Hyperbolic Deep Learning for Foundation Models: A Tutorial

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Abstract

Recent years have seen foundation models, such as large language models (LLMs), vision transformers (ViTs), diffusion generative models, and large multi-modal models, reshape machine learning by driving advancements across training, inference, and deployment. Despite notable successes, the performance of Euclidean foundation models is inherently limited by the representational capacity of Euclidean geometry. Recent studies have shown that input token embeddings exhibit hierarchical structures and scale-free properties, which are poorly captured by Euclidean embeddings due to their linear and uniform scaling. In contrast, hyperbolic spaces, with their exponential volume growth relative to distance, provide a promising alternative for embedding tree-like structures and power-law distributions. This has spurred a surge of novel methods that integrate hyperbolic geometry into foundation model settings, enabling more efficient and effective representation of token distributions. In support of the growing interest in hyperbolic foundation models, this tutorial offers a systematic review of methods and challenges in this vibrant field, especially focusing on being accessible to all audiences. More specifically, we will first give a brief introduction to hyperbolic geometry motivated by non-Euclidean input structures in foundation models and discuss the scope of these models. We will then present unified approaches for hyperbolic deep learning—including essential hyperbolic neural network operations and models—which serve as the building blocks of hyperbolic foundation models. Next, we will comprehensively revisit the technical details for developed hyperbolic foundation models, focusing on hyperbolic LLMs and Transformers, vision foundation models, and multi-modal foundation models. Finally, we discuss several challenges and propose potential solutions, including some preliminary efforts of our own, that may pave the way for future advances in this research area.

CCS Concepts

• **Mathematics of computing** → **Geometric topology**; • **Computing methodologies** → **Machine learning approaches**; **Knowledge representation and reasoning**.

Keywords

Hyperbolic Geometry, Foundation Models, Representation learning, Transformer

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1 Target Audience and Prerequisite

This tutorial targets machine learning researchers of any background, particularly those interested in foundation models, non-Euclidean, and geometric deep learning. We explore advanced applications of hyperbolic geometry in these fields, focusing on large-scale hyperbolic models that extend beyond the concepts covered in previous tutorials [**? ?**]. No special background in differential geometry is required, though familiarity with mathematical concepts such as Riemannian manifolds would be helpful.

2 Tutors

2.1 In-person Presenters

- **Neil He** is a bachelors/master student in the mathematics department at Yale University, advised by Professor Rex Ying. His research focus includes geometry and hyperbolic deep learning, their applications to foundation models, and their theoretical foundations. He is particularly interested in developing theoretical robust deep-learning methods and finding theoretical foundations for empirically observed behaviors.
- **Menglin Yang** is currently an Assistant Professor in AI Thrust at the Hong Kong University of Science and Technology (Guangzhou), HKUST(GZ). He was previously a postdoctoral researcher at Yale University with Prof. Rex Ying

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and received his Ph.D. from The Chinese University of Hong Kong. His research interests include hyperbolic geometric learning and foundation models. He organized tutorials on hyperbolic graph learning at KDD 2023 and ECML-PKDD 2022. He also organized a workshop on the non-Euclidean foundation model at WWW (TheWebConf) 2025.

- **Rex Ying** is an Assistant Professor in the Department of Computer Science at Yale University. His research focus includes geometric deep learning, foundation models with structured data, multi-modal models, graph learning, and trustworthy deep learning. He is interested in the use of graphs and geometry to empower representation learning in expressiveness and trustworthiness in large-scale settings. Rex has built multi-modal foundation models in engineering, natural science, social science and financial domains. He won the best dissertation award at KDD 2022, and the Amazon Research Award in 2024. His research is in part supported by National Science Foundation, Gordon and Betty Moore Foundation, and industry partners such as NetApp, Goldman Sachs, Snap Research and Amazon Research Award.

2.2 Contributors

- **Hiren Madhu** is a PhD student in the Department of Computer Science at Yale University, advised by Professor Rex Ying and Professor Smita Krishnaswamy. His research focus includes geometric deep learning, foundation models with structured data, and their applications to biological data. He is particularly interested in developing novel machine learning methodologies that leverage graph-based and manifold-based approaches for efficient and interpretable representation learning for large biological datasets.
- **Ngoc Bui** is a PhD student in the Department of Computer Science at Yale University, working under the supervision of Professor Rex Ying. His research interests broadly include large language models (LLMs) and their applications, such as personalized assistants and synthetic data generation through simulations.

2.3 Overlap Statement

- Hyperbolic Neural Networks: Theory, Architectures and Applications [**?**] at Washington D.C., USA on August 14th, 2022. While both the proposed tutorial and [**?**] involve hyperbolic deep learning methods, the latter introduces general hyperbolic neural networks such as CNNs and GNNs. The proposed tutorial focuses specifically on hyperbolic foundation models such as LLMs, ViTs and CLIP models, many of which are extremely recent and not included in [**?**].
- Hyperbolic Graph Neural Networks: A Tutorial on Methods and Applications [**?**] at Long Beach, CA, USA on August 6th. While both concern hyperbolic deep learning methods, [**?**] focuses on hyperbolic GNNs and their applications, whereas the proposed tutorial focuses on foundation models such as LLMs, ViTs and CLIP models.

3 Tutorial Content

3.1 Interactive Components

We plan on employing several strategies to encourage audience participation such as

- Interactive Polling: Using software such as Slido or Mentimeter, we plan on incorporating live polls to interact with the audiences.
- Open-ended Discussions: To further increase participation, we plan on posing open-ended questions related to our presentation to facilitate discussions.
- Interactive Q&A: We plan on making Q&A sessions more interactive through software such as Google Moderator.

3.2 Societal Impact

Foundation models [**?**], hyperbolic deep learning models [**?**], and the combination of the two have vast real-world and societal applications: in natural language processing [**? ?**], such as hierarchical natural language concepts capturing [**?**], question answering systems [**? ?**], privacy-preserving text representations [**?**], and multi-document summarization [**?**]; in computer vision [**?**], such as visual and object detection [**? ?**], image segmentation [**?**], and video prediction [**?**]; in natural sciences, such as protein-folding [**?**], single-cell-RNS-seq analysis [**?**], and modeling brain activities [**? ?**]; and in complex networks, such as social networks [**? ? ?**] and recommender systems [**? ? ? ?**]. Our tutorial would facilitate and support future research in the field of hyperbolic foundation models, which have the potential to advance the important areas listed above for wide societal impacts. Additionally, our tutorial would be widely applicable, especially in scenarios requiring large-scale models, given the critical role foundation models have had in recent years for both research and societal issues alike [**? ?**].

3.3 Tutorial Outline

The tutorial will focus on hyperbolic deep learning methods for foundation models and potential future directions. It will be organized as follows, with a 10-minute break:

- Introduction & Motivation (35 minutes)
  - An overview of Euclidean foundation models
  - Brief introduction to hyperbolic geometry and Riemannian manifolds [**? ? ? ?**]
  - Motivation for incorporating hyperbolic geometry into foundation models [**? ? ? ? ? ? ?**]
- Hyperbolic Neural Networks as Building Blocks (40 min)
  - Hyperbolic neural network operations [**? ? ?**]
  - Hyperbolic neural network models [**? ? ? ?**]
- Hyperbolic Foundation Models (60 minutes)
  - Hyperbolic Transformers and LLMs [**? ? ? ? ? ?**]
  - Hyperbolic vision foundation models [**? ? ? ?**]
  - Hyperbolic multi-modal models [**? ? ?**]
- Future Directions and Challenges (35 Minutes)
  - Hyperbolic pre-trained models [**?**]
  - Advanced hyperbolic foundation models [**?**]
  - Hyperbolic deep learning tools and libraries [**?**]

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