25/10/2017

$\binom{n+k-1}{n-1}$

Content review

- Divides, gcd, lcm
- Union, intersection, subset
- Cardinality, power set, Cartesian product
- Venn diagrams, set operation laws (boolean algebras)
- $\lambda, \Sigma^*, \Sigma^{\leq k}, \Sigma^k$
- Lexicographic, lenlex ordering

Functions & relations

Relation R : $R \subseteq A \times B$ ~~$|Pow(R)| = mn$~~ ~~$|R| = mn$~~

$|R| = 2^{mn}$

Converse $R^{\leftarrow} \subseteq B \times A$, $R^{\leftarrow}$

Image $R(x) \subseteq B$

Function $f: A \to B$. ~~$1 \to 0$~~ $1 \to 1$ relation

Image

Injection, surjection, bijection

Inverse f^{-1} $B \to A$ iff. f is bijection

Inverse image $f^{\leftarrow}(x)$ image of x under converse of f exists for any function

Binary relations

Reflexivity, Symmetry, AS, transitivity

partial order: R. AS. T.
Equivalence. R. S. T

glb, lub.

Hasse diagram

Topological sort

Graphs

Basic definition

Eulerian circuit/path, Hamiltonian circuit/path
edge once
0 or 2 vertices of odd.
vertex once

Graph coloring, chromatic number

Planarity

K5. K3,3.

Logic

Conjunction, Disjunction, Negation, wff.

logical equivalence ≡
logical entailment ⊨

CNF/DNF Karnaugh maps

Boolean algebras

'Satisfiable

φ satisfiable
if some assignment makes φ true

φ tautology if all assignment makes φ true

Derive a CNF formula from some literals
$P, \neg P, q, r$.
which is not satisfiable.

~~$(P \wedge \neg P) \wedge (q \wedge r)$~~

$P \wedge \neg P$

It is in CNF b/c it is a conj. of disj.
of literals (p) and ($\neg p$)

It is not satifiable according to to
De Morgan's laws. $p \wedge \neg p = \bot$

Show that logical equivalence is an equivalence relation.

Let $\varphi \equiv \psi$ be arbitrary formulae

(R): $\varphi \equiv \varphi$, $\psi \equiv \psi$
for all formulae φ, ψ.

Take any evaluation v. $v(\varphi) = v(\varphi)$
$\varphi \equiv \varphi$

∴ It is reflective

(S) Suppose $\varphi \equiv \psi$ be arbitrary formulae

for any evaluation $v(\varphi) = v(\psi)$
$v(\psi) = v(\varphi)$
$\psi \equiv \varphi$

∴

(T) Suppose $\varphi \equiv \psi$, $\psi \equiv \theta$ for arbitrary formu

For any evaluation $v(\varphi) = v(\psi)$, $v(\psi) = v$
$v(\varphi) = v(\theta)$
$\varphi \equiv \theta$

Induction and recursion
one or two base cases,
one or two inductive steps

Structural induction

Running time of algorithms.

Big-O

~~Ω~~

Master Theorem

counting

K objects from n w/ replacement K^n

K objects from n without replacements: $\binom{n}{k}$

K balls into n boxes, many balls in boxes.

$\binom{n+K-1}{n-1}$ ~~$\binom{n+K-1}{K-1}$~~

Basic probability and expectation

Sample space, uniform distribution

Conditional probability & independance

not in the exam