ME 542

Assignment – 3

Gauss Elimination and Partial Pivoting

- Students need to save all the programs in a zipped file. Name it with your roll number and submit it on MS TEAMS.
- The programs are to be compiled and checked before submitting.
- Results obtained by your code should be written (do not copy the image file of your run) in a pdf file, and keep this file also in the same zipped folder.

Solve the following systems of linear equations using Gauss elimination with partial pivoting.

(a)

$$2x_1 + x_2 + x_3 = 4$$
$$4x_1 + 3x_2 + 3x_3 + x_4 = 6$$
$$8x_1 + 7x_2 + 9x_3 + 5x_4 = 8$$
$$6x_1 + 7x_2 + 9x_3 + 8x_4 = -2$$

(b) $5x_1 + 6x_2 + 9x_3 = 29, 6x_1 + 9x_2 + 2x_3 = 19, 11x_1 + 9x_2 + 5x_3 = 30$

(c) $5x_1 + 6x_2 + 9x_3 = 29, 6x_1 + 9x_2 + 2x_3 = 19, 11x_1 + 159x_2 + 11.001x_3 = 49.002$

(d) Hilbert matrix, matrix $H = [h_{ij} = \frac{1}{i+j-1}]$ where i is the i −th row and j is the j −column. The vector $b = [b_i = \sum_{j=1}^{n} h_{ij}]$. Keep the size of the matrix as input to the code and check for a 5×5 Hilbert matrix.

- Use different number of significant digits and compare $||x - x^*||_2$ and $||x - x^*||_\infty$, where x is the solution obtained from your code and x^* is the solution with 10 significant digits. Apply the concept of significant digits in backpropagation when the value of the components of vector x is estimated.
- Calculate the exact number of function evaluations for different significant digits and compare. Number of function evaluations includes multiplication and division in every operation.
- Include the results in a tabular form and submit a pdf file along with the source code.

Guidelines

Consider the effect of significant digits in the back-substitution only.

The general form of system of equations is like this.

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n &= b_1 \qquad \ldots (1)\\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n &= b_2 \qquad \ldots (2)\\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \ldots + a_{3n}x_n &= b_3 \qquad \ldots (3)\\
\vdots \qquad \vdots \qquad \vdots \qquad\quad \vdots \qquad \vdots \quad & \qquad\qquad\quad \vdots\\
a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \ldots + a_{nn}x_n &= b_n \qquad \ldots (n)
\end{aligned}
$$

After forward substitution, we get

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n &= b_1 \qquad \ldots (1)\\
a'_{22}x_2 + a'_{23}x_3 + \ldots + a'_{2n}x_n &= b'_2 \qquad \ldots (2)\\
a''_{33}x_3 + \ldots + a''_{3n}x_n &= b''_3 \qquad \ldots (3)\\
\vdots \qquad \vdots \quad & \qquad\qquad \vdots\\
a_{nn}^{(n-1)}x_n &= b_n^{(n-1)} \;\; \ldots (n)
\end{aligned}
$$

After calculating the value of x_n, pass this value to the function

significant_digit (int d, double x)

which will return

return (x_n);

with a required number of significant digits (d).

Then, consider the returned value of x_n and find x_{n-1}value.

Again, call *significant_digit* (int d, double x) and pass x_{n-1}value to get the updated x_{n-1}value with a required significant digits.

Follow the same procedure till you get x_1.

Now collate $(x_1, x_2, \ldots, x_n)$ and estimate the error with respect to x^*. Note that x^* is the solution corresponding to $d = 10$.