Answers:

1.(a)

```
  gcd(288, 120)
= gcd(288 mod 120, 120)
= gcd(48, 120)
= gcd(120 mod 48, 48)
= gcd(48, 24)
= 24
```

(b)

```
  lcm(-91, 52)
= |-91 * 52| / gcd(-91,52)

  gcd(-91, 52)
= gcd(39, 52)
= gcd(13, 53)
= 13
```

hence:

```
  lcm(-91, 52)
= |-91 * 52| / 13
= 364
```

(c)

If n = 0, n + 1 = 1, 1|0, 1|1, so gcd(0, 1) = 1

If n ≠ 0:

```
  gcd(n, n+1)
= gcd(n+1-n, n)
= gcd(1, n)
= 1
```

hence

gcd(n, n + 1) = 1 for n ∈ N

2. (a)

$Pow(\emptyset) = \{\emptyset\}$

$Pow(Pow(\emptyset)) = \{\emptyset, \{\emptyset\}\}$

$card(Pow(Pow(\emptyset))) = 2$

(b)

$A \cap (B \oplus C)$

$= A \cap ((B \cap C^c)) \cup (C \cap B^c))$

$= (A \cap (B \cap C^c)) \cup (A \cap (C \cap B^c))$

$= ((\emptyset \cap B) \cup (A \cap (B \cap C^c))) \cup ((\emptyset \cap C) \cup (A \cap (C \cap B^c)))$

$= (((A \cap A^c) \cap B) \cup (A \cap (B \cap C^c))) \cup (((A \cap A^c) \cap C) \cup (A \cap (C \cap B^c)))$

$= (((A \cap B) \cap A^c) \cup ((A \cap B) \cap C^c)) \cup (((A \cap C) \cap A^c) \cup ((A \cap C) \cap B^c))$

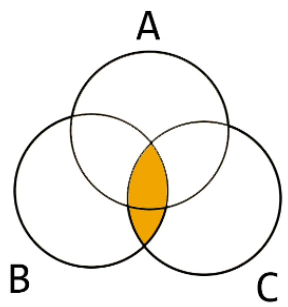
$= ((A \cap B) \cap (A^c \cup C^c)) \cup ((A \cap C) \cap (A^c \cup B^c))$

$= ((A \cap B) \setminus (A \cap C)) \cup ((A \cap C) \setminus (A \cap B))$

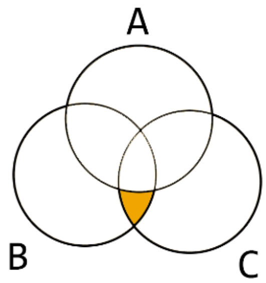
$= (A \cap B) \oplus (A \cap C)$

(c)

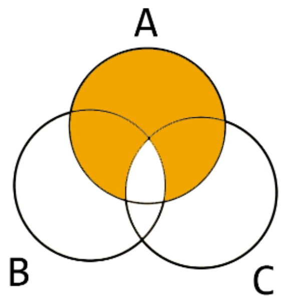
$B \cap C$:



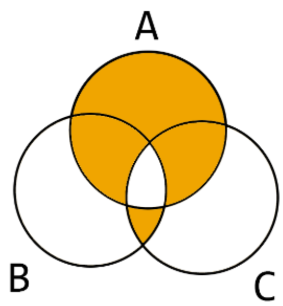
$(B \cap C) \setminus A$:



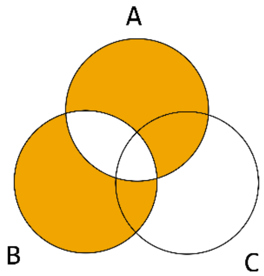
A \ (B ∩ C):



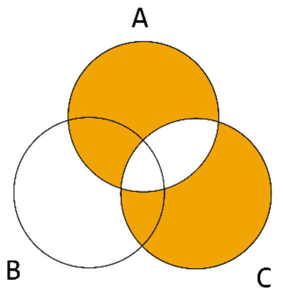
A ⊕ (B ∩ C) = ((B ∩ C) \ A) ∪ (A \ (B ∩ C)):



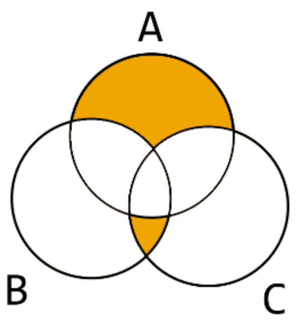
A ⊕ B:



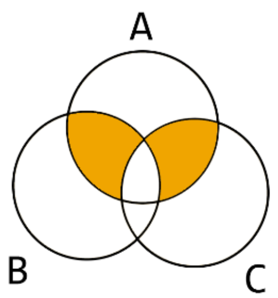
$A \oplus C$:



$(A \oplus B) \cap (A \oplus C)$:



Hence, $(A \oplus (B \cap C)) \setminus ((A \oplus B) \cap (A \oplus C))$:



The same Venn diagram above can also be written as:

$$(A \cap B) \oplus (A \cap C)$$

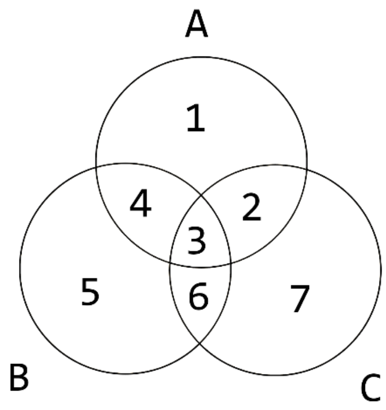
Hence

$$(A \oplus (B \cap C)) \setminus ((A \oplus B) \cap (A \oplus C)) = (A \cap B) \oplus (A \cap C).$$

if $(A \cap B) \oplus (A \cap C) \neq \emptyset$, $(A \oplus (B \cap C)) \neq ((A \oplus B) \cap (A \oplus C))$.

For instance, if A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {2, 3, 6, 7}, then $(A \oplus (B \cap C)) = \{1, 2, 4, 6\}$, and $((A \oplus B) \cap (A \oplus C)) = \{1, 6\}$,

Hence, in this case, $(A \oplus (B \cap C)) \neq ((A \oplus B) \cap (A \oplus C))$



3.

(a) $\Sigma^{\leq 3}$ = {λ, a, aa, aaa, aab, ab, aba, abb, b, ba, baa, bab, bb, bba, bbb}

(b) $\Sigma^{\leq 3}$ = {λ, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb}

4.

(a)

f: f(a) = 0, f(b) = 0, f(c) = 0

f: f(a) = 0, f(b) = 0, f(c) = 1

f: f(a) = 0, f(b) = 1, f(c) = 0

f: f(a) = 0, f(b) = 1, f(c) = 1

f: f(a) = 1, f(b) = 0, f(c) = 0

f: f(a) = 1, f(b) = 0, f(c) = 1

f: f(a) = 1, f(b) = 1, f(c) = 0

f: f(a) = 1, f(b) = 1, f(c) = 1

(b)

(i) For each $b \in B$, we can find m members from A to establish functions. Hence number of functions = n^m

(ii) Relations between A and B: R = A x B. Hence |R| = |A| x |B| = m x n

(c)

Pow(a, b, c) = {∅, a, b, c, ab, ac, bc, abc}

|Pow(a, b, c)| = $2^{|\{a,\ b,\ c\}|}$ = 2^3 = 8

Number of f : {a, b, c} → {0, 1} = n^m = 2^3 = 8

|Pow(a, b, c)| = Number of f : {a, b, c} → {0, 1}

To explain this, for f : {a, b, c} → {0, 1}, each possible function corresponds to some subset A ⊆ Pow(a, b, c), in the way:

for each element x ∈ A, f(x) ⟼ 1, and

for each element y ∈ (Pow(a, b, c) \ A), f(y) ⟼ 0

Hence

the number of all possible functions = card(Pow(a, b, c)) = 8

5.

(a)

(i) (abab, baba)

(v) (λ, bbb)

(b)

(R) In the case of (a, a) ∈ R, for any v ∈ Σ^*, av = av. Hence R is reflexive.

(S) For (a, b) ∈ R, for all v ∈ Σ^*,

(av ∈ L ∧ bv ∈ L) ⊕ (av ∉ L ∧ bv ∉ L) = true.

Since the Boolean " ∧" is commutative:

(bv ∈ L ∧ av ∈ L) ⊕ (bv ∉ L ∧ av ∉ L) = true

Hence in the case of (b, a), for all v ∈ Σ^*, R is symmetric.

(T) Let (a, b) ∈ R and (b, c) ∈ R, hence for all v ∈ Σ^*, v' ∈ Σ^*:

(av ∈ L ∧ bv ∈ L) ⊕ (av ∉ L ∧ bv ∉ L) = true, (bv` ∈ L ∧ cv` ∈ L) ⊕ (bv` ∉ L ∧ cv` ∉ L) = true.

With av ∈ L, bv ∈ L, since v and v' are arbitrary, bv` ∈ L, cv` ∈ L:

Since v and v' are arbitrary, av' ∈ L, bv' ∈ L, bv ∈ L, cv ∈ L also establish, so av ∈ L, cv ∈ L, a R c.

With av ∉ L, bv ∉ L, since v and v' are arbitrary, bv` ∉ L and cv` ∉ L:

Since v and v' are arbitrary, av' $\notin$ L, bv' $\notin$ L, bv $\notin$ L and cv $\notin$ L also establish, so (av $\notin$ L, cv $\notin$ L).

Hence R is transitive.

Concluding from (R), (S), (T), R is an equivalence relation.

(c)

$X = \{w \in \Sigma^*: 3|\text{length}(w)\}$

$Y = \{w \in \Sigma^*: 3|(\text{length}(w) + 1)\}$

$Z = \{w \in \Sigma^*: 3|(\text{length}(w) + 2)\}$