$k - \omega$ SST

Turbulence Model

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February 25^{th}, 2019

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Why SST?

- Two-equation eddy-viscosity turbulence models have a difficult time correctly predicting the location and amount of flow separation when facing adverse pressure gradients
- The $k - \omega$ Shear Stress Transport (SST) turbulence model developed by Florian Menter [Men94] looks to combine the best of the $k - \epsilon$ model with the $k - \omega$ model to improve accuracy and agreement with experimental and direct numerical simulation data

The New Baseline (BSL) Model

- Based on the Wilcox $k-\omega$ model
- Designed to take advantage or the accuracy and robustness of the $k-\omega$ model near the wall and the "freestream independence" of the $k-\epsilon$ model outside of the boundary layer
- Uses a transformed version of the $k-\epsilon$ model, introducing an additional cross-diffusion term
- Each model multiplied by F_1 and $(1-F_1)$, then summed.
  - $F_1 = 1$ in sublayer and logarithmic region of boundary layer
  - $F_1 \rightarrow 0$ in the wake and free stream regions

The Shear Stress Transport (SST) Model

- Exactly the same as the BSL model except that the eddy viscosity is redefined to account for the transport of the principle turbulent shear stress, $\tau =: -\rho\overline{u'v'}$

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BSL/SST Formulation

Original $k-\omega$ model:

$$\frac{\partial(\rho k)}{\partial t}+u_i\frac{\partial(\rho k)}{\partial x_i}=\tau_{ij}\frac{\partial u_i}{\partial x_j}-\beta^*\rho\omega k+\frac{\partial}{\partial x_j}\left[(\mu+\sigma_{k1}\mu_t)\frac{\partial k}{\partial x_j}\right]$$

$$\frac{\partial(\rho\omega)}{\partial t}+u_i\frac{\partial(\rho\omega)}{\partial x_i}=\frac{\gamma_1}{\nu_t}\tau_{ij}\frac{\partial u_i}{\partial x_j}-\beta_1\rho\omega^2+\frac{\partial}{\partial x_j}\left[(\mu+\sigma_{\omega 1}\mu_t)\frac{\partial\omega}{\partial x_j}\right]$$

Transformed $k-\epsilon$ model:

$$\frac{\partial(\rho k)}{\partial t}+u_i\frac{\partial(\rho k)}{\partial x_i}=\tau_{ij}\frac{\partial u_i}{\partial x_j}-\beta^*\rho\omega k+\frac{\partial}{\partial x_j}\left[(\mu+\sigma_{k2}\mu_t)\frac{\partial k}{\partial x_j}\right]$$

$$\frac{\partial(\rho\omega)}{\partial t}+u_i\frac{\partial(\rho\omega)}{\partial x_i}=\frac{\gamma_2}{\nu_t}\tau_{ij}\frac{\partial u_i}{\partial x_j}-\beta_2\rho\omega^2+\frac{\partial}{\partial x_j}\left[(\mu+\sigma_{\omega 2}\mu_t)\frac{\partial\omega}{\partial x_j}\right]+2\rho\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial\omega}{\partial x_j}$$

BSL/SST Formulation

New Baseline (BSL) model:

$$\frac{\partial(\rho k)}{\partial t} + u_i \frac{\partial(\rho k)}{\partial x_i} = \tau_{ij} \frac{\partial u_i}{\partial x_j} - \beta^* \rho \omega k + \frac{\partial}{\partial x_j} \left[(\mu + \sigma_k \mu_t) \frac{\partial k}{\partial x_j} \right]$$

$$\frac{\partial(\rho \omega)}{\partial t} + u_i \frac{\partial(\rho \omega)}{\partial x_i} = \frac{\gamma}{\nu_t} \tau_{ij} \frac{\partial u_i}{\partial x_j} - \beta \rho \omega^2 + \frac{\partial}{\partial x_j} \left[(\mu + \sigma_\omega \mu_t) \frac{\partial \omega}{\partial x_j} \right] + 2\rho(1 - F_1)\sigma_{\omega 2} \frac{1}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j}$$

Where for any constant ϕ_1 in the original model or ϕ_2 in the transformed model:

$$\phi = F_1 \phi_1 + (1 - F_1) \phi_2$$

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BSL Constants

The constants for the BSL model are simply those of the Wilcox $k - \omega$ model and the standard $k - \epsilon$ model.
Set 1 (ϕ_1):

$$\sigma_{k1} = 0.5, \qquad \sigma_{\omega 1} = 0.5, \qquad \beta_1 = 0.0750$$
$$\beta^* = 0.09, \qquad \kappa = 0.41, \qquad \gamma_1 = \beta_1/\beta^* - \sigma_{\omega 1}\kappa^2/\sqrt{\beta^*}$$

Set 2 (ϕ_2):

$$\sigma_{k2} = 1.0, \qquad \sigma_{\omega 2} = 0.856, \qquad \beta_2 = 0.0828$$
$$\beta^* = 0.09, \qquad \kappa = 0.41, \qquad \gamma_2 = \beta_2/\beta^* - \sigma_{\omega 2}\kappa^2/\sqrt{\beta^*}$$

BSL Definitions

Kinematic eddy viscosity:

$$\nu_t = \frac{k}{\omega}$$

Turbulent stress tensor $\tau_{ij} = -\rho\overline{u_i' u_j'}$:

$$\tau_{ij} = \mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij} \right) - \frac{2}{3}\rho k \delta_{ij}$$

BSL Definitions

Blending function:

$$F_1 = tanh(arg_1^4)$$

$$arg_1 = min\left[max\left(\frac{\sqrt{k}}{0.09\omega y};\frac{500\nu}{y^2\omega}\right);\frac{4\rho\sigma_{\omega 2}k}{CD_{k\omega}y^2}\right]$$

where y is the distance from that point to the nearest surface and $CD_{k\omega}$ is the positive portion of the cross-diffusion term given by:

$$CD_{k\omega} = max\left(2\rho\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j};10^{-20}\right)$$

SST Constants and Definitions

The constants for the SST model are identical to the BSL model except for:

- Set 1 (ϕ_1):

$$\sigma_{k1} = 0.85, \quad \sigma_{\omega 1} = 0.5, \quad \beta_1 = 0.0750, \quad a_1 = 0.31$$
$$\beta^* = 0.09, \quad \kappa = 0.41, \quad \gamma_1 = \beta_1/\beta^* - \sigma_{\omega 1}\kappa^2/\sqrt{\beta^*}$$

- And the eddy viscosity:

$$\nu_t = \frac{a_1 k}{max(a_1\omega; \Omega F_2)}$$

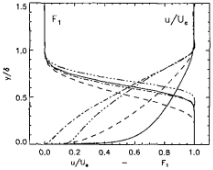
where Ω is the absolute value of the vorticity.

SST Definitions

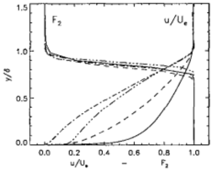
Blending function:

$$F_2 = tanh(arg_2^2)$$

$$arg_2 = max\left(2\frac{\sqrt{k}}{0.09\omega y}; \frac{500\nu}{y^2\omega}\right)$$



(a) Function F_1



(b) Function F_2

Figure: Blending functions F_1 and F_2 vs y/δ for different velocity profiles

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Recommended SST Freestream Boundary Conditions

$$\frac{U_\infty}{L} < \omega_\infty < 10\frac{U_\infty}{L}$$

$$\frac{10^{-5}U_\infty^2}{Re_L} < k_\infty < \frac{10^{-2}U_\infty^2}{Re_L}$$

$$\omega_{wall} = 10\frac{6\nu}{\beta_1(\Delta y_1)^2}$$

$$k_{wall} = 0$$

where L is the approximate length of the computational domain.

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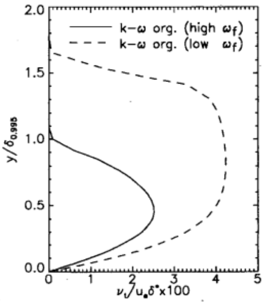
SST Validation

Experiments and flows used for validation and calibration:

- Flat Plate Boundary Layer
- Free Shear Layers
- Adverse Pressure Gradient Flow
- Backward-Facing Step Flow
- NACA 4412 Airfoil Flow
- Transonic Bump Flow

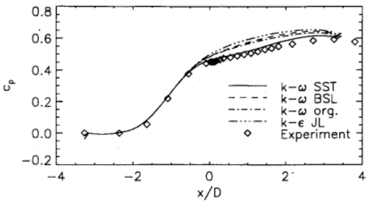
Flat Plate Boundary Layer

Here we see the freestream dependency of the eddy-viscosity of the original $k - \omega$ model and the BSL model[Men94].

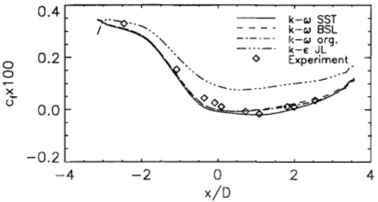


Adverse Pressure Gradient Flow

This flow as experimented by Driver is of a flow around a circuler cylinder at $Re_D = 2.8 \times 10^5$ with a diameter $D = 140mm$ [Dri91].



(a) Wall Pressure

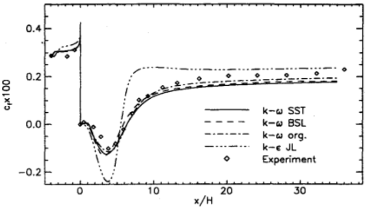


(b) Wall Shear Stress

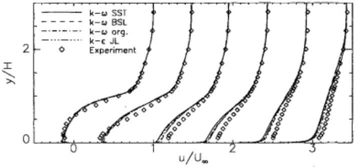
Figure: Distributions for Driver's adverse pressure gradient flow[Men94]

Backward-Facing Step Flow

This flow as experimented by Driver and Seegmiller is over a backward-facing step. The four models produced reattachment lengths of: 6.5(SST), 5.9(BSL), 6.4($k-\omega$), 5.5($k-\epsilon$), and 6.4 for the experiment[Dri].



(a) Wall Shear Stress



(b) Velocity Profiles at locations: $x/H = 2.0, 4.0, 6.5, 8.0, 14.0, 32.0$

Figure: Distributions for Driver and Seegmiller's backward-facing step flow[Men94]

NACA 4412 Airfoil

This flow was experimented around a NACA 4412 airfoil at a 13.87° angle of attack with a $Re_L = 1.52 \times 10^6$ [Col].

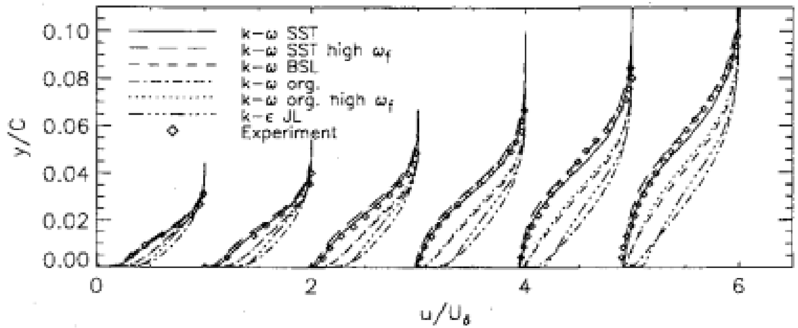


Figure: Velocity profiles on the upper surface of the NACA 4412 airfoil at 13.87° angle of attack at streamwise stations $x/c = 0.675, 0.731, 0.786, 0.842, 0.897, 0.953$[Men94]

Transonic Bump Flow

This final flow was is an axisymmetric transonic shockwave/ turbulent boundary layer experiment around a circular arc by Bachalo and Johnson at a mach number of 0.925[Bac70].

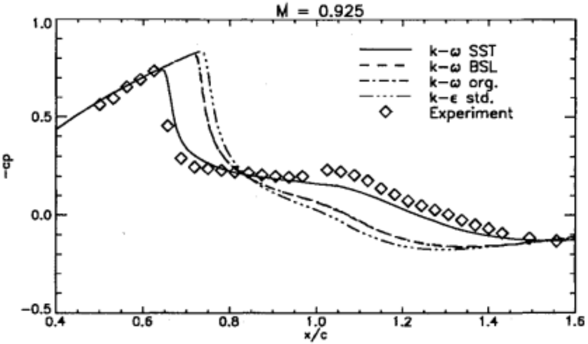


Figure: Surface pressure distributions for transonic bump flow[Men94]

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SST Variations and Extensions

Since Menter's original paper in 1993 many variations and extensions have been made to the SST model. These include:

- SST with Vorticity Source Term (SST-V)
- SST from 2003 (SST-2003)
- SST with Controlled Decay (SST-sust)
- SST with Controlled Decay and Vorticity Source Term (SST-Vsust)
- SST with Rotation/ Curvature Correction (SST-RC)
- SST with Hellsten's Simplified Rotation/ Curvature Correction (SST-RC-Hellsten)

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Conclusions

- Simply better than $K - \omega$ and $k - \epsilon$ for adverse pressure gradients and determining flow separation.
- Does require some addition programming effort
- However, no significant change in computing time or stability

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