ECE 513 - DIGITAL SIGNAL PROCESSING

Cranos M. Williams

Assigned: November 15, 2017

HOMEWORK 10, DUE DECEMBER 1, 2017

1. *Biorthogonal Filter Banks (Linear Phase): (20 Pts)* The following *half-band zero phase* filter is used to create the analysis and synthesis filters for a 2 channel QMF

$$P(z) = (1 + z^{-1})^m (1 + z)^m R(z) \tag{1}$$

where for $m = 4$

$$R(z) = az^3 + bz^2 + cz + d + cz^{-1} + bz^{-2} + az^{-3} \tag{2}$$

Using what we know about $P(z)$, calculate the appropriate values of a, b, and c. Appropriately factor $P(z)$ into the following form

$$P(z) = z^7 H_0(z) G_0(z) \tag{3}$$

where $H_0(z)$ and $G_0(z)$ are both 8-tap symmetric linear phase lowpass filters. Derive the highpass filters using the alias-free condition $H_1(z) = G_0(-z)$ and $G_1(z) = -H_0(-z)$. Using MATLAB, plot the frequency responses of the analysis and synthesis filter pairs. Prove whether or not these filters are perfect reconstruction filters by ensuring $A(z) = 0$ and $T(z)$ is constant magnitude with arbitrary delay.

2. *Orthogonal Filter Banks (Nonlinear Phase): (20 Pts)* Using the polynomial found in Problem 1 and the equation for $P(z)$ in (3), factor $P(z)$ such that $H_0(z)$ is a minimum phase filter and $G_0(z)$ is a maximum phase filter. In this case, $H_0(z)$ is composed of all the zeros inside the unit circle and half the zeros on the unit circle. Adjust the coefficients to ensure a gain of $\sqrt{2}$. Calculate the other lowpass filter, $G_0(z)$, using the following relationship.

$$G_0(z) = z^{-N} H_0(z^{-1}) \tag{4}$$

where N is the order of $H_0(z)$. Derive the highpass filters using the alias-free condition $H_1(z) = G_0(-z)$ and $G_1(z) = -H_0(-z)$. Using MATLAB, plot the frequency responses of the analysis and synthesis filter pairs. Prove whether or not these filters are perfect reconstruction filters by ensuring $A(z) = 0$ and $T(z)$ is constant magnitude with arbitrary delay.

3. *Analysis of Nonuniform Filter Banks: (20 Pts)* In this problem, you will analyze and interpret the frequency response of the Nonuniform Analysis Filter Bank shown in Figure 1.

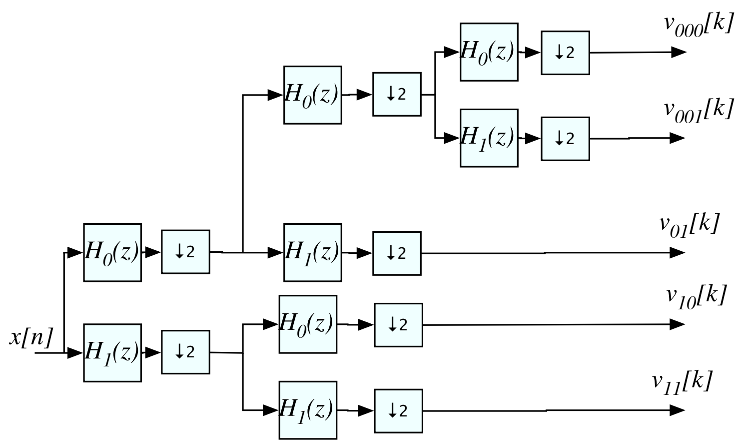

Figure 1: Nonuniform Subband Decomposition

   (a) Use the MATLAB routine `fir1` to design a 41 tap lowpass filter $H_0(z)$ and a 41 tap highpass filter $H_1(z)$. Let $\omega_c = \frac{\pi}{4}$ for the lowpass and $\omega_c = \frac{\pi}{4}$ for the highpass filter. Also, compute and plot their spectrum using MATLAB.
   (b) Using the tree structure in Figure 1, draw the equivalent 5 band analysis filter bank composed of $H_i'(z)$, $i = 0, 1, \ldots, 4$. Write the definitions of $H_i'(z)$ in terms of $H_0(z)$ and $H_1(z)$.
   (c) Assuming ideal magnitude responses of $H_0(z)$ and $H_1(z)$, draw what you would expect $H_i'(z)$, $i = 0, 1, \ldots, 4$ to look like based on your definitions in part b).
   (d) Use MATLAB to calculate $H_i'(z)$, $i = 0, 1, \ldots, 4$ based on your definitions in part b). Plot the magnitude response of these filters on the same plot. Outside of the obvious overlap, how do the sketched magnitude responses in part c) compare to the results given by MATLAB.

4. *Denoising using Subband Decomposition: (40 Pts)*

   (a) Write two functions to implement the analysis and synthesis parts of a QMF shown in Figures 2a and 2b given some arbitrary filters $H_i(z)$ and $G_i(z)$, $i = 0, 1$. Use the MATLAB routine `conv` to implement the filters.

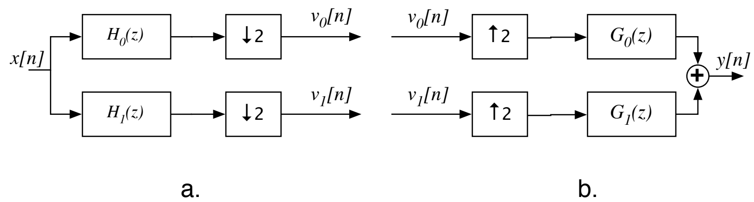

Figure 2: QMF Primitives for Analysis and Synthesis

(b) i. Using the implementation of the analysis part of the QMF written in part a) as a primitive, write a routine to implement the subband decomposition shown in Figure 1.

ii. Using the implementation of the synthesis part of the QMF written in part a), write a routine to implement the subband reconstruction shown in Figure 3.

iii. Using the routine `sampdata` to produce the input $x[n]$, and the orthogonal analysis and synthesis filter banks from Problem 2, run your routine to obtain the output $y[n]$. Verify that the output $y[n]$ is just a delayed version of the input $x[n]$.

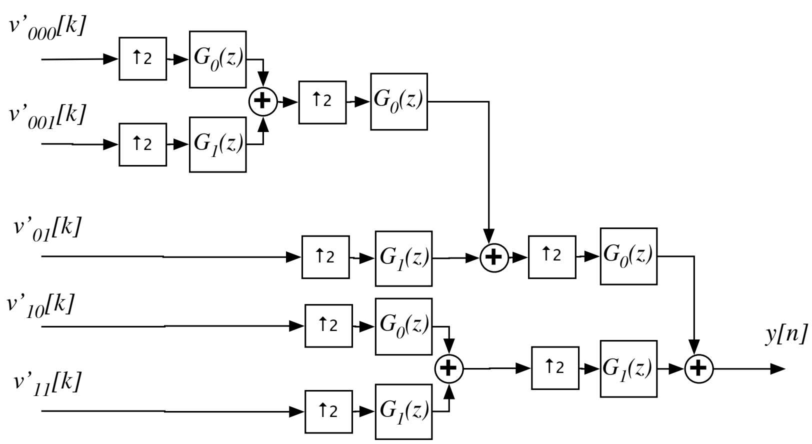

Figure 3: Subband Reconstruction

(c) Denoising can be accomplished by performing thresholding on the resulting subband time signals. Write a routine to perform hard thresholding the resulting subband signals v_i, $i \in \{000, 001, 01, 10, 11\}$. For hard thresholding, if the absolute value of the subband signal at time k for all k is less than the threshold for that level, set it to zero. In other words, set $v_i[k] = 0$ if $|v_i[k]| < \beta_i$, $i \in \{001, 01, 10, 11\}$

for all k, where $v_i[k]$ represents the k^{th} sample of the sequence from the i^{th} decomposition level and β_i is the threshold for the i^{th} subband signal. Notice we do not threshold the lowpass components. Use the following threshold values:

$$\beta_{001} = 0.390$$
$$\beta_{01} = 0.390$$
$$\beta_{10} = 0.390$$
$$\beta_{11} = 0.353$$

(d) Download the file "hwk10_signals.mat" from the course locker. Use the following command to load the data file into your workspace.

```
load hwk10_signals.mat xn_prb2
```

This will load the variable xn_prb2 into your workspace. You can listen to the sequence using the MATLAB command `soundsc`. Plot the signal. Create a new sound signal that contains Additive White Gaussian noise using the following command

```
xn_prb2_noisy = xn_prb2 + 0.1*randn(size(xn_prb2));
```

Plot the noisy signal. Again use the MATLAB command `soundsc` to listen to the sequence, noting that differences between the clean and the noisy signal.

(e) Using the noisy sound signal as the input, implement the subband decomposition, the denoising using hard thresholding, and the subband reconstruction as shown in the Figure below.

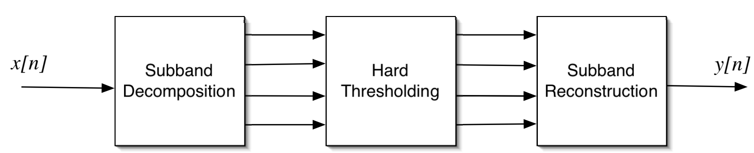

Figure 4: Block Diagram of Denoising using Subband Decomposition

(f) Plot the resulting denoised signal. Using the `soundsc` command, compare the original signal, the noisy signal, and the denoised signal. Does the denoised signal sound better than the noisy signal? How good are the improvements? How does the denoised signal sound compared to the original signal?

(g) Repeat part (e) and (f), assuming

i.

$$\begin{aligned}\beta_{001} &= 0.200\\ \beta_{01} &= 0.130\\ \beta_{10} &= 0.120\\ \beta_{11} &= 0.120\end{aligned}$$

ii.

$$\begin{aligned}\beta_{001} &= 1.900\\ \beta_{01} &= 0.960\\ \beta_{10} &= 0.950\\ \beta_{11} &= 0.930\end{aligned}$$

Listen to the denoised signals with the new thresholding parameters (β) as defined in part (g)-i and (g)-ii and plot them in MATLAB. Explain what will happen by increasing and decreasing β's.