4.Geometrical features of binary objects

4.1.Introduction

This lab work presents some important geometric properties of binary images and the algorithms used for computing them. The properties described are: the area, the center of mass, the elongation axis, the perimeter, the thinness ratio, the aspect ratio and the projections of the binary image.

4.2.Theoretical considerations

After applying segmentation and labeling algorithms to images we obtain a new image in which each object can be referenced separately.

An object ‘i’ is described in the image by the function:

$$I_i(r,c) = \begin{cases} 1, & \text{if } I(r,c) \in \text{object labeled 'i'} \\ 0 & \text{otherwise} \end{cases}$$
$$\text{where } r \in [0 ... Height-1] \text{ and } c \in [0 ... Width-1]$$

The geometric properties of objects can be classified into two categories:

- position and orientation properties: the center of mass, the area, the perimeter, the elongation axis
- shape properties: aspect ratio, thinness ratio, Euler’s number, the projections, the Feret diameters of the objects

4.2.1.Area

$$A_i = \sum_{r=0}^{H-1} \sum_{c=0}^{W-1} I_i(r,c)$$

The area A_i is measured in pixels and it indicates the relative size of the object.

4.2.2.The center of mass

$$\overline{r}_i = \frac{1}{A_i} \sum_{r=0}^{H-1} \sum_{c=0}^{W-1} r I_i(r,c)$$

$$\overline{c}_i = \frac{1}{A_i} \sum_{r=0}^{H-1} \sum_{c=0}^{W-1} c I_i(r,c)$$

The equations above correspond to the row and column where the center of mass is located. This attribute helps us locate the object in a bi-dimensional image.

4.2.3.The axis of elongation (the axis of least second order moment)

$$\tan(2\varphi_i) = \frac{2\sum_{r=0}^{H-1}\sum_{c=0}^{W-1}(r-\overline{r}_i)(c-\overline{c}_i)I_i(r,c)}{\sum_{r=0}^{H-1}\sum_{c=0}^{W-1}(c-\overline{c}_i)^2 I_i(r,c) - \sum_{r=0}^{H-1}\sum_{c=0}^{W-1}(r-\overline{r}_i)^2 I_i(r,c)}$$

If both the nominator and the denominator of the above equation are equal to zero, then the object has a circular symmetry, and any line that passes through the center of mass is a symmetry axis.

For finding the direction of the line (the angle) one must apply the arctangent function. The arctangent is defined on the interval (-∞, +∞) and it takes values in the interval (-π/2, π/2). The evaluation of the arctangent becomes unstable when the denominator of the fraction tends to zero.

The signs of the numerator and of the denominator are important for determining the right quadrant in which the result lays. The arctangent function does not make the difference between directions that are opposed. For this reason, the usage of the function “atan2” is suggested. The “atan2” function has as arguments the numerator and the denominator of such fraction, and it returns a result in the interval (-π, π).

The axis of elongation gives information about how the object is positioned in the field of view, that is its orientation. The axis corresponds to the direction in which the object (seen as a plane surface of constant width) can rotate most easily (has a minimum kinetic moment).

After the φ_i angle is found, the correctness of the resulted value can be validated by drawing the axis of elongation. The axis of elongation will correspond to the line that pass through the center of mass and determines the φ_i angle with Ox axis.

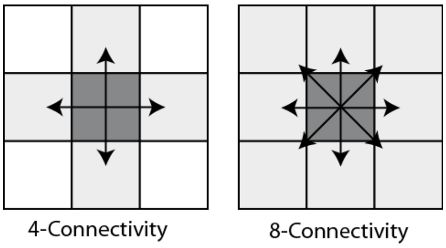
4.2.4.The perimeter

The perimeter of the object helps us determine the position of the object in space and it also gives information about the shape of the object. The perimeter can be computed by counting the number of pixels on the contour (pixels of value 1 and having at least one neighbor pixel of value 0).

A first approach to contour detection is the scanning of the image, line by line and counting the number of pixels in the object that satisfy the condition mentioned above. A main disadvantage of this method is that we cannot distinguish the exterior contour from the interior contours (if they exist they are generated by the holes in the object). As the pixels of digital images represent distributions on a rectangular raster, the length of curves and oblique lines in the image cannot be correctly estimated by counting the pixels. A first correction is given by the multiplication by π/4 of the perimeter that resulted in the previous algorithm. There are other methods for length correction. These methods take into account the type of neighborhood used (4 neighbors, 8 neighbors etc).

Another method for detecting the contour of an object involves the usage of an existing algorithm for edge detection, the thinning of the edges until they become 1 pixel thick and in the end the counting of the resulted edge pixels.

Methods of type “chain-codes” represent complex methods for contour detection and offer a high accuracy.



4.2.5.The thinness ratio (circularity)

$$T = 4\pi\left(\frac{A}{P^2}\right)$$

The function above has the maximum value equal to 1, and for this value we obtain a circle. The thinness ratio is used for determining how “round” an object is. If the value of T is close to 1, the object tends to be round.

The value of the thinness ratio also offers information on how regular an object is. The objects that have a regular contour have a greater value of T than the objects of irregular contours. The value 1/T is called irregularity factor of the object (or compactness factor).

4.2.6.The aspect ratio

This property is found by scanning the image and keeping the minimum and maximum values of the lines and columns that form the rectangle circumscribed to the object.

$$R = \frac{c_{\max} - c_{\min} + 1}{r_{\max} - r_{\min} + 1}$$

4.2.7.The projections of the binary object

The projections give information about the shape of the object. The horizontal projection equals the sum of pixels computed on each line of the image, and the vertical projection is given by the sum of the pixels on the columns.

$$h_i(r) = \sum_{c=0}^{W-1} I_i(r,c) \qquad\qquad v_i(c) = \sum_{r=0}^{H-1} I_i(r,c)$$

The projections are used in applications of text recognition in which the interest object can be normalized.