# MECH 510 – Assignment 3

## Question 2

Stability boundary for the four-stage Runge-Kutta time advance compared against the three-stage and two-stage, as well as with different amplification factors





## Question 3

Eigenvalues for each of the schemes and the schemes combined.



Maximum time-step for RK4 with the combined scheme



## Question 4

# Code

%% MECH 510 - Assignment 3

% Nicholas Earle

clear; clc; close all

phi = 0:2\*pi/100:2\*pi;

%% Second Order Upwind Scheme - 1st Derivative

LU2\_1 = @(phi) -(3 -4\*cos(phi) + cos(2\*phi) + 1i\*(4\*sin(phi) - sin(2\*phi)));

LU2 = LU2\_1(phi);

plot(1/2\*real(LU2), 1/2\*imag(LU2));

xlabel('Real \lambda \*(u/\Deltax)');

ylabel('Imag \lambda \*(u/\Deltax)');

title('Eigenvalues for 2nd order upwind and centred schemes');

grid on;

%% Second Order Centred Scheme - 2nd Derivative

LC2\_2 = @(phi) cos(phi) - 1;

LC2 = LC2\_2(phi);

hold on;

plot(2\*real(LC2), 2\*imag(LC2));

%% Combined

LUC2 = @(phi) -17 + 22\*cos(phi) - 5\*cos(2\*phi) - 5i\*(4\*sin(phi) -sin(2\*phi));

LUC = LUC2(phi);

plot(1/10\*real(LUC), 1/10\*imag(LUC));

legend('2nd Upwind - 1st Deriv.','2nd Centred - 2nd Deriv.', 'Combined');

%% Runge-Kutta Schemes

% Specify x range and number of points

x0 = -3;

x1 = 3;

Nx = 901;

% Specify y range and number of points

y0 = -3;

y1 = 3;

Ny = 901;

% Construct mesh

xv = linspace(x0,x1,Nx);

yv = linspace(y0,y1,Ny);

[phi,y] = meshgrid(xv,yv);

% Calculate z

z = phi + 1i\*y;

g2 = 1 + z + 1/2\*z.^2;

g3 = 1 + z + 1/2\*z.^2 + 1/6\*z.^3;

g4 = 1 + z + 1/2\*z.^2 + 1/6\*z.^3 + 1/24\*z.^4;

b = 1;

% Calculate magnitude of g

gmag2 = abs(g2);

gmag3 = abs(g3);

gmag4 = abs(g4);

% Plot contours of gmag

figure();

hold on;

contour(phi,y,gmag2,[b b], 'r','LineWidth', 2);

contour(phi,y,gmag3,[b b], 'g','LineWidth', 2);

contour(phi,y,gmag4,[b b], 'b','LineWidth', 2);

xlabel('Real \lambda\Deltat');

ylabel('Imag \lambda\Deltat');

title('Stability boundary for RK2, RK3, RK4');

legend('RK2','RK3','RK4');

axis tight;

grid on;

b = 0.2:0.2:1;

figure()

contour(phi,y,gmag4,b,'ShowText', 'on');

xlabel('Real \lambda\Deltat');

ylabel('Imag \lambda\Deltat');

title('RK4 Contour for |\sigma|');

axis tight;

grid on;

%% Max time step for RK4

figure()

contour(phi, y, gmag4, [1 1], '--','LineWidth', 2);

hold on;

plot(real(LUC/10),imag(LUC/10));

plot(0.75\*real(LUC/10), 0.75\*imag(LUC/10));

plot(0.5\*real(LUC/10), 0.5\*imag(LUC/10));

plot(0.633\*real(LUC/10), 0.633\*imag(LUC/10));

xlabel('Real \lambda\Deltat \*(u/\Deltax)');

ylabel('Imag \lambda\Deltat \*(u/\Deltax)');

title('Maximum stable time step for combined scheme with RK4');

legend('RK4 Stability','\Deltat = 1','\Deltat = 0.75','\Deltat = 0.5','\Deltat = 0.633');

grid on;

%% Max amp factor for funky 3 stage time advance

phi = pi/2:pi/200:pi;

f = @(phi) 1/2\*(-3 + 4\*cos(phi) - cos(2\*phi) - 1i\*(4\*sin(phi) - sin(2\*phi)));

sig = @(CFL) max(abs(1 + 97/75\*CFL\*f(phi) + 24/25\*CFL^2\*f(phi).^2 + 16/75\*CFL^2\*f(phi).^3));

cmin = fminsearch(sig, 0.3)

sig\_max = sig(cmin)