MECH511 Programming Assignment 3
Compressible Flow

Nick Earle

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In this assignment I solve the Sod shock tube problem using the Steger-Warming and Roe FDS schemes to a final time of $t = 0.15$. The parameters for the problems, shown in Table 1 below, should result in a strong moving shock in the tube.

$\rho_L = 6$	$\rho_R = 1$
$u_L = 0$	$u_R = 0$
$P_L = 12$	$P_R = 1$

Table 1: Sod shock tube problem initial parameters

The problem was solved in 1 dimension using 200 cells on the interval $(0, 1)$, using both a first order space discretisation with an exlicit Euler time advance and a second order space discretisation with the two stage Runge-Kutta (RK2) time advance. For a second order case multiple flux limiters were tested including minmod, Van Leer, and superbee. All tests were performed using a timestep of 0.001.

The second order discretisations are defined on the cell interface as:

$$U_{i+\frac{1}{2};i} = U_i + \frac{1}{2}\phi(r_i)(U_i - U_{i-1}) \tag{1}$$

$$U_{i+\frac{1}{2};i+1} = U_{i+1} - \frac{1}{2}\phi\left(\frac{1}{r_i}\right)(U_{i+2} - U_{i+1}) \tag{2}$$

where ϕ is the flux limiter of choice, and r_i is defined by:

$$r_i = \frac{U_{i+1} - U_i}{U_i - U_{i-1}} \tag{3}$$

The flux limiters are then defined as:

minmod:

$$\phi_{mm}(r) = max\,(0, min(1, r)) \tag{4}$$

Van Leer:

$$\phi_{vl}(r) = \frac{r + |r|}{1 + |r|} \tag{5}$$

superbee:

$$\phi_{sb}(r) = max\,(0, min(2r, 1), min(r, 2)) \tag{6}$$

The figures below plot ρ, u, P, and T, plotted as RT, for each scheme. Figure 1, below, compares the first order results of Steger-Warming and Roe.

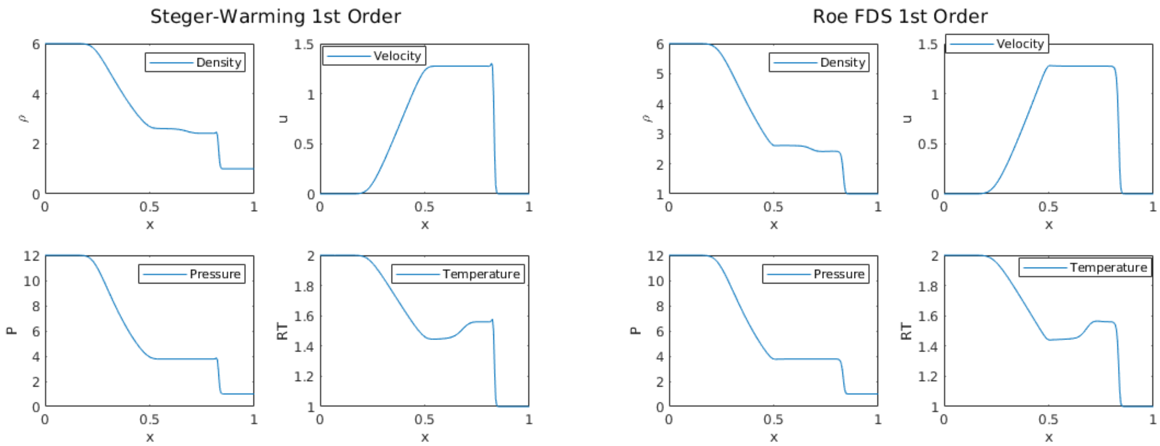

Figure 1: First order Steger-Warming and Roe schemes

The first order schemes produce very similar results, except there is a small overshoot on the velocity with Steger-Warming, which translates to the pressure and temperature plots as well.

Figure 2, below, shows the results of Steger-Warming and Roe using the second order discretisations with RK2 and the minmod flux limiter.

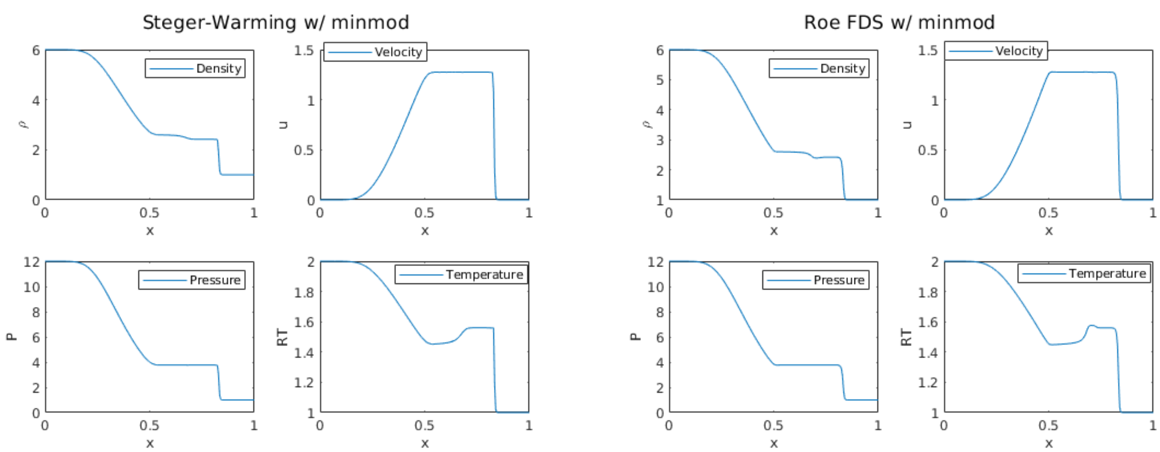

Figure 2: Second order Steger-Warming and Roe schemes using the minmod limiter

The minmod limiter produces very similar results with the only difference being a slight oscillation in the density for the Roe scheme, which again translates into the temperature plot too.

Figure 3 shows the results using the Van Leer flux limiter.

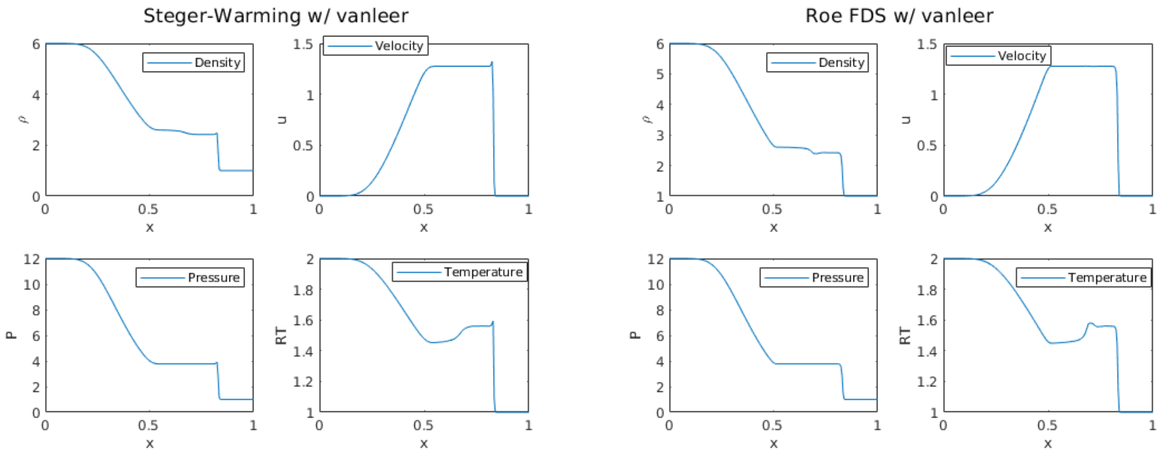

Figure 3: Second order Steger-Warming and Roe schemes using the Van Leer limiter

The Van Leer limiter again produces very similar plots, except that there is a blip in the velocity with Steger-Warming, similar to the first order case, and a slight oscillation in the density with Roe, like the minmod case.

Figure 4 shows the plots when using the superbee flux limiter.

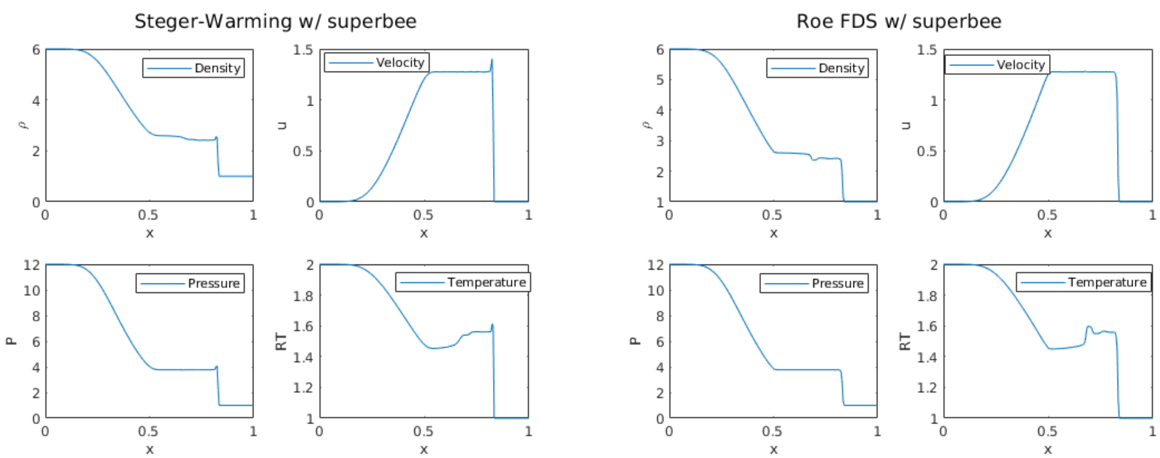

Figure 4: Second order Steger-Warming and Roe schemes using the superbee limiter

Superbee produces results akin to Van Leer, except that both the blip in the Steger-Warming velocity and the oscillations in the Roe density are much larger. Comparing each of the flux limiters, we see that the overshoots increase from minmod to Van Leer to superbee having the largest. As we know, flux limiters are within the Sweby region to be admissable for a second order scheme. Looking at the diagrams of each we see that the minmod limiter reaches a maximum of 1, while Van Leer and superbee both have their limits at 2. However, Van Leer is a gradual increase and only reaches 2 by taking the limit as $r \to \infty$, while superbee increases to 2 when $r > 2$. This distinction could be the cause of the oscillations we see in the plots.