Technical University of Cluj - Napoca
Computer Science Department

Image Processing

(Year III, 2-nd semester)

Binary Images: Morphological Image Processing (V)

Morphologic Image Processing

1. Introduction

- **Morphology**: denotes a branch of biology that deals with the form and structure of animals and plants
- **Mathematical** morphology:
  - **a tool for extracting image components** that are useful **in the representation and description of region shape**, such as boundaries, skeletons, and the convex hulls
  - **for pre or post-processing**, such as morphological filtering, thinning, and pruning
- **Set theory**: is the language of mathematical morphology

2. Some Basic Concepts from Set Theory

- Let A be a ***set*** in Z^2. If $a=(a1,a2)$ is an element of A, then we write

$$a \in A$$

- Similarly, if a is not an element of A, we write

$$a \notin A$$

-The set with no elements is called the ***null*** or empty set and is denoted by the symbol Ø.

-A set is specified by the contents of two braces: {…}. The **elements of the sets** with which we are concerned in this chapter **are the coordinates of pixels** representing objects or other features of interest in an image.

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Some Basic Concepts from Set Theory (2)

- If every element of a set A is also an element of another set B, then A is said to be a ***subset*** of B, denoted as: $A \subseteq B$

- The ***union*** of two sets A and B, is the set of all elements belonging to either A, B, or both: $C = A \cup B$

- Similarly, the ***intersection*** of two sets A and B, is the set off all elements belonging to both A and B: $D = A \cap B$

- Two sets A and B are said to be ***disjoint*** or ***mutually exclusive*** if they have no common elements: $A \cap B = \varnothing$

- The ***complement*** of a set A is the set of elements not contained in A:

$$A^C = \{w \mid w \notin A\}$$

- The **difference** of two sets A and B, denoted A-B, is defined as:

$$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^C$$

- The ***reflection*** of set B, is defined as:

$$\hat{B} = \{w \mid w = -b,\ for + b \in B\}$$

- The ***translation*** of set A by point $z=(z1,z2)$, is defined as:

$$(A)_Z = \{c \mid c = a + z,\ for\ a \in A\}$$

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Some Basic Concepts from Set Theory (3)

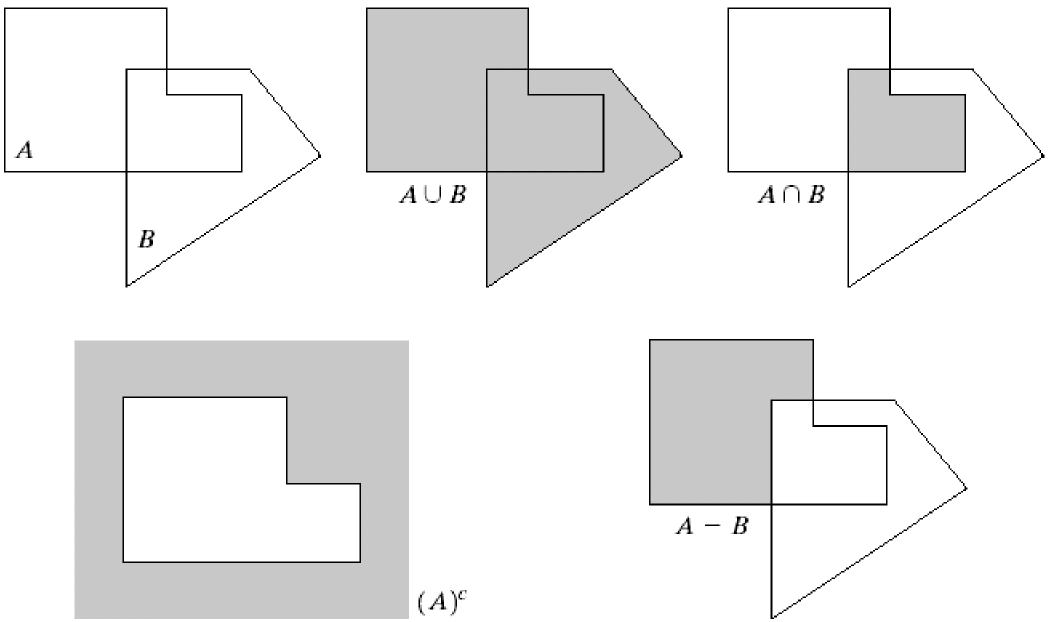

FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

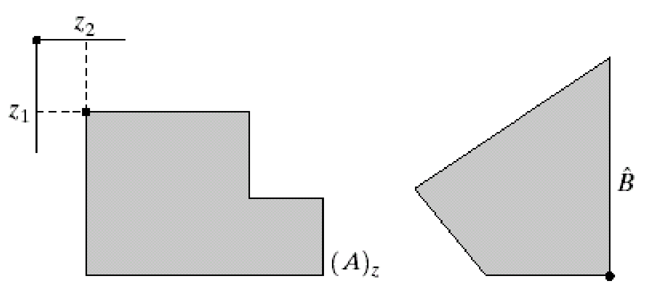

FIGURE 9.2 (a) Translation of A by z. (b) Reflection of B. The sets A and B are from Fig. 9.1.

From "Digital Image Processing" by R. C. Gonzales and R. E. Woods

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3. Logic operations involving Binary Images

- The principal logic operations used in image processing are AND, OR, and NOT (COMPLEMENT). These operations can be combined to form any other logic operation.

- Logic operations are performed on a pixel by pixel basis between corresponding pixels of two or more images (except NOT, which operates on the pixels of a single image).

- The logic operations have a one-to-one correspondence with the set operations, with the limitation that logic operations are restricted to binary variables, which is not the case in general for set operations.

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Logic operations involving Binary Images(2)

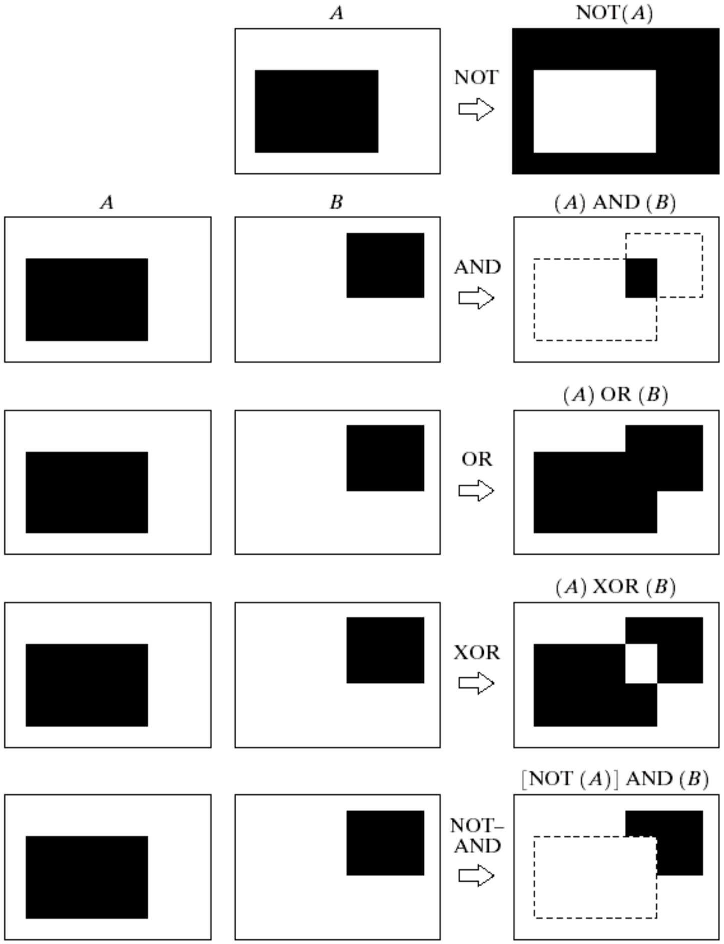


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

From “Digital Image Processing” by R. C. Gonzales and R. E. Woods

4. Dilation and Erosion

Dilation and erosion are two primitive morphological operations.

4.1 Dilation

a) With A and B as sets in Z^2, the ***dilation*** of A by B, denoted $A \oplus B$, is defined as:

$$A \oplus B = \left\{ z \mid (\hat{B})_z \cap A \neq \varnothing \right\}$$

This equation is based on obtaining the reflexion of B about its origin and shifting this reflexion by z. The dilation of A by B then is the set off all displacements z, such that $(\hat{B})_z$ and A overlap by at least one element. Based on this interpretation we can write:

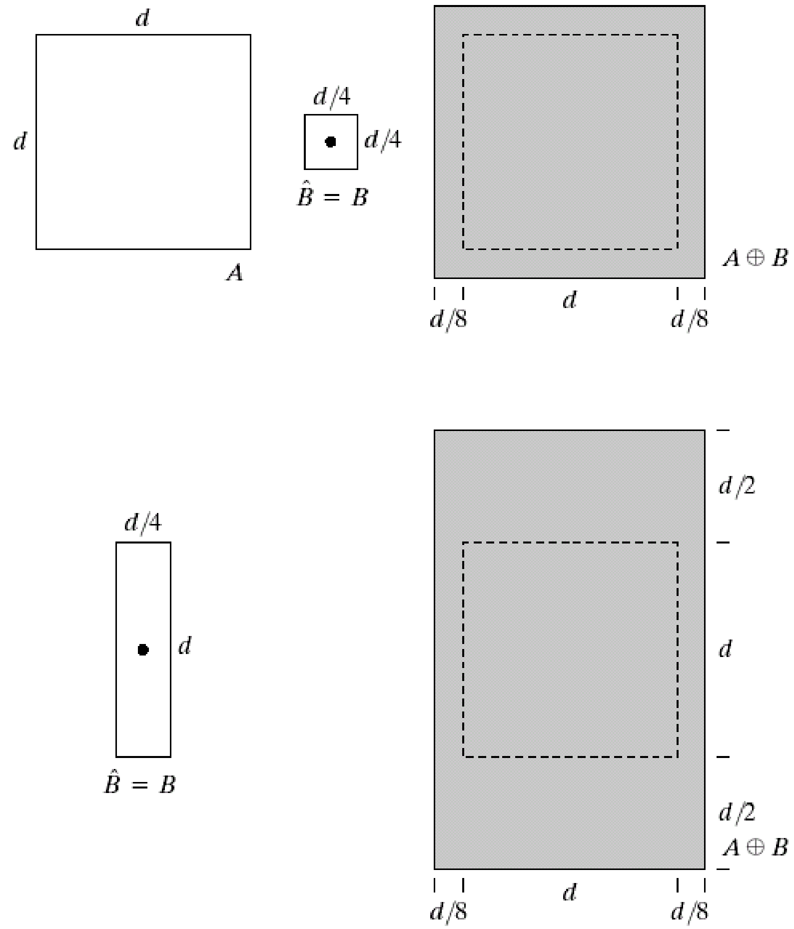
$$A \oplus B = \left\{ z \mid [(\hat{B})_z \cap A] \subseteq A \right\}$$

Set B is commonly referred to as **the structuring element**.

Dilation (2)

a b c
d e

FIGURE 9.4
(a) Set A.
(b) Square structuring element (dot is the center).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element.
(e) Dilation of A using this element.



From “Digital Image Processing” by R. C. Gonzales and R. E. Woods

Dilation (3)

b). Dilation is the morphological transformation that combines two sets using vector addition of set elements. If A and B are sets in Z^2 with elements a and b respectively, $a=(a_1, a_2, \ldots, a_N)$ and $b=(b_1, b_2, \ldots, b_N)$ being element coordinates, then the dilation of A by B is the set of all possible vector sums of pairs of elements, one coming from A and one coming from B.

$$A \oplus B = \left\{ z \in Z^2 \mid z = a + b,\ for\_some\_a \in A\_and\_b \in B \right\}$$

Because addition is commutative, dilation is commutative $A \oplus B = B \oplus A$
In practice the sets A and B are not thought of symmetrically.

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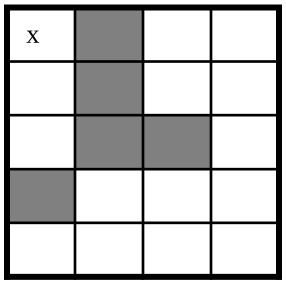
Dilation (4)

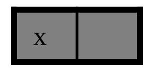
A={(0,1), (1,1), (2,1), (2,2),(3,0)}; B={(0,0), (0,1)}

$A \oplus B=\{(0,1), (1,1), (2,1), (2,2), (3,0), (0,2), (1,2), (2,2), (2,3), (3,1)\}$

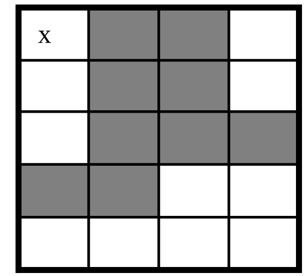
A



B



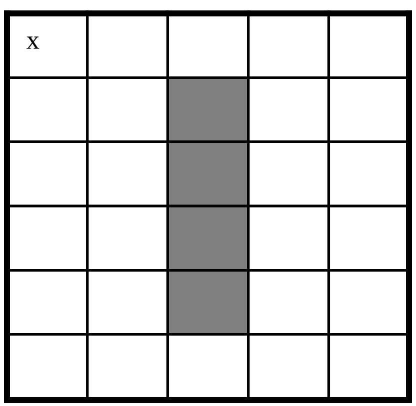
A ⊕ B



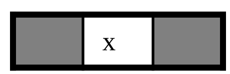
A={(1,2), (2,2), (3,2), (4,2)}; *B={(0,-1), (0,1)}*

$A \oplus B = \{(1,1), (2,1), (3,1), (4,1), (1,3), (2,3), (3,3), (4,3)\}$

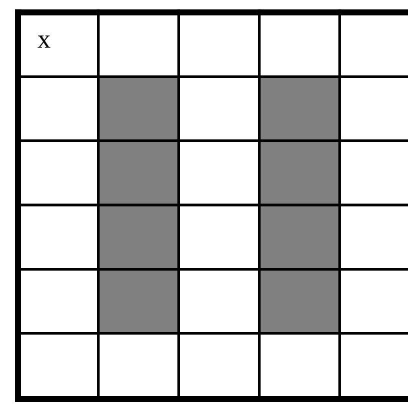
A



B



A ⊕ B



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Dilation (5)

c) The dilation operation can be represented as a union of translates of the structuring element.

$$A \oplus B = \bigcup_{a \in A} B_a$$

If *B* has a center on the origin, then the dilation of *A* by *B* can be understood as the locus of the points covered by *B* when the center of *B* moves inside *A*.

The dilation process is performed by laying the structuring element on the image and sliding it across the image in a manner similar to convolution. The difference is in the operation performed. It is best described in a sequence of steps:

1. If the origin of the structuring element coincides with a '0' in the image, there is no change; move to the next pixel.

2. If the origin of the structuring element coincides with a '1' in the image, perform the OR logic operation on all pixels within the structuring element.

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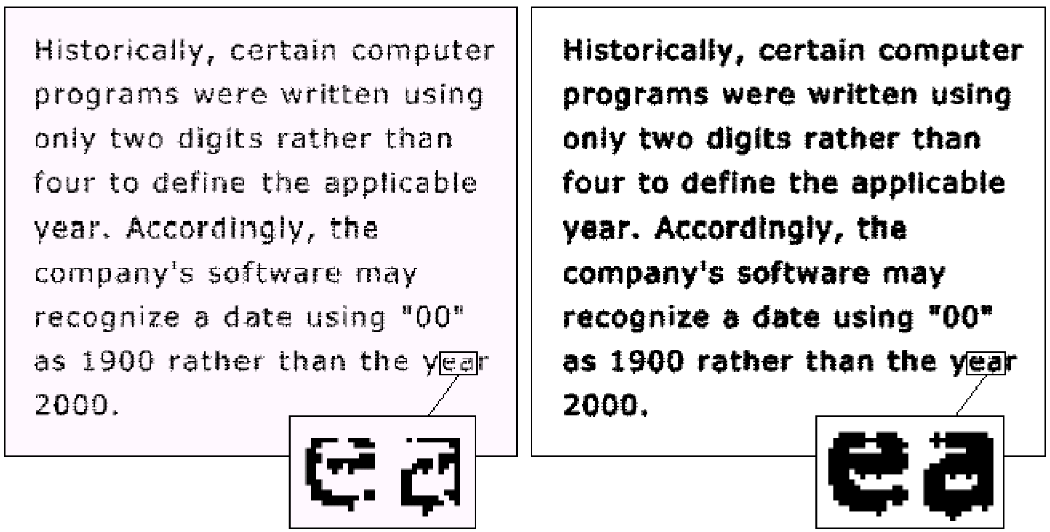
Dilation (6)

One of the simplest applications of dilation is for bridging gaps.



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

From "Digital Image Processing" by R. C. Gonzales and R. E. Woods

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4.2 Erosion

a) With A and B as sets in Z^2, the ***erosion*** of A by B, denoted $A\Theta B$, is defined as:

$$A\Theta B = \{z \mid (\hat{B})_Z \subseteq A\}$$

The erosion of A by B is the set of all points z such that $\hat{B}$ translated by z, is contained in A.

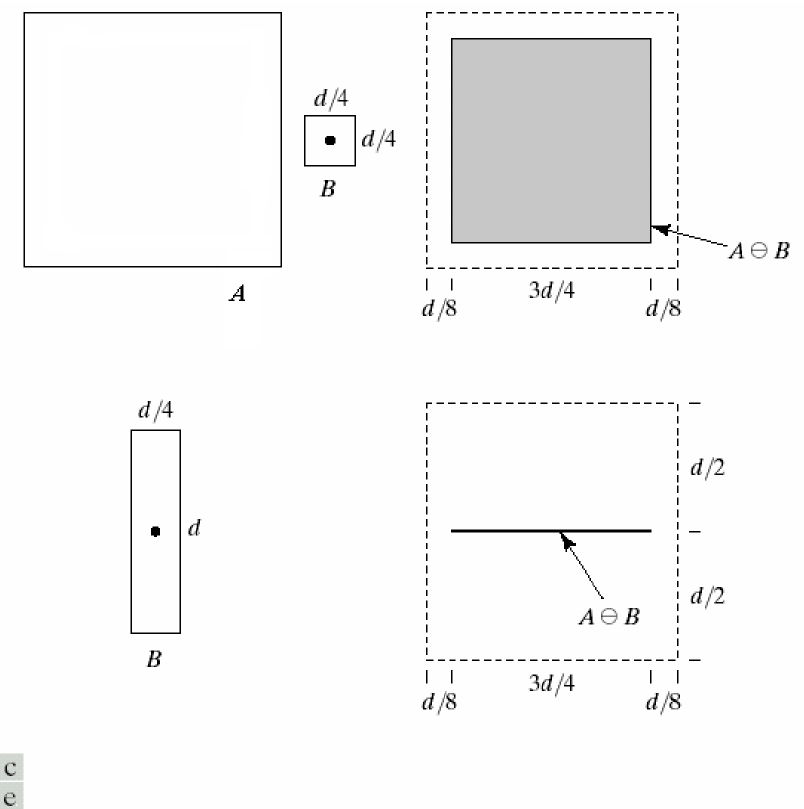

FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.

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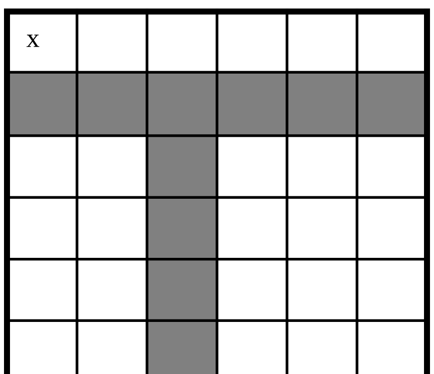
Erosion (2)

b) Erosion is the morphological transformation that combines two sets by using containment to its basis set. If A and B are sets in Z^2, then the erosion of A by B is the set of all elements x for which $x+b \in A$ for every $b \in B$.

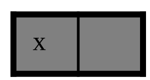
$$A \Theta B = \{ x \in Z^2 \mid x+b \in A, \; for\_every\_b \in B \}$$

A={(1,0), (1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (3,1), (4,1), (5,1)}
B={(0,0), (0,1)}
A ΘB={(1,0), (1,1), (1,2), (1,3), (1,4)}

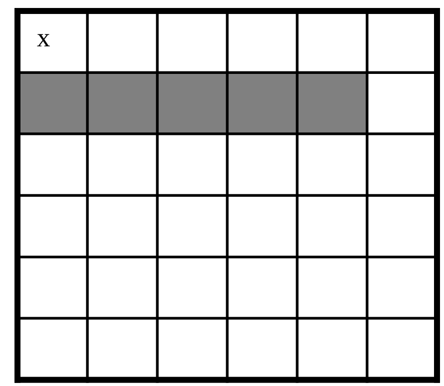
A



B



A ΘB



Erosion (3)

c) Erosion can be viewed as a morphological transformation that combines two sets by using vector subtraction of set elements. Expressed as a difference of elements a and b, erosion becomes

$$A\Theta B=\{x \in Z^2 | \textit{for every } b \in B, \textit{ there exists an } a \in A \textit{ such that } x=a-b\}$$

The erosion process is similar to dilation, but we turn pixels to '0', not '1 '. As before, slide the structuring element across the image and then follow these steps:

1. If the origin of the structuring element coincides with a '0' in the image, there is no change; move to the next pixel.

2. If the origin of the structuring element coincides with a '1' in the image, and any of the '1' pixels in the structuring element extend beyond the object ('1' pixels) in the image, then change the '1' pixel in the image to a '0'.

$\in$

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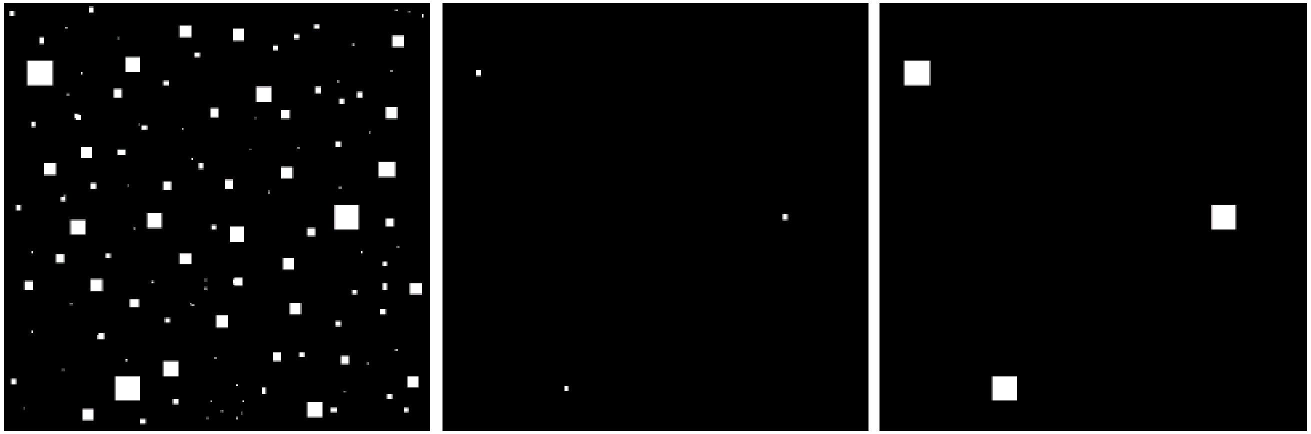
Erosion (4)

One of the simplest uses of erosion is for eliminating irrelevant detail from a binary image. It is used in conjunction with dilation.
Dilation and erosion are duals of each other with respect to set complementation and reflection.

$$(A \ominus B)^C = A^C \oplus \hat{B}$$



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

From "Digital Image Processing" by R. C. Gonzales and R. E. Woods

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5. Opening and Closing

As we have seen, dilation expands an image and erosion shrinks it.

The ***opening*** of set A by structuring element B, denoted $A \circ B$, is defined as

$$A \circ B = (A \Theta B) \oplus B$$

Thus, the opening ***A*** by B is the erosion of A by B, followed by a dilation of the result by B. Opening generally **smoothes the contour of an object**, **breaks narrow isthmuses**, and **eliminates thin protrusions**.

The ***closing*** of set A by structuring element B, denoted $A \bullet B$ is defined as

$$A \bullet B = (A \oplus B) \Theta B$$

Thus, the closing of set A by B is simply the dilation of A by B, followed by the erosion of the result by B. Closing also tends to **smooth sections of contours** but, as opposed to opening, it generally **fuses narrow breaks and long thin gulfs**, **eliminates small holes**, and **fills gaps in the contour**.

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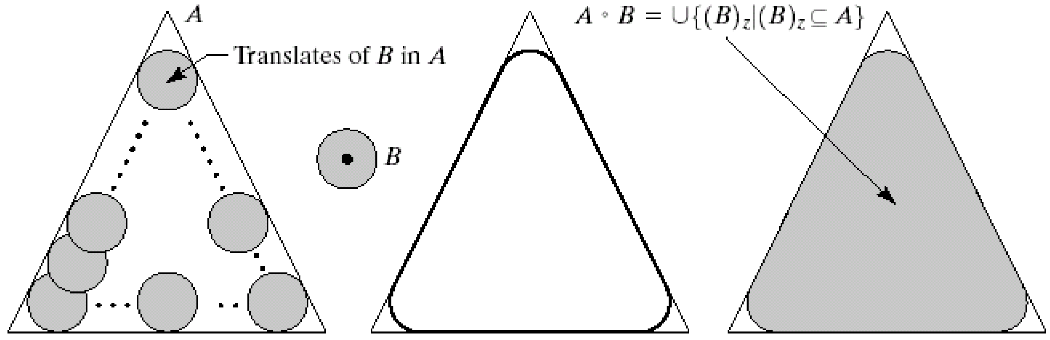
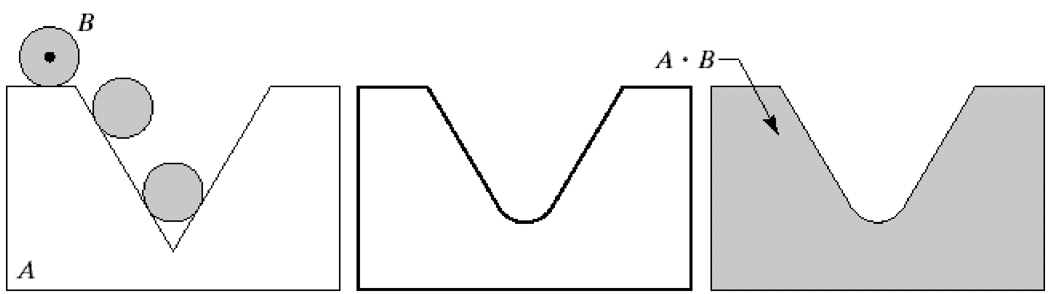


a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

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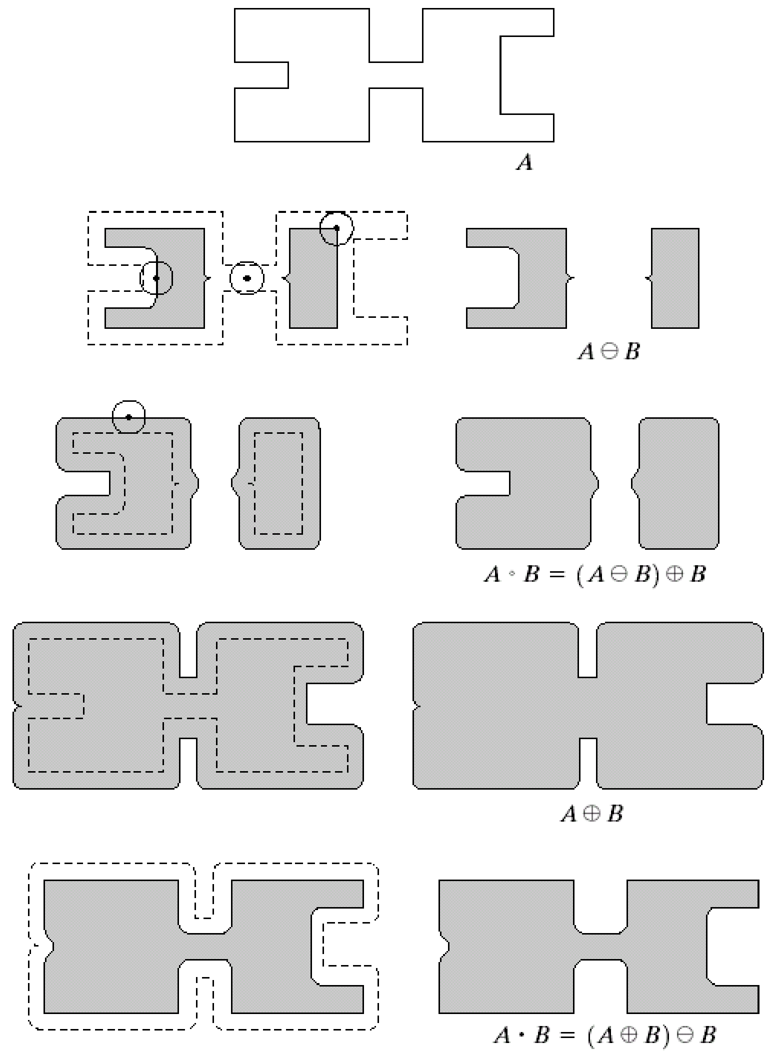

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

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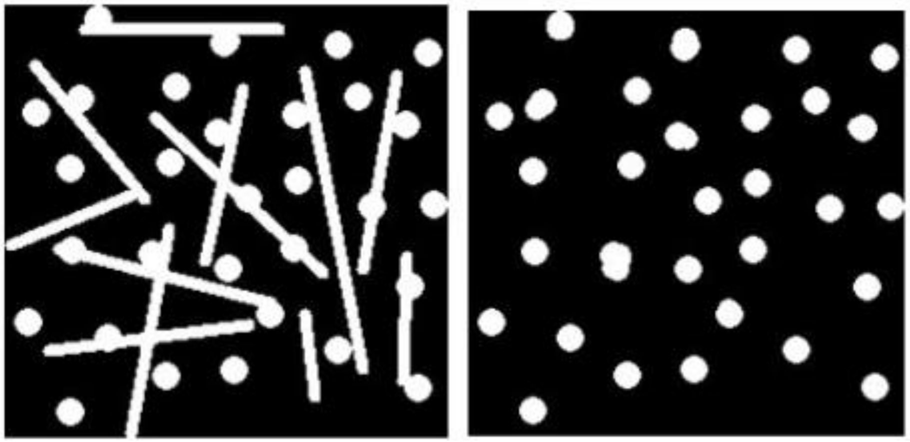
Opening:
-Extracting the horizontal and vertical lines
-The results of an opening with a 3x19 vertically and 19x3 horizontally oriented structuring element

Opening:
-Separate out the circles from the lines so that they can be counted.
-Opening with a disk shaped structuring element 11 pixels in diameter



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6. Hit-or-Miss Transform

The ***hit-or-miss transform*** is a natural operation to **select out pixels that have certain geometric properties**, such as **corner points**, **isolated points**, or **border points**, and that performs **template matching, thinning, thickening**, and **centering**.

This transform is accomplished by using **intersections of erosions**.
Let ***J*** and ***K*** be two structuring elements that satisfy $J \cap K = \varnothing$. Let be **B={J,K}**. **The hit-or-miss transformation of a set *A* by *B* is denoted by**
$A \circledR B = (A \Theta J) \cap (A^C \Theta K)$.

Ex: The hit-or-miss transform used to find **the upper right-hand corner points** of a set A.
The J structuring element locates all pixels of A that have south and est neighbors that are also parts of A. The K structuring element locates all pixels of A^C that have south and est neighbors in A^C. Notice that J and K are displaced from one another. K is J translated by one pixel to the northeast. The pixels that K locates can be thought of as all pixels of A^C that are candidates for being an exterior border pixel to a corner pixel of A.

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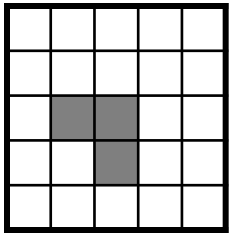
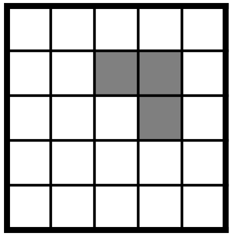
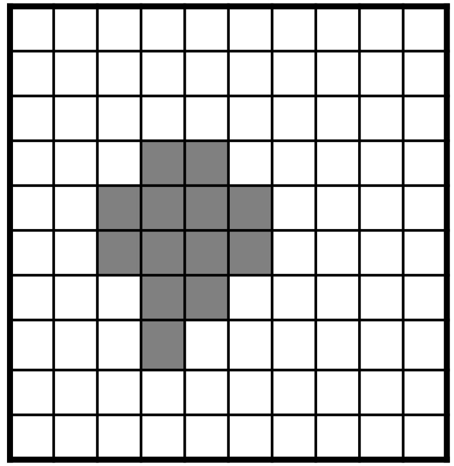
J



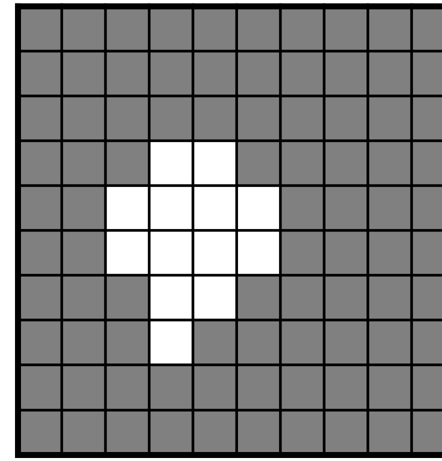
K



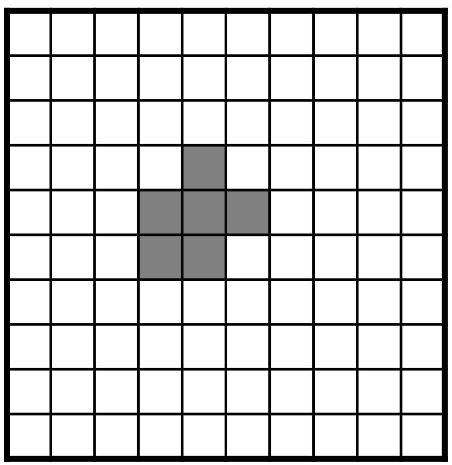
A



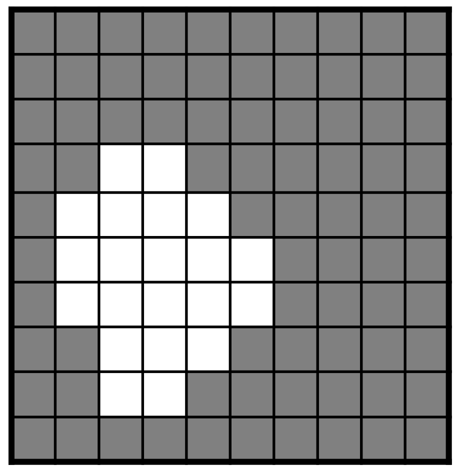
A^C

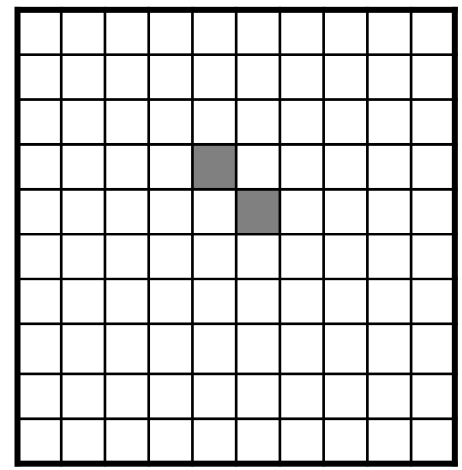


$A \Theta J$



$A^C \Theta K$





$A®B=(A\,\Theta J)\cap(A^C\Theta K)$

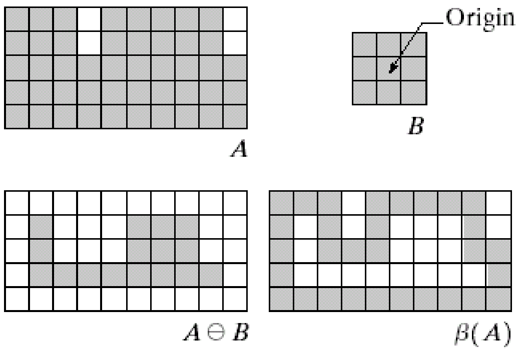
7. Some Basic Morphological Algorithms

7.1 Boundary extraction

The boundary of a set *A*, denoted by $\beta(A)$, can be obtained by first eroding *A* by *B* and then performing the set differences between *A* and its erosion. That is,

$$\beta(A)=A-(A\Theta B)$$

where *B* is a suitable structuring element.



a b
c d

FIGURE 9.13 (a) Set *A*. (b) Structuring element *B*. (c) *A* eroded by *B*. (d) Boundary, given by the set difference between *A* and its erosion.

From "Digital Image Processing" by R. C. Gonzales and R. E. Woods

7.2 Region Filling

Next it is presented a simple algorithm for region filling based on set dilations, complementation, and intersections.

Beginning with a point p inside the boundary, the objective is to fill the entire region with 1's. If we adopt the convention that all non-boundary (background) points are labelled 0, then we assign a value of 1 to p to begin. The following procedure fills the region with 1's:

$$X_k = (X_{k-1} \oplus B) \cap A^C, k = 1,2,3,\ldots$$

where $X_0=p$, and B is the symmetric structuring element. The algorithm terminates at iteration step k if $X_k= X_{k-1}$. The set union of X_k and A contains the filled set and its boundary.

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Region Filling (2)

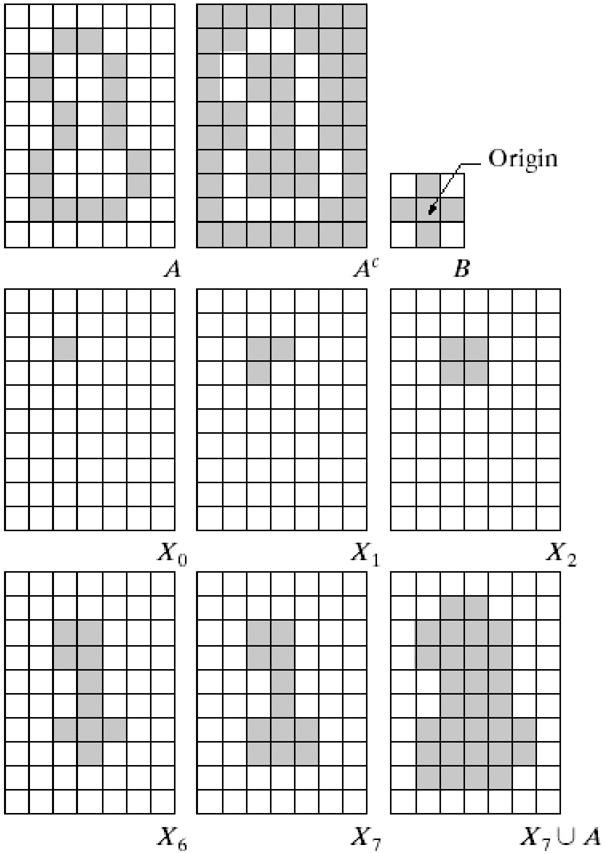


FIGURE 9.15 Region filling. (a) Set A. (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

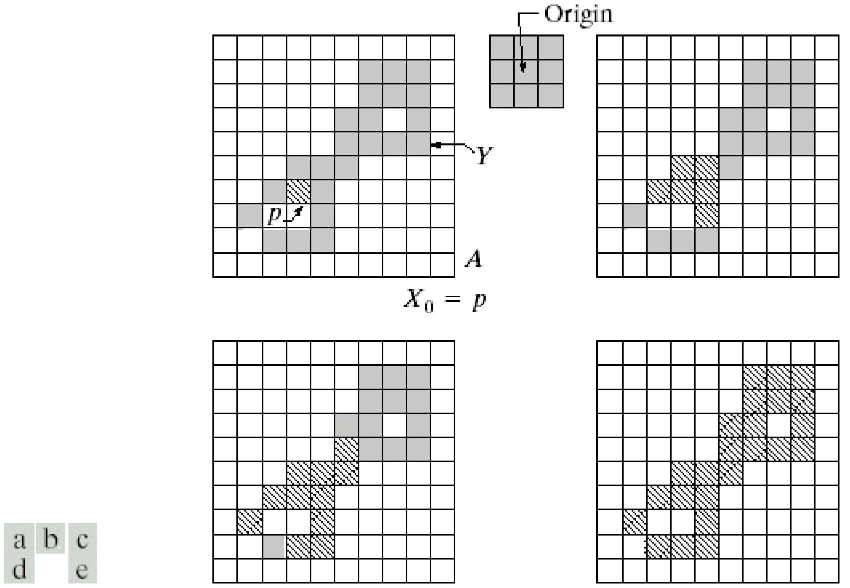
From "Digital Image Processing" by R. C. Gonzales and R. E. Woods

7.3 Extraction of Connected Components

Let Y represent a connected component contained in a set A and assume that a point p of Y is known. Then the following iterative expression yields all the elements of Y:

$$X_k = (X_{k-1} \oplus B) \cap A, k = 1,2,3,\ldots$$

where $X_0=p$, and B is a suitable structuring element. If $X_k= X_{k-1}$, the algorithm has converged and we let $Y= X_k$.



From "Digital Image Processing" by R. C. Gonzales and R. E. Woods

FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

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7.4 Thinning

The thinning of a set A by a structuring element B, denoted

$A \otimes B = A - (A \circledR B)$

$= A \cap (A \circledR B)^C$.

For thinning A symmetrically a sequence of structuring elements is necessary:

$B = \{B^1, B^2, B^3, \ldots, B^n\}$

where B^i is a rotated version of B^{i-1}

$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$

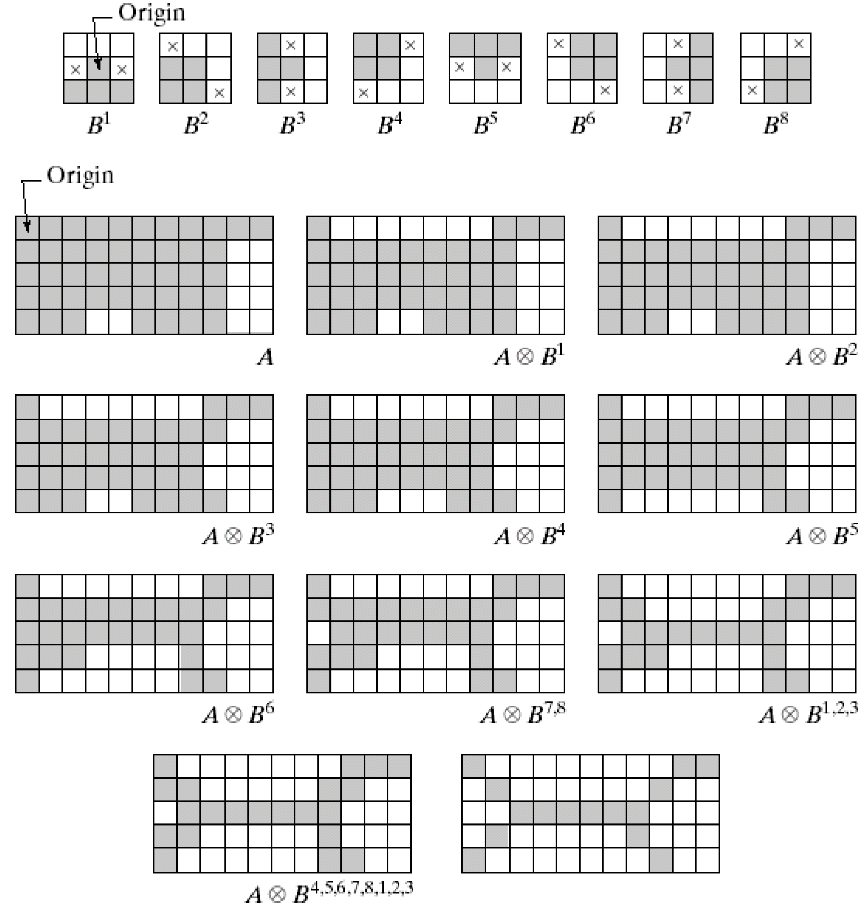

FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first element again (there were no changes for the next two elements). (k) Result after convergence. (l) Conversion to m-connectivity.

From "Digital Image Processing" by R. C. Gonzales and R. E. Woods

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Other Morphological algorithms:

-Convex Hull detection, Thickening, Skeletons, Pruning

References:

Rafael C. Gonzales, Richard E. Woods, “Digital Image Processing”, *Prentice Hall*, 2002

Robert M. Haralick, Linda G. Shapiro, “Computer and Robot Vision”, *Addison-Wesley Publishing Company*, 1993

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TABLE 9.2 Summary of morphological operations and their properties.

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Translation	$(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$	Translates the origin of A to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)

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Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c.
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Region filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Fills a region in A, given a point p in the region. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Finds a connected component Y in A, given a point p in Y. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$.	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)

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Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0's and 1's reversed.

TABLE 9.2
Summary of morphological results and their properties. *(continued)*

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Skeletons	$S(A) = \bigcup_{k=0} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosion of A by B. (I)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.

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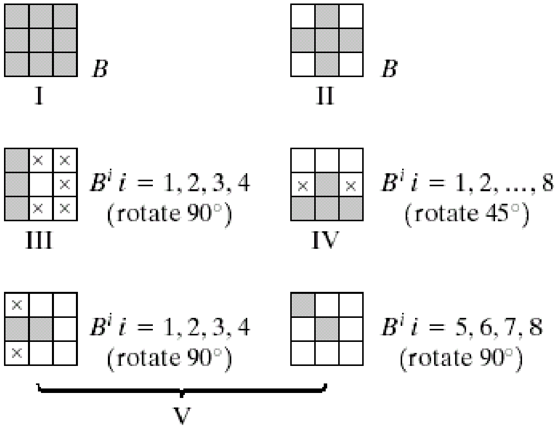

FIGURE 9.26 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.

From "Digital Image Processing" by R. C. Gonzales and R. E. Woods