2.1.12 Group Work Check whether or not the following functions
$$u$$
 are in $L^2(\Omega)$ with the given domains Ω :

(a) the Heaviside step function

$$\|u\|_{L^2}^2 = \int_{-\infty}^{\infty} u(x)^2 dx = \int_{-\infty}^{\infty} (1^2 dx)^2 dx = \int_{-\infty}^{\infty} (1^2 dx)^2 dx$$

over $\Omega =]-1,1[$

$$u(x) = \begin{cases} 0 & x < 0 \\ 1 & x \ge 0 \end{cases}$$
$$-1,1[$$

$$c\geq 0$$

on over $\Omega=$

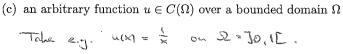
over
$$\Omega = \mathbb{R}$$

(b) the Heaviside step function over
$$\Omega = \mathbb{R}$$

$$\|u\|_{L^{2}}^{2} = \int_{-\infty}^{\infty} u(x)^{2} dx = \int_{0}^{\infty} 1^{2} dx = \infty$$
12 no!

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$$\mathrm{main}\ \Omega$$



 $\|u\|_{L^{2}}^{2} = \int \frac{1}{x^{2p}} dx < \infty \iff 2p \angle 1 \iff p \angle \frac{1}{2}$

 $\|u\|_{L^{2}}^{2} = \int \frac{1}{|x|^{2p}} dx = \int \int \frac{1}{\sqrt{x^{2p}}} v dr dp \ \angle \omega = \sum_{n=1}^{\infty} \frac{1}{2p-1} x dr dp$

(d) an arbitrary function
$$u \in C(\bar{\Omega})$$
 over a bounded domain Ω

$$i \in C(S)$$

(e) $u(x) = \frac{1}{|x|^p}$ over the interval $\Omega =]-1,1[$ (where p > 0)

a assumes its max & min over the compact set I => a is bounded on I

(f) $u(x) = \frac{1}{|x|^p}$ over the unit disk $\Omega = B(0,1) \subset \mathbb{R}^2$

(g) $u(x) = \frac{1}{|x|^p}$ over the unit ball $\Omega = B(0,1) \subset \mathbb{R}^3$

$$u \in C(\Omega)$$