

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = \left( \left( \frac{\partial u}{\partial x} \right)^2 + 2 \frac{\partial v}{\partial x} \frac{\partial u}{\partial y} + \left( \frac{\partial v}{\partial y} \right)^2 \right)$$

$$u = x^3 - 3xy^2$$

$$v = -3x^2y + y^3$$

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2$$

$$\frac{\partial u}{\partial y} = -6xy$$

$$\frac{\partial v}{\partial x} = -6xy$$

$$\frac{\partial v}{\partial y} = -3x^2 + 3y^2$$

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = - \left( (3x^2 - 3y^2)^2 + 2(-6xy)(-6xy) + (-3x^2 + 3y^2)^2 \right)$$

$$= (9x^4 - 18x^2y^2 + 9y^4) + 72x^2y^2 + (9x^4 - 18x^2y^2 + 9y^4)$$

$$= -18(x^4 + 2x^2y^2 + y^4)$$

$$S = -18(x^2 + y^2)^2$$

