EVALUATING SOLUTIONS TO A CROWDED ELEVATOR SYSTEM WITH STOCHASTIC SIMULATION

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ABSTRACT

A textbook on mathematical problem solving provides as an example a hypothetical office building with too few elevators to carry all employees to their offices on time at the beginning of the workday. We present a model to evaluate solutions to this problem by performing a stochastic simulation on a computer. We model the time of arrival of each employee to the office as a random variable. We use interpolation to select a parameter for the probability density function to best match the employee arrival times of the system. We use the simulation to compare the effectiveness of several solutions to the problem and provide commentary on their benefits and drawbacks. Although we address a hypothetical problem, the work could potentially be put to use in real buildings.

Index Terms— elevator, simulation, office building, modeling

1. INTRODUCTION

We present an interpretation of a problem described by J. Walton and B. Davidson in the book *Solving Real Problems with Mathematics*, volume 2 [3]. The problem describes an office building in which employees have difficulty reaching their offices on time at the beginning of the workday because they are limited by the capacity of the elevators. For logistic reasons, the company cannot increase the number or capacity of its elevators, so the problem requires a solution by other means. The authors present several solutions that involve restricting the floors on which certain elevators can stop. They evaluate their solutions by estimating the average time for each elevator trip and calculating the resulting number of late employees. In this report, in order to produce a more exact model, we implement a computer simulation of the elevator system and use it to evaluate various solutions.

We begin by reviewing the details of the problem, as stated by Walton & Davidson [3]. We then provide a description of the simulation, including the selection of probability distribution for randomized employee arrival times. We use the simulation to evaluate solutions to the problem and finish with conclusions and directions for further investigation.

2. MODEL

Walton and Davidson [3] detail the elevator problem as follows. The workday at the hypothetical *Titanic Insurance Co. Ltd.* begins at 9:00 am. There are three elevators, each with a capacity of ten employees. Under existing conditions, any elevator can travel to any floor. There are four floors above the ground floor, each housing 10, 30, 100, and 100 employees, respectively, for a total of 240 employees. Under existing conditions, 60 employees are late on a typical day. In the lobby, an elevator takes 25 seconds to board its employees. An elevator takes 10 seconds to travel between floors and takes 15 seconds to stop on a floor above the lobby. If any employee arrives in the lobby as the doors of an available elevator are closing, then the doors will reopen and the elevator waits for an additional five seconds.

In order to provide solutions to the elevator problem, a few basic assumptions are necessary. Given that most employees are entering the building rather than leaving it in the morning, we assume that employees only board the elevator on the ground floor and only exit the elevator on the floor where they work. To simplify matters, we do not allow passengers to prematurely close the doors of the elevator; the elevator will always wait the amount of time detailed in the previous paragraph. The elevator only stops at floors on which its passengers work. The elevator stops at relevant floors in increasing order until it has no more passengers; it will then return to the lobby.

We assume that all employees arrive at the office building before 9:00 am. We treat the time of arrival of an employee to the office building as a random variable. The number of minutes before 9:00 am that an employee arrives is taken as a sample from the chi-squared distribution. The positive support of the chi-squared distribution agrees with the assumption that all employees arrive before 9:00 am. As a single-parameter family of distributions [2], the chi-squared distribution facilitates running the simulation with a range of parameter values in order to choose the value that best matches the existing conditions. Increasing the number of degrees of freedom of the chi-squared distribution moves the mean further from zero, so that the typical employee arrives earlier with increasing degree of freedom. Parameter selection will be discussed in the methods section.

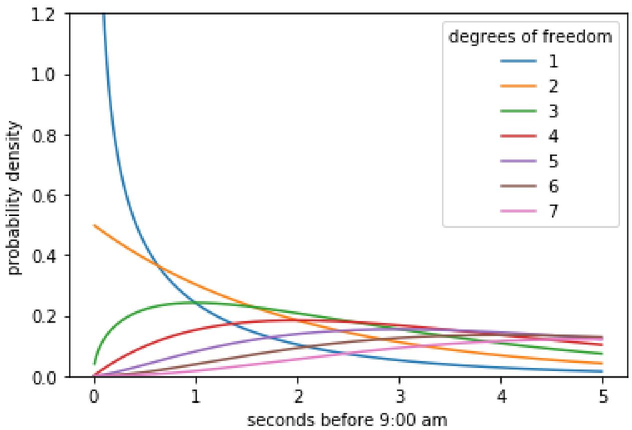

Fig. 1. Graph of the probability density function of the chi-squared distribution [2], shown with several parameter inputs. With increasing degrees of freedom, the distribution moves further from zero.

We create a computer simulation to model the system described so far. The simulation is implemented with an object-oriented design pattern, with classes `controller`, `building`, `elevator`, and `employee`. The simulation runs in discrete time steps, with each step representing one second. The controller object keeps track of time and repeats the simulation after every step, starting when the first employee arrives and ending at 9:00 am. The building object belongs to the controller and contains all of the elevators and employees. An elevator has a location and a collection of employees, with behavior dictating when it moves, loads, and unloads. Each employee has an arrival time and location. At its arrival time, an employee enters the lobby and is available to board an elevator. At 9:00 am, the simulation stops and returns the number of employees not yet delivered to their office floors. Code is available from the authors upon request.

3. METHODS

With the simulation in hand, we can test any potential solution and compare it to the original conditions. In order to fairly compare with the original system, we need to choose a parameter value for the chi-squared distribution that results in 60 late employees. In order to do so, we run the simulation with a range of parameter values, ranging from 1 to 801 degrees of freedom, with intervals of 100. Each value is the average number of late employees over 30 trials of the simulation. To interpolate between the samples, we use C^2 rational quadratic splines [1] with zero derivative at the endpoints. The advantage of C^2 rational quadratic splines over other interpolation methods in this case is that it preserves monotonicity [1], which we expect to hold given that the number of late employees should steadily decrease as the average arrival time gets earlier and earlier. The value at the first node is 240, the total number of employees, and the value at the last node is 0. The choice of zero derivative at both endpoints reflects the impossibility of further change outside the sampling interval.

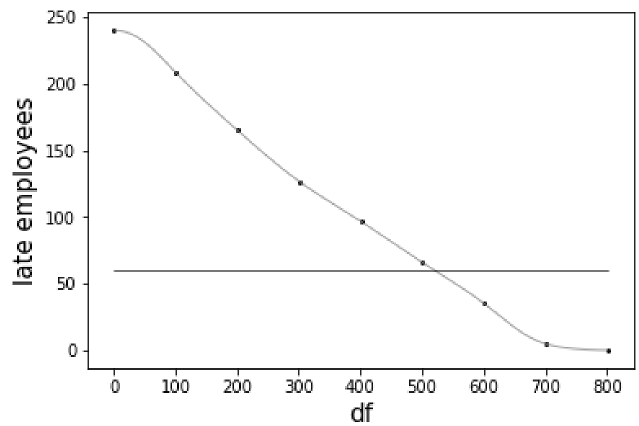

Fig. 2. Number of late employees under existing conditions, calculated at different parameter values. *df* is the number of degrees of freedom for the chi-squared distribution, which determines when employees arrive at the lobby. At each node, the number of late employees is averaged over 30 trials. The grey curve intersecting the points is produced by C^2 rational quadratic splines [1]. The horizontal grey line has height 60, the number of late employees according to the problem statement.

For the parameter to use in subsequent simulation, we use the closest integer to the point where the splines evaluate to 60. Solutions to the problem, as we will see in the next section, consist of restrictions on the floors to which an elevator can travel. To evaluate a solution, we implement the corresponding restriction or alteration in the simulation and take an average number of late employees over 100 trials.

4. SOLUTION

Walton and Davidson [3] provide a potential solution to the problem, which we will call *Strategy 1*.

- **Strategy 1:** One elevator can visit any floor. The second elevator can only visit the third floor. The last elevator can only visit the fourth floor.

We also provide another similar solution, which we will call *Strategy 2*.

- **Strategy 2:** One elevator can visit any floor. The other two elevators are both only able to visit the third and fourth floors.

These solutions aim to minimize the number of stops that an elevator takes by reducing the number of floors it has to service. They take advantage of the fact that some of the floors, floors three and four, have significantly more employees than other floors.

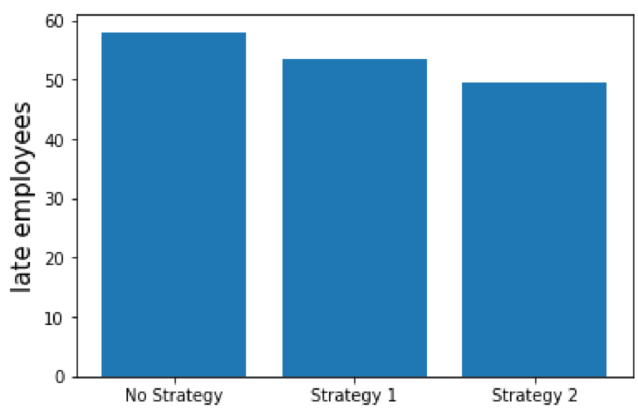

Fig. 3. The number of late employees for each of three elevator solutions. *No Strategy* refers to the existing conditions, wherein any elevator can visit any floor. Strategies 1 and 2 restrict the floors that each elevator can visit and are detailed in the main text. Each solution is averaged over 100 trials.

The results using *Strategy 1* and *Strategy 2* are shown in Figure 3. *Strategy 1* performs slightly better than the existing conditions, and *Strategy 2* performs slightly better yet, bringing the number of late employees down to about 50. The results are underwhelming, with many late employees remaining. This solution, however, has its benefits. It requires no expansions on the building infrastructure, and everyone can still use the elevator.

Another solution suggested by Walton and Davidson [3] is to make the employees on the first and second floors take the stairs, restricting the use of the elevators to the third and fourth floors. Figure 4 shows the performance of this solution, where all three elevators can all visit the third and fourth floors, but no others. This solution far outperforms *Strategy 1* and *Strategy 2*, though it has its disadvantages. The employees who have to take the stairs are likely to be unhappy.

It is interesting to see what would happen if it were possible to add more elevators. Figure 5 shows the results with 4 and 5 elevators, where any elevator can visit any floor. With 4 elevators, the problem is significantly reduced, and with 5

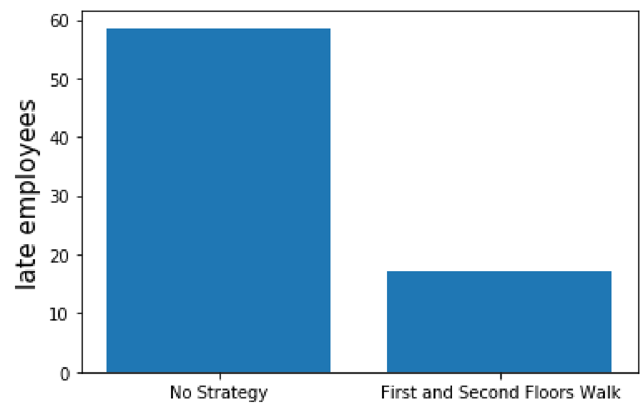

Fig. 4. The number of late employees for two elevator solutions, averaged over 100 trials. *No Strategy* refers to the existing conditions. In the right column, employees on the first and second floors take the stairs, so the elevator only services the third and fourth floors. All three elevators can visit both the third and fourth floors.

elevators, there are never any late employees. Of course, the problem statement forbids this solution as an option.

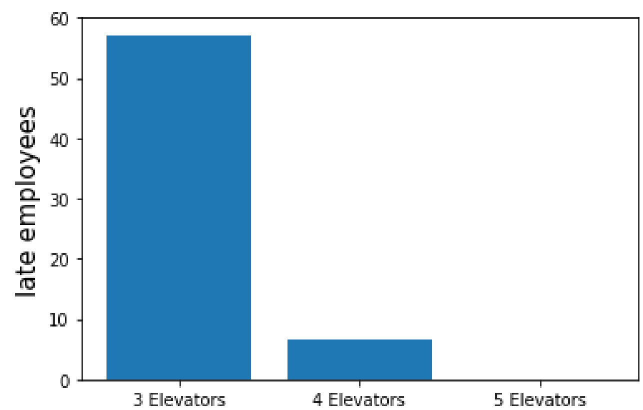

Fig. 5. The number of late employees for differing numbers of elevators, averaged over 100 trials. In each case, any elevator can visit any floor.

It is worth mentioning that there is a solution that requires no mathematics, though it may be against the spirit of the problem. Simply require employees to reach their offices on time, even if it means taking the stairs. Alternatively, after some time, say 8:55 am, stop the elevator service, so that employees who arrive too close to 9 am have to take the stairs. In this case, no employees will be late as a result of the elevator system, and anyone who arrives early enough will have a chance to use the elevator regardless of the floor on which they work. This solution does have its disadvantages. Firstly, it requires some employees to take the stairs. Also, by stopping the elevators early, we fail to make full use of them as a resource.

5. CONCLUSION

Using a stochastic simulation of the elevator system, we evaluate solutions to the elevator problem as posed by Walton and Davidson [3]. Restricting the floors to which an elevator can travel results in modest improvement. Reducing the number of floors serviced by the elevators results in a greater improvement, though there are still late employees. Although it is not an allowed solution, increasing the number of elevators solves the problem.

There are several directions for further investigation.

To improve the computer simulation, it would be worthwhile to implement a graphical component, so that the user can see a visual representation of the elevator system as it functions. This functionality would promote better understanding and could lead to more effective solutions to the problem. Given that the basic simulation is already implemented and functioning, adding a graphical component would be a simple matter of extension.

Another improvement to the computer simulation could include a thorough unit testing. Although we are fairly certain that it works as intended, mistakes are a possibility. Given that the validity of our results depends on the proper functioning of the simulation, it would be worthwhile to be certain.

In this report we only investigate two of the solutions that involve restricting the floors to which an elevator can travel. There are many more options, and there is no guarantee that any of the solutions presented here are optimal. Finding the true optimal solution would be an interesting challenge in combinatorial optimization, since a brute-force search through all possible solutions would probably be computationally prohibitive.

The assumption of the chi-squared distribution of employee arrival times is convenient for the purposes of this report, though it is worth investigating how well it actually describes real employee arrival times. In order to best represent human behavior, a statistical analysis of a real office building and its employee arrival times could suggest a more realistic distribution.

Although this report addresses a hypothetical problem from a textbook, there is probably application for the work presented here. It could be worthwhile to find an actual office building facing a similar problem and use this model to evaluate potential solutions.

6. REFERENCES

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