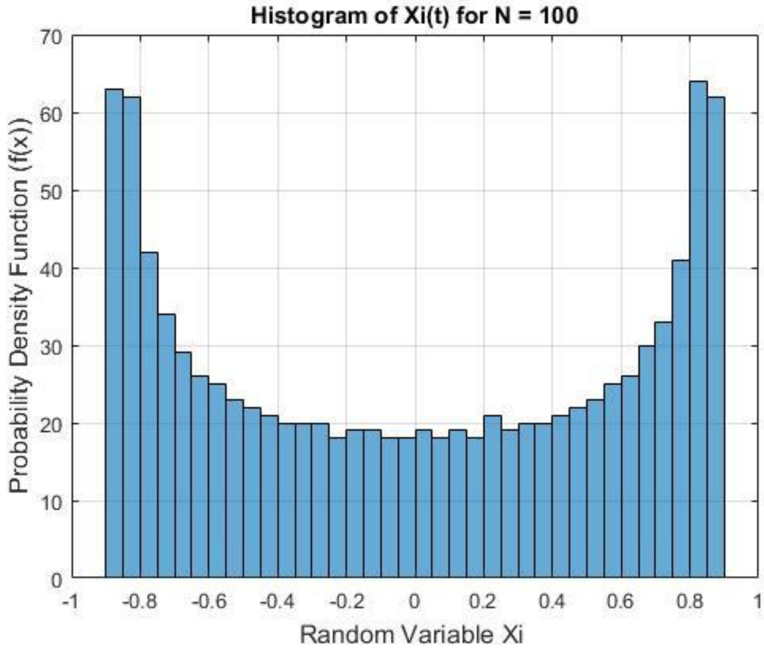

Fig. 1: Histogram of one reflected waveform for N = 100 samples.

6. ***Simulate Received Waveform X(t)***:

$$X(t) = \sum_{i=1}^{N} X_i(t)$$

At the receiver, we have a set of received waveforms. Fig. 3 shows the histogram of this set when samples are taken across time. We fix the time and plot the different realizations of the received signals. As expected, the received waveform is normally distributed by Central Limit Theorem.

X(t) is a sequence of random variables, one for each instant in time. X(t) is said to be a stochastic gaussian process if the joint distribution of the sequence of random variables is gaussian. This implies that each random variable has a gaussian distribution. In the histogram we show that the distribution of X(t) at t = 1 is gaussian.

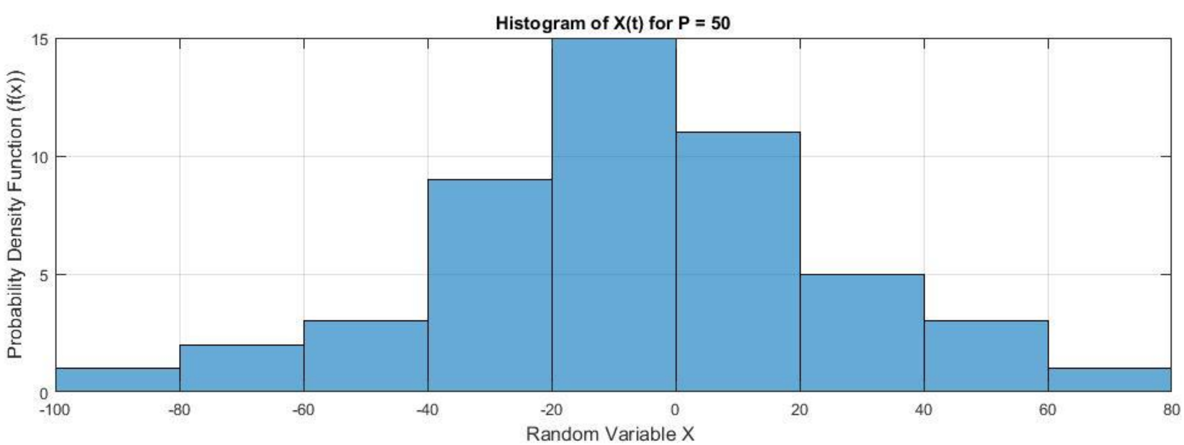

Fig. 2. Histogram for X(t) for a fixed time instant t = 1.

7. *Simulating U_i, V_i and A_i*

A_i can be simulated using the following relation: $Ai = \sqrt{(Ui^2 + Vi^2)}$

Where U_i *and* V_i are given to be normally distributed with a mean value zero and variance 1. From Fig. 5, we can conclude that modeling of Ai as Rayleigh distributed is consistent with the expected result.
The Histograms of these random variates are given in Fig. 3-5

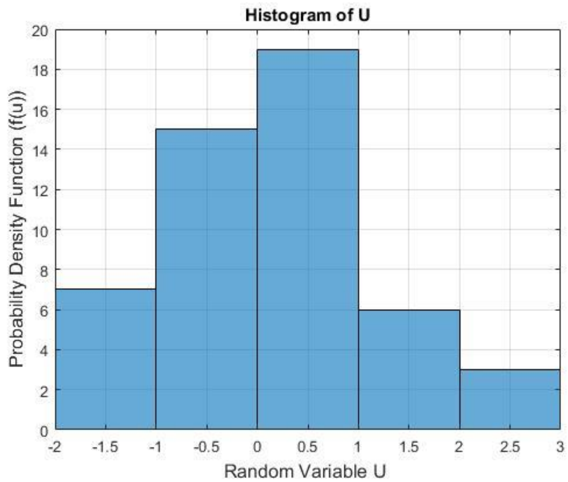

Fig. 3. Histogram for Normally Distributed U

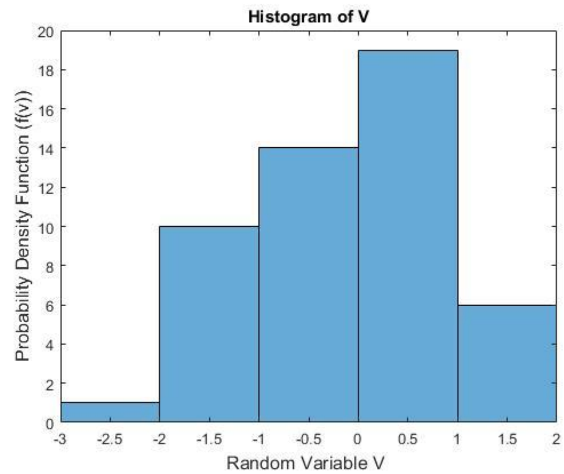

Fig. 4. Histogram for Normally Distributed V

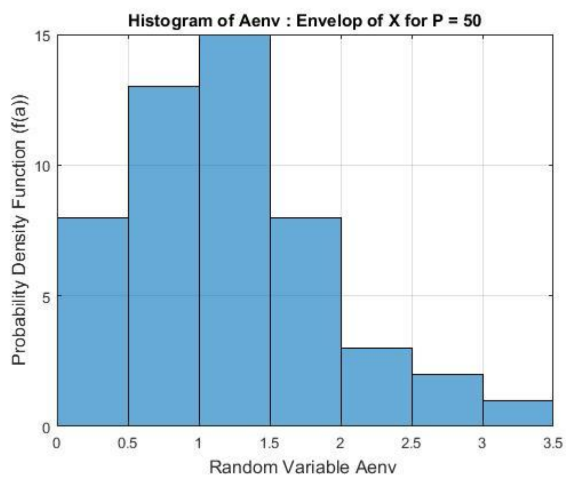

Fig. 5. Histogram for the Envelope of X(t): Rayleigh Distributed