Luiss Guido Carli University

Masters in Data science and management

Optimizing Financial Portfolio management using integrated Fuzzy logic, Game theory, and Reinforcement learning: Python

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1. MPT: Modern Portfolio Theory
2. DQN: Deep Q-Networks
3. RL: Reinforcement learning
4. DDPG: Deep Deterministic Policy Gradient
5. PPO:Proximal policy optimization
6. CAPM: Capital Asset Pricing Model
7. Q-Learning: Quality Learning
8. VaR: Value at Risk
9. POM: Portfolio Financial Management
10. PFM: Portfolio Financial Management
11. GT: Game Theory
12. FL: Fuzzy Logic
13. ETFS: Exchange Trade Funds
14. SQ:Square, Inc. Class A Common Stock
15. AAPL:Apple Inc. - Common Stock
16. MSFT:Microsoft Corporation - Common Stock
17.MSF: Microsoft -common stock
18. NASDAQ: National Association of Security Dealers Automated Quotations
19. AMZN: Amazon inc-common stock
20. Yfinance: Yahoo Finance
21. S& P: Standard & Poor's (a financial services company that creates indices like S&P 500)
21. GSPC: S&P 500 index (used as a ticker symbol for the index)
22. EDA: Exploratory Data analysis
23. CSV: Comman Separated Values
24. KDE: Kernel Density Estimation
25. ROI : Return on investment
26. RSI: Relative Strength Index
27. EMA: Exponential Moving Average

28. SMMA: Smoothed Moving Modifed Average
29. RS: Relative Strength

Abstract

This Thesis presents an innovative approach for managing investments that relies on fuzzy logic, game theory and reinforcement learning combined model to help make choice about how to allocate resources intelligently in unsteady markets. The aim is develop and analyse dynamic framework that adopts investment strategies as they change in the market and depending on how different investors interact. In order to handle ambiguity and imprecision in market data, this study uses a mixed model that combines different methods: fuzzy logic is used to provide a clear understanding of uncertain information, game theory helps in analyzing competitive & cooperative behaviors of market players while reinforcement learning technique continually upgrades investment strategies as it interacts with the market setting using Python programming language. The framework is tested on historical financial data to evaluate its performance in terms of returns, risk management, and resilience during periods of market volatility. The results are expected to demonstrate the feasibility and effectiveness of integrating these diverse methodologies, by providing valuable insights into their combined benefits for portfolio optimization, this research seeks to contribute to academic researches of the interdisciplinary nature of finance, data science, investment,and also portfolio mangers and investors in the porfolio.

CHAPTER 1
INTRODUCTION

1.1 Background and Motivation

Financial markets are the center of global economy and they act as systems with which capital is allocated or valued through pricing of assets or facilitating trade. Nonetheless, these systems are subject to uncertainty and complex nature due to different factors such as economic signals and human decisions. It takes smart tactics that can change with the form of the market in order to effectively handle investment portfolios in such surroundings while still balancing risks with returns. Normal methods utilized include modern portfolio theory (MPT) with its forms of mean-variance optimization whose aim is constructing varied portfolios from past asset returns as well as correlations. Although they are commonly used, the majority of these models expect that the conditions of the markets remain constant and the investors will act logically yet this may not always be the case. Additionally, strategies that are guided by the MPT approach do not provide the best results in times of unstable and more volatile market because they are influenced by the variables that are required as inputs in their calculations and model estimation errors can make them produce bad results.

In the world of investment, financial portfolio management is a key factor that plays a significant role in how successful and robust investments are. In other words, it focuses on managing a group of assets with a view to increasing returns and minimizing risks. Nevertheless, There are a lot of changes and uncertainties in the dynamic places portfolio managers are working in because they are always changing.

Balancing risk and reward is the main objective of managing finances through portfolios. The idea behind this is not just earning money by saving more than one earns in order to increase it with time. Asset allocation, risk assessment, and periodic re-balancing are some of the main components of portfolio management. They are crucial in determining how a portfolio performs and bounces back from losses.

The financial markets are riddled with uncertainties that come in different shapes including economic instability, geopolitical tensions, and sudden market volatility. Such uncertainties, in turn, make investing harder as far as managing one's portfolio as they disrupt existing strategies hence asset managers are required to keep on changing their course all the time so they can succeed or at least break-even in this hostile environment. The traditional portfolio optimization models struggle in providing effective solutions under conditions of uncertainty as they more often underestimate than overestimate risks and returns.

The management of portfolios under uncertain market conditions faces many challenges which require creative ways and advanced methods, not least among them being problems of predicting future values for assets (compounded by their volatilities), determining how such values will behave with time, and specifying associations between different classes of investments; also there is increased danger from rare occurrences threatening most conventional criteria of danger.

Recognizing constraints to usual Portfolio Management styles, there becomes a pressing need for mixed advanced ones able to move with ease in this uncertain market. In this context, using fuzzy logic, game theory and reinforcement learning together makes promises of increased portfolio management efficacy. These advanced methods have the ability to respond to changing market trends, discover non-linear connections and improve decision formation when there is high confusion of actions.

Financial portfolio management operates within a dynamic landscape characterized by uncertainties, which creates significant challenges for conventional approaches. Theoretical foundations as well as practical applications related to optimizing portfolio management under uncertainty through integration of fuzzy logic, game theory, and reinforcement learning would thus be discussed in the following chapters. The purpose of this research is to clarify inventive approaches which increase efficiency and robustness of portfolios under various conditions in markets.

1.2 problem statement

The challenge of optimizing financial portfolio management while other investors are making loss, investors trying to outdo each other and the market is unstable is real in investment management. This problem lies in the necessity to develop and handle portfolios with a given intensity of volatility in addition to strategic reactions from other market players.

There is a degree of variation or dispersion in returns of financial instrument or market index. While asset prices move unpredictably, the antecedent to uncertainty in investment decisions introduces risk and ambiguity in investment decisions as there is no steady increase in return on investment or profits for any asset class and this could lead an investor to gain wealth overnight or lose everything one has when dealing with such instruments.. Each investor in financial markets is not an isolated player. They are strategic actors that influence market dynamics as well as asset prices. These interactions include information acquisition, trading strategies and herding hence must be accounted for in portfolio management actions. A problem exists with regard to portfolio management amidst volatile markets. It involves designing strategies which lessen the risks while capturing the opportunities that exists within the market. Portfolio management must thus take into consideration these strategic interactions.

1.2.1 **Key components of the Problem**

1. Risk management
2. Asset valuation
3. Game-theoretic considerations
4. Adaptive-Decision making

1.2.2 **Objective**

The purpose of solving this problem is to create innovative investment management approaches that can move smoothly through unstable market conditions and communicate effectively with various other investors for the purpose of maximizing returns after adjusting for risk. What we want to achieve by using sophisticated nuggets from disciplines like fuzzy logic, game theory, and reinforcement learning is that these will help to augment both stability as we know it as well improve overall returns when taking into account volatile market times as well as those characterized by more uncertainty.

1.3 Research purpose

The purpose of this research is to investigate how integrating fuzzy logic, game theory, and reinforcement learning can effectively address the challenges encountered in optimizing portfolio management, particularly in volatile markets with strategic interactions among investors. By examining the synergistic effects of these methodologies, the research aims to contribute to the development of innovative approaches that enhance decision-making, adaptability, and performance in dynamic and uncertain market environments.

The Research purpose involve:

- · **Enhancing Decision-making under uncertainty**:

  One key aspect of portfolio management in volatile markets is the need to make informed decision amidst uncertainty. Fuzzy logic offers flexible framework for reasoning with uncertain and imprecise information, allowing the incorporation of qualitative assessments alongside quantitative data. By integrating fuzzy logic into portfolio management, decision-makers can better capture the nuanced relationships between market variables and investment decisions, leading to more robust and adaptive strategies.

- · **Modeling strategic interactions among investors**:

  In competitive market environments, strategic interactions among investors play crucial role in shaping asset prices and market dynamics. Game theory provides a formal framework for analyzing these interactions and predicting their outcomes. By integrating game-theoretic models into portfolio management, researchers can gain insights into the strategic behaviors of market participants and designs strategies that exploit inefficiencies or mitigate risks arising from competitive pressures.

- · **Adaptive learning and optimization:**

  The dynamic nature of financial markets necessitates adaptive learning and optimization techniques to continuously update portfolio strategies based on evolving market conditions. Reinforcement learning offers a principled approach to adaptive decision-making, allowing agents to learn optimal strategies through and error interactions with their environment. By integrating reinforcement learning into portfolio management, researchers can develop algorithms that adaptively optimize portfolio strategies in response to changing market dynamics and investor objectives.

- · **Synergies and complementarities**:

  When you combine fuzzy logic with game theory as well as reinforcement learning, scientists can easily utilize the area of common interest as well as the enhancement characteristics between them in solving the matters of better portfolio management. Fuzzy logic hence allows for flexible modeling and dealing with uncertainties in reasoning while at the same time game theory explains the strategic investor relationships. Reinforcement learning closes the gap as it allows for adaptive optimization.

- · **Practical Implications and Applications**:

  The investigation has the purpose of creating real-life application for the abstract ideas, so that those busy managing assets can gain from it. By combining these three very different approaches in the research process they are most likely to generate specific guidelines on how to manage portfolios within an uncertain environment. The number of possible investment strategies is infinite. Although this is true for the number of views of the investment strategies' market performance, there are methods through which investors can evaluate these strategies today. One of the methods through which investors can assess their strategies today is by seeking the opinions of others who are active in this field. These strategies may include dynamic asset allocation, risk management techniques and adaptive trading strategies that capitalize on emerging opportunities while mitigating risks associated with uncertainty and strategic interactions among investors.

Ultimately, this research seeks to investigate how combining fuzzy logic, game theory and reinforcement learning, would offer a new solution to solving the problem of portfolio management in turbulent markets with strategic interactions among investors. By examining the synergies and complementarities aimed at using these methodologies, the research is designed to improve understanding on portfolio management and offer innovative ways on how high performance and protection from collapsing values can be achieved despite changing and unclear market conditions.

1.4 significance of the study

The proposed approach of integrating fuzzy logic, game theory and reinforcement learning into portfolio management has the potential to significantly impact portfolio performance and risk management in several key ways. By examining the potential implications of this approach, we can elucidate its significance in enhancing decision-making, adaptability, and resilience in volatile markets.Referring to the possible future consequences of this combination gives us the ability to understand why it is so important for choosing right and staying flexible while being able to recover quickly during periods of high uncertainty.

1.5 Research questions

Research questions are designed to investigate theoretical foundations, methodological strategies, empirical outcomes and practical implications of computational methodology in financial portfolio management. This framework is useful for observing the complex dynamics of financial markets and the role that advanced computer techniques play in enhancing the quality of fund management decisions. This is the list of questions which will help to dive more into the topic:

1. How may financial portfolio management strategies be optimized in the face of uncertain market conditions by utilizing integrated fuzzy logic, game theory, and reinforcement learning techniques?
2. What are the greatest challenges while managing their financial portfolios that investors must grapple with most especially adjusting dynamically to turbulent market conditions? How can integrated computational approaches assist investors to overcome these obstacles?
3. What game theory, reinforcement learning, and fuzzy logic can do for portfolio optimization and how they compare in terms of relative benefits and drawbacks is a question that has been bothering investors?
4. How do investors' strategic relationships influence the decisions made concerning their investments, and how can game-theoretic models help understand and anticipate such relationships aiming at enhancing the performance of investment portfolios?
5. Applying game theory, reinforcement learning and integrated fuzzy logic techniques to real world portfolio management situations have what practical consequences? up?

1.6 Organization of the thesis

1. **Introduction**: covers the background, problem statement, research purpose, research questions and overview of the thesis structure.
2. **Literature review**: Review existing applications of all these methods in the thesis topic like fuzzy logic, game theory, and reinforcement learning.
3. **Theoretical Framework**: Give an overview of the fuzzy, logic, game theory, reinforcement learning and their integration.
4. **Methodology**: Lists the computational framework, algorithm development, data collection process and data source as this thesis will use secondary data.
5. **Results and discussion**: Presents the findings
6. **Conclusion**: Conclude the work by showing its implication of portfolio management.
7. **References**: Lists all resources used in the thesis

Lists all resources used in the thesis

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction to Fuzzy logic

Fuzzy Logic is an approach based on 'degrees of truth' rather than a binary logic, on which PC working is based. Fuzzy Logic was introduced by (Zadeh (1965)), an American Mathematician that was working on Natural Language. The description of all the informations in our world cannot be described only by 0,1. Fuzzy Logic is an extension of Boolean Logic as it allows to give a broader explanation to the phenomena. 0,1 are the extreme cases of truth, but there are many intermediate degrees. Fuzzy Logic is very useful to imitate human way of thinking.

Comparison between boolean and fuzzy logic

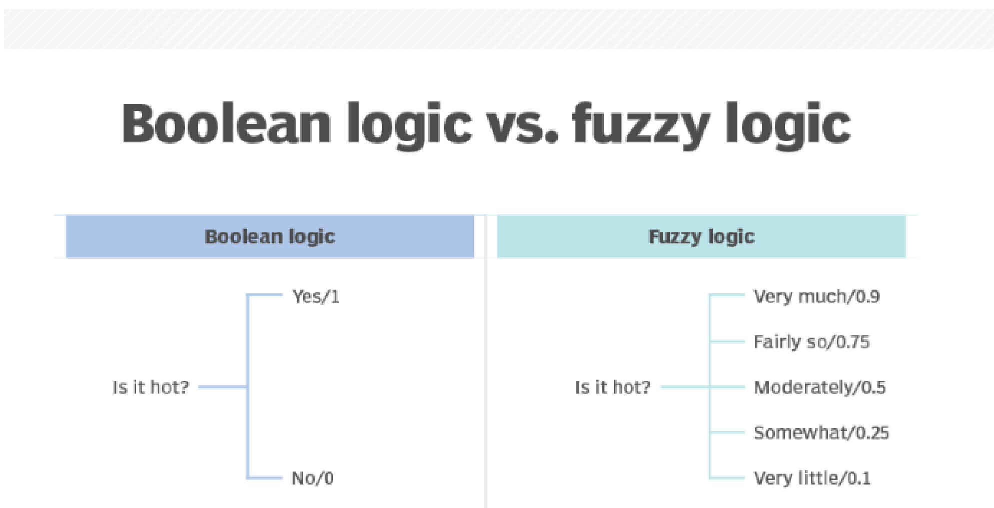

Figure 2.1: boolean vs fuzzy logic(1)

Application of fuzzy logic:

• **Natural Language Processing**: understanding human instinct by artificial intelligence

• **Medicine**: interpretation of sets of medical findings or diagnosis of diseases in Western medicine.
• **Aerospace**: automatic control systems in aerodynamics, computational fluid dynamics, aeroelasticity and so on.

• **Finance**: to understand and model traders irrational behaviour in the field of investments.

• **Dishwashers**: to check the level of water consumption and possibly reduce it.

Principles of fuzzy logic:

• **Fuzzy sets**: are collections of objects with a continuum of grades of membership. Unlike classical sets, which are binary in nature, fuzzy sets allow for degrees of membership ranging from 0 to 1 (Klir & Yuan).

• **Membership functions**: Functions that define the degree to which an element belongs to a fuzzy set, typically ranging from 0 to 1.

• **Fuzzy rules**: Logical statements that use fuzzy sets to describe how to make decisions bases on the input data.

• **Fuzzy Inference System(FIS)**: A system that applies fuzzy logic to map inputs to outputs using a set of fuzzy rules.

Fuzzy Process

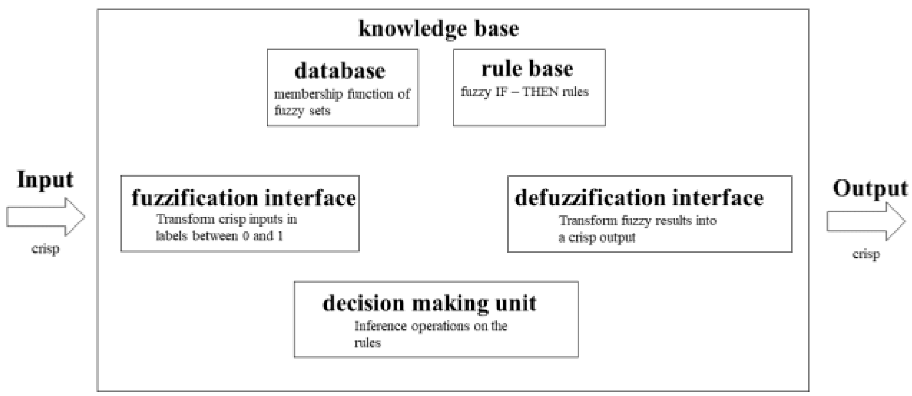

Figure 2.2: Fuzzy Process

2.2 Fuzzy logic in finance

Due to the ability to handle uncertainty and imprecision associated with financial decision-making processes, fuzzy logic has attracted a lot of attention in the field of finance. This subsection examines the areas where fuzzy logic has been applied in financial decision making and also discusses how it has been used effectively in dealing with ambiguity across different financial sectors.

2.2.1 Applications of fuzzy logic in financial decision-making

Risk Management

Fuzzy logic is used in applying risk management techniques, such as determination and control of credit, market, and operational risks. By permitting the inclusion of expert judgements and qualitative risks alongside quantitative statistics, fuzzy logic is increasing the precision and robustness levels in risk assessments.

By reviewing paper by Laszlo Pokaradi from University of Debrecen, Hungary on Fuzzy logic-based risk assessment. They defined as risk as measure of harm or loss associated with human activity. It is a combination of the likelihood and consequence of a specified hazard being realized (Pokoradi, 2002). Determining the context and acceptability of a risk is the aim of risk assessment and characterization, frequently by comparison with other risks that are comparable. With the use of fuzzy logic the outcome of an action can be expressed not only in term of being true or false, it can also gave meaning like very true, fairly true, moderately false.

Category	**Description**
Catastrophic	Complete mission failure, death, or loss of system
Critical	Major mission degradation, severe injury, occupational illness, or major system damage
Moderate	Minor mission degradation, injury, minor occupational illness, or minor system damage
Negligible	Less than minor mission degradation, injury, occupational illness, or minor system damage

Table 2.1: Severity categories

2*		Frequency			
		Frequent (EH)	Likely (H)	Occasional (M)	Seldom (L)
4*Severity	Catastrophic	EH	EH	H	M
	Critical	EH	H	H	M
	Moderate	H	M	M	L
	Negligible	M	L	L	L

Table 2.2: Probaility

EH: Extra High, **H**:High, **M**:Medium, **L**:Low

They combined the severity categories, probability categories to make a table of sample risk assessment matrix. By applying fuzzy logic through its fuzzy process from fuzzification, inference, composition and Defuzzification. below is the figure of the process:

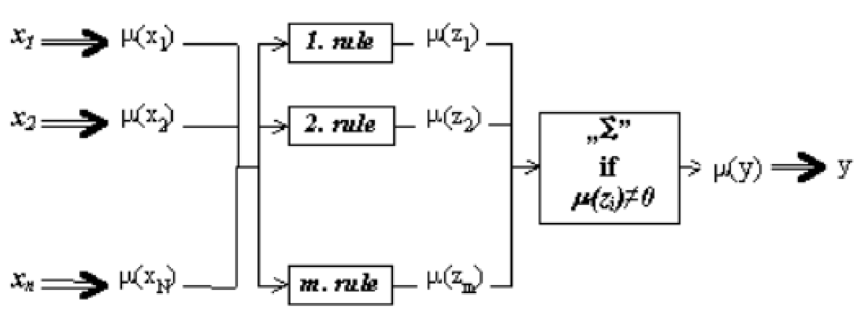

Figure 2.3: Fuzzy logic application

This is used to manage special helicopter mission, air staff assessed it risk using fuzzy logic.

- **Fuzzification**:Membership defined are applied to the actual values to get the degree of truth.
- **Inference**:these are the fuzzy rules used obtained from the table of risk assessment matrix
- **Rule(A)**: if severity is critical and probability is occasional then **risk is high**
- **Rule(B):** if the severity is moderate and probability is occasional then risk is medium
- **Rule(C):** if severity is critical and probability is seldom then risk is medium
- **Rule(D)**: if severity is moderate and probability is seldom then **risk is low**

Using these rules, the risk- result of rule can be high, medium and low, these values got using graphing.

Rule (A) – risk-is-high: $\mu(z_A) = min(0.75; 0.7) = 0.7;$

Rule (B) – risk-is-medium: $\mu(Z_B)min(0.25; 0.7) = 0.25;$

Rule (C) – risk-is-medium: $\mu(Z_C))min(0.75; 0.3) = 0.3;$

Rule (D) – risk-is-low: $\mu(Z_D)min(0.25; 0.3) = 0.25;$

where the risk is medium, they used the max among the two which is 0.3. then the results are:

- $\mu(risk - is - high) = 0.7\mu(risk - is - medium) = 0.3$

- $\mu(risk - is - low) = 0.25$

Defuzzification This create a crisp from fuzzy conclusion set by using the weighted mean of maximum, this method gives the average weighted by their degree of truth of support values at which all the membership functions that apply reach their maximum value.

Z = $\frac{\sum_{i=1}^{n}(w_i \cdot M_i)}{\sum_{i=1}^{n} w_i}$

Then, Z = $\frac{0.25 \cdot 0.85 + 0.3 \cdot 3.5 + 0.7 \cdot 7}{0.25 + 0.3 + 0.7} = 4.93$

The degree of risk is investigated undesired event is 4.93 $\mu(risk - is - medium) = 1$

The air unit commander could use this result to accept the risk during the next step of risk management process. Balancing assessed risk and the probable benefit of the sorti, **the leader permitted the mission and did not prescribe to investigate any risk control tools**.

Portfolio Optimization

The use of fuzzy logic in portfolio optimization functions allows for the integration of subjective criteria and restrictions in constructing a portfolio by investors. In this regard, portfolio optimization models based on fuzzy logic can accommodate uncertain inputs like predicted returns, estimations of risk (volatilities) or correlation coefficients between different assets which results in more life-like and adaptable portfolio strategies.

Financial Forecasting

Fuzzy logic has found usage in financial processes such as forecasting stocks prices, exchange rates or various economic indicators. These models acknowledge uncertainty and complexity in financial markets, generating forecasts that are more dependable and resilient than those produced by the conventional modules.

Credit Score and Rating

Credit risk assessment models use fuzzy logic techniques to appraise the creditworthiness of borrowers and rate financial instruments. By combining various parameters as well as nonlinear dependencies, fuzzy logic allows to increase accuracy and interpretability of credit risk evaluations.

Trading and Investment strategies

Traders and investors have used fuzzy logic to build adaptive trading and investment tactics that adjust depending on market shifts as well as investors' tastes. These trading systems leverage fuzzy rules and linguistic variables to allow for decision-making based on expert knowledge and qualitative judgments, hence making trading strategies more responsive and efficient.

2.2.2 Conclusion

Fuzzy logic has a special place in finance because it gives direction on how to muddle through ambiguity and vague points in financial decision-making procedures. It covers numerous areas such as risk management, portfolio optimization, credit scoring, financial forecasting, and trading strategies. Fuzzy logic-based models enables the integration of quantitative and qualitative information efficiently, adaptability of financial decisions and their robustness; in the end improving fiscal stability and performance through risk management.

2.3 Game theory in financial markets

2.3.1 Introduction

Since the emergence of game theory, it has become highly effective in studying how traders interact in a strategic way and comprehending the way capital markets operate.

2.3.2 Applications of Game theory in Financial Markets

A formal framework for analyzing strategic interactions among rational decision-makers in financial markets, Game theory provides. It has been used to model different scenarios, and to mention a few; it has been applied to competition among fund managers, strategic trading in stock markets and bargaining between buyers and sellers in auctions. Some people have used game theory to study the microstructure about mall financial markets. That includes market makers behavior, order flow dynamics and prices mechanisms. Also, game theory models gathered information on such topics as strategic interaction between liquidity providers and informed traders. Respectively, their findings have made it clear why the market sometimes cannot be liquid enough and shares cannot be fairly priced(commas not needed).

Game theory gives us the opportunity to understand information flow in financial markets and how information spreads in these markets. Research has leveraged the signaling game model as well as the rational expectations equilibrium model to explore how information asymmetries affect prices of assets, volumes of trade and market efficiency. This knowledge contributes to understanding strategic actions taken by individuals who seek strategic positions, as well as market manipulations that happen quite often. Strategic trading models such Kyle model have provided insights into how informed traders exploit their information advantage and interact with liquidity providers to maximize profits.

In financial markets, there are implications for both the regulatory policies applied and the structure of markets from a game theory perspective. Models of market competition, such as the Bertrand and Cournot competition models, have been used to analyze the effects of market structure on price competition, market efficiency and welfare outcomes. Game theory also provides insights into the design of regulatory mechanisms to mitigate market manipulation and promote fair and efficient trading environments.

2.3.3 **Role of Game Theory in Modeling market Dynamics**

Studying tricks used by players in markets and creating policies to control them is a key factor in game theory. While looking at market manipulation described in game theory, such as those practiced by manipulative traders as they interface with regulatory authorities, it becomes possible to determine how best to go about ensuring that our markets remain both honest as well stable.

2.3.4 **Conclusion**

In conclusion, game theory serves as a valuable tool for modeling strategic interactions among investors and understanding market dynamics in financial markets. Its applications span various domains, including strategic trading, market microstructure, information dynamics, and regulatory policies. Simply put, one of the many uses of game theory in financial markets is to model strategic interactions among investors thereby understand the market dynamics. This is applicable in different areas such as strategic trading, market micro structure, information dynamics as well as regulatory policies.

2.4 Reinforcement learning in Portfolio management

Reinforcement learning(RL) has emerged as a powerful technique for optimizing investment strategies in portfolio management. This sub-chapter intend to provides a summary of the applications of reinforcement learning in portfolio management and highlights its effectiveness in learning optimal policies for asses allocation, risk management, and trading decisions.

Reinforcement learning in Portfolio optimization:

Reinforcement learning techniques are applied to optimize portfolio construction and re-balancing decisions. RL agents learn to select assets, allocate capital, and re-balance portfolios based on investment objectives, risk constraints, and market conditions. By optimizing portfolio performance metrics such as Sharpe ratio, maximum draw down or cumulative wealth, RL-based portfolio optimization approaches aim to achieve superior risk-adjusted returns compared to traditional optimization methods.

Conclusion

In conclusion, reinforcement learning has shown promising results in optimizing investment strategies across various aspects of portfolio management. They are dynamic asset allocation, risk management, trading strategies, market prediction, and portfolio optimization among others. Portfolio managers can enhance portfolio performance, mitigate risks, and capitalize on market opportunities in dynamic and uncertain financial markets by harnessing RL algorithms to learn from historical data, adapt to changing market conditions, and optimize decision making processes.

2.4.1 **Review of papers**

Weijs (2018) compares Campbell & Viceira (2002), which uses a Vector Autoregressive (VAR) model to predict future returns, with Weijs's reinforcement learning method to portfolio optimization. Weijs employs three US resources: Treasure Bill for three months, The 5-year Treasury Note and the NYSE, NASDAQ, and AMEX weighted average. The data covers a wide time frame, from 1954 to 2016. The findings indicate a 33% increase in cumulative return over the Campbell technique, a 33% decrease in volatility, and a 3% decrease in turnover.

Kim & Khushi (2020) has produced an extremely intriguing work that mixes Transformers with Reinforcement learning. Because the Sortino ratio is included, the return is risk-adjusted. The agent approximates policies using a deep neural network that includes Transformers layers. The training and evaluation of the model is conducted on nine Dow Jones firms, one for each sector. Data covers the years 2000–2020, or 20 years. The model uses 50-day sequences. The findings indicate a 43% cumulative return from 2018 to 2020 and an annualized Sharpe ratio of 0.64.

These are some of the past attempted reviewed research papers, but they are many as this topic is applicable to a wide range of sectors and they are almost applicable to all sectors. Starting with finance, energy, operations and others.

2.5 Attempted Integrated approaches in Finance and other fields

2.5.1 **Introduction**

Integrated approaches that combine fuzzy logic, game theory and reinforcement learning have gained attention in various field, including finance, engineering and artificial intelligence. This subchapter discusses previous attempts, if any, to integrate these methodologies and their applications in finance and other relevant domains.

2.5.2 **Hybrid Fuzzy Logic-Game theory Models**

Researchers such as Lotfi A.Zadeh, the pioneer of fuzzy logic, advocate for integration of fuzzy logic and game theory to model decision-making in uncertain and strategic environments. Zadeh proposed the concept of fuzzy games to handle imprecise information and subjective judgments in strategic interactions. While theoretical developments have been promising, practical applications of hybrid fuzzy logic-game theory models in finance are still limited. One attempt was made by Huanxiu Guo and peide Liu in their paper ”A Fuzzy game theory Approach to portfolio optimization” where they proposed a Fuzzy Game theory framework for portfolio optimization. However, further research is needed to fully explore the potential integrated models in financial decision making.

CHAPTER 3
THEORETICAL FRAMEWORK

3.1 introduction

Financial management is best done with a strong theory to solve dynamic markets unpredictably It is from this chapter that we will investigate integrated approaches to portfolio management. this part analyze how fuzzy logic, game theory, and reinforcement learning can be used for solving problems related to financial portfolio management.

3.2 Fuzzy Rule

Fuzzy rule provides a mathematical framework for reasoning with uncertainty and imprecision, allowing for more flexible and intuitive representation of knowledge and decision-making processes. In the context of financial portfolio management, fuzzy logic offers valuable tools for handling uncertain market conditions, imprecise data, and subjective investor preferences.

3.2.1 Principles of Fuzzy Logic Relevant to Financial Portfolio Management

1. **Fuzzy Sets and Membership Functions**: Fuzzy logic logic introduces the concept of fuzzy sets, where elements can belong to a set with varying degrees of membership between 0 and 1. Linguistic variables like "high", "medium", and "low" can be represented using fuzzy sets in financial portfolio management. The extent to which an element belongs to a fuzzy set is defined by membership functions, thus enabling uncertainty and vagueness in making decisions to be modeled.

2. **Fuzzy Rule-Based System**s: In fuzzy rule based systems, expert knowledge and heuristic rules are encoded as "if-then" rules with linguistic variables being associated with linguistic terms and fuzzy sets. The fuzzy rule based systems that input market conditions and economic indicators into asset allocation or risk level allocation comprise these relationships. This is particularly applicable with regard to fuzzy rule-based systems in portfolio management. They are able to incorporate qualitative assessments and domain expertise in order to facilitate decision-making in times of uncertainty.

3. **Fuzzy Inference Systems**: Fuzzy input data is converted using fuzzy inference systems into fuzzy output data based on fuzzy rules. The method itself involves four main steps: Fuzzification, Evaluation of Rules, Aggregation and Defuzzification. Fuzzification takes crisp input data and converts them into fuzzy sets by making use of membership functions, while application rule evaluates each rule's firing strength or degree of activation using fuzzy rules. Aggregation combines the outputs of multiple rules, and defuzzification converts fuzzy

output data back into crisp values. In financial portfolio management, fuzzy interference systems can applied to transform qualitative assessment and expert judgment into financial decision. This makes it easier to optimize and manage risks.

3.2.2 Conclusion

In financial portfolio management, there are many benefits of using fuzzy logic. Portfolio managers can apply fuzzy rule based systems, such as those in fuzzy control system and fuzzy inference systems to improve decisiveness through dynamic decision-making especially during market instability (or volatility). This actually (or practically) helps them select the right assets that are less riskier than others on average (or generally).

3.3 Game Theory Concepts Applicable to Modeling Investors' Interactions

When it comes to understanding investor behavior and interactions in financial markets, game theory offers a powerful way of representing strategic interaction among rational decision-makers. The following are game theory concepts critical for analyzing investors interactions:

1. **Players**: When we talk about people involved in financial markets, we're referring to investors, traders and institutions-these guys are considered game players.Each of these guys has his/her goals, tastes and tactics which affect his/her actions in terms of holding, selling or buying assets.

2. **Strategies**: In games you can take strategies that are actions or decisions that players can make. They involve buying or selling certain assets, changing investment portfolios or using particular trading methods for strategies in the financial market.

3. **Payoffs**: The payoffs could be defined as either losses or rewards that are linked to certain sets of strategies chosen by game participants; payoffs are commonly quantifiable in trading platforms and investment fields as they take forms of profit or loss made during a trading period, portfolio changes over time or accomplishment of particular investment targets, particularly on finance markets

4. **Information Sets**: In a financial market there are two broad types of information sets; the public information set- consisting largely of market data, economic indicators- and the private information set comprised primarily of insider information, proprietary trading strategies.

5. **Nash Equilibrium**: Nash Equilibrium is a term used in game theory to describe a condition where no player has anything to gain from changing their strategy individually as long as all others keep their strategies constant. In financial markets, Nash equilibrium refers to a situation in which the strategies of participants do not contradict each other and there is no way for any single player to increase their own performance without changing their approach.

6. **Mixed Strategies**: Some games allow for players to use strategies that are based more on likelihoods rather than hard-and-fast rules, for instance, there is an element of randomness to them. While pure strategies involve one or more specific actions selected from the available options, mixed strategies deal with the likelihoods that are attached to these actions. In financial markets, investors mix probabilities when trading because they want to make sure their decisions are not too predictable; this would allow someone else take advantage of the situation such as insider trading which is illegal.

3.4 Reinforcement Learning Algorithms Suitable for Dynamic Investment Strategy

Reinforcement learning method presents a good way of crafting adaptive investment strategies which are capable of changing with the market dynamics so as to improve portfolio performance. Below are some algorithms of reinforcement learning viable for a changing investment strategy:

- **Q-Learning**: Q-Learning is a model-free algorithm used in RL that learns how to choose actions optimally based on the estimated values of different actions within given states. In investment strategy adaptation, Q-Learning is used to make a determination on what will be the best trading actions to take (e.g., by, sell, hold) based on the current state of the market and historical data. By iteratively updating Q-values via exploration as well as exploitation, Q-learning helps the agent in learning solid investment strategies that increase overall revenues hence limiting chances of incurring losses.
- **Deep Q-Networks(DQN):** Deep Q-networks (DQN) extend Q-learning to handle high-dimensional state spaces by approximating Q-values using deep neural networks. In dynamic investment strategy adaptation, DQN can be employed to learn complex patterns and relationships in market data, allowing the agent to make informed trading decisions based on a rich representation of the market environment. By leveraging deep learning techniques, DQN can capture nonlinearities and temporal dependencies in financial time series data, leading to more accurate and effective investment strategies.
- **Policy Gradient Methods**: Policy gradient methods directly learn a policy function that maps states to actions by maximizing the expected cumulative reward. Unlike value-based methods such as Q-learning, Policy gradient methods parameterize the policy function and update its parameters using gradient ascent. In the context of investment strategy adaptation, policy gradient methods can be used to learn a stochastic policy that specifies the probability distribution over actions given the current state of the market. By optimizing the policy parameters through gradient ascent, policy gradient methods enable the agent to learn adaptive trading strategies that maximize long-term returns.
- **Proximal Policy Optimization (PPO)**: Proximal policy optimization (PPO) is a state of-the-art policy gradient algorithm that addresses the stability and convergence issues associated with traditional policy gradient methods. PPO employs a clipped objective function and a trust region constraint to ensure stable and monotonic policy updates. In dynamic investment strategy adaptation, PPO can be applied to learn robust and adaptive trading policies

that balance exploration and exploitation in uncertain market conditions. By leveraging the advantages of policy gradient methods while mitigating their drawbacks, PPO enables the agent to learn effective investment strategies with improved sample efficiency and convergence properties.

- **Deep Deterministic Policy Gradient (DDPG)**: Deep deterministic policy gradient (DDPG) is an off-policy actor-critic algorithm that combines deep Q-learning with policy gradient methods. DDPG learns a deterministic policy function that maps states to actions and utilizes a separate target network to stabilize training. In the context of investment strategy adaptation, DDPG can be employed to learn continuous action policies for portfolio management tasks, such as asset allocation and risk management. By leveraging the strengths of both value-based and policy-based approaches, DDPG enables the agent to learn flexible and adaptive investment strategies that maximize long-term returns while controlling risk.

3.5 Theoretical foundation for integrating fuzzy logic, Game theory, and Reinforcement Learning in Portfolio Management

Integration of fuzzy logic, game theory, and reinforcement learning provides a comprehensive theoretical framework for addressing the challenges of portfolio management in dynamic and uncertain market environments. Each of these methodologies brings unique strengths and when combined can enhance decision-making processes and improve portfolio performance. Below is the theoretical foundation for these approaches:

- **Fuzzy Logic for Uncertainty Handling**: Fuzzy logic is well-suited for handling uncertainty and imprecision in decision-making processes. In portfolio management, fuzzy logic can be applied to represent qualitative assessments, linguistic variables and subjective preferences of investors. By encoding expert knowledge and heuristic rules into fuzzy-rule based systems, fuzzy logic enables portfolio managers to make informed decisions under uncertain market conditions.

- **Game theory for strategic interactions:** Game theory provides a formal framework for modeling strategic interactions among rational decision-makers. In financial markets, investors engage in strategic behaviors such as competition, cooperation, and negotiation. By applying game theory concepts such as Nash equilibrium, mixed strategies, and sequential games, portfolio managers can analyze strategic interactions among market participants and anticipate their actions to formulate optimal investment strategies.

- **Reinforcement Learning for Adaptive Decision-Making**: Reinforcement learning offers a dynamic approach to learning optimal decision-making policies through interaction with the environment. In portfolio management, reinforcement learning algorithms can adaptively adjust investment strategies based on feedback from market data and portfolio performance. By continuously exploring and exploiting different trading actions, reinforcement learning enables portfolio managers to learn from past experiences and improve decision-making over time.

3.5.1 **Benefits of Integration**

- **Comprehensive Decision-Making**: Integration fuzzy logic, game theory, and reinforcement learning enables portfolio managers to make comprehensive decisions that account for uncertainty, strategies interactions, and adaptive learning
- **Robustness and Flexibility**: By combining the strengths of multiple methodologies, integrated approach offer robust and flexible decision-making frameworks that can adapt to diverse market conditions and investor preferences.
- **Improved Performance**: Integration of fuzzy logic, game theory, and reinforcement learning has the potential to enhance portfolio performance, optimize risk-adjusted returns, and achieve superior outcomes compared to traditional approaches.
- By integrating fuzzy logic, game theory, and reinforcement learning, portfolio managers can develop dynamic and adaptive investment strategies that capitalize on market opportunities, mitigate risks, and achieve long-term success in dynamic and uncertain financial markets.

3.6 Portfolio Optimization

In finance, portfolio optimization is the process of creating an investment portfolio with the goal of maximizing returns within a certain risk level or minimizing risks at a given level of returns. Analysis of multiple investment avenues including stocks, bonds, real estate, commodities, involves selecting highest possible amount of money distribution between these assets. Normally done through examining such aspects like previous performance data, volatility levels, correlations between different types of investments or even considering individuals risks and goals when it comes to return on their investments. Mathematically speaking, mean-variance optimization is one of the key methods used in the field of portfolio optimization. The objective is to enhance the expected return from a portfolio while minimizing its variance at the same time. Another technique would involve maximizing Sharpe ratio in order to achieve risk-adjusted performance on securities that make up the investor's market index for example S&P500.

3.6.1 **Asset return**

An asset is usually referred to by investors as something which can be procured or disposed. Assets include Stocks, bonds, and exchange-traded funds(ETFS). When the market first opens, the opening price is the price at which a product is sold while closing price refers to the value of an asset at the end of the trading session. Two fundamental definitions should be given total return on asset and rate of return on asset which are denoted R and r respectively. Total return:

$$R = \frac{\text{Closing Price}}{\text{Opening Price}}$$

Rate of return:

$$r = \frac{\text{closing price} - \text{opening price}}{\text{closing price}}$$

The two notions are related by:

$$R = 1 + r$$

Frequently the shorter expression return means the rate of return because after hours trading, close price at day t and opening price at day and opening price of a day t+1 can differ. A cumulative return on an investment is the amount gained or lost by the investment throughout time, regardless of the length of time involved. Given a sequence of rate of returns ri for i = 1, . . . , n their cumulative return is defined as

$$C_r = \frac{Y_n}{\prod_{i=1}^{n}(1 + r_i)}1$$

3.6.2 **Portfolios**

An investment portfolio is a collection of assets, or financial investments. Portfolios are sometimes referred to as master assets in some literary works. Let us suppose we have n available assets. we form the portfolio by investing a specific amount in each asset, we indicate the amount investing in asset i for $i = 1, 2, \ldots, n$ as X_{0i} such that

$$\sum_{i=1}^{n} X_{0i} = X_0.$$

If we are allowed to sell an asset, some of the X_{0i}'s can be negative; otherwise, we restrict X_{0i}'s to be non-negative. The amounts invested can be expressed as fractions of the total investment. Therefore, we write: $X_{0i} = W_i X_0$, $i = 1, 2, \ldots, n$, where W_i is the weight or fraction of asset i in the portfolio. Clearly:

$$\sum_{i=1}^{n} W_i = 1$$

and some $w_i's$ can be negative if short selling is allowed. let R_i denote the total return of asset i, then the amount of money generated at the end of the period by the ith asset is

$$R_i X_{oi} = R_i w_i Xo$$

The total amount received by this portfolio at the end of the period is therefore

$$\sum_{i=1}^{n} R_i W_i X_0$$

.

As a result we find that the overall total return of the portfolio is the weighted sum of the returns of its components:

$$R = \frac{\sum_{i=1}^{n} R_i W_i X_0}{Xo} = sum_{i=1}^{n} R_i$$

Equivalently, since

$$\sum_{i=1}^{n} w_i = 1$$

, we have

$$r = \sum_{i=1}^{n} w_i r_i$$

3.6.3 **Asset Volatility**

Asset volatility capture the degree of dispersion of returns around the mean or average return of an asset, the high volatility indicate that there is high probability of large prices swings which shows high risk. We use the notation E to indicate the expected value of a random variable x, for convenience E(x) is often denoted as $\bar{x}$. Always **mean** or **mean value** are used for the expected value. The expected value of a random variable provides useful summary of the probable nature of the variable, but to measure the degree of possible deviation from the mean, the useful measure is the **Variance**. Given a random z with expected value $\bar{z}$, their differences is $z - \bar{z}$ is also random, but with the expected value of zero, Mathematical proof:

$$E(z - \bar{z}) = E(z) - E(\bar{z}) = \bar{z} - \bar{z} = 0$$

The value of $(z - \bar{z})^2$, is a useful when measuring how much z tends to change to change from its expected value. In general for any random variable z its variance is defined as:

$$\textit{Var(z)} = E[(z - \bar{z})^2]$$

Mathematically variance is represented by symbol σ^2, there is it is written this way $\sigma^2 = \textit{var(z)}$. the square root of the variance is denoted as σ and it is called standard deviation. It has the same units as the quantity z and is another measure of how much the variable is likely to deviate from its expected value.

$$\textit{var(z)} = \sqrt{E[(z - \bar{z})^2]}$$

In finance, and risk management the term **Volatility** is used either as variance or standard deviation but mainly as standard deviation.

3.6.4 **Portfolio Covariance**

When considering two or more random variables, as assets returns in portfolio, their mutual dependence can be summarised by their **covariance**. let z_1 and z_2 be two asset returns with expected returns $\bar{z_1}$ and $\bar{z_2}$, the covariance of these assets is expressed as follows:

$$\text{cov}(z_1, z_2) = \text{E}[(z_1 - \bar{z}_1)(z_2 - \bar{z}_2)]$$

The covariance of two assets x and y is denoted by σ_{xy}, for assets z_1 and z_2 can be denoted this way: $\text{cov}(z_1, Z_2) = \sigma_{12}$, by symmetry property $\sigma_{12} = \sigma_{21}$. If two assets are uncorrelated it does not show that they are independent, it just show that knowing return of one assets give no information about other. correlation measure linear association and if two assets are related, correlation can not work to distinguish from independent case.

3.6.5 Sharpe Ratio

Sharpe ratio is used to help investors understand an investor's return against risk. The ratio is the average return earned in excess of the risk-free rate per unit of volatility or total risk. The higher the Sharpe ratio, the risk-adjusted return of the the investment.

$$S_r = \frac{r - r_f}{\sigma_r}$$

Where:

- **r**: rate of return
- r_f: Risk- free return
- σ_r: Standard deviation (Volatility)

Some Sharpe ratio limitations:

1. Using standard deviation as measure of risk, this shows that returns are normally distributed but financial markets shows large number of surprising drops or spikes in prices.
2. It focus on volatility without focusing on its direction, it can not differentiate upside and downside trends. it consider large losses and benefits almost the same.
3. Sharpe ratios use historical returns and volatility. The decisions based on the ratio take future performance to be the same as past, and it is not always true.

3.6.6 Relative Strength Index

One of the most used technical indicators is the Relative Strength Index(RSI). It used to measure the price movement velocity and magnitude. The momentum is the rate at which price rises or falls. The RSI generates values between 0 and 100. Traditionally, when the RSI is above 70, it suggests that the asset is overbought, meaning the price may be due for a reversal or correction. Conversely, when the RSI is below 30, it indicates oversold conditions, suggesting that the price may be due for a bounce or reversal upwards. A down period is characterized by close being lower than the previous period's close. We calculate U (Upward change) and D (Downward change) as the follows for an up period:

$$U = \text{Close}_{\text{now}} - \text{Close}_{\text{previous}} \tag{3.1}$$

$$U = \text{Close}_{\text{previous}} - \text{Close}_{\text{now}} \tag{3.2}$$

Relative strength can be calculated using average gain over average loss

$$\text{RS} = \frac{\text{Average Gain}}{\text{Average Loss}}$$

$$\text{RSI} = 100 - \frac{100}{1 + \text{RS}}$$

where RS is the Relative Strength.

I will use Sharpe ratio, Portfolio Covariance and volatility of the portfolio which in this case stand for risk, as the volatility increase also the risk increase.

CHAPTER 4
METHODOLOGY

4.1 Game Theory Framework for Portfolio Management

Game theory,a multidisciplinary area situated at the nexus of economics and mathematics, provides a strong framework for examining strategic relationships and decision-making procedures across a range of industries, including finance. Game theory clarifies the interactions of rational agents, like investors and decision-makers, and offers important insights into equilibrium outcomes and market dynamics. Game theory provides a potent toolkit for comprehending the intricacies of investor behavior, market competitiveness, and resource allocation in financial markets, where uncertainty and strategic behavior are common. Game theory gives investors the ability to predict and react to the activities of other market participants, which can result in better outcomes and more informed decision-making. Examples of these principles include strategic dominance and Nash equilibrium. Furthermore, game-theoretic models can provide insight on phenomena like herding behavior, pricing competition, and strategic alliances by illuminating the incentives and motivations that drive market actors. All things considered, game theory is a fundamental component of contemporary finance theory, offering a strict analytical framework that improves our comprehension of market dynamics and guides strategic decision-making.

HOW GAME THEORY IS USED IN PORTFOLIO MANAGEMENT:

4.1.1 Modeling Strategic Interactions

Game theory allow us to model the strategic interactions among investors in financial markets. Investors make decisions based not only on their own objectives but also on their expectations of how other investors might behave. By representing these interactions as strategic games, we can analyze the choices made by investors and their impact on market outcomes.

4.1.2 Analyzing Market Dynamics

Game theory provides tools for analyzing the dynamics of financial markets. Through strategic games, we can study how changes in investors behaviors, market conditions, and other external factors influence the asset prices, trading volumes, and market efficiency. By understanding these dynamics, portfolio managers can make decisions effectively about asset allocation, risk management, and trading strategies.

4.1.3 Identifying Nash Equilibrium Outcomes

One of the key concepts in game theory is Nash equilibrium, where no player has incentive to unilaterally deviate from their chosen strategy. In the context of portfolio management, Nash equilibrium outcomes represent stable states where investors' strategies are mutually consistent, and market prices reflect rational expectations. By identifying equilibrium outcomes, we can gain insights into long-term dynamics and stability of financial markets.

4.1.4 **Addressing Strategic Interactions**

Strategic interactions among investors can lead to complex phenomena such as herding behaviors, market bubbles, and price manipulation. Game theory provides framework for understanding these phenomena and developing strategies to mitigate their impact on portfolio performance. By analyzing the incentives and strategies of different market participants, portfolio managers can anticipate market trends, identify opportunities, and manage risks more effectively.

4.1.5 **Enhancing Decision-making**

By incorporating game theory concepts into portfolio management, we can enhance decision-making process and improve portfolio performance. Game theory helps us understand the strategic interactions that drive market dynamics, enabling us to develop more robust investment strategies, optimize portfolio allocations, and adapt to changing market conditions. By leveraging insights from game theory, portfolio managers can make more informed decisions that maximize returns while minimizing risks.

In summary, game theory provides a strong framework for understanding strategic interactions in financial markets and enhanced portfolio management practices. By modeling investor behaviors, analyzing market dynamics, and identifying equilibrium outcomes, game theory helps portfolio managers navigate the complexities of financial markets and make more effective investment decisions.

4.2 Key Game Theory Models Related to Portfolio Management

4.2.1 **Prisoner's Dilemma**

a. The prisoner's dilemma is a classic model in game theory that illustrates the tension between individual rationality and collective welfare in the prisoner's dilemma, two individuals must decide whether to cooperate with each other or act in their self-interest, knowing that their outcome depend on the simultaneous decisions of both parties.

b. In portfolio management, the prisoner's dilemma can be applied to situations where investors must decide whether to cooperate with each other (e.g., By sharing information or coordinating investment strategies) or act in their own self-interest (e.g., by withholding information or pursuing individual gains)

c. The prisoner's dilemma highlights the challenges of cooperation and coordination among investors in financial markets. While cooperation may lead to better collective outcomes(e.g., Market efficiency, reduced volatility), individual investors may have incentives to defect and pursue their own interests, leading to sub-optimal outcomes for group as whole.

4.2.2 Nash Equilibrium in portfolio Management

Conceptual understanding

Nash equilibrium is a fundamental concept in game theory that describes a stable state in a strategic interaction where no player has an incentive to unilaterally deviate from their chosen strategy. In the context of portfolio management, Nash equilibrium represents a situation where each investor portfolio allocation is optimal given the allocations of others investors.

Illustrative example

To understand Nash equilibrium in portfolio management, consider this simplified example of two investors, Eric and Espoir, who must decide how to allocate their investment between two asset: stock A and stock B. Each investor aims to maximize their expected return while considering the actions of the other investor. In this example, the payoffs are represented in percentage terms:

	A	B
Eric	8%, 8%	10%, 6%
Espoir	6%, 10%	9%, 9%

Table 4.1: Investment Strategies

In the payoff matrix:

. the first entry in each cell represents the expected return for Eric.

. The second entry in each cell represents the expected return for Espoir.

. For example, if both Eric and Espoir invest in Stock A, Eric earns an expected return of 8%, and Espoir an expected return of 8%.

Nash equilibrium Analysis

. To identify Nash equilibrium, we analyze each player's strategy and ensure that no player can improve their payoff by unilaterally changing their strategy. . In the example, the Nash equilibrium occurs when both players choose their optimal strategies, and no player has incentive to deviate. . By examining the payoff matrix, we can see that both Eric and Espoir have a dominant strategy: to invest in stock A. regardless of the other player's choice, investing in Stock A yields a higher expected return for both players. . Therefore, the Nash equilibrium in this example is for both Eric and Espoir in Stock A.

Implications for Portfolio Management

. The concept of Nash equilibrium in portfolio management highlights the importance of strategic decision-making and the interdependence of investors' choices. . Portfolio managers can use game-theoretic models to analyze the strategic interactions among investors and identify equilibrium outcomes that provide insights into market dynamics and investor behaviors. . BY understanding Nash equilibrium, portfolio managers can anticipate market behaviors, optimize investment strategies, and adapt to changing market conditions more effectively.

In summary, Nash Equilibrium provides a valuable framework for understanding strategic interactions among investors in portfolio management. Through analysis modeling, portfolio managers can identify equilibrium outcomes that inform decision-making and enhance portfolio performance in dynamic and uncertain market conditions.

4.2.3 Glosten-Milgrom Model

The Glosten-Milgrom model describes a single asset market, where informed traders interact with a market maker, in the presence of noise traders Touzo et al. (2021).

Let V represent the true unknown value of the asset. The informed traders receive private signals s about the true value of the asset. These signals are drawn from a known probability distribution $F(s|V)$. Uninformed traders only have access to public information, denoted as I, which could include past prices or other general market data.

Market prices, denoted as P, are determined as a function of the true value of the asset V and the information available to all traders, both informed and uninformed This is the simple price formation equation:

$$P = V + \beta I + \epsilon$$

where:

- P represents the market price.
- V is the true value of the asset.
- α and β are parameters representing the weights assigned to the true value and public information, respectively.
- I is the public information available to all traders.
- ϵ represents random noise or shocks.

Informed Traders' Trading Strategy:

Informed traders decide when and how much to trade based on their private information s and their beliefs about their true value of the asset.

A simple trading strategy could involve trading whenever the private signal s exceeds a certain threshold, indicating a deviation from the current market price.

Market Equilibrium:

Equilibrium in the Glosten-Milgrom Model occurs when prices fully reflect all available information, both public and private.

At the equilibrium, there is no incentive for informed traders to further exploit their private information, as prices accurately reflect the true value of the asset.

Purpose of Glosten-Milgrom Model:

In finance, the Glosten-Milgrom model is a useful tool for determining how markets function. It illustrates the relationship between prices, buying and selling, and information in financial markets.

4.3 Data Collection and selection criteria

In conducting this portfolio management analysis, secondary data was used. Historical data was sourced from publicly available datasets obtained from NASDAQ. The dataset includes historical daily prices for all tickers trading on NASDAQ, the data are stored in separate folders for ETFS and stocks.

4.3.1 Source of Historical Financial Data

The dataset contains historical daily prices for all tickers trading on NASDAQ. NASDAQ stands for the National Association of Securities Dealers Automated Quotations. Here is its breakdown:

Feature	**Description**
Stock Exchange	Electronic marketplace for buying and selling securities
Rank (by Market Capitalization)	2nd largest in the world
Rank (by Trading Volume)	Most active in the U.S.
Trading System	Electronic (no open outcry auctions)
Focus	Large technology companies
Founded	1971
Location	New York City, USA
Owner	Nasdaq, Inc.
Indexes	Includes Nasdaq-100 (tracks top 100 non-financial companies)

4.3.2 Criteria for selection of financial assets

I decided to use four stocks trends which is Apple Inc , Amazon Inc, Microsoft Inc-common stock, Square Inc-common stock and their ticker are APPL, AMZN, MSFT, and SQ respectively. The selection of these stocks dataset was guided by several key criteria aimed at ensuring the relevance and effectiveness of portfolio management strategy. These are the reasons why this dataset can be used in this research:

1. Asset liquidity: these stocks are one of the most actively traded stocks on NASDAQ characterized by high liquidity and trading volume. This ensures of buying and selling shares with significant price impact, making it an ideal candidate for portfolio management analysis.

2. Availability of Historical Data: The dataset includes historical daily prices for Apple In Spanning up to May 19,2024. This extensive historical dataset provides ample data points for analysis, allowing for robust modeling and evaluation of investment strategies over time.

3. Relevance for Portfolio Objectives: The inclusion of these stocks in the portfolio aligns with the investment objectives and risk tolerance of the portfolio. As a leading technology company with a diversified product portfolio and strong financial performance, Apple Inc. Offers potential for long-term growth and stability, making it a strategic choice for portfolio diversification.

4. Market Capitalization: These stocks are one the largest publicly traded companies by market capitalization, with a significant presence in global financial markets. Its size and market dominance contribute to portfolio stability and mitigate risks associated with smaller companies.

By using the dataset I aim to develop and optimize a portfolio management strategy that maximizes returns while effectively managing risks using Game theory.

4.4 Algorithm Development

This chapter outlines my methodology for designing algorithms that optimize returns while effectively managing risks. I'll explore how to integrate game theory to capture the dynamics of investor interactions within the market.

4.4.1 Methodology overview

My approach is built on a strong foundation in portfolio optimizations, risk management and game theory. leveraging a multi-disciplinary perspective, drawing insights from finance, mathematics, and computer science.

4.4.2 Integration of Game Theory

Game theory plays a crucial role in my algorithms. By formulating strategic games, i can model the competitive and cooperative behaviors of investors, considering factors like risk tolerance, access to information, and overall market sentiment.

4.4.3 Optimization Techniques

My algorithms incorporate various optimization techniques to achieve the optimal balance between risk and return. This includes strategies for asset allocation, diversification methods, and risk-adjusted performance measures. I leverage optimization algorithms like linear programming, quadratic programming, and even evolutionary algorithms to identify the best portfolio allocations.

4.5 Algorithm to be used

1. **Calculate Expected Return**: The calculation of expected return will be done using the Capital Asset Pricing Model (**CAPM**).

$$E_{Ri} = R_f + \beta_i(E_{Rm} - R_f)$$

where:

E_{Ri} = expected return of investment
R_f = risk-free rate
β_i = beta of the investment
$(E_{Rm} - R_f)$ = market risk premium

The reason of using this in calculating expected return is to make sure that a stock is fairly valued and the beta in the formula measure how much the risk the investment will add to portfolio.

2. **Calculate volatility**: This is risk in other words as the market volatility get higher also that is the risk rising.

3. **Calculate Sharpe ratio or Negative Sharpe ratio**:

4. c**ombining with this method of using scipy.optimize.minimize** : This will focus on reducing risk while achieving a specified return level, aligning with the concept of mean-variance optimization introduced by Harry Markowitz.

5. **Set Up the Constraints**: The sum of the weights should be 1 (fully invested portfolio). The expected portfolio return should be equal to a target return.

6. **Diversification**: add diversification to limit the variance of weights

7. : **Define the Bounds**: Weights should be between 0 and 1 for a long-only portfolio.

CHAPTER 5

RESULTS AND DISCUSSION

This chapter presents the research findings that are the result of the thorough approach described in Chapter 4. The investigation starts with a thorough data analysis that includes visualization strategies to offer distinct insights into the underlying patterns and trends. The next sections explore portfolio strategy optimization using the data analysis, covering cooperative and non-cooperative methods. The outcomes present the optimized portfolios' performance data side by side with industry standards like the S&P 500 Index. The chapter provides a comprehensive overview of portfolio management strategies through a thorough analysis, illuminating the consequences of these strategies for investors and pointing out possible directions for further research and development.

5.1 Analysis and data visualization

The start date of the data is1999-01-04 and the end date is 2024-05-10 I have these 4 stocks(symbols): SQ:Square, Inc. Class A Common Stock, AAPL:Apple Inc. - Common Stock and MSFT:Microsoft Corporation - Common Stock, and MSF: Microsoft -common stock.

Before performing Portfolio optimization, i want to know how the portfolio assets are performing in the market and then, know how to make the optimization.

5.1.1 Loading data and data preparation

THE FIRST PART OF THE CODE:

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import yfinance as yf #to be used in fetching risk-free-rate in a given per
import seaborn as sns # for data visualization
from scipy.optimize import minimize
```

The purpose of this part of the code is to be ready to use python especially libraries which allow to manipulate, visualize, and then optimize. the second part of the code will be importing data to be used which was already saved from NASDAQ.

```
# Load your data
data = pd.read_csv('stock_data.csv', parse_dates=['date'])

# Pivot the data to have symbols as columns for close prices
data_pivot = data.pivot_table(index='date', columns='symbol', values='close
```

```
# Display the first few rows to check the data
print(data_pivot.head())
```

Visualization of Asset price movement over time

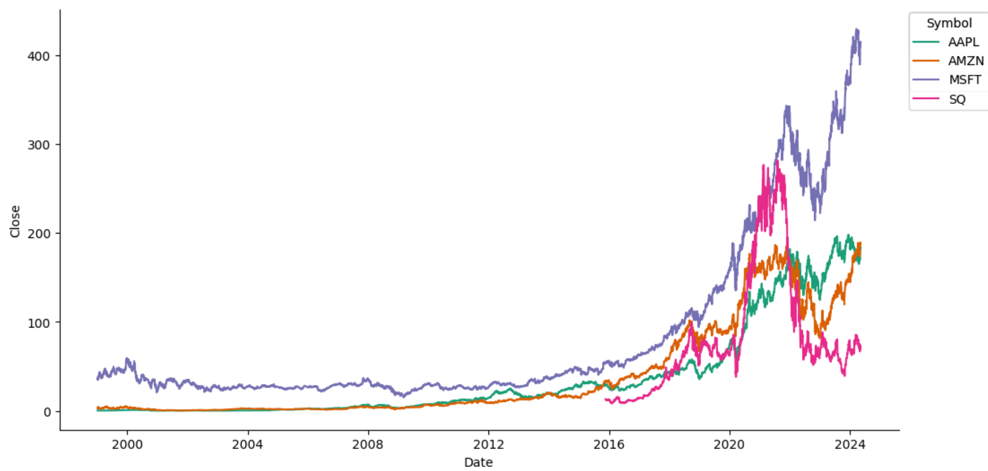

Figure 5.1: stock price change

Key Interpretation on this time series graph:

- The x-axis represents the Date of the portfolio.
- The y-axis represents the price of the portfolio.
- These lines shows change of price over time which is date
- The Microsoft Inc. - Common Stock is the stock which performed better based on the closing price, its line is blue.
- The least performing stock is : Square, Inc. Class A Common Stock.

Correlation between stocks

```
# Descriptive Statistics
descriptive_stats = returns.describe()
# Correlation Analysis
correlation_matrix = returns.corr()
# part 2
# Create a heatmap
sns.heatmap(correlation_matrix, annot=True, cmap="coolwarm")
```

```
# Add labels and title
plt.xlabel("Features")
plt.ylabel("Features")
plt.title("Correlation Heatmap")

# Show the plot
plt.show()
```

RESULTS:

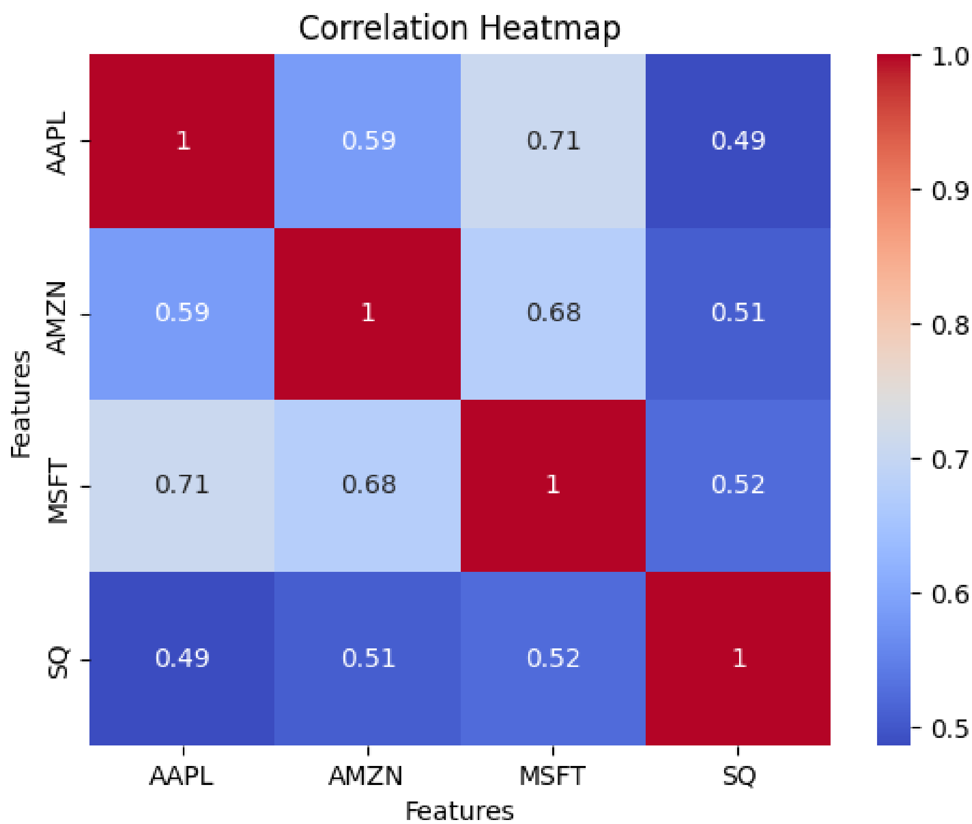

Figure 5.2: correlation of stocks

- The heatmap shows that these assets or stocks in the portfolio are highly correlated
- Their correlation value are over 0.65 and others are even above 0.9

Auto-correlation of daily returns

This is to analyse the linear relationship between today and yesterday return.

```
# Calculate daily returns (percentage change)
dataset['daily_return'] = dataset['close'].pct_change() * 100
```

```
# Analyze autocorrelation of daily returns
autocorrelation = dataset['daily_return'].autocorr()

# Print results
print(f"Autocorrelation of Daily Returns (Lag 1): {autocorrelation:.4f}")
```

RESULTS: Auto-correlation of Daily Returns (Lag 1): -0.0758

INTERPRETATION:

- Auto-correlation Value: -0.0758 (negative value)
- Strength of Relationship: Weak negative.
- A negative auto-correlation suggests a tendency for opposite movements between today's return and yesterday's return. So, if yesterday's return was positive, there might be a slight tendency for today's return to be negative, and vice versa.

Relative strength index

As seen from the table the AAPL has the highest average Relative strength index. The average RSI of 44.70 for Apple inc suggests that the stocks are neither oversold nor overbought and it is the for all except the Square, Inc. Class A Common Stock was oversold, next step will be to analyse the trend of Relative strength index.

Symbol	RSI (Relative Strength Index)
AAPL	48.700616
AMZN	40.106443
MSFT	50.479485
SQ	39.642304

Table 5.1: RSI Index

VISUALIZATION OF RELATIVE STRENGTH INDEX

- Used range from November 2024 to May 2025
- This is the graph showing trend of RSI, the intention is to check where or whether the stocks was oversold or overbought on period of time.
- AAPL(Apple inc): the RSI went above 70 for like two days, this indicate that it was overbought for short period of time in the period between November and May.
- The same for amazon December close to January it was overbought for short period of time
- Approaching may AMZN, APPL, SQ were oversold the RSI went below 20 for most part of April

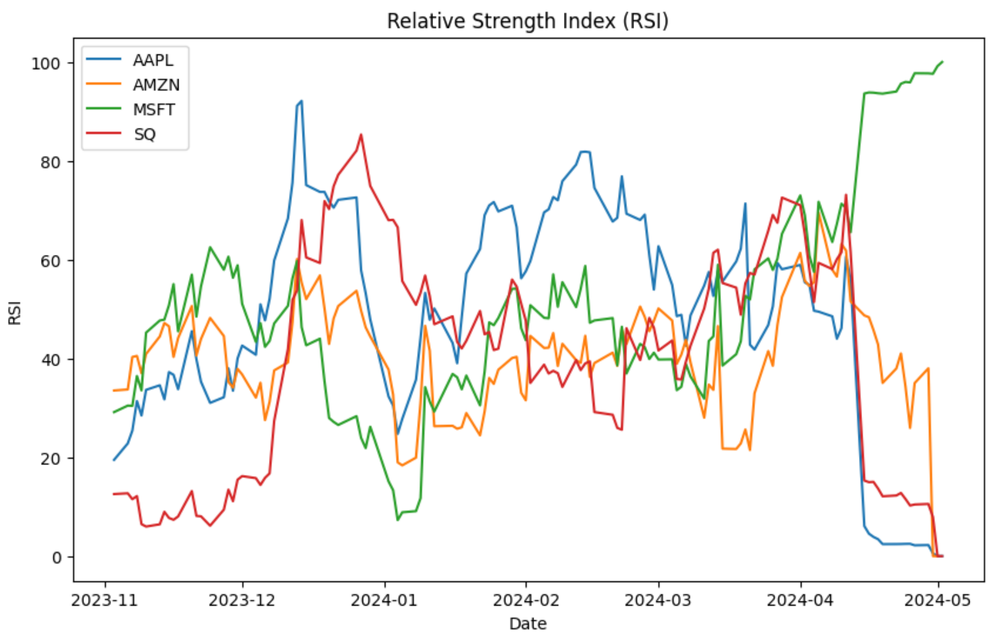

Figure 5.3: Relative Price Index

5.2 Optimization

After The tasks of data cleaning, analysis, visualization, the task to follow is optimization of portfolio which is made by four assets, the used method in getting expected return is CAPM which is capital asset pricing model. The steps in defining function for portfolio optimization was this way:

- Defining portfolio performance which is made of weights, expected returns, and covariance matrix
- Defining negative sharp ratio using weights, expected returns, covariance matrix, and risk free rate
- defining optimize portfolio which is made of expected returns and covariance matrix
- defining constraints, bound and initial guess

Code:

```
def portfolio_performance(weights, expected_returns, cov_matrix):
   # Calculate portfolio return
    portfolio_return = np.sum(expected_returns * weights)
    # Calculate portfolio volatility
    portfolio_volatility = np.sqrt(np.dot(weights.T, np.dot(cov_matrix, weig
    return portfolio_return, portfolio_volatility

def negative_sharpe_ratio(weights, expected_returns, cov_matrix,
```

```
risk_free_rate=0.001):
    p_return, p_volatility = portfolio_performance(weights,
    expected_returns,cov_matrix)
    # Calculate Sharpe Ratio
    sharpe_ratio = (p_return - risk_free_rate) / p_volatility
    return -sharpe_ratio  # Negative because we minimize in optimization

def optimize_portfolio(expected_returns, cov_matrix):
    num_assets = len(expected_returns)
    args = (expected_returns, cov_matrix)
    # Constraints: sum of weights = 1
    constraints = ({'type': 'eq', 'fun': lambda x: np.sum(x) - 1})
    # Bounds: weights between 0 and 1 (long-only portfolio)
    bounds = tuple((0, 1) for asset in range(num_assets))
    # Initial guess (equal distribution)
    init_guess = num_assets * [1. / num_assets]

    result = minimize(negative_sharpe_ratio, init_guess, args=args,
                      method='SLSQP', bounds=bounds,
                constraints=constraints)
    return result
optimized_result = optimize_portfolio(expected_returns, cov_matrix)

# Get the optimized weights
optimized_weights = optimized_result.x

print("Optimized Weights:\n", optimized_weights)

# Calculate optimized portfolio performance
optimized_return, optimized_volatility = portfolio_performance(optimized_we
print("Optimized Portfolio Return:", optimized_return)
print("Optimized Portfolio Volatility:", optimized_volatility)
```

Optimize results

- **Optimized Weights**: [0.10440239 0.1540988 0.10575557 0.63574324]
- Optimized Portfolio Return: 0.0002
- **Optimized Portfolio Volatility**: 0.032917573172

Interpretation:

The portfolio optimal weights for Apple(AAPL) is almost zero, and it is the same for Amazon(AMZN) and Microsoft(MSFT)

Square inc (SQ) has nearly 100% which shows that the model might have favored the SQ asset

Portfolio expected return and volatility are very low it indicate conservative or low risk portfolio

Investment allocation: The optimization process has determined that the best allocation (to maximize sharp ratio or minimize negative sharp ratio) is to invest 100% in the fourth asset Square Inc(SQ), completely ignoring the other assets.

The return: The portfolio is expected to yield a return of 0.00146

The portfolio's risk is approximately 0.0329

REASONS FOR THESE INTERPRETATION:

1. **Expected Returns**: The fourth asset may have a much higher expected return compared to its risk (volatility) than the other assets.
2. **Covariance Matrix**: The other assets might be highly correlated with each other and/or the fourth asset, or they may have higher individual volatility's, making them less attractive when optimizing for the Sharpe ratio.
3. Constraints and Bounds: The constraints (sum of weights = 1) and bounds (weights between 0 and 1) allow for such an allocation where one asset is heavily favored if it provides the highest risk-adjusted return.

This result suggest that the stock of Square Inc(SQ) is the most efficient in terms of maximizing the Sharpe ratio and maximizing the return.

VISUALIZATION OF OPTIMIZATION

- As seen in this figure which shows relationship between Sharpe ratio, Return, and Volatility.
- As the return increase the volatility and Sharpe ratio also rise.
- The red star show a point where there might be the best risk-adjusted return.

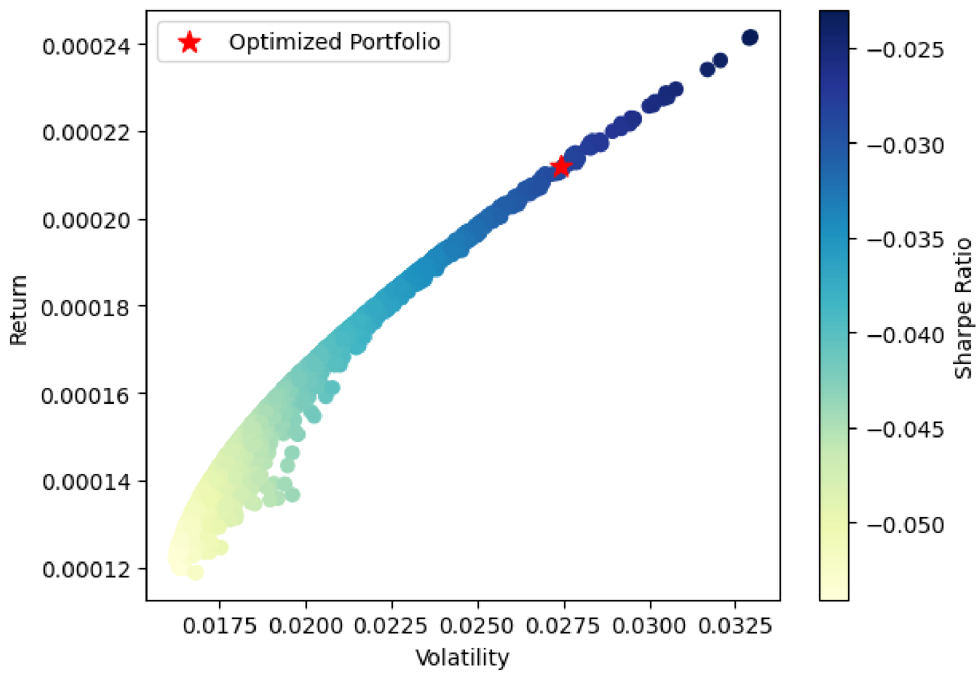

Figure 5.4: Optimized Portfolio

5.3 ANALYSIS OF THE OPTIMIZED PORTFOLIO

1. Annualized Return: 0.3335
2. Annualized Volatility: 0.4351
3. Sharpe Ratio: 0.7642
4. Maximum Drawdown: -3.1192

INTERPRETATIONS:

- **Annualized Return**: The average rate of return on the portfolio over a one-year period is indicated by this value, 0.3335. in order words, if you had made an investment in this portfolio, you would have anticipated an average yearly return of about 33.35%.
- **Annualized Volatility**: This number, 0.4351, represents the degree of risk or uncertainty related to your portfolio's annual returns. It calculates how much the returns in your portfolio deviate from the average return. Greater risk is indicated by increased volatility, which also suggests larger return variability.
- **Sharpe Ratio**: A measurement of risk-adjusted return is the Sharpe Ratio. It determines the excess return, or return over the risk-free rate, for each unit of volatility, or risk. Better risk-adjusted performance is indicated by a greater Sharpe Ratio. In this case, a Sharpe Ratio of 0.7642 indicates that your portfolio produces an excess return of roughly 0.7642 units for

each unit of risk (volatility). It represents favorable risk-adjusted returns.

- **Maximum Drawdown**: Maximum Drawdown, usually represented as a percentage, calculates the biggest decrease in the value of your portfolio over a given time period, from a peak to a trough. At worst, your portfolio dropped by around 3.1192% from its peak value for the time under review, as indicated by a figure of -3.1192.
- Overall, the positive Sharpe Ratio suggests that the optimized portfolio has produced attractive returns with relatively minimal risk. But while evaluating the risk-return profile of the portfolio, investors should be mindful that there have been times of substantial losses, as indicated by the Maximum Drawdown.

5.3.1 Analysis of each stock in the portfolio

STOCKS	Annualized return	Volatility	Sharpe ratio	Maximum Drawdown
AAPL	0.0269	0.0303	0.8534	0.8534
AMZN	0.0399	0.0506	0.7692	-0.1032
MSFT	0.0295	0.0290	0.9826	-0.0437
SQ	0.2372	0.3722	0.6346	-0.678

Table 5.2: Stock performance(optimized)

INTERPRETATION AND IMPLICATIONS:

1. **AAPL (Apple Inc.):**
    - **Annualized Return**: The average annual return on Apple stock has been slightly above 2.69% And it is a not-so-good but still okay return on investment (ROI).
    - **Annualized Volatility**:Apple stocks do not have large price fluctuations, which are constant . Looking at it from the investor 's point of view, it is advisable to invest in a stock with lower risks than a highly volatile one.
    - **Sharpe Ratio**: The risk-adjusted returns on Apple stock appear to be appealing, as shown by its 0.8534 Sharpe Ratio. investors can achieve high returns compared to amount of risk taken.
    - **Maximum Drawdown**:When you put your money into this stock, you can expect to lose as much as 4.75% at most, as the maximum drawdown shows.
2. **AMZN (Amazon Inc.):**
    - Compared to Apple, Amazon's stock can produce marginally higher return, but it has also shown more volatility. Higher downside risk is indicated by a larger maximum drawdown and a slightly lower Sharpe Ratio, both of which are indicators of increased volatility.
3. **MSFT (Microsoft Corporation):**

- Similar gains are provided by Microsoft shares, however with somewhat less volatility than Apple. Better risk-adjusted returns are indicated by a higher Sharpe Ratio. Additionally, the maximum drawdown is comparatively small, indicating less likelihood of a decline.

4. **SQ (Square Inc.):**
   - Compared to the other assets (Stocks), square stock has a lower Sharpe Ratio and notably more volatility, which makes it stand out among the others. There is also a substantially greater maximum drawdown, which suggests a higher chance of downside.

5.4 COMPARE WITH BENCHMARK

A benchmark is a standard that is used to measure how much the value of an asset or some other metric has changed over time. Benchmarks are used in investing to evaluate the performance of financial instruments such as stocks, mutual funds, exchange-traded funds, portfolios, and others. Market benchmarks are indices, or market proxies, that are constructed from a variety of securities, assets, or other instruments in order to simulate the performance of a stock, fund, or other investment of a similar kind and composition(1).

The S&P 500 Index is a weighted market capitalization index that includes 500 of the biggest US publicly traded firms. It is frequently used as a standard for evaluating the general performance of the US stock market. Investors usually cling on this standard to assess how the entire U.S. stock market has been performing on average. Most of the time, investors use the SP 500 Index to measure the returns generated by their shares in relation to the rest of the market.

5.4.1 Code to get S&P 500

This the lines of code starting from retrieving data of S&P 500 using its ticker symbol "GSPC". The were retrieved using API of yahoo finance(Yfinance) in python. The next part is calculating return using the "Adj close" and get the annualized return, volatility, Sharpe ratio and Maximum Drawdown.

```
# Set start and end dates
start_date = "2015-11-19"
end_date = "2024-05-12"

# Retrieve historical price data for the benchmark (GSPC/S&P 500 Index)
benchmark_data = yf.download("^GSPC", start=start_date, end=end_date)

# Calculate benchmark returns
benchmark_returns = benchmark_data['Adj Close'].pct_change().dropna()

# Assuming you already have the optimized weights and returns
# Calculate portfolio returns
```

```
portfolio_returns = returns.dot(optimized_weights)

# Calculate performance metrics for the portfolio
portfolio_annualized_return = np.mean(portfolio_returns) * 252
portfolio_annualized_volatility = np.std(portfolio_returns) * np.sqrt(252)
portfolio_sharpe_ratio = (portfolio_annualized_return - risk_free_rate)
/ portfolio_annualized_volatility
portfolio_cumulative_returns = np.cumprod(1 + portfolio_returns)
portfolio_drawdown = (portfolio_cumulative_returns / np.maximum.accumulate
(portfolio_cumulative_returns)) - 1
portfolio_max_drawdown = portfolio_drawdown.min()

# Calculate performance metrics for the benchmark
benchmark_annualized_return = np.mean(benchmark_returns) * 252
benchmark_annualized_volatility = np.std(benchmark_returns) * np.sqrt(252)
benchmark_sharpe_ratio = (benchmark_annualized_return - risk_free_rate) /
benchmark_annualized_volatility
benchmark_cumulative_returns = np.cumprod(1 + benchmark_returns)
benchmark_drawdown = (benchmark_cumulative_returns / np.maximum.accumulate
(benchmark_cumulative_returns)) - 1
benchmark_max_drawdown = benchmark_drawdown.min()

# Print performance metrics
print("Portfolio Performance Metrics:")
print(f"  Annualized Return: {portfolio_annualized_return:.4f}")
print(f"  Annualized Volatility: {portfolio_annualized_volatility:.4f}")
print(f"  Sharpe Ratio: {portfolio_sharpe_ratio:.4f}")
print(f"  Maximum Drawdown: {portfolio_max_drawdown:.4f}")

print("\nBenchmark Performance Metrics:")
print(f"  Annualized Return: {benchmark_annualized_return:.4f}")
print(f"  Annualized Volatility: {benchmark_annualized_volatility:.4f}")
print(f"  Sharpe Ratio: {benchmark_sharpe_ratio:.4f}")
print(f"  Maximum Drawdown: {benchmark_max_drawdown:.4f}")
```

5.4.2 RESULTS

This subsection include results obtained from comparing optimized portfolio to the benchmark. the table is below the interpretation.

Interpretation:

- **Returns**: In terms of annualized return, the optimized portfolio has beaten the benchmark by a considerable margin, returning over 33.35% as opposed to the SP 500 Index's 12.58%.

- **Volatility**:When compared to the benchmark, the optimized portfolio's volatility is higher, suggesting more fluctuations in returns and also the higher the return the high the risk associated to it.
- Risk-adjusted returns (Sharpe Ratio): The optimized portfolio's Sharpe Ratio (0.7644) is greater than the benchmark's (0.6796), indicating that the portfolio has produced better risk-adjusted returns.
- **Maximum drawdown**: The optimized portfolio has a larger maximum drawdown (-68.99%) compared to the benchmark (-33.92%), indicating higher downside risk for the portfolio.
- The overall performance of the optimized portfolio has been stellar, surpassing the benchmark S&P 500 Index. Nevertheless, it has higher volatility, which implies it carries more risks and is prone to larger potential drawdowns. It is therefore very important that investors in order to compare how well the optimized portfolio has done vis-à-vis the benchmark critically evaluate their risk tolerance and investment objectives carefully.

Results:

	Annualized Return	Annualized Volatility	Sharpe Ratio	Maximum Drawdown
OPTIMIZED	0.3335	0.4350	0.7644	-0.6899
BENCHMARK	0.1258	0.1836	0.6796	-0.3392

Table 5.3: comparison with benchmark

5.5 Game Theory

Shapley value is a solution concept used in game theory that involves fairly distributing both gains and cost to several actors working in a coalition(5). In this case it is going to be used to ensure that the returns or loss are shared equally among investors. I used four investors in this study to check the most profitable between either cooperative or non cooperative strategy. **Values** is the relative contribution of each investor to the overall performance of the portfolio.

5.5.1 Cooperative

	Shapley values	Distributed Annual returns
Investor 0	1.25112468e-05	0.0196
Investor 1	2.13067592e-05	0.0335
Investor 2	1.25506589e-05	0.0197
Investor 3	1.65594793e-04	0.2605

Table 5.4: Cooperative results

Interpretation:

- Investor 3 has the highest Shapley value indicating that the contribution of this investor is significantly larger than the other investors.
- Investor 0, 1, and 2 have small Shapley values, it shows that their contribution is less.
- Investor 3 as the one with highest Shapley value also get the highest return of 0.2605 and it is shared accordingly to all other investors.

5.5.2 Non cooperative

Intro: The individual portfolio were optimized independently, finding individual weights of each investor.

Code

```
def individual_sharpe_ratio(weights, individual_return, cov_matrix,
risk_free_rate=0.001):
    p_return, p_volatility = portfolio_performance(weights, individual_retu
    sharpe_ratio = (p_return - risk_free_rate) / p_volatility
    return sharpe_ratio

def optimize_individual_sharpe(expected_returns, cov_matrix, index):
    num_assets = len(expected_returns)
    args = (expected_returns[index], cov_matrix)
    constraints = ({'type': 'eq', 'fun': lambda x: np.sum(x) - 1})
    bounds = tuple((0, 1) for asset in range(num_assets))
    init_guess = num_assets * [1. / num_assets]
     # add diversification for constaint(limiting the variance of weights)
    def diversification_constraint(weights):
        return 0.05 - np.var(weights)
        # example threshold to promote diversification

    # Combine all constraints
    constraints = (
        constraints,
        {'type': 'ineq', 'fun': diversification_constraint}
    )

    result = minimize(negative_sharpe_ratio, init_guess, args=args,
                      method='SLSQP', bounds=bounds,
                    constraints=constraints)
    return result.x
```

```
# Optimize individual Sharpe ratios for each investor
individual_weights = []
for i in range(len(expected_returns)):
    individual_weights.append(optimize_individual_sharpe(expected_returns,
    cov_matrix, i))
# print individual weights
print("Individual Weights (Non-Cooperative Game Theory):")
for i, w in enumerate(individual_weights):
    print(f"Investor {i}: {w}")

# Calculate and print the portfolio performance for each investor
for i, weights in enumerate(individual_weights):
    p_return, p_volatility = portfolio_performance(weights,
    expected_returns, cov_matrix)
    print(f"Investor {i} - Return: {p_return:.4f}, Volatility:
    {p_volatility:.4f}, Sharpe Ratio: {sharpe_ratio:.4f}")
```

RESULTS:

This table contain results obtained from the above code

	Annualized Return	Volatility	Sharpe Ratio	Weights
Investor0	0.0002	0.0274	0.7642	0.1053
Investor1	0.0002	0.0274	0.7642	0.1053
Investor2	0.0002	0.0274	0.7642	0.1053
Investor3	0.0002	0.0274	0.7642	0.1053

Table 5.5: Results from Non cooperative

Interpretation:

- With extremely small exceptions, each investor's weights are nearly equal. This implies that an equivalent allocation was made for each investment as a result of the optimization procedure.
- The return are also the same for all investors and also Sharpe ratio of negative value

Comparison between Cooperative and non cooperative Since the cooperative approach takes into account the unique effects of each investor, it may result in a more equitable and balanced allocation of returns.

CHAPTER 6
CONCLUSIONS

6.1 Implications for Financial Portfolio Management

Optimizing asset allocation is essential to portfolio management in order to achieve desired risk-adjusted returns. This thesis looked at several portfolio optimization strategies and evaluated its performance by comparing it to the benchmark (S&P 500 Index) which is the benchmark used worldwide to assess if the optimized portfolio is effective. The results were impressive as the optimized portfolio nearly outperformed the benchmark. Also applying the game theory approaches like cooperative and non cooperative game theory-based strategy, where as the cooperative give the highest risk-adjusted return compared to non cooperative and also returns depend on the contribution of the investor in the portfolio based on the shapely Value. Below there is the implication of this whole work on investors and portfolio managers.

For portfolio managers

Although the optimized portfolio carries a larger risk, it also has higher potential gains. It is vital for managers to guarantee that their clients are cognizant of the heightened volatility and possibility of substantial losses. Aggressive investors with a high risk tolerance and a long investing horizon may find this portfolio appropriate. The cooperative approach offers a structure for allocating returns in a fair manner according to each investor's participation, which can raise investor satisfaction and perceived fairness and also from this research the returns are higher than the non cooperative approach.

For investors

Investors must evaluate their investment objectives and risk tolerance. Due to its strong returns, the optimized portfolio may appeal to those who can tolerate significant volatility and probable drawdowns. Those who are getting close to retirement or have a reduced risk tolerance, however, might favor a less volatile portfolio.

Although the optimized portfolio has a higher risk profile and substantial downsides, it has a bigger potential return. Portfolio managers should think about ways to reduce risks while retaining strong returns and make sure that their investment strategies match the risk tolerance and financial objectives of their clients. Investors should assess their risk tolerance carefully and consider whether a diversified approach comprising both steady benchmark investments and optimized techniques might be beneficial in order to mitigate volatility and potential losses.

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