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UBC MATH521 – Research Project Proposal

A Comparison of Structured Elliptic, Hyperbolic, and Parabolic Mesh Generation and Unstructured Smoothing

Objectives and Significance

My research project will compare the aspects of elliptic, hyperbolic, and parabolically generated meshes, looking at the mathematics behind their creation and the structure of the meshes that are generated on simple geometries. This will be followed by a comparison of the performance of each of these meshes for different applications, and geometries. For instance, it is widely known that elliptic meshes produce better results for internal flows while hyperbolic meshes are better for external flows, why? I will then look at how each mesh handles more complex geometries, especially corners, and how unstructured meshes can be improved using smoothing techniques based on each type of governing equation. This can then be related to the spring force smoothing technique as explained by Numerow and Ollivier-Gooch in two dimensions of which hopefully improvements can be made using an elliptic or hyperbolic technique instead.

Literature Review

The fundamentals of grid generation by use of partial differential equations first came to be in the late 1960s and early 1970s with papers published by Winslow [1] and Thompson [2], in which both use elliptically generated meshes to solve problems. Steger and Chausse [3] later developed a process for mesh generation by use of a hyperbolic equation, which proved to be much better for convex corners. Thompson has since published two textbooks on the subject which describe the basics of structured grid generation all the way to some algorithms for smoothing three-dimensional unstructured meshes [4] [5]. Since then, others have looked at the comparison of each scheme for different applications, namely Falsifoon et al. [6], and some have looked at further developing algorithms to handle even the most complex geometries and mesh smoothing [7].

BIBLIOGRAPHY

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