2.1.17 Group Work Looking at the properties ①-③ which are required for the well-posedness of (W*), can you identify the "right" function spaces for the solution u, the test functions v and the right hand side f in the preliminary weak form (W*)?

① u, v are differentiable if they are in $H^1(\Omega)$

②
$$\left| \int_\Omega \nabla u \cdot \nabla v \, dx \right| = \left| (\nabla u, \nabla v)_{L^2(\Omega)^d} \right| \overset{\text{Cauchy Schwarz}}{\leq} \|\nabla u\|_{L^2(\Omega)^d} \|\nabla v\|_{L^2(\Omega)^d} < \infty \quad \text{if } \nabla u, \nabla v \in L^2(\Omega)^d$$

③
$$\left| \int_\Omega f v \, dx \right| = \left| (f, v)_{L^2(\Omega)} \right| \overset{\text{Cauchy Schwarz}}{\leq} \|f\|_{L^2(\Omega)} \|v\|_{L^2(\Omega)} < \infty \quad \text{if } f, v \in L^2(\Omega)$$

Therefore, we need

$$u \in H^1(\Omega) \qquad v \in H^1(\Omega) \qquad f \in L^2(\Omega)$$

(Recall that, additionally, $u = g$ and $v = 0$ on $\partial\Omega$).