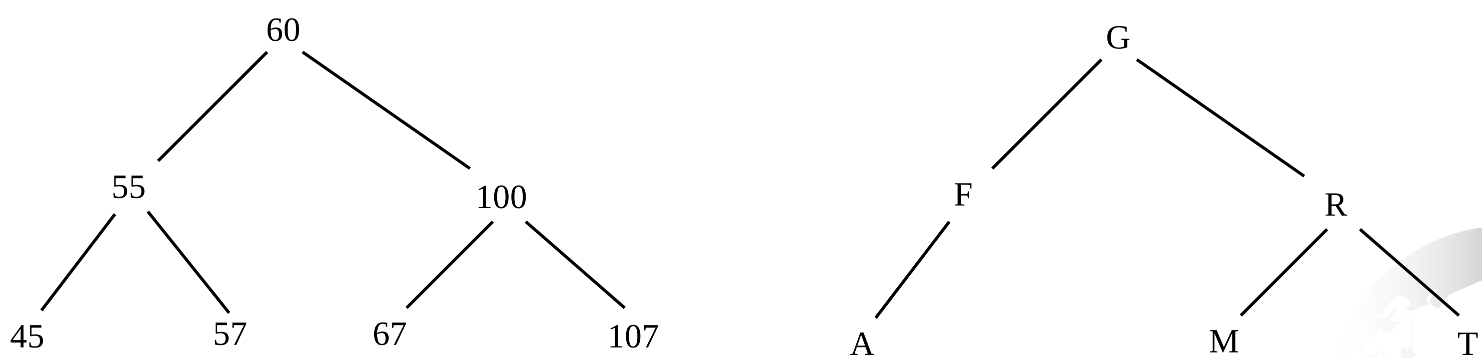
Binary Search Trees

Objectives

- To design and implement a binary search tree (§25.2).
- To represent binary trees using linked data structures (§25.2.1).
- To search an element in binary search tree (§25.2.2).
- To insert an element into a binary search tree (§25.2.3).
- To traverse elements in a binary tree (§25.2.4).
- To delete elements from a binary search tree (§25.3).
- To display binary tree graphically (§25.4).
- To create iterators for traversing a binary tree (§25.5).
- To implement Huffman coding for compressing data using a binary tree (§25.6).

Binary Trees

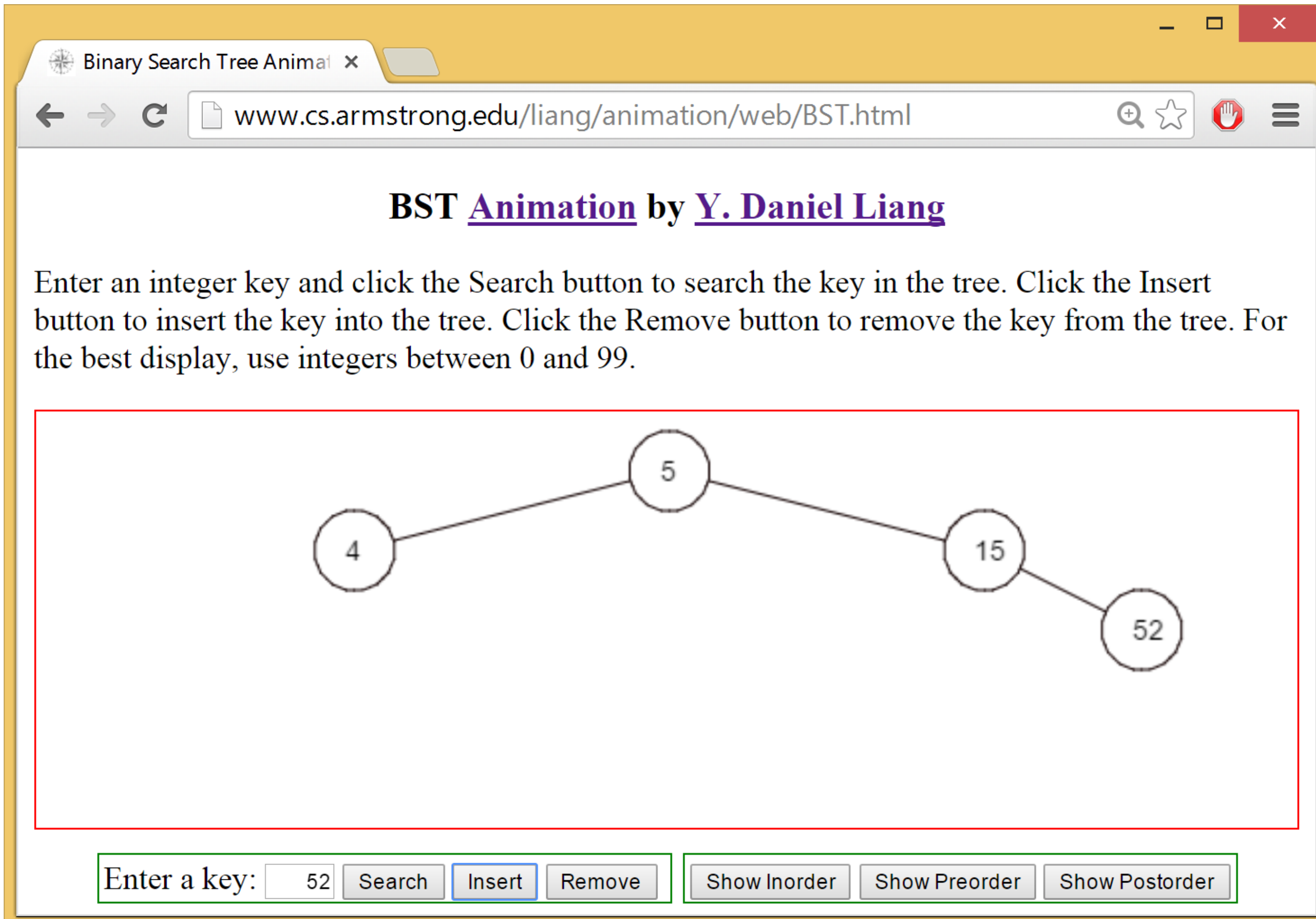
A list, stack, or queue is a linear structure that consists of a sequence of elements. A binary tree is a hierarchical structure. It is either empty or consists of an element, called the *root*, and two distinct binary trees, called the *left subtree* and *right subtree*.



(A) (B)

See How a Binary Search Tree Works

https://liveexample.pearsoncmg.com/dsanimation/BSTeBook.html

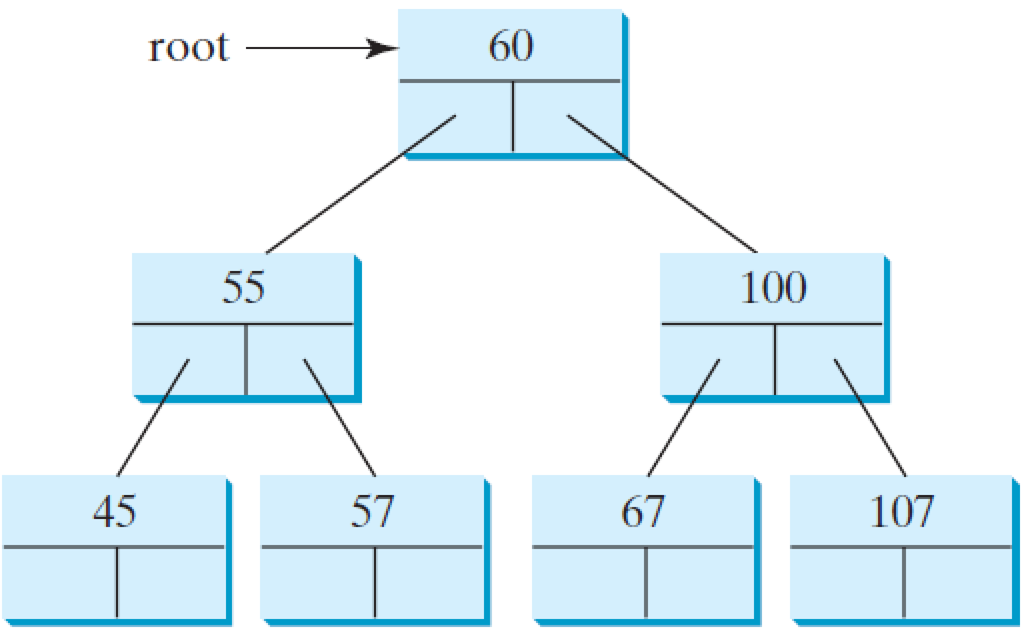


Binary Tree Terms

The root of left (right) subtree of a node is called a *left (right) child* of the node. A node without children is called a *leaf*. A special type of binary tree called a *binary search tree* is often useful. A binary search tree (with no duplicate elements) has the property that for every node in the tree the value of any node in its left subtree is less than the value of the node and the value of any node in its right subtree is greater than the value of the node. The binary trees in Figure 25.1 are all binary search trees. This section is concerned with binary search trees.

Representing Binary Trees

A binary tree can be represented using a set of linked nodes. Each node contains a value and two links named *left* and *right* that reference the left child and right child, respectively, as shown in Figure 25.2.




```
class TreeNode<E> {
  E element;
  TreeNode<E> left;
  TreeNode<E> right;

  public TreeNode(E o) {
    element = o;
  }
}
```

Searching an Element in a Binary Search Tree

```
public boolean search(E element) {
  TreeNode<E> current = root; // Start from the root
  while (current != null)
    if (element < current.element) {
      current = current.left; // Go left
    }
    else if (element > current.element) {
      current = current.right; // Go right
    }
    else // Element matches current.element
      return true; // Element is found
  return false; // Element is not in the tree
}
```

Inserting an Element to a Binary Search Tree

If a binary tree is empty, create a root node with the new element. Otherwise, locate the parent node for the new element node. If the new element is less than the parent element, the node for the new element becomes the left child of the parent. If the new element is greater than the parent element, the node for the new element becomes the right child of the parent. Here is the algorithm:

Inserting an Element to a Binary Tree

```
if (root == null)
  root = new TreeNode(element);
else {
  // Locate the parent node
  current = root;
  while (current != null)
    if (element value < the value in current.element) {
      parent = current;
      current = current.left;
    }
    else if (element value > the value in current.element) {
      parent = current;
      current = current.right;
    }
    else
      return false; // Duplicate node not inserted

  // Create the new node and attach it to the parent node
  if (element < parent.element)
    parent.left = new TreeNode(elemenet);
  else
    parent.right = new TreeNode(elemenet);

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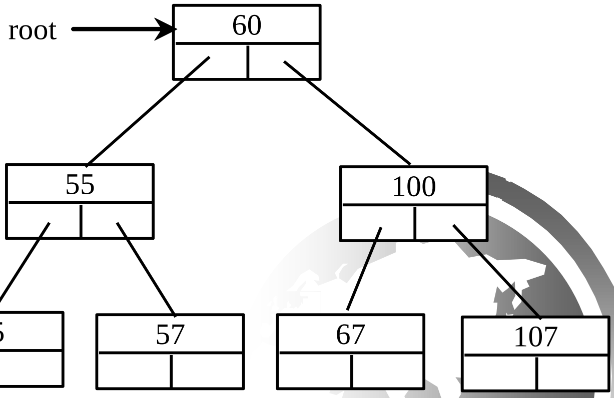
}
```

Insert 101 into the following tree.



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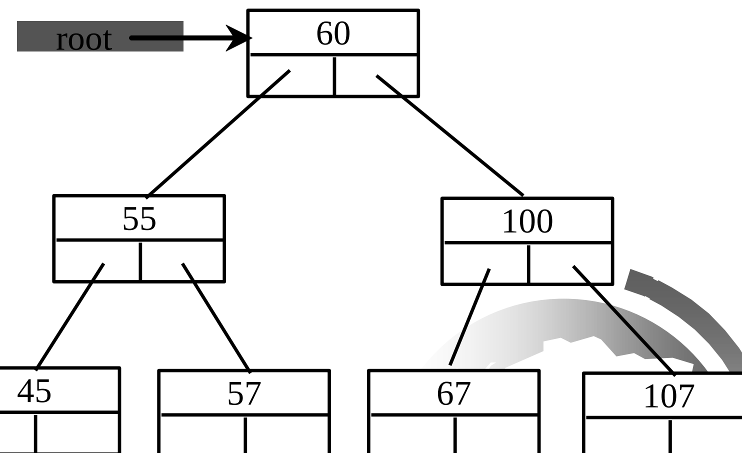

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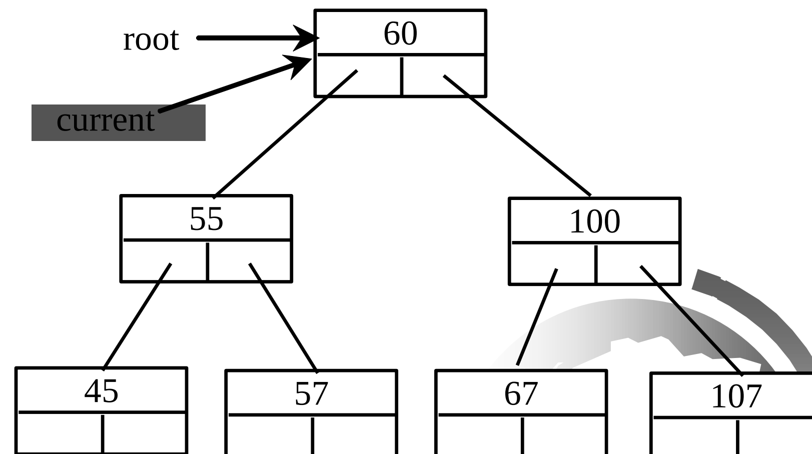

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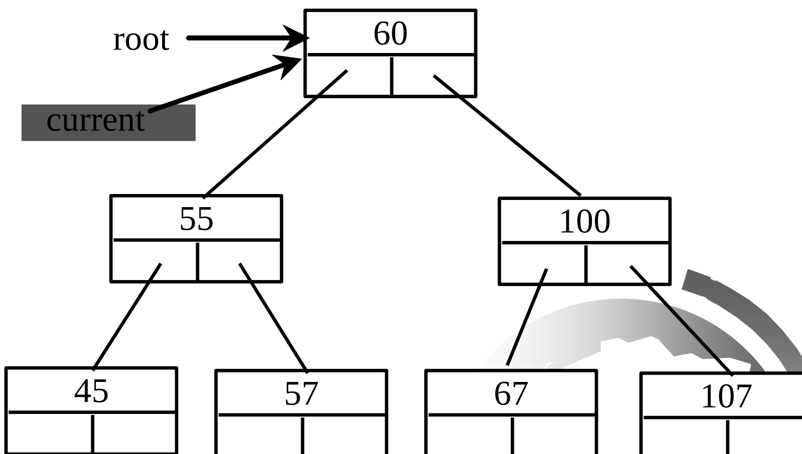

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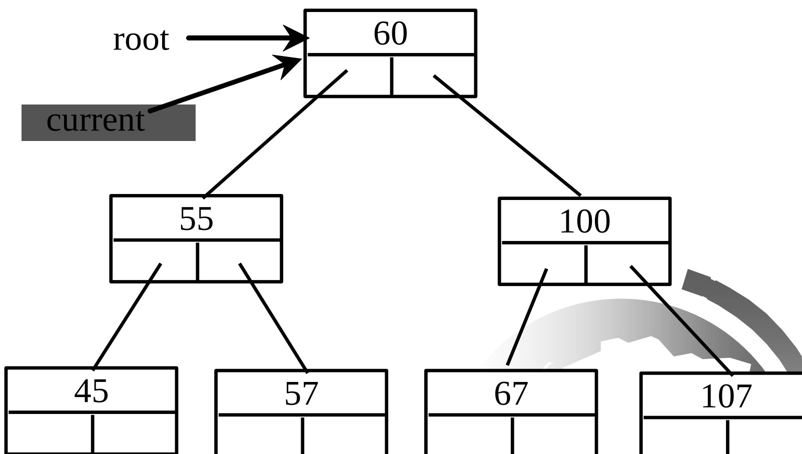
Insert 101 into the following tree.

101 < 60?



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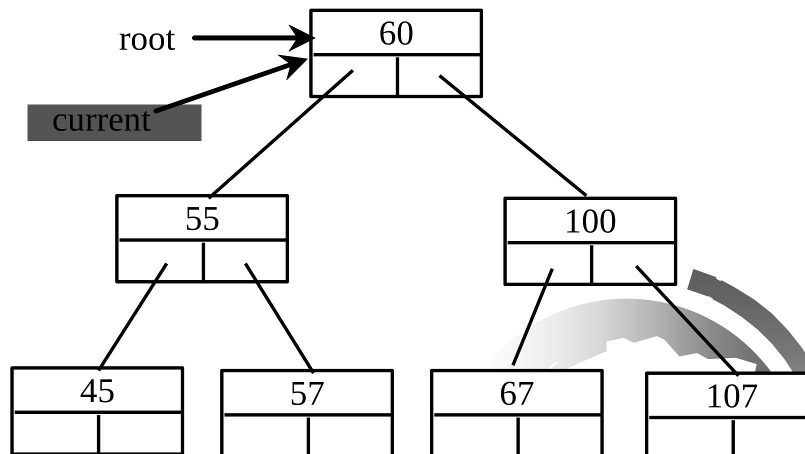

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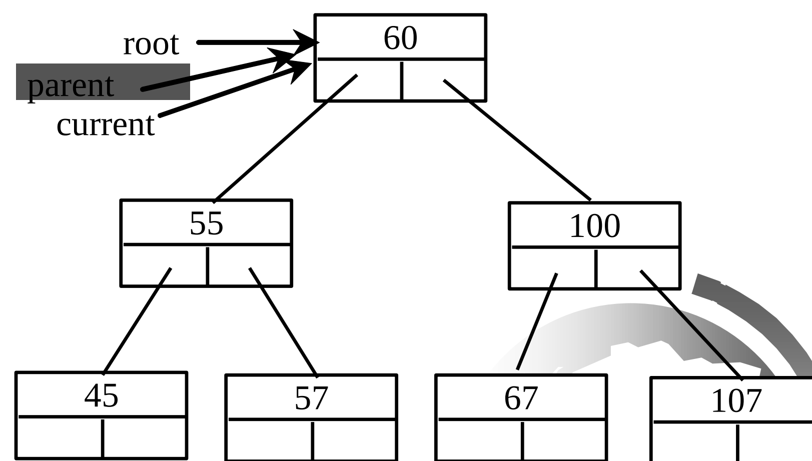

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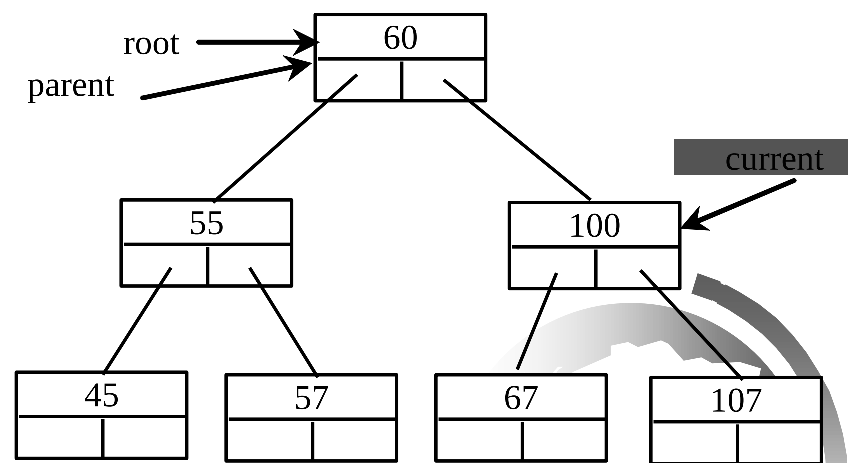

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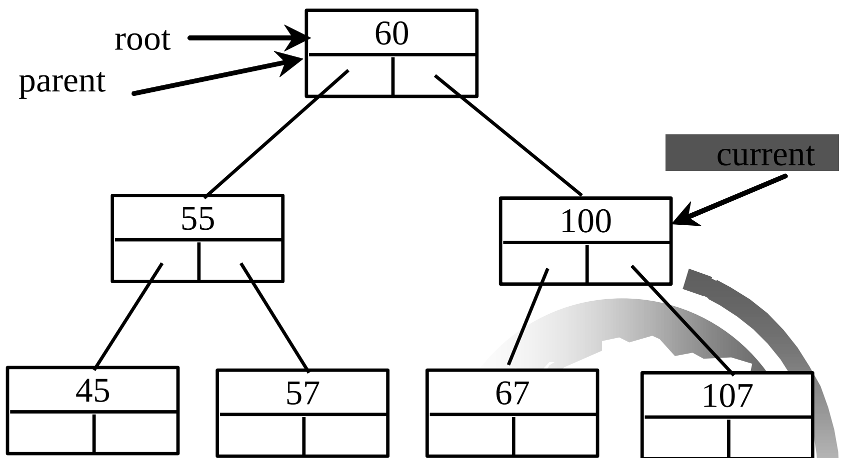

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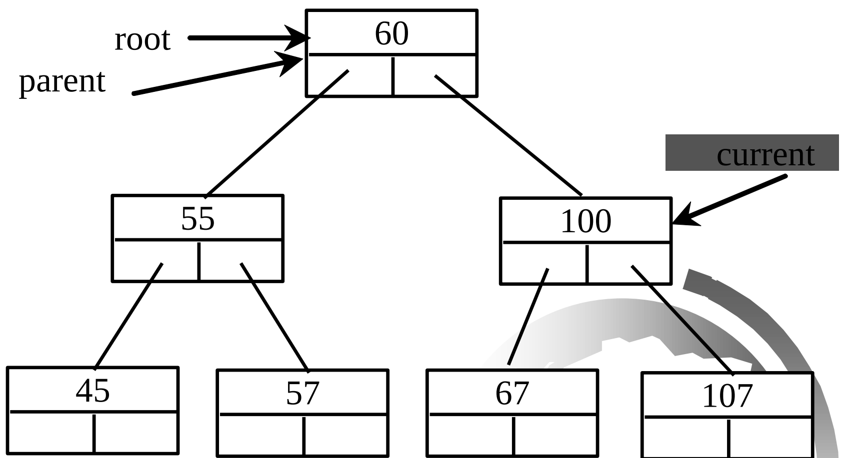

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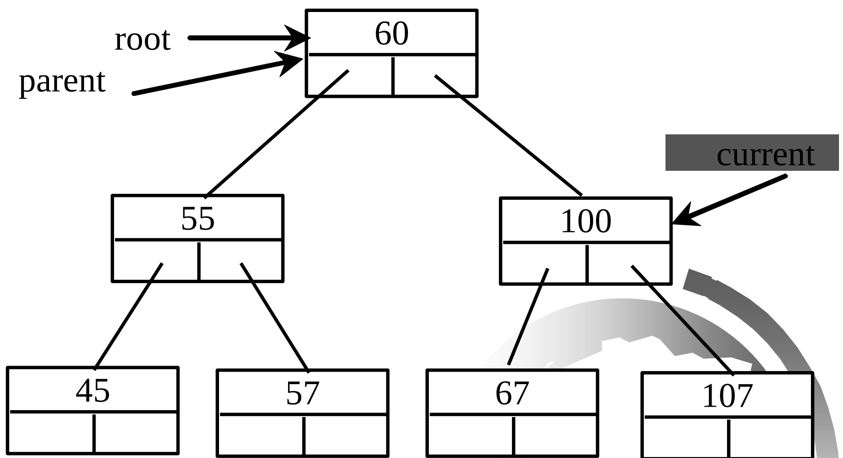

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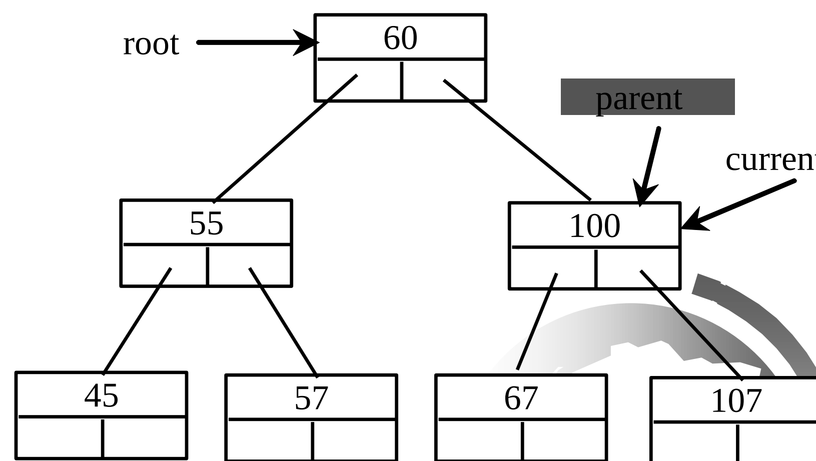

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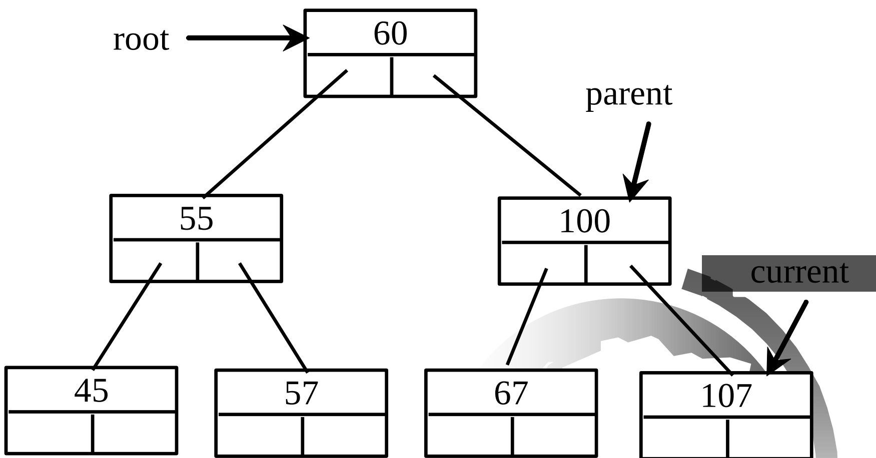

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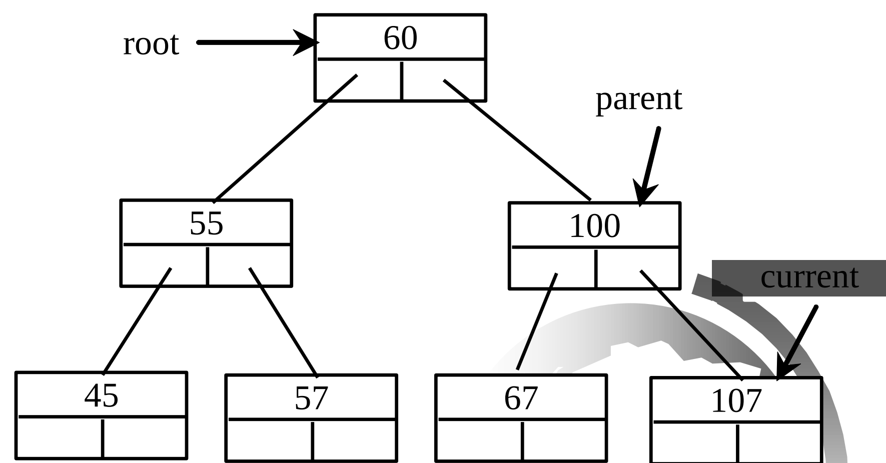

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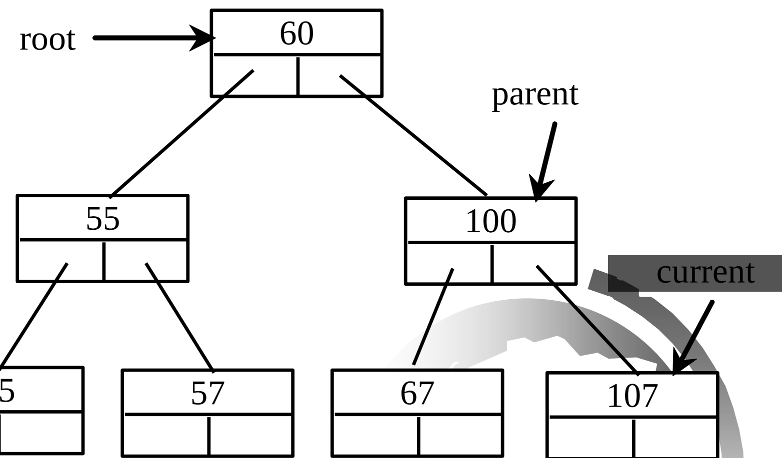
101 < 107 true



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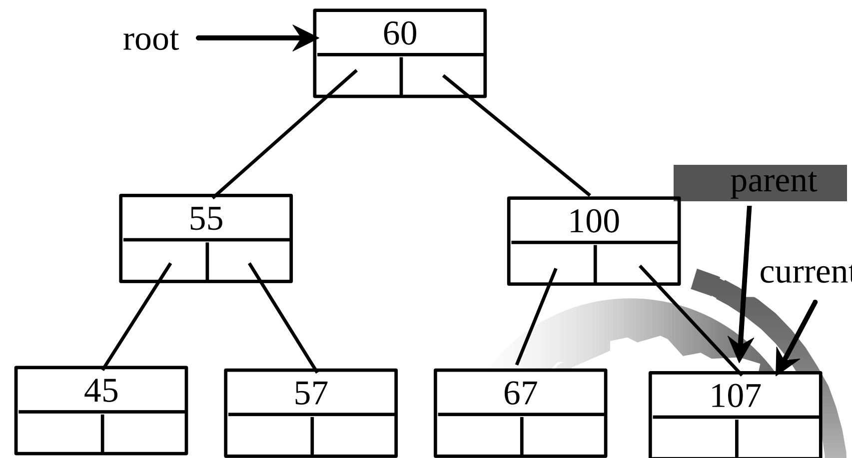

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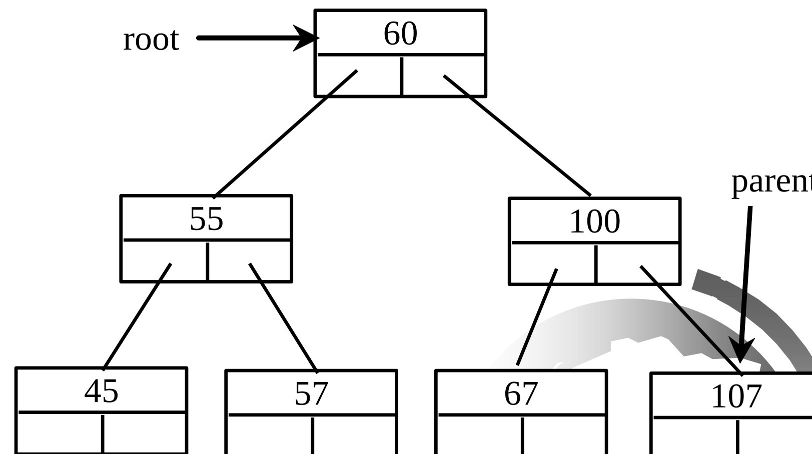

Since current.left is null,current becomes null

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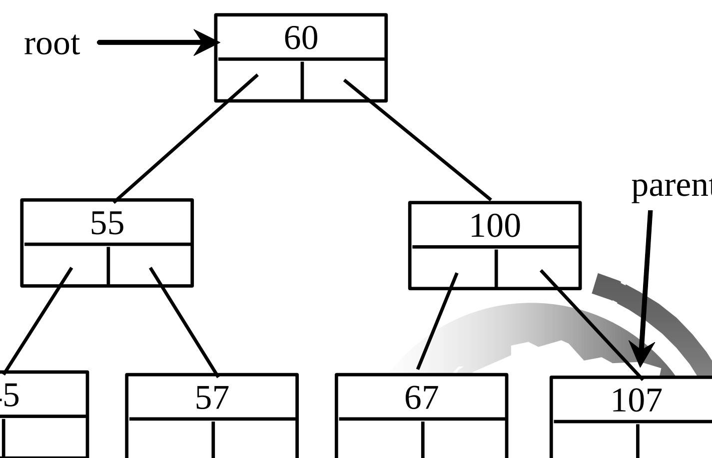

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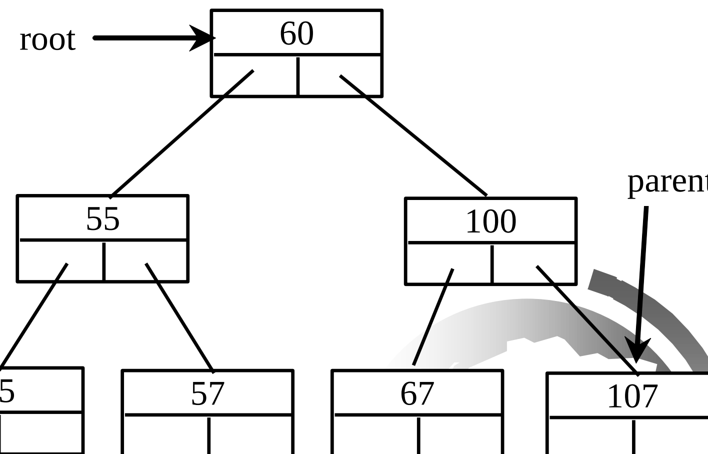

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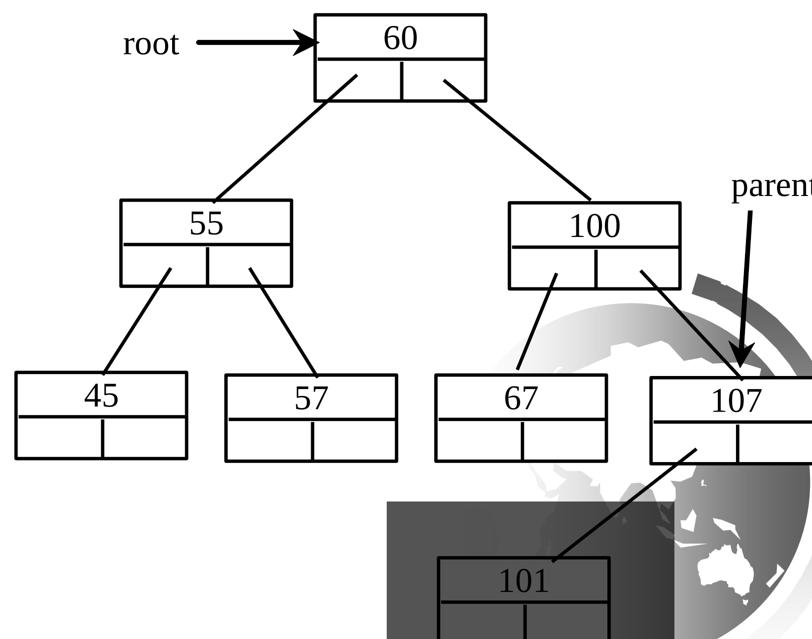

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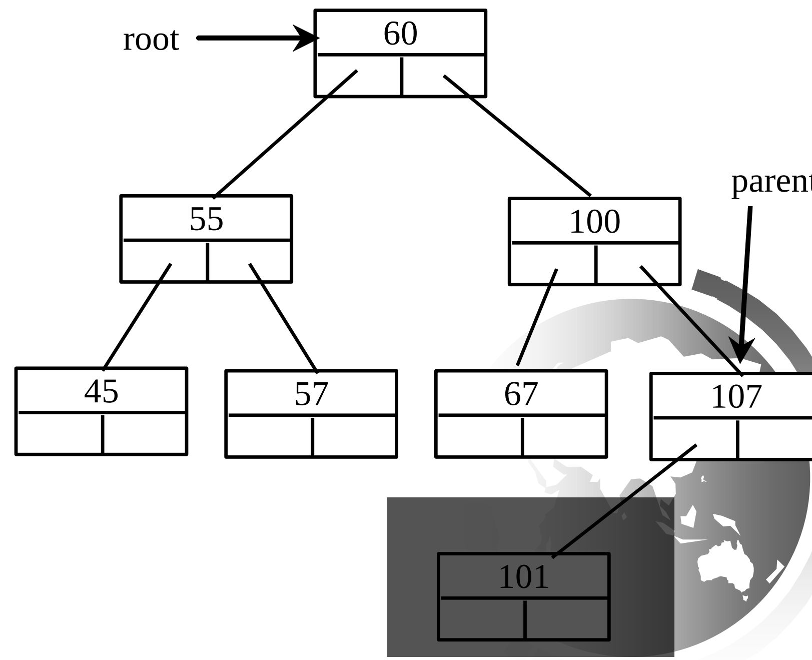
    }
    else
      return false; // Duplicate node not inserted

  // Create the new node and attach it to the parent node
  if (element < parent.element)
    parent.left = new TreeNode(elemenet);
  else
    parent.right = new TreeNode(elemenet);

  return true; // Element inserted
}
```

101 < 107 true

Insert 101 into the following tree.

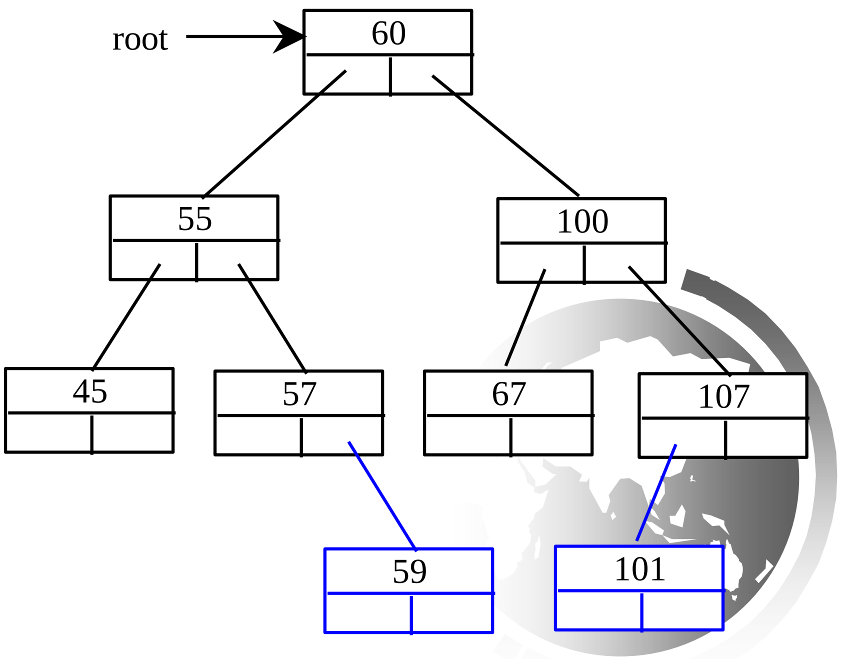


Inserting 59 into the Tree

```
if (root == null)
  root = new TreeNode(element);
else {
  // Locate the parent node
  current = root;
  while (current != null)
    if (element value < the value in current.element) {
      parent = current;
      current = current.left;
    }
    else if (element value > the value in current.element) {
      parent = current;
      current = current.right;
    }
    else
      return false; // Duplicate node not inserted

  // Create the new node and attach it to the parent node
  if (element < parent.element)
    parent.left = new TreeNode(elemenet);
  else
    parent.right = new TreeNode(elemenet);

  return true; // Element inserted
}
```



Tree Traversal

Tree traversal is the process of visiting each node in the tree exactly once. There are several ways to traverse a tree. This section presents *inorder*, *preorder*, *postorder*, *depth-first, and breadth-first* traversals.

The inorder traversal is to visit the left subtree of the current node first recursively, then the current node itself, and finally the right subtree of the current node recursively.

The postorder traversal is to visit the left subtree of the current node first, then the right subtree of the current node, and finally the current node itself.

Tree Traversal, cont.

The preorder traversal is to visit the current node first, then the left subtree of the current node recursively, and finally the right subtree of the current node recursively.

Tree Traversal, cont.

The breadth-first traversal is to visit the nodes level by level. First visit the root, then all children of the root from left to right, then grandchildren of the root from left to right, and so on.

For example, in the tree in Figure 25.2, the inorder is 45 55 57 59 60 67 100 101 107. The postorder is 45 59 57 55 67 101 107 100 60. The preorder is 60 55 45 57 59 100 67 107 101. The breadth-first traversal is 60 55 100 45 57 67 107 59 101.

The Tree Interface

«interface»
java.lang.Collection<E>

«interface»
Tree<E>

`+search(e: E): boolean`	Returns true if the specified element is in the tree.
`+insert(e: E): boolean`	Returns true if the element is added successfully.
`+delete(e: E): boolean`	Returns true if the element is removed from the tree successfully.
`+inorder(): void`	Prints the nodes in inorder traversal.
`+preorder(): void`	Prints the nodes in preorder traversal.
`+postorder(): void`	Prints the nodes in postorder traversal.
`+getSize(): int`	Returns the number of elements in the tree.
`+isEmpty(): boolean`	Returns true if the tree is empty.
`+clear(): void`	Removes all elements from the tree.

Override the add, isEmpty, remove, containsAll, addAll, removeAll, retainAll, toArray(), and toArray(T[]) methods defined in Collection using default methods.

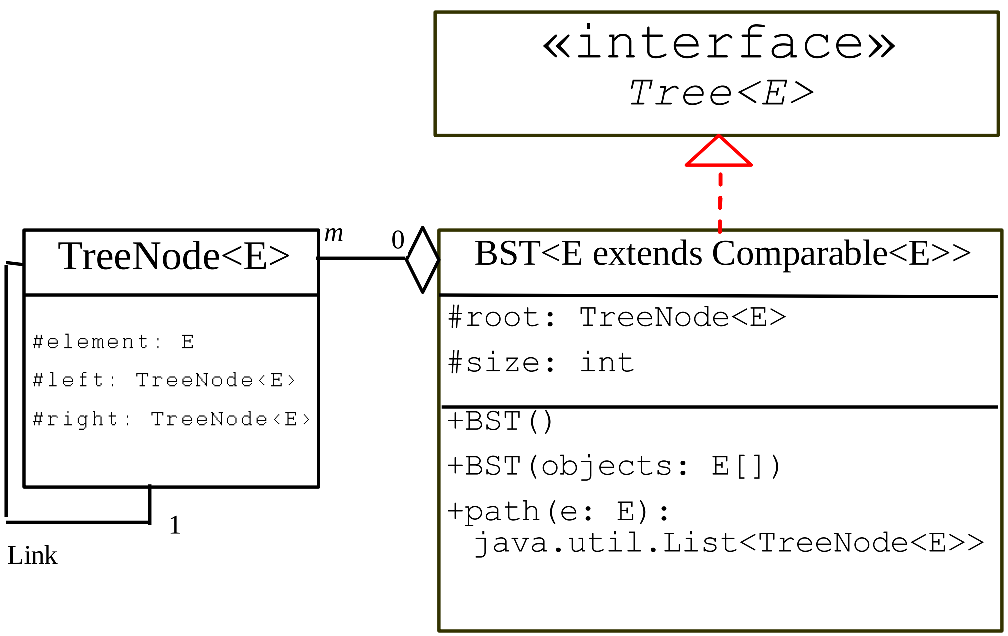
The Tree interface defines common operations for trees.

Tree

The BST Class

Let's define the binary tree class, named BST with A concrete BST class can be defined to extend AbstractTree.

BST



The root of the tree.

The number of nodes in the tree.

Creates a default BST.

Creates a BST from an array of elements.

Returns the path of nodes from the root leading to the node for the specified element. The element may not be in the tree.

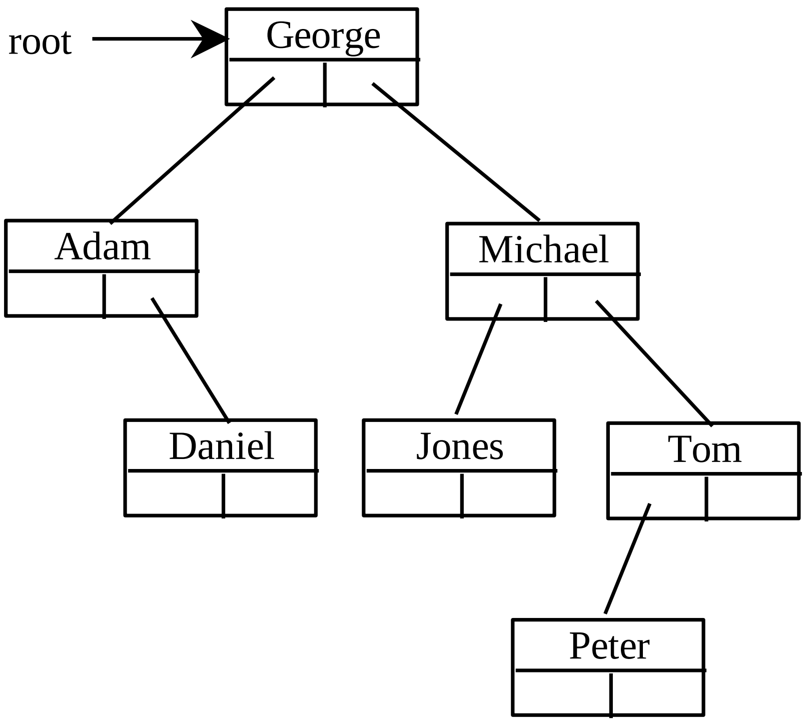
Example: Using Binary Trees

Write a program that creates a binary tree using BST. Add strings into the binary tree and traverse the tree in inorder, postorder, and preorder.

TestBST

Run

Tree After Insertions



Inorder: Adam, Daniel George, Jones, Michael, Peter, Tom

Postorder: Daniel Adam, Jones, Peter, Tom, Michael, George

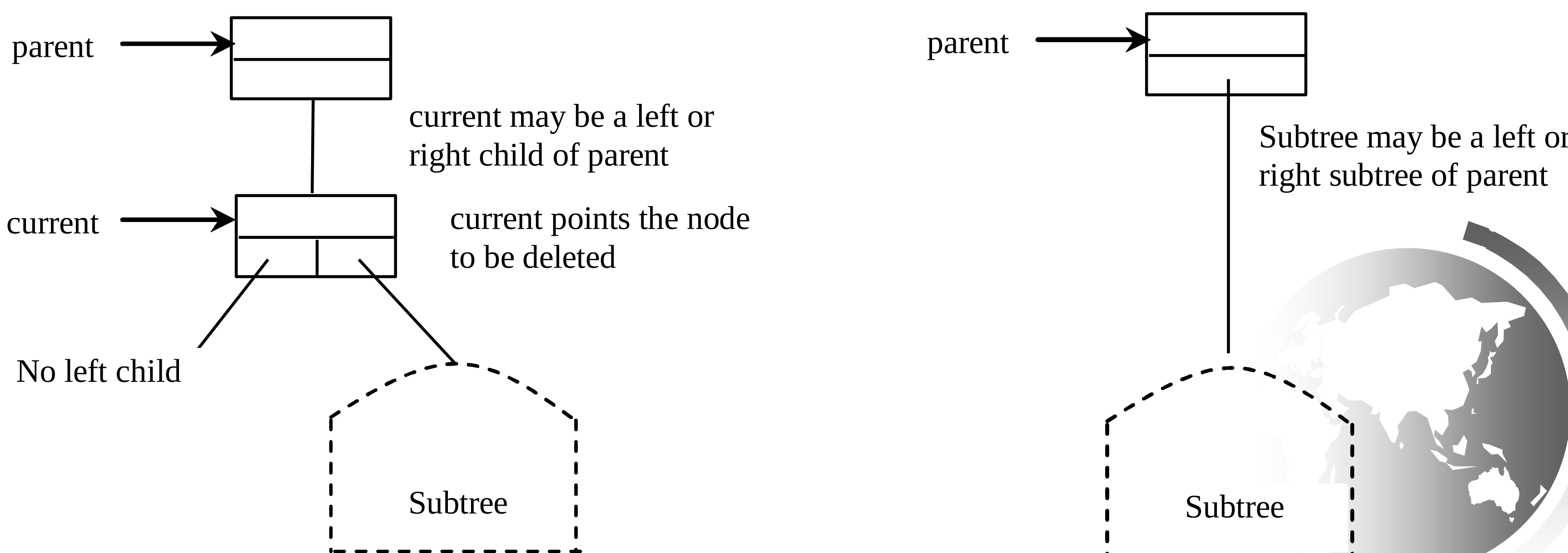
Preorder: George, Adam, Daniel, Michael, Jones, Tom, Peter

Deleting Elements in a Binary Search Tree

To delete an element from a binary tree, you need to first locate the node that contains the element and also its parent node. Let current point to the node that contains the element in the binary tree and parent point to the parent of the current node. The current node may be a left child or a right child of the parent node. There are two cases to consider:

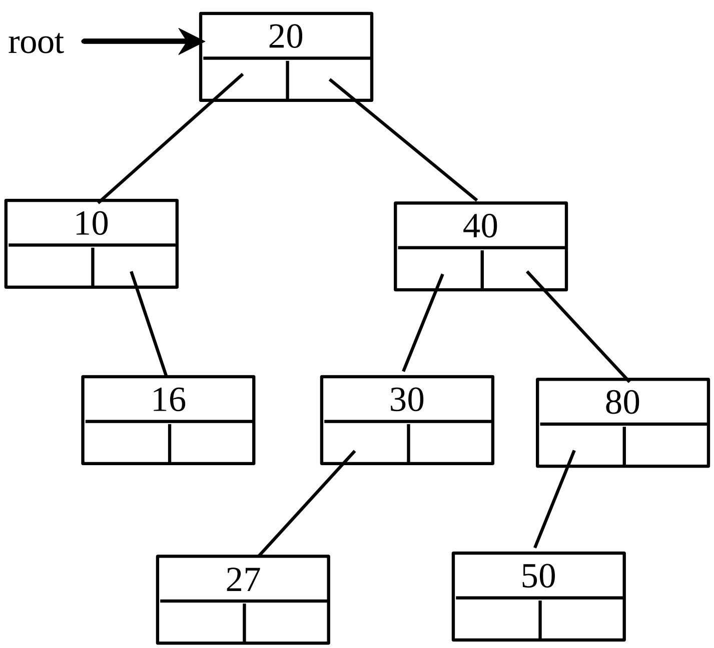
Deleting Elements in a Binary Search Tree

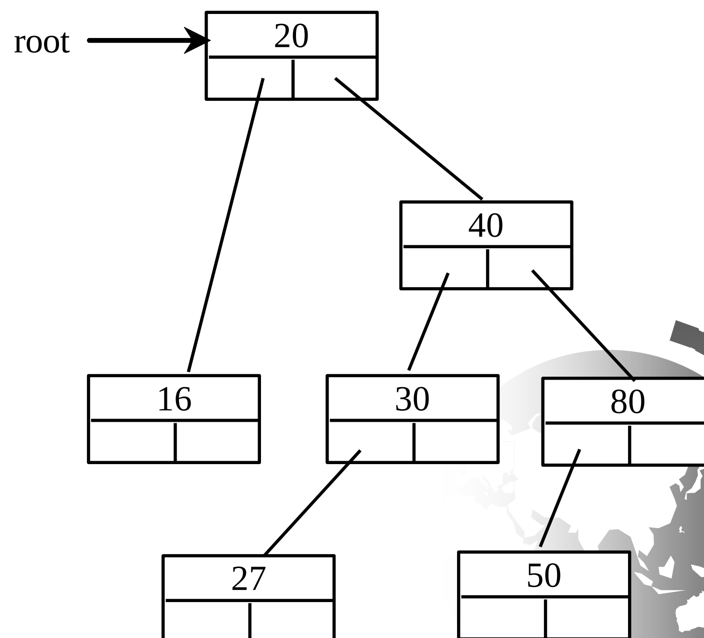
Case 1: The current node does not have a left child, as shown in this figure (a). Simply connect the parent with the right child of the current node, as shown in this figure (b).



Deleting Elements in a Binary Search Tree

For example, to delete node 10 in Figure 25.9a. Connect the parent of node 10 with the right child of node 10, as shown in Figure 25.9b.



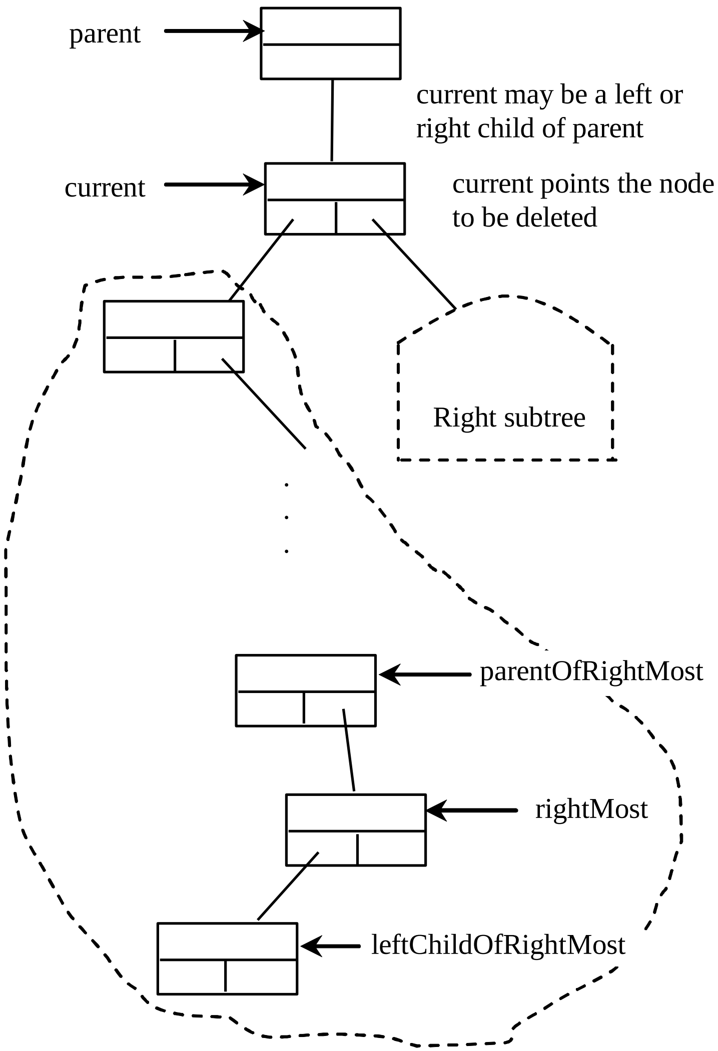


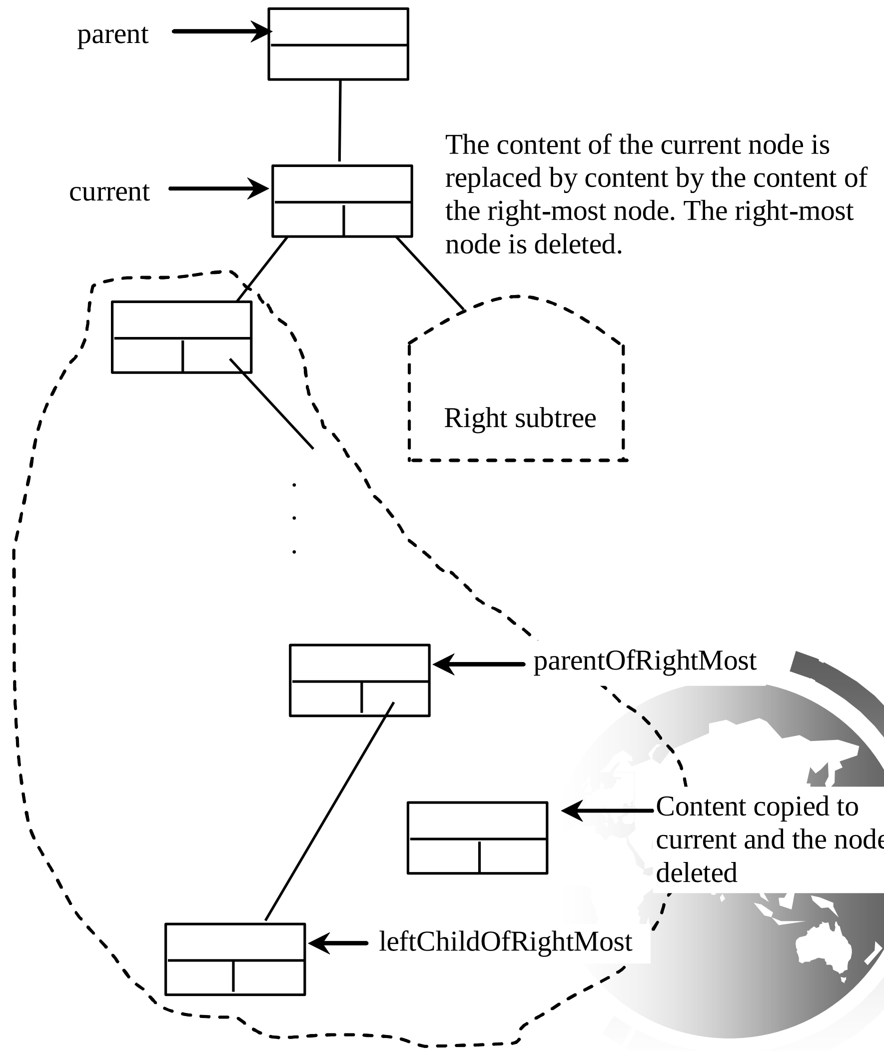
Deleting Elements in a Binary Search Tree

Case 2: The current node has a left child. Let rightMost point to the node that contains the largest element in the left subtree of the current node and parentOfRightMost point to the parent node of the rightMost node, as shown in Figure 25.10a. Note that the rightMost node cannot have a right child, but may have a left child. Replace the element value in the current node with the one in the rightMost node, connect the parentOfRightMost node with the left child of the rightMost node, and delete the rightMost node, as shown in Figure 25.10b.

Deleting Elements in a Binary Search Tree

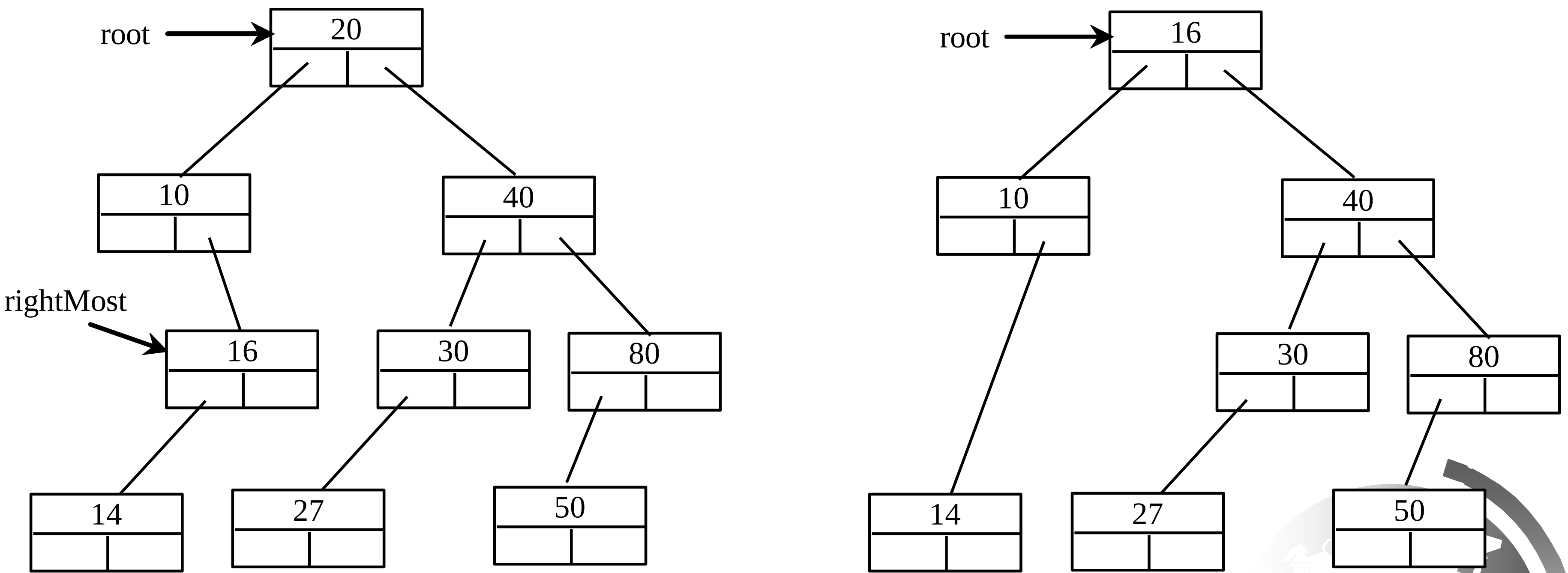
Case 2 diagram



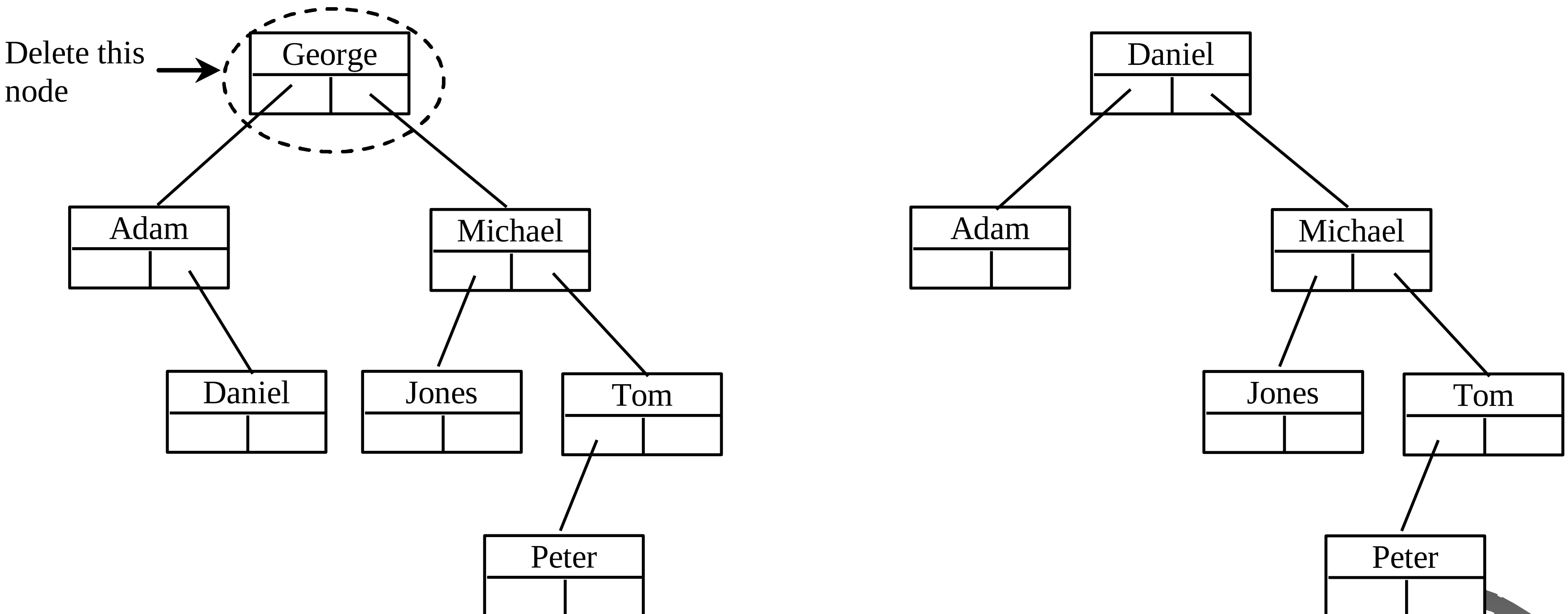


Deleting Elements in a Binary Search Tree

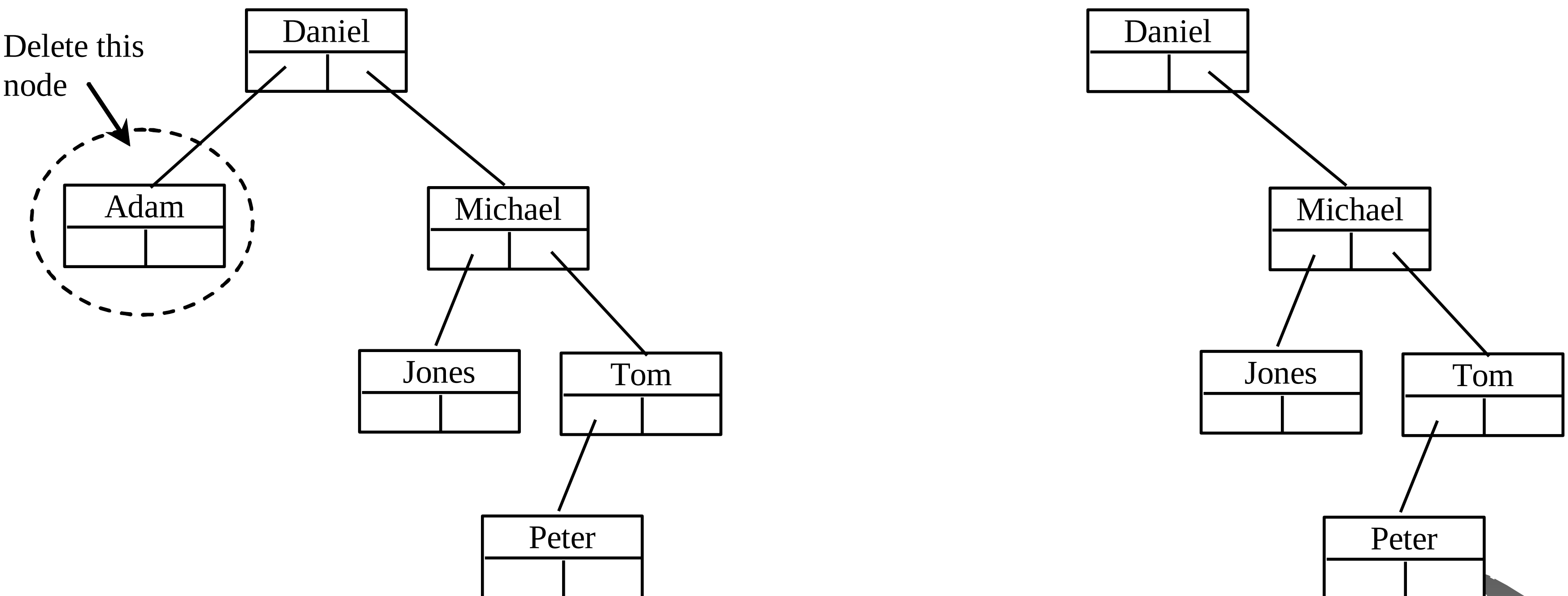
Case 2 example, delete 20



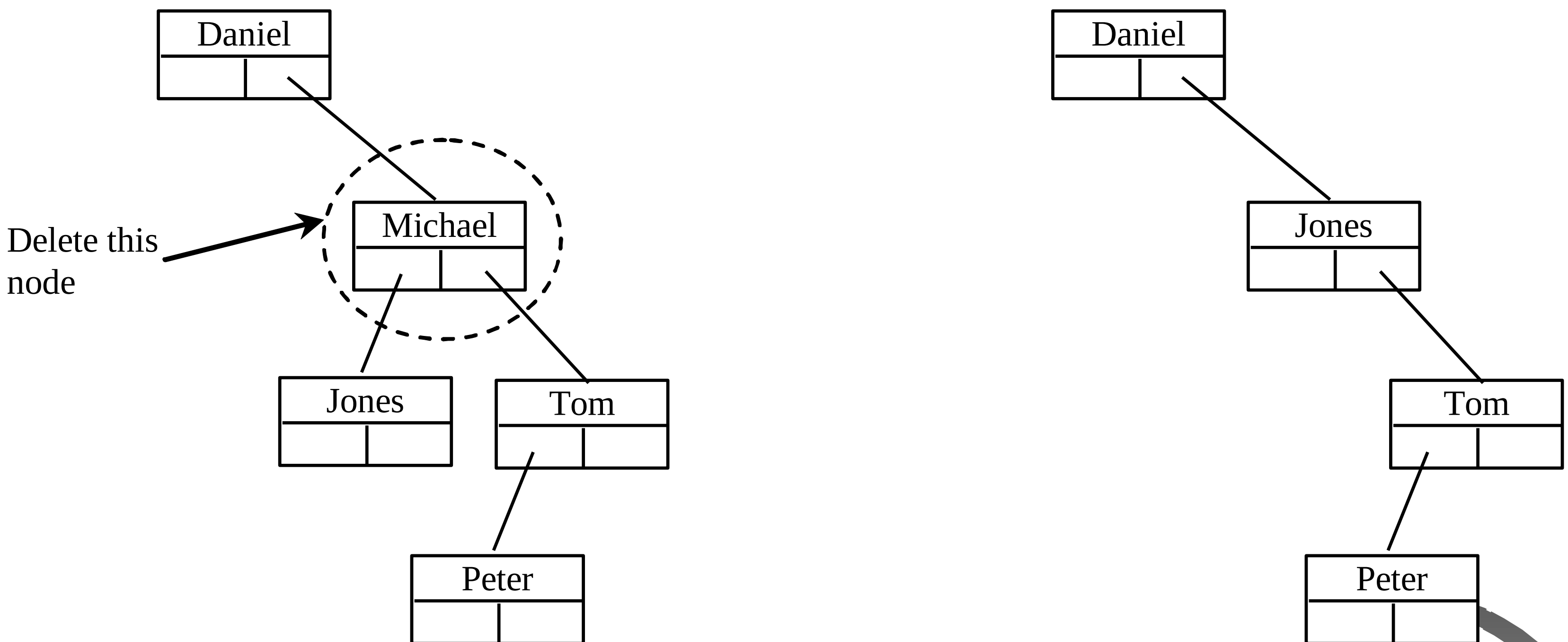
Examples


Delete this node
George
Adam
Michael
Daniel
Jones
Tom
Peter
Daniel
Adam
Michael
Jones
Tom
Peter

Examples


Delete this node
Daniel
Adam
Michael
Jones
Tom
Peter
Daniel
Michael
Jones
Tom
Peter

Examples



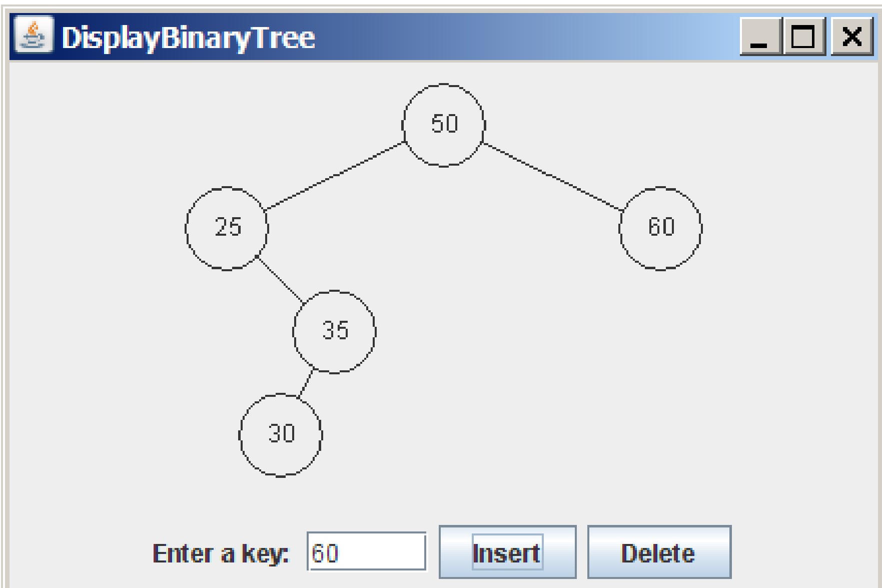
TestBSTDelete

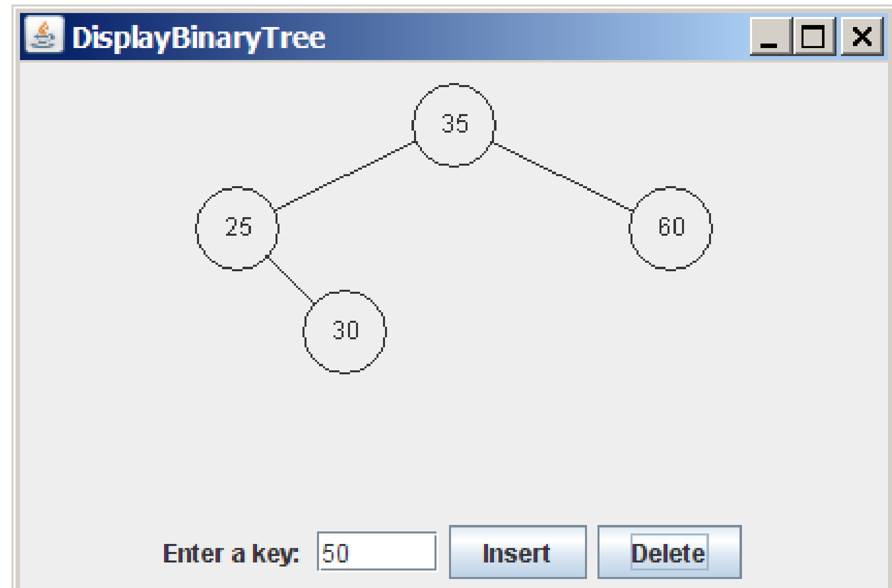
Run

binary tree time complexity

It is obvious that the time complexity for the inorder, preorder, and postorder is O(n), since each node is traversed only once. The time complexity for search, insertion and deletion is the height of the tree. In the worst case, the height of the tree is O(n).

Tree Visualization





BTView BSTAnimation Run

Iterators

An *iterator* is an object that provides a uniform way for traversing the elements in a container such as a set, list, binary tree, etc.

«interface» *java.util.Iterator<E>*	
+hasNext(): boolean	Returns true if the iterator has more elements.
+next(): E	Returns the next element in the iterator.
+remove(): void	Removes from the underlying container the last element returned by the iterator (optional operation).

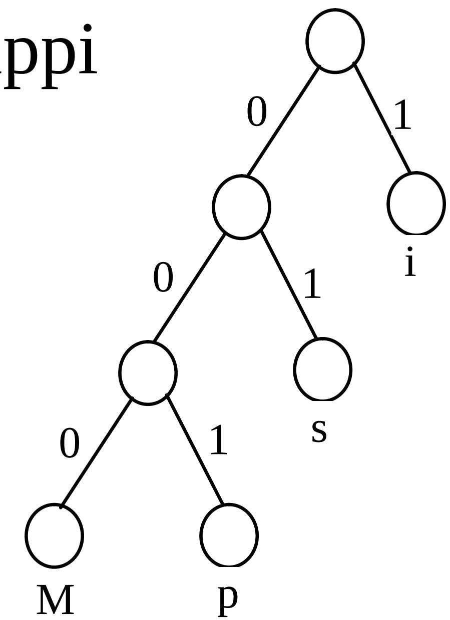
TestBSTWithIterator

Run

Data Compression: Huffman Coding

In ASCII, every character is encoded in 8 bits. Huffman coding compresses data by using fewer bits to encode more frequently occurring characters. The codes for characters are constructed based on the occurrence of characters in the text using a binary tree, called the *Huffman coding tree*.

Mississippi



Character	Code	Frequency
M	000	1
p	001	2
s	01	4
i	1	4

000101011010110010011

21 bits

Constructing Huffman Tree

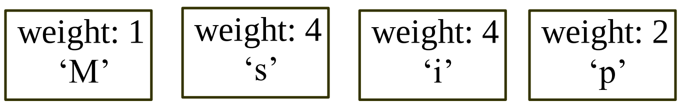
To construct a *Huffman coding tree*, use a greedy algorithm as follows:

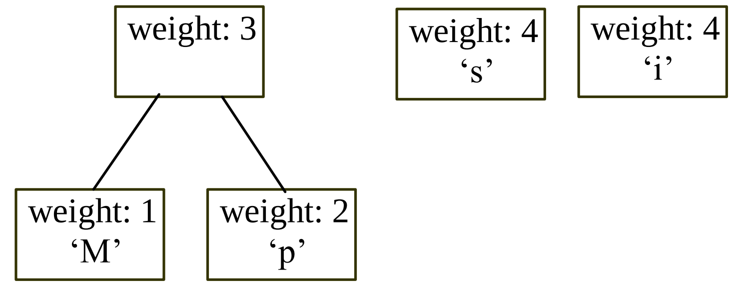
- Begin with a forest of trees. Each tree contains a node for a character. The weight of the node is the frequency of the character in the text.
- Repeat this step until there is only one tree:

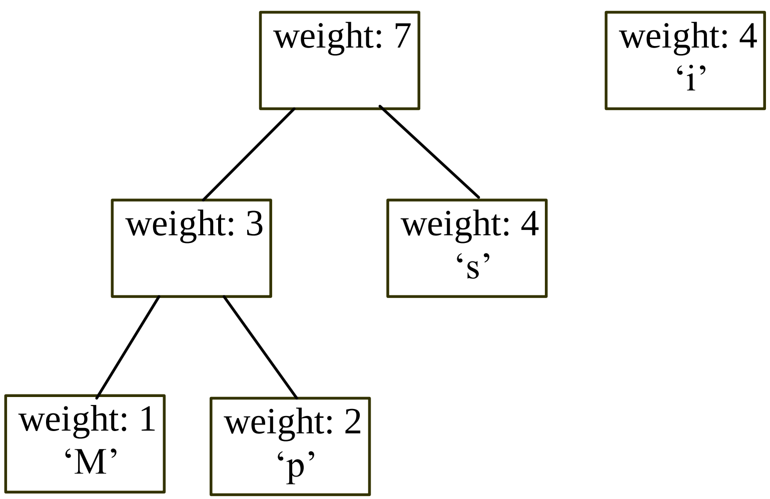
  Choose two trees with the smallest weight and create a new node as their parent. The weight of the new tree is the sum of the weight of the subtrees.

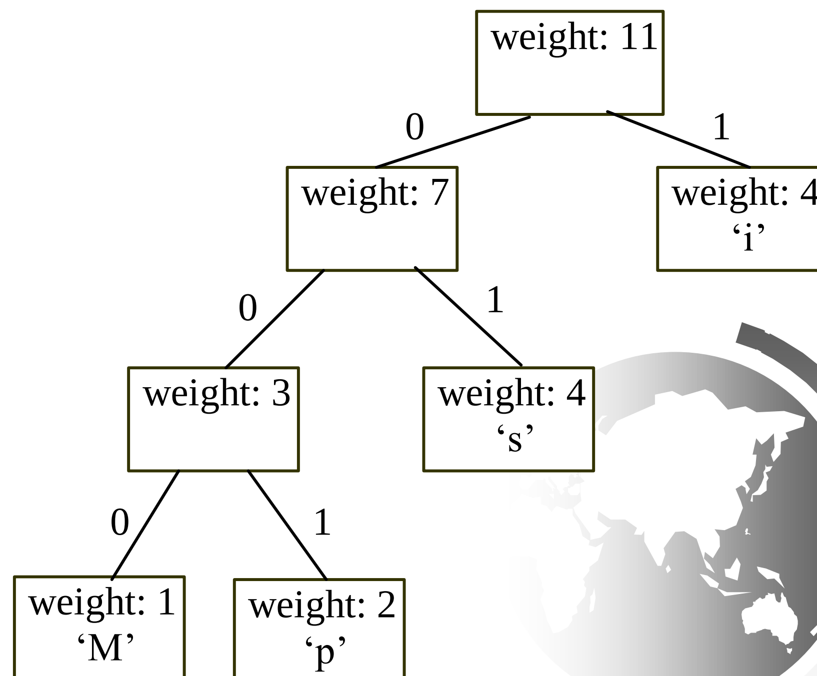
Constructing Huffman Tree

Mississippi









Constructing Huffman Tree

HuffmanCode

Run