MATH521: Numerical Analysis of Partial Differential Equations
Winter 2018/19, Term 2

Due Date: Thursday, 7 March 2019
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Homework Assignment 8

Please submit the following files as indicated below: source code PDF file image file video file

5 marks | Spend about six hours working on your project and then document your progress here! You may wish to write up a little report that you can later re-use for your written submission. This assignment shall give you an opportunity to receive feedback on your work. Please address all of the following questions:

(a) Think about what you have learnt in this course so far. What applies to your project? Remember to always use proper terminology.

(b) Do some research on some aspect of your project we have not covered in this course. E.g. if you are solving a nonlinear problem for your project, find out how to apply iterative methods such as Newton's method to this PDE. If your project involves numerical computations, begin with the implementation in FEniCS or your software of choice.

(c) What are you planning to do next? Do you have any questions on how to proceed?

Hint: If you are working on a time-dependent problem, I suggest to replace the time derivative with a reactive term for the purpose of this assignment (this is how we will solve these equations!). E.g. instead of the equation

$$\frac{\partial u}{\partial t} - \Delta u + u^3 = f$$

consider

$$au - \Delta u + u^3 = f$$

with some $a > 0$.

My project is concerned with the generation and performance of structured elliptic and hyperbolic meshes and their comparison to the unstructured meshes that we have become familiar with in class. I will delve into reason why using a structured mesh may be in your best interest even though for many geometries they will likely be more difficult to implement. So far I have done much research into how each of these meshes are actually generated, the advantages/ disadvantages of each, and developing ways to test each one, as outlined below. While we have covered finite difference and finite element methods extensively in class, my testing will focus on and use finite volume methods instead as they are still the preferred method for CFD applications (my field) (although on structured grids FV and FE are very similar), and structured meshing is still used whenever possible in CFD.

A lot of the research that I have completed so far is looking into the fundamentals of each of these mesh generation methods, as well as how elliptic generators are used for unstructured smoothing to improve quality. I won't write each system of equations here, however one interesting thing that I have found is the continued use of finite differences in the application of both elliptic and hyperbolic mesh generators to be used with finite volume and finite element solvers.

The next steps of my project are to build my test cases in which I will evaluate performance and quality of each mesh. For these tests I will be generating meshes on simple and complex geometries and performing fluid flow tests. While a lot of mesh testing is a rather qualitative proposition and somewhat subtle, I hope to use mesh metrics to obtain a more quantitative comparison of each. Some of the metrics I intend to use for the tests include minimum and maximum angle and for structured meshes the smoothness (i.e. are there any discontinuities) for mesh quality. For both elliptic and hyperbolic generators, the boundary conditions play a huge role in the overall smoothness, as well as forcing functions. One other aspect of concern is how mesh refinement affects the quality of each mesh. For performance, this can include the computation time to generate each mesh, which may include additional time if the mesh generated for some geometries is invalid or terribly poor. As well, I will look to test the accuracy of each mesh comparing results to an experimental test case. For this I intend to use the backward-facing step flow, as there is much experimental data to be used, and it does provide a challenge with both a convex and concave corner without being overly complex, as well it contains flow reversal which I presume the elliptic mesh will have a harder time capturing.

One aspect of unstructured mesh generation that may become of concern is the differences between schemes, such as Delaunay triangulation and advancing front/ layer methods. If there proves to be a large discrepancy between the two, I will do my best to test and report it accordingly.

Moving forward I would like to know if you think I am missing any important points of comparison as well as if you would like to see any tests or results that I have not mentioned.

Your Learning Progress | 0 marks, but -1 mark if unanswered | What is the most substantial new insight that you have gained from this course this week? Any *aha moment*?

This week I learned to make sure that I have all of the necessary libraries installed for Fenics. I was missing C++, trying to run on WSL Ubuntu, even though it is installed on my machine. Very confusing error but managed to get it fixed.