HW Week 4

Ex 7: Acme Comp – $138 bln & net profit $8 bln
Nadir Softw – $87 bln & net profit $5 billion
Quixote – $114 billion & 13 bill

a) F Acme Comp's revenue is larger. ✓
b) T both propositions true ✓
c) ~~A~~ Quixote – $F \wedge T \Rightarrow$ ~~A~~ T.
d) ~~F~~ $p \to q$ – F. ✗
e) ~~A~~. $T \vee T \equiv T$. ✓

Ex 11: (p) – It's below freezing
(q) – It's snowing

a)

	p	q	p ∧ q (and)	
a)	T	T	T	$p \wedge q$ ✓
b)	T	F		$p \wedge \neg q$ ✓
c)	F	F		$\neg p \wedge \neg q$ ✓
d)				$p \vee q$ ✓
e)				$p \to q$ ✓

$(p \vee q) \wedge (p \to \neg q)$ ✗ $p \vee q$ $p \vee \neg q$
biconditional. ✗ $p \leftrightarrow q$

Ex 29) How many rows in the truth table?

a) $p \to \neg p$ 2. ✓ $2^1 = 2$

b) $(p \vee \neg r) \wedge (q \vee \neg s)$ 16. ✓ $2^4 = 16$

c) $q \vee p \vee \neg s \vee \neg r \vee \neg t \vee u$ 24 ✗ $2^5 = 64$

d) $(p \wedge r \wedge t) \leftrightarrow (q \wedge t)$ 16. ✓ $2^4 = 16$

Formula = 2^n
n = number of vars.

Aside quiet. Only the number of unique vars matter?

Ex: 31: Construct a truth table for each of these propositions.

$2^2 = 4$ rows

a) $p \wedge \neg p$

b) $p \vee \neg p$

c) $(p \vee \neg q) \rightarrow q$

d) $(p \vee q) \rightarrow (p \wedge q)$

e) $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

f) $(p \rightarrow q) \rightarrow (q \rightarrow p)$

p	q	¬p	¬q	p ∧ ¬p	p ∨ ¬p	(p ∨ ¬q) → q	p ∨ ¬q
T	T	F	F	F	T	T ✓	T
T	F	F	T	F	T	F	T
F	T	T	F	F	T	T	F
F	F	T	T	F	T ?	F	T

e → contrapositive always same value.

(p → q)	↔	(¬q → ¬p)
T ✓	≡	T
F ✓	≡	F
F T	≡	F T
T ✓	≡	F

only F if P is F & Q is F

T
T
T
T

f

(p → q)	→	(q → p)
T		T
F		T
T		F
T		T

T
F T
F F
T

d

(p ∨ q)	→	(p ∧ q)
T		T
F		F
F		F
F		F

T ✓
F ✓
F ✓
X F T

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

Ex. 7

a) $p \wedge q \rightarrow$ – Jan is not rich & is not happy ✓

b) Carlos will not bicycle or run tom. and not

c) Mei doesn't walk or take bus to class and doesn't

d) Ibrahim is not smart & not hard working ✓

Eg: Show that each st-t is tautology by using truth tables

a) $(p \wedge q) \rightarrow p$

$\neg(p \wedge q) \equiv (\neg p \vee \neg q) \vee p$

$\equiv (\neg p \vee p) \vee q \quad = (\neg p \vee p) \vee q$

T ∨

p	q	$p \wedge q$	$p \wedge q \rightarrow p$	
T	T	T	T	$T \rightarrow T = T$
T	F	F	T	$F \rightarrow T = T$
F	T	F	T	$F \rightarrow F = T$
F	F	F	T	$F \rightarrow F = T$

If both p & q are T, then P is T

the premise is false, so the implication is T.

Only if $p \wedge q$ part is T, we check if p is also T.
If the if $(p \wedge q)$ part is false, the whole statement is automatically true.

b) $p \rightarrow (p \vee q)$

p	q	$p \vee q$	$p \rightarrow (p \vee q)$	
T	T	T	T	$T \rightarrow T$ ✓ (T)
T	F	T	T	$T \rightarrow F$ ✓ (T)
F	T	T	T	$F \rightarrow F$ ✓ (T)
F	F	F	T	$F \rightarrow F$ ✓ (T)

c) $\neg p \rightarrow (p \rightarrow q)$

p	¬p	q	p → q
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T

¬p → (p → q)

F → T = T ✓

F → F ✓

T → T ✓

T → T ✓

d) $(p \wedge q) \rightarrow (p \rightarrow q)$

p	q	p ∧ q	p → q	(p ∧ q) → (p → q)
T	F	T	T	T → T T ✓
T	F	F	F	F → F T ✓
F	T	F	T	F → T T ✓
F	T	F	T	F → T T ✓

11) Show it's a Tautology without using a truth table.

$(p \wedge q) \rightarrow p$

$\neg(p \wedge q) \vee p \equiv \neg p \vee \neg q \vee p$

$(\neg p \vee p) \vee \neg q$

(Tautology) $T = \vee \neg q$

Problem 11

15) a) $\forall n (n^2 \geq 0)$ True.

b) $\exists n (n^2 \geq 2)$ F

c) $\forall (n^2 \geq n)$ T.

d $\exists n (n^2 < 0)$ F