Assignment 1

Mech 510

Due: Monday, October 1, 2018

1. (10 marks) In this problem, you will use the control volume averages in control volumes $i-1$, i, and $i+1$ to construct a more accurate flux approximation to the flux for the wave equation at $i+\frac{1}{2}$.

   (a) What order of accuracy would you expect to get for this number of control volumes and why? (No fair changing the answer to this part of the question after you do the derivation!)

   (b) Find a linear combination of these control volume averages which gives the most accurate possible approximation to the solution at $x_{i+\frac{1}{2}}$. What is the leading-order term in the truncation error in this expression?

   (c) Find the analogous flux at $x_{i-\frac{1}{2}}$ (using data from control volumes $i-2$, $i-1$, and i)? You may be able to do this by inspection, without having to repeat the derivation. What is the leading-order term in its truncation error?

   (d) Perform the flux integral for control volume i to obtain a finite-volume approximation to the space term in the wave equation.

2. (a) (10 marks) Suppose that you want to apply the following boundary condition for Poisson's equation:

$$\frac{\partial T}{\partial x} = -a\left(T - T_\infty\right)$$

   This corresponds physically to a free convection boundary condition. This boundary condition can be applied in at least two ways. First, one can compute the wall temperature T_w in terms of the temperature in the last interior cell $\overline{T}_{i,1}$ and the farfield temperature T_∞, then compute a boundary flux $\left.\frac{\partial T}{\partial y}\right|_{i,\frac{1}{2}}$ by using one-sided differences. Second, one can use ghost cells, computing $\overline{T}_{i,0}$ in terms of known quantities, and then applying the usual central-difference flux formula. For each of these approaches, derive the boundary flux $\left.\frac{\partial T}{\partial y}\right|_{i,\frac{1}{2}}$, including any auxiliary variables you need to compute it.

3. (a) (10 marks) This question centers around the accuracy of the simplest extension of our approximation of the Laplacian to a non-uniform mesh in one dimension, for which $\Delta x_{i-1} < \Delta x_i < \Delta x_{i+1}$.

   i. Find an approximation to the Laplacian flux $\frac{\partial T}{\partial x}$ at $i+\frac{1}{2}$ using data from control volumes i and $i+1$. What is the leading-order error for this approximation (including not just the order but the coefficient)?

   ii. Repeat part a for the flux at $i-\frac{1}{2}$.

   iii. Combine your results to find an approximation to the Laplacian term $\frac{\partial^2 T}{\partial x^2}$ for control volume i. Using solution expansions about the center of control volume i, find the leading order error in this approximation, including its coefficient. Assume that $\Delta x_{i+1} = (1+s)\Delta x_i$ and $\Delta x_{i-1} = \Delta x_i/(1+s)$, with $s < 1$; this condition says that the cell size grows by a fixed factor between control volumes.

   iv. What is the least restrictive condition on the ratio of cell sizes under which your result from part c can be made to be second-order accurate?