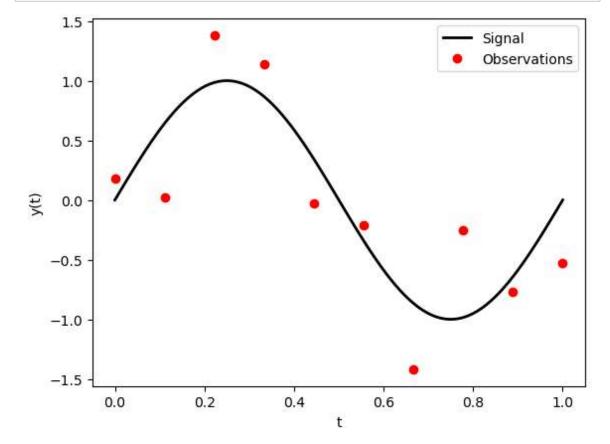
Data Inference and Applied Machine Learning DIAML: Tutorial 7b Copyright (c) 2023 Patrick E. McSharry (patrick@mcsharry.net (mailto:patrick@mcsharry.net))

Fitting a polynomial to a sinusoid Study the effect of noisy measurements, over-fitting and evaluation using in-sample and out-of-sample approaches

```
In [4]: # Importing libraries
import numpy as np
import matplotlib.pyplot as plt
```

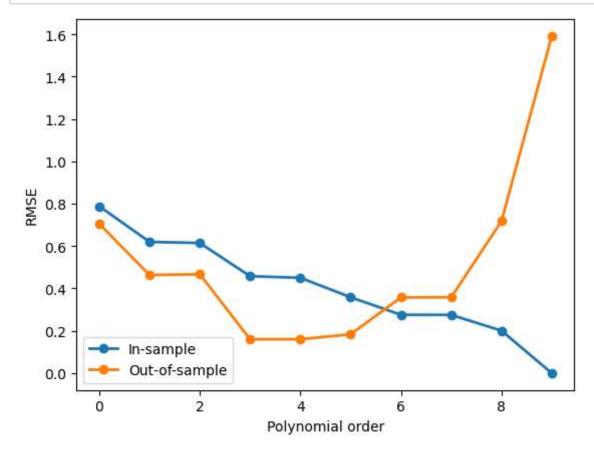
```
In [8]: # Data generating process
T = 10
A = 0.5
t = np.linspace(0,1,T)
noise = A*np.random.randn(T,)
x = np.sin(2*np.pi*t)
y = x + noise
```

```
In [10]: # Visualization
    tp = np.linspace(0,1,100)
    yp = np.sin(2*np.pi*tp)
    plt.plot(tp, yp, color='black', linewidth=2)
    plt.plot(t, y, 'ro', markerfacecolor='red')
    plt.xlabel('t')
    plt.ylabel('y(t)')
    plt.legend(['Signal', 'Observations'])
    plt.show()
```



```
# Loop over polynomials of increasing order and complexity
In [22]:
           RMSE = []
           RMSEp = []
           plt.figure(figsize= (10,10))
           for i in range(1, 11):
                plt.subplot(5,2,i)
                b = np.polyfit(t, y, i-1)
               yphat = np.polyval(b,tp)
                plt.plot(tp, np.sin(2*np.pi*tp), color='black', linewidth=2)
                plt.plot(t, y, 'ro', markerfacecolor='red')
               plt.plot(tp, yphat, color='blue', linewidth=2)
                plt.title(f'Order: {i-1}')
                plt.xlabel('t')
               plt.ylabel('y(t)')
               yhat = np.polyval(b,t)
                E = yhat -y
                Ep = yphat - yp
                RMSE.append(np.sqrt(np.mean(E**2)))
                RMSEp.append(np.sqrt(np.mean(Ep**2)))
           plt.tight_layout()
           plt.show()
                                Order: 0
                                                                              Order: 1
              -1
                        0.2
                                Order: 2
                                                                              Order: 3
              -1
                                                            -1
                        0.2
                                      0.6
                                             0.8
                                                    1.0
                                                               0.0
                                                                      0.2
                                                                             0.4
                                                                                    0.6
                                                                                           0.8
                                                                                                  1.0
                 0.0
                                                                              Order: 5
                                Order: 4
                                                         X(t)
              -1
                                                            -1
                 0.0
                        0.2
                               0.4
                                      0.6
                                             8.0
                                                    1.0
                                                               0.0
                                                                      0.2
                                                                             0.4
                                                                                    0.6
                                                                                           0.8
                                                                                                  1.0
                                Order: 6
                                                                              Order: 7
                                                         y(t)
               0
                                                            0
              -1
                                                            -1
                 0.0
                        0.2
                                      0.6
                                             8.0
                                                               0.0
                                                                      0.2
                                                                                           0.8
                                                                                                  1.0
                                Order: 8
                                                                              Order: 9
                                                           0.0
                                                        € -2.5
                                                          -5.0
                                                    1.0
                                                                                                  1.0
                 0.0
                        0.2
                               0.4
                                      0.6
                                             0.8
                                                               0.0
                                                                      0.2
                                                                             0.4
                                                                                    0.6
                                                                                           0.8
```

```
In [24]: # Monitor effect of complexity on in-sample and out-of-sample error
polyorder = np.arange(0,10)
plt.plot(polyorder, RMSE, 'o-', linewidth=2)
plt.plot(polyorder, RMSEp, 'o-', linewidth=2)
plt.xlabel('Polynomial order')
plt.ylabel('RMSE')
plt.legend(['In-sample', 'Out-of-sample'], loc='lower left')
plt.show()
```



```
In [ ]:
```