Signal to Noise Ratio

The use of SNR (Signal to Noise Ratio) is a confusing topic, since

- There is a range (about 10) definitions.
- Can a single number really classify how good an image is.
- In practice image quality is very strongly image dependant.

For signal independent additive noise,

$$f(i,j) = s(i,j) + n(i,j)$$

Signal & Noise variances are:

$$\begin{aligned} \sigma_s^2 &= \langle |s(i,j) - \langle s(i,j)\rangle|^2 \rangle \\ \sigma_n^2 &= \langle |n(i,j)|^2 \rangle \end{aligned}$$

Define SNR by

$$\mathrm{SNR} = \frac{\sigma_s}{\sigma_n}$$

Signal to Noise Ratio I

Noting that the signal and the noise are uncorrelated, we have

$$\sigma_f^2 = \sigma_s^2 + \sigma_n^2$$

so we can write the SNR as:

$$\mathrm{SNR} = \sqrt{\frac{\sigma_f^2}{\sigma_n^2} - 1}$$

To calculate SNR, need 2 of σ_s, σ_f, or σ_n.

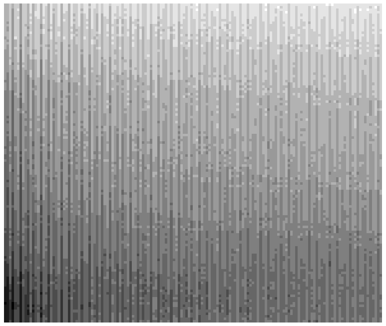
Calculation of SNR for Single Image

From single image can **only** find σ_f.

If region of image with **NO** signal can estimate σ_n from that region.

This method works for an image that contains a large region of "water" or "sky" where these is not signal.

Whole Image



Piece of Sky

Sky region shows typical CCD array fixed pattern noise.

Calculated vales are $\sigma_f^2 = 5287$ and $\sigma_n^2 = 1.85$, so that

$$\mathrm{SNR} \approx 53.4$$

Calcualte SNR for Multiple Images

Assume that we have **two** realisation of same scene, so:

$$
\begin{aligned}
f(i,j) &= s(i,j) + n(i,j) \\
g(i,j) &= s(i,j) + m(i,j)
\end{aligned}
$$

which is equivalent to two image of the same scene take at different times.

The noise in each realsiation have the same PDF, so that

$$
\begin{aligned}
\langle n(i,j) \rangle = \langle m(i,j) \rangle &= 0 \\
\langle |n(i,j)|^2 \rangle = \langle |m(i,j)|^2 \rangle &= \sigma_n^2
\end{aligned}
$$

The two noise realisations were measured at *different* times, so they are uncorrelated, so:

$$
\langle n(i,j)\, m(i,j) \rangle = 0
$$

In both cases the noise is uncorrelated with the signal, so that

$$
\langle s(i,j)\, n(i,j) \rangle = \langle s(i,j)\, m(i,j) \rangle = 0
$$

Calcualte SNR for Multiple Images I

If we then define the *Normalised* Correlation between $f(i,j)$ and $g(i,j)$ as:

$$r = \frac{\langle (fg - \langle f \rangle \langle g \rangle) \rangle}{[\langle |f - \langle f \rangle|^2 \rangle \langle |g - \langle g \rangle|^2 \rangle]^{1/2}}$$

Then be expanding, collecting terms, and cancelling all zero terms, *it-can-be-shown* that

$$r = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_n^2}$$

to we can form the SNR from

$$\mathrm{SNR} = \sqrt{\frac{r}{1-r}}$$

So allowing direct calculation of the SNR independant of the type of the signal.

Calcualte SNR for Multiple Images II

If there is more than two realiastion available a better estimate for SNR can be found by forming the normalised correlation between pairs of images and averaging.

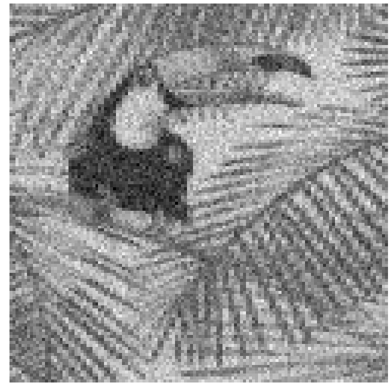
In practice this measure of SNR looks about right for most images. For examples:

SNR	$>$	20	Little visible noise
SNR	$\approx$	10	Some noise visibile
SNR	$\approx$	4	Noise clearly visible
SNR	$\approx$	2	Image severly degraded
SNR	$\approx$	1	Is there an image?

Digital Simulation

Digitally simulated “noisy” images:

$\mathrm{SNR} = 1$



$\mathrm{SNR} = 2$

$\mathrm{SNR} = 4$

$\mathrm{SNR} = 8$

Images formed by addition of Gaussian random noise.