Week 6 HW

① How many elements does each of these sets have where a and b are distinct elements?

number of elements → To power set has elem-s ↓

a) $P(\{a, b, \{a, b\}\})$ // 3 ⇒ $2^3 = 8$

b) $P(\{\emptyset, a, \{a\}, \{\{a\}\}\})$ // ~~2~~4 ⇒ $2^4 = 16$

c) $P(P(\emptyset))$ // 0 ~~1~~ ⇒ $2^0 = 1$

② How many diff-t elements does A^n have when A has m elements & n is a positive integer?

$|A| = m$

$$|A^n| = \underbrace{m \cdot m \cdot m}_{n\ \text{times}} \ldots = m^n$$

$A^2 = A^2 \times A^2$

$A^2 = 2 \cdot 2 = 4$

$A^3 = 2 \cdot 2 \cdot 2 = 8$

③ Find the $\overbrace{\text{truth set}}^{\text{set of all } x}$ of each of these predicates where the domain is the set of integers.

a) $P(x): x^2 < 3$ $\{x \in \mathbb{Z} \mid x^2 < 3\} = \{-1, 0, 1\}$

b) $Q(x): x^2 > x$ $\{x \in \mathbb{Z} \mid \cancel{x^2 > x}\} = \{-\infty ; \infty\}$

Truth set: ⇒ $\{x \in \mathbb{Z} \mid x^2 > x\} = \mathbb{Z} - \{0, 1\} = \{\ldots -2, -1, 2, 3, 4, \ldots\}$

all negative int-s & all nonnegative int-s other than 0 and 1.

c) $R(x): 2x + 1 = 0$ $x = -0{,}5$ ⇒ this is not an integer

$\{x \in \mathbb{Z} \mid 2x + 1\} =$ ~~$\{\infty [-0{,}5]$~~ ...

Since no integer makes it true, the truth set is the empty set. $\emptyset$ (the empty set).

④ Describe a procedure for listing all the subsets of a finite set.

Finite set - a set with a countable number of elements. (3, 5 ...

Subsets - any group of elements you can pick from the set incl.

- ✓ The empty set $\emptyset$
- ✓ subsets with 1, 2, 3 elements ...
- ✓ The full set itself.

If a set has ~~2~~ n elements then it has 2^n subsets.

2 elements $2^2 = 4$

3 elements $2^3 = 8$

For each element, think IN or OUT. List all possibilities.

Each subset corresponds to a binary number from 0 to $2^n - 1$

1 - element included

0 - element not included.

Ex: $S = \{a, b, c\}$

$n = 3$

List all k from 0 to 7 (since $2^3 = 8$);

k	Binary of k	Subset
0	000	$\emptyset$
1	001	$\{c\}$
2	010	$\{b\}$
3	011	$\{b, c\}$
4	100	$\{a\}$
5	101	$\{a, c\}$
6	110	$\{a, b\}$
7	111	$\{a, b, c\}$

⑤ Show that if A, B, and C are sets, then $\overline{A\cap B\cap C} = \overline{A}\cup\overline{B}\cup\overline{C}$.

a) by showing each side is a subset of the other side
b) using a membership table.

a) $\overline{A\cap B\cap C} = \overline{A}\cup\overline{B}\cup\overline{C}$ → not all three sets at the same time → not in A or not in B or not in C.

$\overline{A}$ means the complement of A (element not in A)
$\cap$ - intersection (common elements)
$\cup$ - union (all elements in any of the sets)

1) We must show that every element on the left side is also on the right side,
2) Every element on the right side is also on the left side
→ double-inclusion proof.

If we pick a random element X from the ~~[illegible]~~: $\overline{A\cap B\cap C}$
It means X is not in ~~[illegible]~~ inside these 3 sets together.

That is at least ~~of~~ one of these must be true
$X \notin A$ X is not in A
$X \notin B$ x is not in B
$X \notin C$ => X is not in C
Thus: x belongs to $\overline{A}$ or $\overline{B}$ or $\overline{C}$
↳ $X \in \overline{A}\cup\overline{B}\cup\overline{C}$
So everything on the left side is also on the right side.

If we pick a random e. X from $\overline{A}\cup\overline{B}\cup\overline{C}$
$X \in \overline{A}$ or
$X \in \overline{B}$ or
$X \in \overline{C}$ => X is not in $\overbrace{A \text{ or } B}^{\text{not in}} \overbrace{\text{or } C}^{\text{not in.}}$.

So, X is not in all three sets at once
$X \notin A\cap B\cap C$
thus $X \in \overline{A\cap B\cap C}$

=> A way to remember is :

NOT (A AND B AND C) is the same as
(NOT A) OR (NOT B) OR (NOT C)

b) Membership ~~Truth~~ table

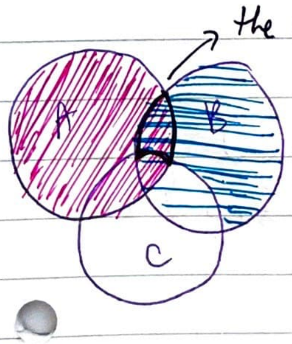
x in A	x in B	x in C	x in $A \cap B \cap C$	~~x in $\{\overline{A \cap B \cap C}\}$~~			~~$\neg A \cup \neg B \cup \neg C$~~	
				x in $\overline{A \cap B \cap C}$	x in $\bar{A}$	x in $\bar{C}$	x in $\bar{B}$	x in $\bar{A} \cup \bar{B} \cup \bar{C}$
T	T	T	T	F	F	F	F	F
T	T	F	F	T	F	T	F	T
T	F	T	F	T	F	F	T	T
T	F	F	F	T	F	T	T	T
F	T	T	F	T	T	F	F	T
F	T	F	F	T	T	T	F	T
F	F	T	F	T	T	F	T	T
F	F	F	F	T	T	T	T	T

Problem 6 : Draw the Venn diagrams for each of these combinations of the sets A, B and C.

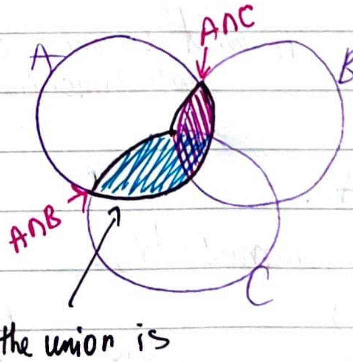
a) $A \cap (B - C)$

A intersects B - C.

=> find el-s that are in A, in B but not in C.

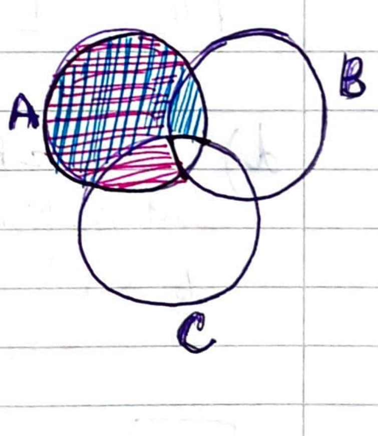


b) $(A \cap B) \cup (A \cap C)$



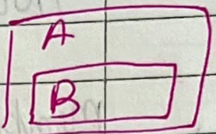
the union is

c) $(A \cap \bar{B}) \cup (A \cap \bar{C})$



Problem 7: What can you say about sets A and B if we know that

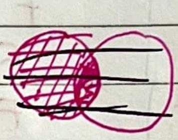
a) $A \cup B = A$. ⇒ $B \subseteq A$



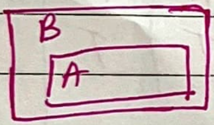
↳ everything that's in A or B (or Both).

$A \cup B = A$ means no extra elements were added by doing the union.
So, everything in B must already be in A.
So, B is a subset of A
$B \subseteq A$

b) $A \cap B = A$ ⇒ $A \subseteq B$



↳ this means the elements that are in both A & B. (common elements whet A and B share).

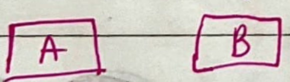


A is inside B.
So the common part is exactly A.

A & B have no elements in common.

c) $A - B = A$. ⇒ $A \cap B = \emptyset$

this means everything in A, but removing anything that is also in B.



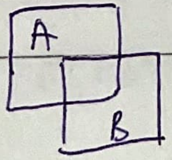
A and B are completely disjoint — not touching.

d) $A \cap B = B \cap A$.

Intersection is commutative

A = {1, 2, 3}
B = {2, 3, 4}

then $A \cap B = \{2, 3\}$
$B \cap A = \{2, 3\}$



e) $A - B = B - A \Rightarrow A = B$

the A, B sets must be identical.

$A = \{1, 2, 3\}$ $A - B = \{\}$

$B = \{1, 2, 3\}$ $B - A = \{\}$

$A - B = B - A$ – is true.

⑧ Suppose that A, B and C are sets such that $A \oplus C = B \oplus C$. Must it be the case that $A = B$?

The symmetric difference between two sets A and B – written as $A \oplus B$ - means : The elements that are in exactly one of A or B, but not both.

~~Ex~~: $A = \{1, 2, 3\}$

$C = \{3, 4\}$. // exclude 3 as it's in both A & C

$A \oplus C = \{1, 2, 4\}$.

$B = \{1, 2, 3\}$ // Same as A

$B \oplus C = \{1, 2, 4\}$.

If $A \oplus C = B \oplus C$ and you undo the XOR with C, you get back the original set.

$(A \oplus C) \oplus C = A$.

$A = B$.