HW Week 3

$\frac{a^m}{a^n} = a^{m-n}$

1) Simplify the expression:

$$\log_2\left(\frac{8\sqrt{2}}{16}\right) + \log_2(32) - 2\log_2(4)$$

a) $\log_2\left(\frac{\sqrt{2}}{2}\right) = \frac{2^{\frac{1}{2}}}{2^1} = 2^{\frac{1}{2}-1} = 2^{-\frac{1}{2}}$

$\log_2\left(2^{-\frac{1}{2}}\right) = -\frac{1}{2}$ $\log_a a^m = m$

b) $\log_2(32) = 5$ $2\cdot2\cdot2\cdot2\cdot2$

c) $2\log_2 4 = 2\cdot 2 = 4.$

Plug in: $-\frac{1}{2} + 5 - 4 = -\frac{1}{2} + 1 = \frac{1}{2}$

(2) Solve for x

$$\log_3(x-1) + \log_3(x+1) = 2$$

$\log_3\left((x-1)\cdot(x+1)\right) = 2.$

$(x-1)(x+1) = x^2 - \cancel{x} + \cancel{x} - 1 = x^2 - 1$

$x^2 - 1 = 3^2$

$x^2 - 1 = 9$

$x^2 = 10$ $x = \pm\sqrt{10}$

check $\sqrt{10}$

$x - 1 > 0 \Rightarrow \sqrt{10} - 1 > 0$ True

$x + 1 > 0 \Rightarrow \sqrt{10} + 1 > 0$ True

$-\sqrt{10}$

$x - 1 > 0$ $-\sqrt{10} - 1 \not> 0$ false.

$x + 1 > 0$ $-\sqrt{10} + 1 \not> 0$ false.

$x = \sqrt{10}$ - is the answer.

3) Compound interest exercises.

Init. investment \$10000 P

Annual interest 6% r $0{,}06$ compounded quaterly $n=4$.

How $\underbrace{\text{many years}}_{t}$ for the investment to grow to at least \$20000 A?

$$f(t_{1th}) = 10000 \times (1+0{,}06)^t \; \cancel{\geq 20.000} = 10.600.$$

$$f(t_{min}) \geq 20000$$

$\frac{6}{100} \cdot \frac{1}{4} = \frac{6}{400} = \frac{3}{200}$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$20000 = 10000\left(1+\frac{\frac{6}{100}}{4}\right)^{4t} \Rightarrow 20000 = 10000(1+0{,}015)^{4t} \qquad /10000$$

$$2 = 1{,}015^{4t}$$

$$2 - \cancel{\log} \quad \ln(2) = \ln\left((1{,}015)^{4t}\right)$$

$$\ln(2) = 4t\ln(1{,}015)$$

$$t = \frac{\ln(2)}{4\ln(1{,}015)} = \frac{0{,}6931}{4 \times 0{,}014889} = 11{,}64 \text{ years.}$$

4) Radiactive Decay Exercises

$$N(t) = N_0 e^{-kt}$$

N_0 - initial amount

k - decay constant

t - time in years

half-life is 5years

decay constant - k -?

$$N_{t\frac{1}{2}} = \frac{N_0}{2}$$

$$\frac{N_0}{2} = N_0 e^{-kt\frac{1}{2}}$$

$$\frac{1}{2} = e^{-k \times 5}$$

$$\ln\frac{1}{2} = -5k \qquad -0{,}6931 = -5k$$

$$k = \frac{0{,}6931}{5} \approx 0{,}1386 \text{ years.}$$

6) Find the vector in the direction from point A (1,2,3) to point B (4,6,9)

$\vec{AB} = \langle \overset{x_2-x_1}{4-1}, \overset{y_2-y_1}{6-2}, \overset{z_2-z_1}{9-3} \rangle \Rightarrow \underline{\vec{AB} = \langle 3, 4, 6 \rangle}$ ✓

1. Magnitude

$|\vec{AB}| = \sqrt{\{\overset{3^2}{9} + \overset{4^2}{16} + \overset{6^2}{36}\}} = \sqrt{61} = 7,81$

2. Unit vector : $u^\wedge = \frac{1}{\sqrt{61}} \langle 3, 4, 6 \rangle = \left\langle \frac{3}{\sqrt{61}}, \frac{4}{\sqrt{61}}, \frac{6}{\sqrt{61}} \right\rangle$

7) $\vec{v} = 7i - 2j + 4k$. Express the vector in its matrix form & find it's magnitude.

$\vec{v}\ |7 \ \ 2 \ \ 4|$

$|\vec{v}| = \sqrt{49+4+16} = \sqrt{69} \approx 8,307$ ✗

$\vec{v} \begin{vmatrix} 7 \\ -2 \\ 4 \end{vmatrix}$

8) Given vectors $\vec{a} = (2, -1, 3)$ and $\vec{b} = (-1, 4, 2)$ compute $3\vec{a} - 2\vec{b}$;

$3(2, -1, 3)$

$3\vec{a} = \langle 6, -3, 9 \rangle$

$2(-1, 4, 2)$

$2\vec{b}\ \langle -2, 8, 4 \rangle$

$3\vec{a} - 2\vec{b}\ \cancel{\vec{ab}} = (\overset{8}{\cancel{4}}, -11, 5)$ Answer

9) Find the angle between vectors
$\vec{p} = \langle 1, 2, 3 \rangle$ and $\vec{q} = \langle 4, -5, 6 \rangle$

$pq = 4 - 10 + 18 = 12$

$|p| = \sqrt{1+4+9} = \sqrt{14} \approx 3,74$ $\quad |q| = \sqrt{16+25+36} = 8,77.$ $\quad \sqrt{77}$

$\cos(\theta) = \frac{pq}{|p||q|} = \frac{4-10+18}{3.74 \cdot 8,77} = \frac{12}{32,80} = 0,365$

$\theta = \cos^{-1}(0,365) \approx 68,56^\circ.$

10) $\vec{u} = \langle 2, -1, 4 \rangle \quad \vec{v} = \langle -8, 4, -16 \rangle$
if the vectors are orthogonal.

$u \cdot v = (2 \cdot (-8)) + (-1)(4) + (4) \cdot (-16) = -16 - 4 - 64 = -84.$
$u \cdot v \neq 0.$ not orthogonal.

11) $A = \begin{vmatrix} 2 & -1 \\ 0 & 3 \end{vmatrix} \quad B = \begin{vmatrix} 4 & 5 \\ -2 & 1 \end{vmatrix}$

$2A - 3B = ?$

$\begin{vmatrix} 4 & -2 \\ 0 & 6 \end{vmatrix} - \begin{vmatrix} 12 & 15 \\ 6 & 3 \end{vmatrix} = \begin{vmatrix} -8 & -\overset{17}{\cancel{13}} \\ -6 & 3 \end{vmatrix}$

Problem 12:

$C = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \quad D = \begin{vmatrix} 5 & 6 \\ 7 & 8 \end{vmatrix} \quad E = CD$

$E = \begin{vmatrix} 5 & 12 \\ 21 & 32 \end{vmatrix} \begin{vmatrix} 1\cdot5+2\cdot7 & 6\cdot1+8\cdot2 \\ 3\cdot5+4\cdot7 & 6\cdot3+8\cdot4 \end{vmatrix} = \begin{vmatrix} 19 & 22 \\ 43 & 50 \end{vmatrix}$

15, 28, 43, 18, 32, 50

Problem 13: Use Gaussian elimination to solve the system

$$\begin{cases} x+y+z=6 \\ 2x-y+3z=14 \\ -3x+2y-2z=-10 \end{cases}$$

$\begin{vmatrix} 1 & 1 & 1 & 6 \\ 2 & -1 & 3 & 14 \\ -3 & 2 & -2 & -10 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 & 6 \\ 3 & 1 & 1 & 14 \\ -5 & 1 & -1 & -10 \end{vmatrix}$

$R_2 \to$

$R_3 + 2R_1$

$-3 \; -2\cdot1$

$-2 \; -2$

$\begin{vmatrix} 1 & +1 & 1 & 6 \\ 2 & -1 & 3 & 14 \\ -3 & 2 & -2 & -10 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 6 \\ 2 & -1 & 3 & 14 \\ 1 & 1 & -1 & -10 \end{vmatrix}$

$R_2 - R_2 + 1R_1$

$-1 - 2$

$R_3 = R_3 + 2R_1$

~~$x+y+z=6$~~

~~$2x-y+3z=16$~~

~~$-2x+y-z=-10$~~

$-1-2\cdot1=$

$3-2$

$-3+3$

$2+3$

$-1+3=$

$\begin{vmatrix} 1 & 1 & 1 & 6 \\ 2 & -1 & 3 & 14 \\ -3 & 2 & -2 & -10 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 14 \\ 0 & 5 & 1 & -10 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \\ 5 & 0 & 3 \end{vmatrix}$

$R_2 = R_2 - 2R_1$

$R_3 = 3R_1$

$R_3 = R_1 - 5R_1$

$5 - 5$

$2 - 5$

$R_2 = \left(R_2 * \frac{5}{-3}\right) + R_3$

$\Rightarrow \begin{vmatrix} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & \frac{14}{3} \\ 0 & 0 & -\frac{1}{3} & 10 \end{vmatrix}$

$$\left|\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & \frac{12}{3} \\ 0 & 0 & -\frac{24}{3} & -8 \end{array}\right|$$

$$-\frac{24}{3}z = -8$$

$$z = \frac{8}{1}\left(-\frac{\cancel{24}}{2}\right) = -\frac{24}{2} = \cancel{-18}, -7$$

$$-3y + 7 = \frac{12}{3}$$

$$-3y = 2 + 7 = 9$$

$$y = -3.$$

$$x + (-3) + (-7) = 6$$

$$x - 10 = 6$$

$$x = 16$$

Problem 14: RREF

$\cancel{-1} \to 2(-3)$

$$B = \left|\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & -1 \end{array}\right| \xrightarrow[R_1 = R_1 + R_3 \times 1]{\cancel{R_1 = R_1 + 3R_2}} \left|\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 2 & 6. \\ 0 & 0 & 1 & -1 \end{array}\right| =$$

$$\xrightarrow{R_2 = R_2 - 3R_3} \left|\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & -1 \end{array}\right| \xrightarrow{R_1 = R_1 - 2R_2} \left|\begin{array}{ccc|c} 1 & 0 & 0 & -17 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & -1 \end{array}\right|$$