UBC

MATH521: Numerical Analysis of Partial Differential Equations
Winter 2018/19, Term 2

Due Date: Thursday, 14 February 2019
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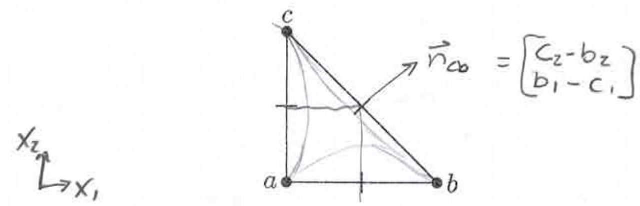
Homework Assignment 6

Please submit the following files as indicated below: source code PDF file image file video file

Question 1 | 2 marks | Given three points $a, b, c \in \mathbb{R}^2$ that are not collinear (not all on one line) and that are sorted in anticlockwise order, we define

$$T = \Delta(a, b, c) \qquad \text{(the triangle with these vertices)}$$
$$P = P_2(T)$$
$$L = \left\{ p \mapsto p(a),\quad p \mapsto p(b),\quad p \mapsto p(c),\quad p \mapsto \frac{\partial p}{\partial n}\left(\frac{a+b}{2}\right),\quad p \mapsto \frac{\partial p}{\partial n}\left(\frac{b+c}{2}\right),\quad p \mapsto \frac{\partial p}{\partial n}\left(\frac{c+a}{2}\right) \right\} \subset P^*$$



(a) Show that given any data for

$$p(a), \quad p(b), \quad p(c), \quad \frac{\partial p}{\partial n}\left(\frac{a+b}{2}\right), \quad \frac{\partial p}{\partial n}\left(\frac{b+c}{2}\right) \quad \text{and} \quad \frac{\partial p}{\partial n}\left(\frac{c+a}{2}\right)$$

there exists a unique interpolant $p \in P$.

1st Hint: Where does a parabola with $p(a) = 0$ and $p(b) = 0$ have its vertex?

2nd Hint: There is a video on checking unisolvence in the Media Gallery.

$P(a) = 0$

$P(b) = 0$

$\frac{\partial P}{\partial X_2}\left(\frac{a+b}{2}\right) = 0$

$P(c) = 0$

$\frac{\partial P}{\partial X_1}\left(\frac{a+c}{2}\right) = 0$

$\frac{\partial P}{\partial \vec{n}_{ca}}\left(\frac{b+c}{2}\right) = 0$

If we try to set each $p_i = 0$ we can certainly find a function, such as a paraboloid, that satisfies each vertex and each midpoint derivative on X_1 and X_2, and this would work form any rectangular domain, except the normal derivative cannot also be zero for the midpoint of the cb edge, therefore the only function to satisfy zeros across the board is the zero function. (??)

(b) Now let Ω^h be a domain with a regular triangulation $\mathcal{T}^h$ such that

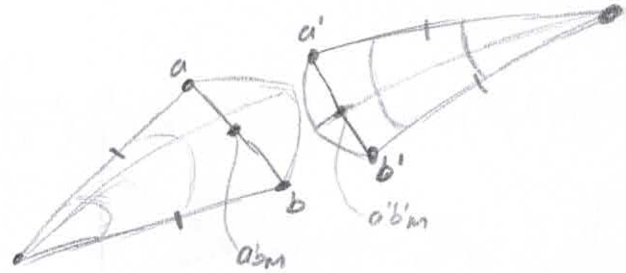
$$\bar{\Omega}^h = \bigcup_{T \in \mathcal{T}^h} T.$$

Show that the space

$$V^h = \left\{ v^h : \bar{\Omega}^h \to \mathbb{R} \;\middle|\; v^h\big|_T \in P_2(T), v^h \text{ is continuous in all vertices}, \frac{\partial v^h}{\partial n} \text{ is continuous in all edge midpoints} \right\}$$

is not H^1-conforming by giving me a specific example of a function $v^h \in V^h$ that has a jump across a triangle edge.

(Think similar to two cone halves)



Connected, each vertex is continuous and the derivatives are defined at each midpoint but the values between ab + a'b' are opposite. The derivative is the same at ab_M and $a'b'_M$ but values opposite. {again ??}

Question 2 | 3 marks | We will now complete our finite-element solver for the linear elasticity problem

$$\begin{aligned} -c\Delta u + au &= f \quad \text{in } \Omega \\ u &= g \quad \text{on } \partial\Omega. \end{aligned} \tag{1}$$

(a) Remove lines 1-10 from `discretiseLinearElasticity.m` and uncomment the sections of code that are currently commented out. Complete the missing commands, including the subfunction `assembleStiffness`. Also inspect the `assembleLoad` subfunction. Remember that you may use the code from last week's model answers if you are unsure whether your own code works correctly.

(b) Write a script `hw6.m` which

- solves the linear elasticity problem on Ω^h, which you may choose from `kiwi.mat`, `maple.mat`, `pi.mat`, `ubc.mat`. You may also select your own data for $f(x_1, x_2)$, $g(x_1, x_2)$, a and c.

  Hint: You have to set `GammaD = @(x1,x2) true(size(x1))`. For debugging, you might want to use `video.mat` and check the sparsity patterns of the various matrices.

- calculates the L^2, H^1 and energy norms

$$\|u^h\|_{L^2} = \sqrt{\int_{\Omega^h} |u^h|^2 \, \mathrm{d}x} \qquad \sqrt{\bar{\bar{u}}^T M \bar{\bar{u}}}$$

$$\|u^h\|_{H^1} = \sqrt{\|u^h\|_{L^2}^2 + \|\nabla u^h\|_{L^2}^2} = \sqrt{\int_{\Omega^h} |u^h|^2 \, \mathrm{d}x + \int_{\Omega^h} |\nabla u^h|^2 \, \mathrm{d}x} \qquad \sqrt{\bar{\bar{u}}^T M \bar{\bar{u}} + \bar{\bar{u}}^T K \bar{\bar{u}}}$$

$$\|u^h\|_B = \sqrt{B(u^h, u^h)} = \sqrt{c\int_{\Omega^h} |\nabla u^h|^2 \, \mathrm{d}x + a\int_{\Omega^h} |u^h|^2 \, \mathrm{d}x} \qquad \sqrt{c\bar{\bar{u}}^T K \bar{\bar{u}} + a\bar{\bar{u}}^T M \bar{\bar{u}}}$$

  of the solution, where B is the bilinear form corresponding to the elliptic operator

- creates undistorted plots of the mesh, the force f and the solution u^h (including the boundary points). Post your plots of f and u^h in the discussion forum!

(c) What problem do you solve numerically when you set `GammaD = @(x1,x2) false(size(x1))`? Analyse the code to infer its weak formulation:

All points are free $\rightarrow$ $g = f$ on $\partial\Omega$

$B(x,v) = \langle f, v\rangle_{V^*,V}$

$A = -cK + aM$

$b = f$

$B(x,v) = (A^T x) v \qquad \langle f, v\rangle_{V^*,V} = b^T v$

So that you don't get bored during the break... Install FEniCS and ParaView on your computer and bring it with you to our first class after the break on Tuesday 26 February. Please make sure everything is set up and running before that date. Both FEniCS and ParaView are free and open source software.

FEniCS on Ubuntu Linux or Windows 10 Follow the instructions here: `https://fenicsproject.org/download/`.

FEniCS on other Linux distributions, older versions of Windows or macOS It will be easiest to use FEniCS on Docker. Follow the installation instructions here: `https://fenics-containers.readthedocs.io/en/latest/quickstart.html`.

ParaView on Linux ParaView should already be included in the official repositories of your distribution.

ParaView on other operating systems Download it here: `https://www.paraview.org/download/`.

If you need help with troubleshooting, there is a discussion thread on Canvas.

Your Learning Progress | 0 marks, but -1 mark if unanswered | What is the one most important thing that you have learnt from this assignment?

Ask more questions, not anything new, but rings very true of the moment! :)

What is the most substantial new insight that you have gained from this course this week? Any *aha moment*?

I'm still not sure, last week and the notes from this week are all still very confusing for me, and I'll have to sit down and figure them out sometime soon.