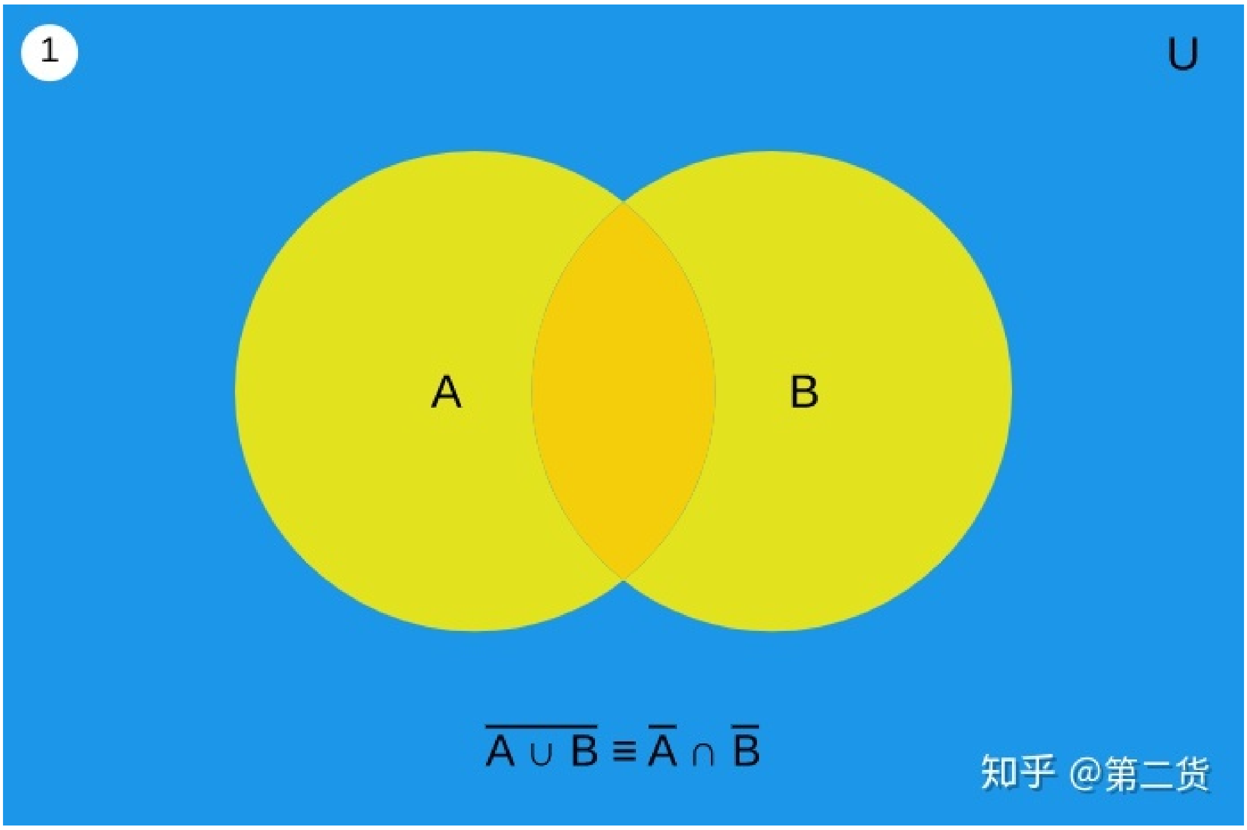
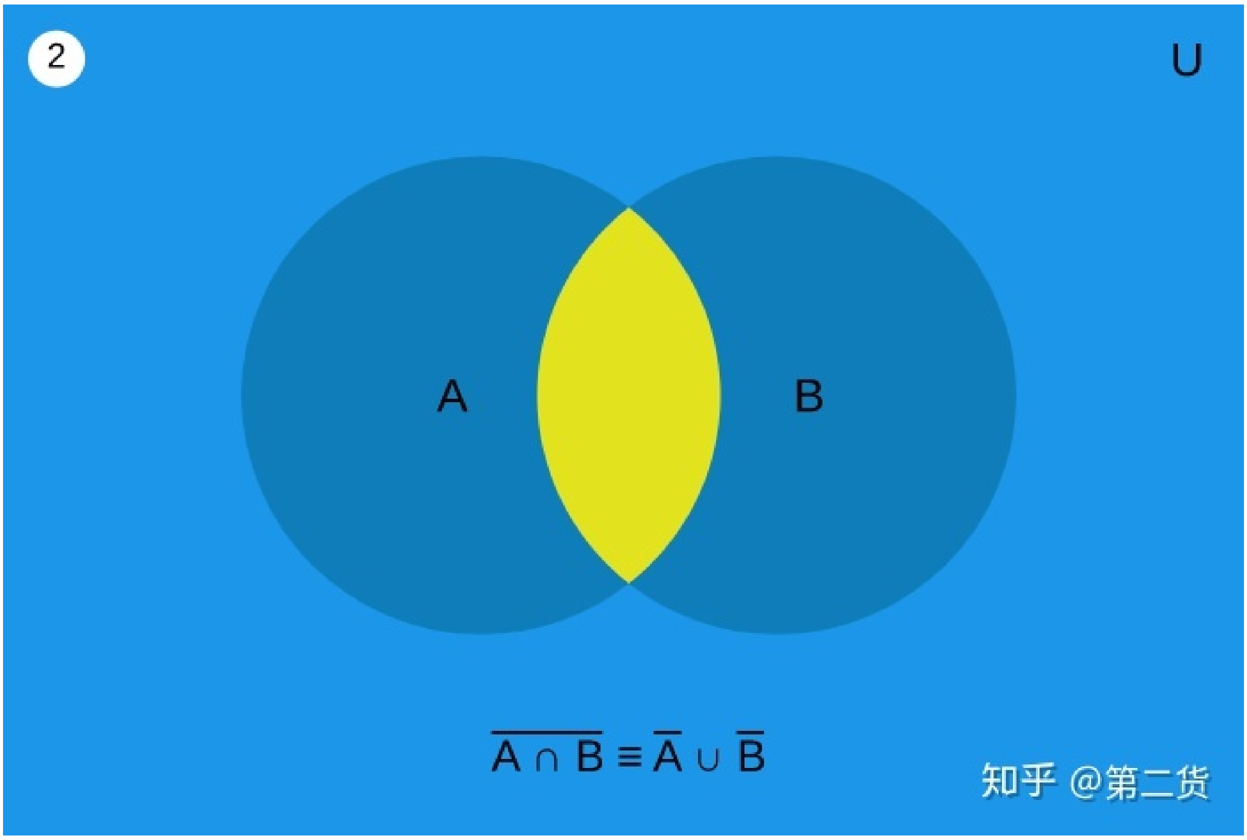
德摩根定律De Morgan's laws





一、在集合论和布尔代数的德摩根定律证明

德摩根定律，其中c表示补集:

1) (⋃nSn)c=⋂nSnc(\bigcup_{n}^{} S_{n})^{c} = \bigcap_{n}^{} S_{n}^{c} 2) (⋂nSn)c=⋃nSnc(\bigcap_{n}^{} S_{n})^{c} = \bigcup_{n}^{} S_{n}^{c}

现在证明第一个公式：

Step 0： x∈(⋃nSn)cx \in(\bigcup_{n}^{} S_{n})^{c} ， 具有任意性x具有任意性x具有任意性

⇒x∉⋃nSn=S1∪S2...∪Sn\Rightarrow x \notin\bigcup_{n}^{} S_{n}^{}=S_{1}\cup S_{2} ...\cup S_{n}

⇒x∉S1x∉S2...x∉Sn\Rightarrow x \notin S_{1} \ \ \ x \notin S_{2} \ \ \ ... \ \ \ x \notin S_{n}

⇒x∈S1cx∈S2c...x∈Snc\Rightarrow x \in S_{1}^{c} \ \ \ x \in S_{2}^{c} \ \ \ ... \ \ \ x \in S_{n}^{c}

⇒x∈S1c∩S2c...∩Snc=⋂nSnc\Rightarrow x \in S_{1}^{c} \cap S_{2}^{c} ... \cap S_{n}^{c} =\bigcap_{n}^{} S_{n}^{c} = 右边

即 (⋃nSn)c⊂⋂nSnc(\bigcup_{n}^{} S_{n})^{c} \subset \bigcap_{n}^{} S_{n}^{c}

Step 1： x∈⋂nSnc=S1c∩S2c...∩Sncx \in \bigcap_{n}^{} S_{n}^{c} = S_{1}^{c} \cap S_{2}^{c}... \cap S_{n}^{c} ， 具有任意性x具有任意性x具有任意性

⇒x∈S1cx∈S2c...x∈Snc\Rightarrow x \in S_{1}^{c} \ \ \ x \in S_{2}^{c} \ \ \ ... \ \ \ x \in S_{n}^{c}

⇒x∉S1x∉S2...x∉Sn\Rightarrow x\notin S_{1} \ \ \ x\notin S_{2} \ \ \ ... \ \ \ x\notin S_{n}

⇒x∉S1∪S2...∪Sn=⋃nSn\Rightarrow x \notin S_{1} \cup S_{2} ... \cup S_{n} = \bigcup_{n}^{} S_{n}

⇒x∈(⋃nSn)c\Rightarrow x \in (\bigcup_{n}^{} S_{n})^{c} =左边

即 ⋂nSnc⊂(⋃nSn)c\bigcap_{n}^{} S_{n}^{c} \subset (\bigcup_{n}^{} S_{n})^{c}

由Step 0、Step 1 => (⋃nSn)c=⋂nSnc(\bigcup_{n}^{} S_{n})^{c} = \bigcap_{n}^{} S_{n}^{c} ，即得证；第二个公式的证明类似

特别地，当n=2时，德摩根公式一般化为
(A∪B)c=Ac∩Bc(A∩B)c=Ac∪Bc(A \cup B)^{c} = A^{c} \cap B^{c} \ \ \ \ (A \cap B)^{c} = A^{c} \cup B^{c}

https://www.zhihu.com/question/339501136/answer/1108754202

二、逻辑学德摩根定律证明

（一）证明： ¬(A∨B)=(¬A)∧(¬B)\neg(A\vee B)=(\neg A)\wedge(\neg B)

¬(A∨B)\neg(A\vee B) ⇔\Leftrightarrow A或B其中一个为真是假的 ⇔\LeftrightarrowA和B都不为真

⇒\Rightarrow (¬A)∧(¬B)(\neg A)\wedge(\neg B)

（二）证明： ¬(A∧B)=(¬A)∨(¬B)\neg(A\wedge B)=(\neg A)\vee(\neg B)

¬(A∧B)\neg(A\wedge B) ⇔\Leftrightarrow A和B都为真是假的 ⇔\Leftrightarrow A或B至少其中一个假的是真的

⇒\Rightarrow (¬A)∨(¬B)(\neg A)\vee(\neg B)