Camera frame ↔ image plane transformation

Camera frame ⇒ image plane transformation

(*projection / normalization*) : $P = [X_C, Y_C, Z_C]^T$ [metric units] ⇒ $p = [u, v]^T$ [pixels]

1. Transform $P = [X_C, Y_C, Z_C]^T \Rightarrow p = [x, y, -f]^T$

Fundamental equations of the *perspective camera model* normalized with *1/Z*:

$$\begin{bmatrix} x \\ y \end{bmatrix} = f \begin{bmatrix} X_c / Z_c \\ Y_c / Z_c \end{bmatrix} = f \begin{bmatrix} x_N \\ y_N \end{bmatrix}$$

f – focal distance [metric units]

2. Transform **p** $[x, y]^T$ [metric units] ⇒ image coordinates $[u, v]^T$ [pixels]

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} D_u \cdot x \\ D_v \cdot y \end{bmatrix} + \begin{bmatrix} u_0 \\ v_0 \end{bmatrix}$$

Du, Dv - coefficients needed to transform metric units to pixels: Du = 1 / dpx; Dv = 1 / dpy

1 + 2 ⇒ projection equation:

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = A \cdot \begin{bmatrix} x_N \\ y_N \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} f_X & 0 & u_0 \\ 0 & f_Y & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

A – is the camera matrix:

f_X – is the focal distance expressed in units of horizontal pixels:

$$f_X = f \cdot D_u = \frac{f}{dpx}$$

f_Y – is the focal distance expressed in units of vertical pixels:

$$f_Y = f \cdot D_v = \frac{f}{dpy}$$

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Camera frame $\leftrightarrow$ image plane transformation

Image plane transformation $\Rightarrow$ camera frame

(*reconstruction*) : p = [u, v]^T [pixels] $\Rightarrow$ P = [X_C, Y_C, Z_C]^T [metric units]

$$\begin{bmatrix} x_N \\ y_N \\ 1 \end{bmatrix} = A^{-1} \cdot \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

Notes:

With one camera we cannot measure depth (Z). We can determine only the projection equation / normalized coordinates:

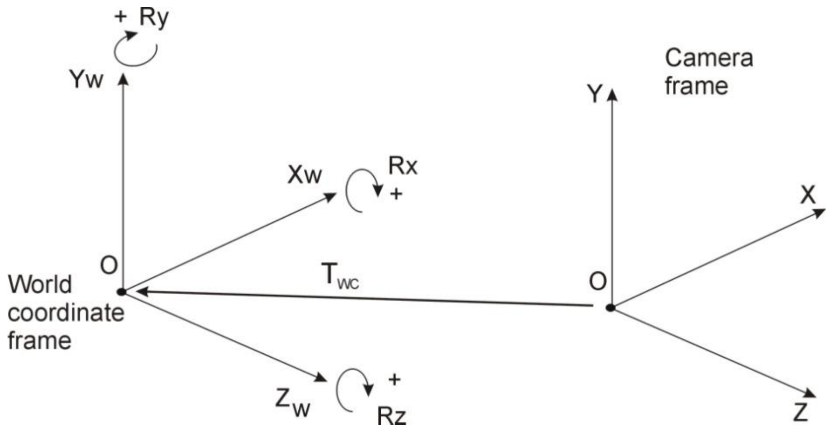
$$\begin{bmatrix} x_N \\ y_N \end{bmatrix} = \begin{bmatrix} X_C / Z_C \\ Yc / Z_C \end{bmatrix}$$

To measure the depth (Z) a stereo system (2 cameras) is needed

Camera frame ↔ world reference frame transformation

Direct mapping (world ⇒ camera)

XX$_W$ = [X_W , Y_W , Z_W]^T **(**world coordinate system - WRF) ⇒ **XX**$_C$ = [X_C , Y_C , Z_C]^T (camera coordinate system – CRF)



$$\mathbf{XX}_C = \mathbf{R}_{WC} \cdot \mathbf{XX}_W + \mathbf{T}_{WC}$$

where:

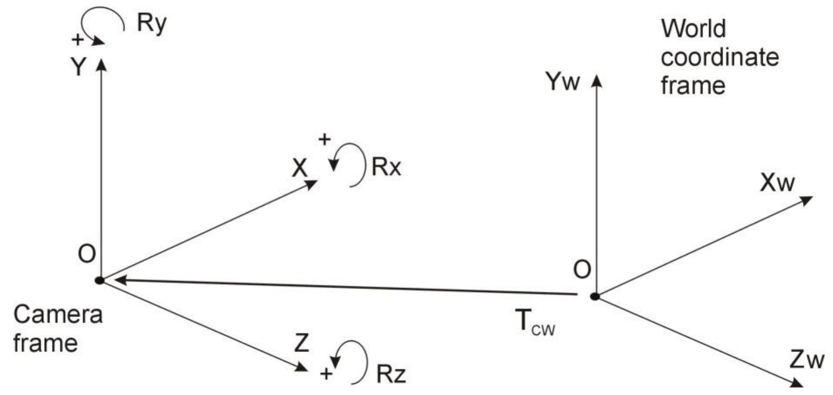
$\mathbf{T}_{WC}$ = [Tx, Ty, Tz]^T – world to camera translation vector;
$\mathbf{R}_{WC}$ – world to camera rotation matrix:

Camera frame ↔ world reference frame transformation

Inverse mapping (camera ⇒ world)

$\mathbf{XX}_C = [X_C, Y_C, Z_C]^T$ (camera coordinate system – CRF) ⇒ $\mathbf{XX}_W = [X_W, Y_W, Z_W]^T$ (world coordinate system - WRF)

$$\mathbf{XX}_W = \mathbf{R}_{WC}^{-1} \cdot (\mathbf{XX}_C - \mathbf{T}_{WC})$$



Rotation matrix is orthogonal [Trucco1998]:

$$\mathbf{R} \cdot \mathbf{R}^T = \mathbf{R}^T \cdot \mathbf{R} = 1 \Rightarrow \mathbf{R}^T = \mathbf{R}^{-1}$$

$$\mathbf{XX}_W = \mathbf{R}_{WC}^{T} \cdot (\mathbf{XX}_C - \mathbf{T}_{WC}) = \mathbf{R}_{CW} \cdot (\mathbf{XX}_C + \mathbf{T}_{CW})$$

where:

$\mathbf{T}_{CW} = [T_X\ T_Y\ T_Z]^T$ – camera to world translation vector $\quad T_{CW} = -T_{WC}$

$\mathbf{R}_{CW}$ – camera to world translation vector $\quad R_{CW} = R_{WC}^{T}$

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Rotation Matrix

World-to-camera

$$\mathbf{R}_{WC} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} \mathbf{n}^{XW} & \mathbf{n}^{YW} & \mathbf{n}^{ZW} \end{bmatrix} = \begin{bmatrix} n_X^{XW} & n_X^{YW} & n_X^{ZW} \\ n_Y^{XW} & n_Y^{YW} & n_Y^{YW} \\ n_Z^{XW} & n_Z^{YW} & n_Z^{YW} \end{bmatrix}$$

$\mathbf{n}^{XW} = \begin{bmatrix} n_X^{XW} & n_Y^{XW} & n_Z^{XW} \end{bmatrix}^T$ – normal vector of **OX**$_W$ axis in the CRF

$\mathbf{n}^{YW} = \begin{bmatrix} n_X^{YW} & n_Y^{YW} & n_Z^{YW} \end{bmatrix}^T$ – normal vector of **OY**$_W$ axis in the CRF

$\mathbf{n}^{ZW} = \begin{bmatrix} n_X^{ZW} & n_Y^{ZW} & n_Z^{ZW} \end{bmatrix}^T$ – normal vector of **OZ**$_W$ axis in the CRF

Camera-to-world

$$\mathbf{R}_{CW} = \mathbf{R}_{WC}^{\ T} = \begin{bmatrix} r_{11} & r_{21} & r_{31} \\ r_{12} & r_{22} & r_{32} \\ r_{13} & r_{23} & r_{33} \end{bmatrix} = \begin{bmatrix} \mathbf{n}^{XC} & \mathbf{n}^{YC} & \mathbf{n}^{ZC} \end{bmatrix} = \begin{bmatrix} n_X^{XC} & n_X^{YC} & n_X^{ZC} \\ n_Y^{XC} & n_Y^{YC} & n_Y^{YC} \\ n_Z^{XC} & n_Z^{YC} & n_Z^{YC} \end{bmatrix}$$

$\mathbf{n}^{XC} = \begin{bmatrix} n_X^{XC} & n_Y^{XC} & n_Z^{XC} \end{bmatrix}^T$ – normal vector of **OX**$_C$ axis in the WRF

$\mathbf{n}^{YC} = \begin{bmatrix} n_X^{YC} & n_Y^{YC} & n_Z^{YC} \end{bmatrix}^{YT}$ – normal vector of **OY**$_C$ axis in the WRF

$\mathbf{n}^{ZC} = \begin{bmatrix} n_X^{ZC} & n_Y^{ZC} & n_Z^{ZC} \end{bmatrix}^T$ – normal vector of **OZ**$_C$ axis in the WRF

Rotation Matrix ↔ Rotation Vector

Rotation vector – Rotation matrix

$$\mathbf{r} = [\theta, \psi, \gamma]^T \qquad \mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$\mathbf{R}x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \qquad \mathbf{R}y = \begin{pmatrix} \cos\psi & 0 & \sin\psi \\ 0 & 1 & 0 \\ \sin\psi & 0 & \cos\psi \end{pmatrix} \qquad \mathbf{R}z = \begin{pmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{R} = \mathbf{R}x\mathbf{R}y\mathbf{R}z$$

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Rotation Matrix ↔ Rotation Vector

Rotation vector

$\mathbf{r}_{WC} = [R_X\ R_Y\ R_Z]^T$ (R_X - pitch, R_Y - yaw, R_Z - tilt / roll)

$r_{WC} \Rightarrow R_{WC}$ transform:

$$r_{11} = \cos(R_Y)\cos(R_Z)$$
$$r_{12} = \sin(R_X)\sin(R_Y)\cos(R_Z) - \cos(R_X)\sin(R_Z)$$
$$r_{13} = \cos(R_X)\sin(R_Y)\cos(R_Z) + \sin(R_X)\sin(R_Z)$$
$$r_{21} = \cos(R_Y)\sin(R_Z)$$
$$r_{22} = \sin(R_X)\sin(R_Y)\sin(R_Z) + \cos(R_X)\cos(R_Z)$$
$$r_{23} = \cos(R_X)\sin(R_Y)\sin(R_Z) - \sin(R_X)\cos(R_Z)$$
$$r_{31} = -\sin(R_Y)$$
$$r_{32} = \sin(R_X)\cos(R_Y)$$
$$r_{33} = \cos(R_X)\cos(R_Y)$$

$R_{WC} \Rightarrow r_{WC}$ transform:

$$R_Y = \arcsin(r_{31})$$

If $\cos(R_Y) \neq 0$:

$$R_X = \operatorname{atan2}\left(-\frac{r_{32}}{\cos(R_Y)}, \frac{r_{33}}{\cos(R_Y)}\right)$$
$$R_Z = -\operatorname{atan2}\left(-\frac{r_{21}}{\cos(R_Y)}, \frac{r_{11}}{\cos(R_Y)}\right)$$

If $\cos(R_Y) = 0$:

$$R_X = \operatorname{atan2}(r_{12}, r_{22})$$

$$R_Z = 0$$

3D (world) ⇒ 2D (image) mapping using the Projection Matrix

Projection matrix

$$\mathbf{P} = \mathbf{A} \cdot \left[\mathbf{R}_{WC} \mid \mathbf{T}_{WC}\right]$$

The projection equation of a 3D world point [X_W , Y_W , Z_W] expressed in normalized coordinates :

$$s \cdot \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} x_S \\ y_S \\ z_S \end{bmatrix} = \mathbf{P} \cdot \begin{bmatrix} X_W \\ Y_W \\ Z_W \\ 1 \end{bmatrix}$$

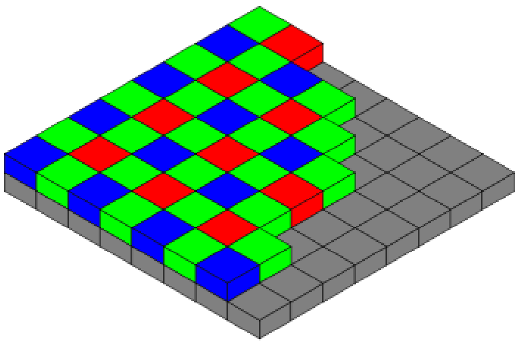
$s=z_S$ – scaling factor

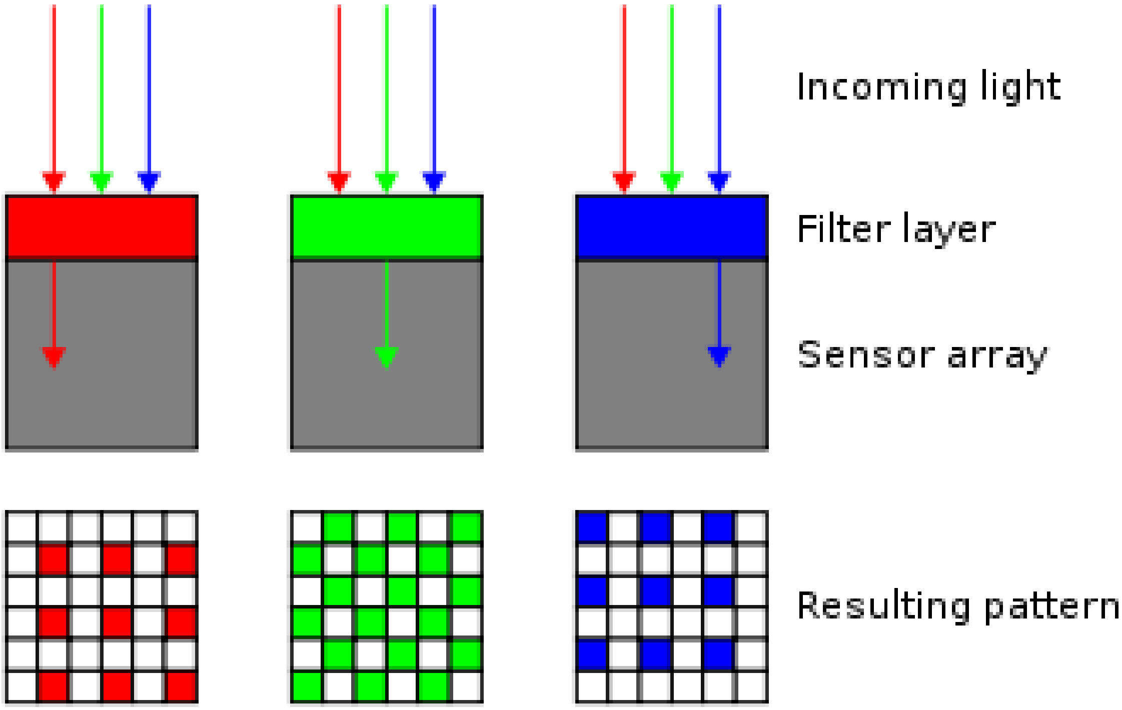
Obtaining the 2D image coordinates from normalized coordinate

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} x_S / z_S \\ y_S / z_S \end{bmatrix}$$

Imaging sensors

Demosaicing Bayer pattern: Bilinear interpolation





$$G = (G_n + G_w + G_e + G_s)/4$$
$$R_4 = (R_{nw} + R_{ne} + R_{se} + R_{sw})/4$$
$$R_{2c} = (R_n + R_s)/2$$
$$R_{2l} = (R_w + R_e)/2$$

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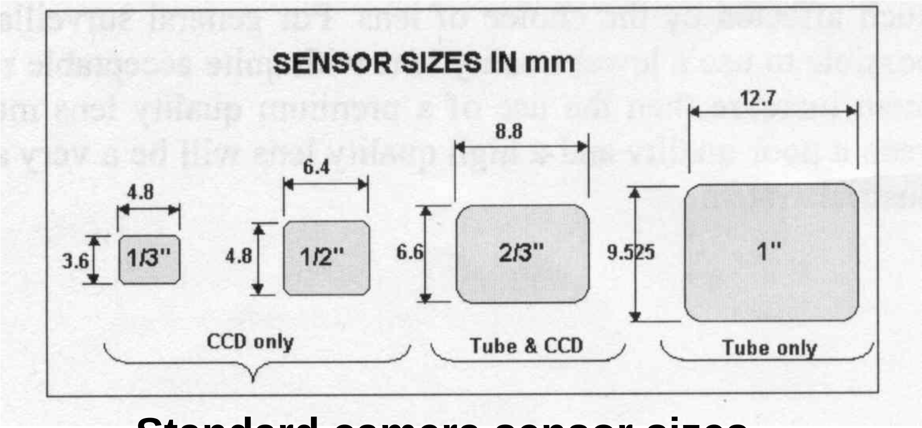
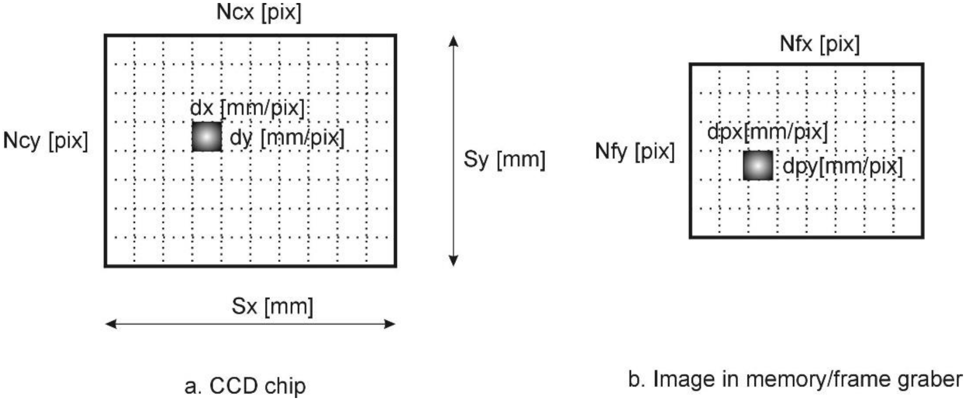
Imager parameters

Imager (sensor) parameters



Standard camera sensor sizes



Parameters of the imager and image in memory

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