HW week 2 DM

1) $S = 5+9+13+\dots+89$

$\sum_{k=0}^{99} = a_2 + (a_1+d)$

$a_1 = 5$
$d = 4$

$a_n = a_1 + (n-1)d$
$89 = 5 + (n-1)\cdot 4$
$89 = 5 + 4n - 4$
$4n = 89 - 5 + 4$
$n = \frac{88}{4} = 22$

$\sum_{k=1}^{22} [5 + (k-1)\times 4]$

2) $\sum_{k=3}^{15} (2k+1)$

$k = 1$
~~S~~

$\sum_{k=1}^{13} (2k+1) - 2$

let $j = k-2$

$\sum_{j=1}^{13} [2(j+2)+1] = \sum_{j=1}^{13} (2j+4+1) = \sum_{j=1}^{13} (2j+5)$

3) $a_1 = 12 \quad a_n = a_{n-1} + d$

if $a_{10} = 57 \quad d = ?$

$a_{25} = ?$

a_{10}: $57 = a_9 + d$.

$d = 57 - a_9$

$a_2 = a_1 + d$

$a_2 = 12 + d$

$a_n = a_1 + (n-1)d$
$57 = 12 + 9d$
$9d = 45$
$d = 5$

$a_{25} = a_1 + 24 \times 5$
$a_{25} = 12 + 120$
$a_{25} = 132$

4 Find the sum of multiples of 7 between 100 & 1000

$a_n = a_1 + (n-1)d$

$a_n = 7 + (n-1) \cdot 7$

$\sum_{n=7}^{100} = 7 + 42 = 49.$

First multiple $a_1 = 7 \times 15 = 105$
Last multiple $a_n = 7 \times 142 = 994$

$n = \frac{a_n - a_1}{d} + 1 = \frac{994 - 105}{7} + 1 = 128.$

$S = \frac{n}{2}(a_1 + a_n) = \frac{128}{2}(105 + 994) = 70336.$

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5 $S = \sum_{k=1}^{n} (3k+2)$

$n = ?$
$S = 2650$

$2650 =$

$a_n = a_1 (n-1) d$

$2850 = \cancel{7},5($

$n = \frac{a_n - a_1}{d} + 1 =$

$n = 5.$

1) $S_n = \frac{n}{2}[a_1 + a_n]$
→ arithmetic series. – to find the sum of the first n terms

2) If you don't know a_n:

$S_n = \frac{n}{2}(2a_1 + (n-1)d)$

$\cancel{2650 = \frac{n}{2}(2 \cdot 1 + (n-1) \times d)} =$

GPT: $a_1 = 3 \times 1 + 2 = \underline{5}$
$a_2 = 3 \times 2 + 2 = 8$
$d = 3$

$S_n = \frac{n}{2}(2 \cdot a_1 + (n-1)d) = \cancel{\frac{n}{2} \times 2 + \frac{n}{2}(n-1) \times 3} =$

$2650 = \frac{n}{2}(2 \times 5 + (n-1)(3)) = \frac{n}{2} \times (10 + 3n - 3) =$

$\cancel{5n + \frac{3}{2}}$ $\frac{n}{2}(7 + 3n)$

$2650 = \frac{n}{2}(3n + 7)$

$\times 2$

$5300 = n(3n + 7)$

$3n^2 + 7n - 5300 = 0$

$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ → use this formula.

Where $a = 3$
$b = 7$
$c = -5300$

1) $\sqrt{b^2 - 4ac} = \sqrt{7^2 - 4 \times 3 \times (-5300)} =$
$= \sqrt{49 + 63600} = \sqrt{63649} = 252$

2) $n = \frac{-7 \mp 252}{2 \times 3}$;

$n = \frac{-7 + 252}{6}$ or $n = \frac{-7 - 252}{6}$

this is negative - so not possible

$n = \frac{245}{6} = 40,83$ → n must be an int $= 41$.

Problem #6 mean formula.

$a_5 = 20$
$a_{15} = 60$
$a_{10} =$ show that the 10^{th} term is the arithmetic mean of the 5^{th} & 15^{th} terms.

$a_n = a_1 + (n - 1)d$

$a_5 = a_1 + (5 - 1)d$

$20 = a_1 + 4d$

$4d = 20 - a_1$

$d = \frac{20 - a_1}{4}$

$a_5 = a_1 + 4d$
$a_{15} = a_1 + 14d$
$a_{10} = a_1 + 9d$

~~60 = a_1~~ $a_{10} =$

$a_5 - a_{15} = 18d$
$40 = 18d$
$d = \frac{40}{18}$

$a_{10} = a_1 + 9d$
$= a_1 + 9 \times \frac{40}{18} = \boxed{\frac{40}{2} + a_1}$

$a_5 \Rightarrow a_1 + 4d$ 　　　 $20 = a_1 + 4d$

$a_{15} \Rightarrow a_1 + 14d$ 　　　 $60 = a_1 + 14d$

Subtract eq. 1 from 2.

$$60 - 20 = (a_1 + 14d) - (a_1 + 4d)$$

$$40 = 14d - 4d = 10d$$

$$d = 4$$

$$a_5 = a_1 + 4d$$

$$20 = a_1 + 4 \times 4$$

$$a_1 = 4$$

$$a_{10} = a_1 + 9d$$

$$a_{10} = 4 + 36 = 40$$

$$\cancel{a_{15} = a_1 + 14d}$$

$$\cancel{60 = a_1 + 14 \times 4}$$

$$\text{Mean} = \frac{20 + 60}{2} = \frac{80}{2} = 40.$$

Problem 7:

A staircase has 20 steps

$a_1 = 5$

$d = 0,5$

$n = 20$

$$S_n = \frac{n}{2}\left(2a_1 + (n-1)d\right)$$

$$S_n = 10\left(10 + 19 \times 0,5\right) =$$

$$= 10\left(10 + 9,5\right) = \underline{195,}$$

$a_2 = a_1 + 0,5$

$5,5 = 5$

Problem #8

$a_1 = 11$

$d = 3$

$S_n > 1000 \quad n = ?$

$S_n = \frac{n}{2}(2a_1 + (n-1)d)$

$\frac{n}{2}(2a_1 + (n-1)d) > 1000$

$\frac{n}{\cancel{2}}(\cancel{22} + (n-1)3) > 1000$

$\cancel{11n + 3n - 3 > 1000}$

$\cancel{14n > 1003}$

$\cancel{n > \frac{1003}{14}}$

$S_n = \frac{n}{2}[22 + 3n - 3]$

$S_n = \frac{n}{2}(3n + 19)$

$S_n > 1000$

$\frac{n(3n+19)}{2} > 1000$

$n(3n+19) > 2000$

$3n^2 + 19n - 2000 > 0$

$3n^2 + 19n - 2000 = 0$

$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$n = \frac{-19 \pm \sqrt{19^2 - 4 \times 3 \times (-2000)}}{2 \times 3} = \frac{-19 \pm \sqrt{361 + 24000}}{6} = \frac{-19 \pm 156{,}03}{6}$

$n = \frac{-19 + 156{,}03}{6} \approx 22{,}84$. $n = 23$. (integer.)

$n = 23$

$S_{23} = \frac{23(3 \times 23 + 19)}{2} = \frac{23(69+19)}{2} = \frac{23 \times 88}{2} = 1{,}012$

$S_{23} > 1000$ $\quad$ $n = 23$