Week 9 HW

Problem 1.9. You have 3 shirts & 4 pants
How many outfits can you make ?

$\cancel{3+4=7}$ $3 \times 4 = 12.$

Sh_1 P_1 P_2 P_3 P_4
Sh_2 P_1 P_2 P_3 P_4
Sh_3 P_1 P_2 P_3 P_4

Problem 1.11 How many ways can we form a licence plate if there are 7 char-s, none of which is the letter O, the first is a number from 0-9, the second letter, and the remaining 5 can be either digit or letter.

~~# char 1~~ A-Z (not inc. O)
\# 2 → 0-9 10 choices
\# 3 → A-Z (not inc. O) 25 choices
\# 4
\# 5 A-Z (not inc. O) ~~or~~ 0-9. $10+25=35$ ch.
\# 6
\# 7

$\cancel{7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$10 \times 25 \times 35 \times 35 \times 35 \times 35 \times 35 = 10 \times 25 \times 35^5 =$
$= 13,130,468,750.$

Here each character doesn't depend on other characters.

Problem 1.16 ; In the lottery, 25 balls numbered 1 through 25 are placed in a bin. Four balls are drawn one at a time & their numbers are recorded.
The winning combination consists of 4 balls with the numbers in order they are selected.

How many winning combinations are there, if:
a) Each ball is discarded after it's removed
b) each ball is replaced in the bin after it's removed & before the next draw?

1 2 3 4

a) $25 \times 24 \times 23 \times 22 = 303600$.

$\cancel{\frac{25!}{4!}} =$ b) $25 \times 25 \times 25 \times 25 = 25^4 = 390\,625$.

Problem 2.2 ; The Mublian alphabet has only 5 letters. & every word has $\leq$ 3 letters. How many words?

$$\cancel{r = 3 \quad n = 5 \quad P(n,r) = \frac{n!}{(n-r)!} = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 60.}$$

X

1) 1 letter words case 5

b) 2 letter words

letter 1 → 5 choices
letter 2 → 5 choices

$5 \times 5 = 25$

c) 3 letter words

letter 1 → 5
letter 2 → 5 = 125.
letter 3 → 5

Answer - $5 + 25 + 125 = 155$ ✓

Problem 2.7: The Smith family has 4 sons & 3 daughters. In how many ways they can be seated on a chair such that, at least 2 boys are next to each other?

GGG B B B B
GGG B B B B
GBGGB B B
GBB GGBB

GBB B GG B
GBB B B GG

6 variations.

$6 \times 4! \times 2! = 6 \times 4 \cdot 3 \cdot 2 \times 2 = 288.$

The only way 2 boys can seat together is BGBGBGB where there are 4! orderings for boys & 3! orderings for girls $4! \times 3! = 144$.

$7! =$

Total $= 7\cancel{!} \times 6\cancel{!} \times 5\cancel{!} \times 4\cancel{!} \times 3\cancel{!} \times 2\cancel{!} = 5040$

$5040 - 144 = 4896$

Problem 2.9: How many sequences $x_1, x_2, x_3, x_4, x_5, x_6, x_7$ can be formed, in which all the $x_i > 0$ and < 6 and no two adjacent x_i are equal?

0 1 2 . 3 4 5 ~~6~~

We have 5 choices for x_1 & have 4 choices for the remaining 6 nums.

$5 \times 4^6 = 20,480$

Problem 2.14 : Math club has 20 members & 3 officers (Pres, Vice Pr. & Treasurer).
How many ways we can fill the offices if Ali refuses to serve as an officer if Brenda is also an officer?

$20 \times 19 \times 18 = 6840$ → if we ignore Ali & Brenda

Constructive counting.

3 choices for Ali
2 choices for Brenda
18 choices for the 3rd Treasurer.

$3 \times 2 \times 18 = 108$

Now subtract: $6840 - 108 = 6732$

3.4 How many distinct permutations arrangements are in PAPA

We pretend all letters are diff-t - $P_1 A_1 P_2 A_2$

For the 2Ps each possibility is counted $2!$

2 As $= 2!$

$2! \times 2! = 4$ ways

Therefore, there are $4!$ ways to arrange 4 letters (if all were unique).

Number of distinct arrangements $= \frac{4!}{2! \times 2!} = 6$

repeated Ps, repeated As

6 - distinct arrangements

We Multiply their factorials to remove all overcounts. since these repetitions are independent.

Combinatorics key concepts

Situation	What to ask yourself	What to do	Formula/Example.
Choosing between options (either/or)	Do I pick one of multiple options?	ADD	Soup or salad: $3+4=7$
Doing one thing and another	Do I pick 1 and then another?	Multiply	3 soups $\times$ 4 salads = 12 combos.
Arranging all items (no repeats)	All items are unique?	Use $n!$	ABCD $\rightarrow 4! = 24$
Arranging items with some repeated	Are some letters & items repeated?	Divide by factorials of repeats.	PAPA $\rightarrow \frac{4!}{2! \times 2!} = 6$
Choosing r items from n (order doesn't matter)	Is order irrelevant?	Use combinations	$\binom{n}{r} = \frac{n!}{r!(n-r)!}$
Choosing r items from n (order matters)	Is order important?	Use permutations	$\frac{n!}{(n-r)!}$
Repeated choices allowed	Can I choose the same item more than once	Use power rule	Password of 3 letters from A-Z $\rightarrow 26^3 = 17\,576$

Problem ~~4~~.3 : In the lottery 48 balls are numbered from 1 to 48, and 6 are chosen. How many diff. sets of winning numbers are there? (The order of the numbers doesn't matter).

$n = 48 \quad r = 6$

$$\binom{48}{6} = \frac{48!}{6!\,\underbrace{(n-r)!}_{42}} = \frac{48 \cdot 47 \cdot 46 \cdot 45 \cdot 44 \cdot 43 \cdot \cancel{42!}}{6!\ \cancel{42!}} =$$

$$= \frac{48 \cdot 47 \cdot 46 \cdot 45 \cdot 44 \cdot 43}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \underline{12\,271\,512}$$

Problem 5,4 12 players in the team → 2 of them Bob & Yogi refuse to play together
How many starting lineups of 5 players can we make if it can't contain both Bob & Yogi.

$$\cancel{P(n,r) = P(12,5) = \frac{12!}{7!}} =$$

3 Cases to solve this:

1) Bob starts & Yogi doesn't

Bob + $\binom{10}{4}$ remaining 11 players − Yogi

Case 2) Yogi starts & Bob doesn't $\binom{10}{4}$

Case 3) They both don't start $\binom{10}{5}$

So total nr of lineups: $\frac{10}{4} + \frac{10}{4} + \frac{10}{5} = 210 + 210 + 252 = 672$

We can solve also by complimentary counting:

If no restrictions = ~~in~~ $\binom{12}{5}$

then we subtract the lineups that are not allowed + $\binom{10}{3}$ − the coach must chose the remaining 3 players from 10.

~~672~~ − $\binom{12}{5} - \binom{10}{3} = 792 - 120 = 672$