Sequential Labeling

Iterative Algorithm (Haralick 1981)

- no auxiliary storage to produce the labeld image from the binary image.
- useful in environments whose storage is severely limited.

1. initialization step
2. repeat

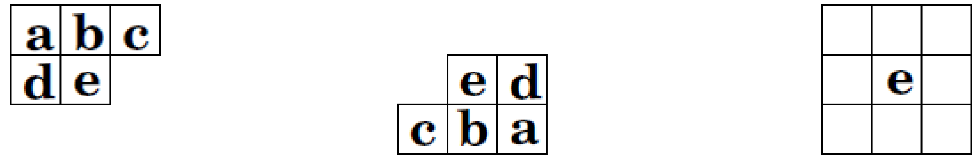
    top-down & left-right label propagation

    bottom-up & right-left label propagation

   until no changes occur

```
procedure Iterate;
"Initialization of each 1-pixel to a unique label"
for L:=1 to NLINES do
        for P:=1 to NCOLUMNS do
                if  I(L,P) =1
                then LABEL(L,P):=NEWLABEL()
                else LABEL(L,P):=0
        end for
end for;
```

Sequential Labeling



"Top-down passes; Bottom up passes; 8-connected neighborhood"

"**procedure** Iterate – page 2"

"Iteration of top-down followed by bottom-up passes"

```
repeat
"Top-down passes"
CHANGE:=false;
 for L:=1 to NLINES do
          for P:=1 to NCOLUMNS do
                    if  LABEL(L,P)<>0 then
                              begin
                                        M:=MIN(LABELS(NEIGHBORS(L,P)U(L,P)));
                                        if M<> LABEL(L,P)
                                        then CHANGE:=true;
                                        LABEL(L,P):=M
                              end
          end for
 end for;
```

Sequential Labeling

“**procedure** Iterate – page 3”

“Bottom-up pass”

```
for L:= NLINES to 1 by –1 do
        for P:= NCOLUMNS to 1 by –1 do
                if  LABEL(L,P)<>0 then
                        begin
                                M:=MIN(LABELS(NEIGHBORS(L,P)U(L,P)));
                                if M<> LABEL(L,P)
                                then CHANGE:=true;
                                LABEL(L,P):=M
                        end
        end for
end for;

until CHANGE:=false

end Iterate
```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Sequential Labeling

Example (N4)

	1	1		1	1	
	1	1		1	1	
	1	1	1	1	1	

1. Initial image

	1	2		3	4	
	5	6		7	8	
	9	10	11	12	13	

2. Initialization

	1	1		3	3	
	1	1		3	3	
	1	1	1	1	1	

3. Top-down & left-right label propagation

	1	1		1	1	
	1	1		1	1	
	1	1	1	1	1	

4. Bottom-up & right-left label propagation

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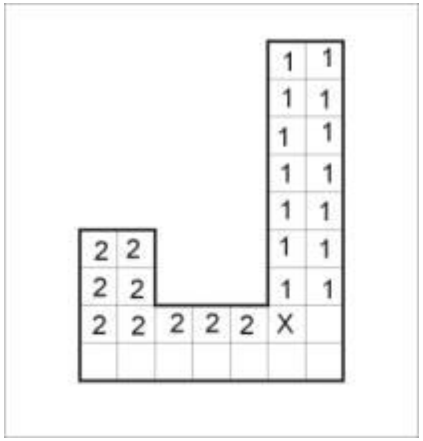
Classical Algorithm

- Based on the classical connected components algorithm for graphs.
- 2 passes through the image but requires a large global table for recording equivalences.

1. First pass: performs label propagation, much as described above.
   - Whenever a situation arises in which two different labels can propagate to the same pixel, the smaller label propagates and each such equivalence found is entered in an equivalence table (e.g. (1,2) $\rightarrow$ EqTable).
   - Each entry in the equivalence table consists of an ordered pair, the values of its components being the labels found to be equivalent.
   - After the first pass, the equivalence classes are found.
   - Each equivalence class is assigned a unique label, usually the minimum (or oldest) label in the class.



2. A second pass through the image performs a translation, assigning to each pixel the label of the equivalence class of its 1-st pass label.

Classical Algorithm

```
procedure Classical
"Initialize global equivalence table"
EQTABLE:=CREATE();

"Top-down pass 1"
for L:= 1 to NLINES do
        "Initialize all labels on line L to zero"
        for P:= 1 to NCOLUMNS do
                LABEL(L,P):=0
        end for
        "Process the line"
        for P:=1 to NCOLUMNS do
                if I(L,P):= 1 then
                        begin
                                A:= NEIGHBORS((L,P));
                                if ISEMPTY(A)
                                then M:=NEWLABEL()
                                else M:= MIN(LABELS(A));
                                LABEL(L,P):=M;
                                for X in LABELS(A) and X<>M
                                        ADD(X, M, EQTABLE)
                                end for;
                        end
        end for
end for;
```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Classical Algorithm

```
"Find the equivalence classes"
EQCLASSES:=Resolve(EQTABLE);

"Find the equivalence label of an equivalence class"
for E in EQCLASSES
        EQLABEL(E):= min(LABELS(E))
end for;

"Top-down pass 2"
for L:= 1 to NLINES do
        for P:= 1 to NCOLUMNS do
                if I(L,P) = 1
                then LABEL(L,P):=EQLABEL(CLASS(LABEL(L,P)))
        end for
end for
end Classical
```

- RESOLVE - algorithm for finding the connected components of the graph structure, defined by the set of equivalences (EQTABLE) defined in pass 1.
- The main problem with the classical algorithm is the global equivalence table (large images with many regions, the equivalence table can become very large)

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Classical Algorithm

Example (N4)

1					1	1
		1	1			1
		1				1
		1				1
1	1	1				1
	1	1		1		1
	1	1	1	1		1
				1	1	1

1. Initial image

1					2	2
		3	3			2
		3				2
		3				2
4	4	3				2
	4	3		5		2
	4	3	3	3		2
				3	3	2

2. Top down (pass 1)

EQTABLE:
(4, 3), (3, 5), (3, 2) …

EQCLASSES:
1: {4, 3, 5, 2}
2: (6,8,9, …}
….
n: {….}

EQLABEL:
1, 2
2, 6
…
n, x

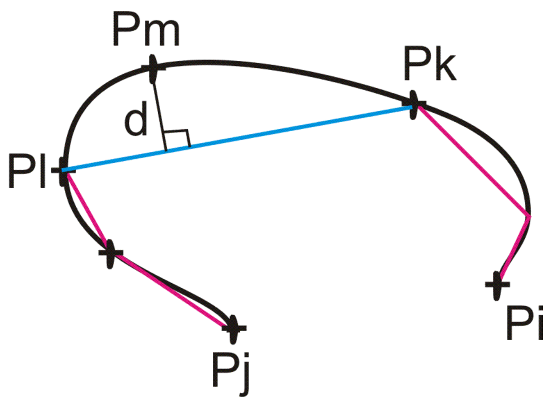
Polygonal Approximation

Polygonal approximation of contours

Curve C: $f(x,y)=0 \Rightarrow$ polygon that closely approximates C with an error smaller than ε and having a number of vertices as small as possible**:**

- Any polygonal fitting algorithm requires that the data points be subdivided into groups, each one of them to be approximated by a side of the polygon.
- The first simplification of the polygon fit problem is to draw a line between the endpoints of each group rather than search for the optimal solution.
- If the approximation error is too big the group could be split in two and so one until the error becomes acceptable.
- Let Q a contour consisting of P_i (x_i,y_i) where i=1,2,…,n, and ε the error threshold.

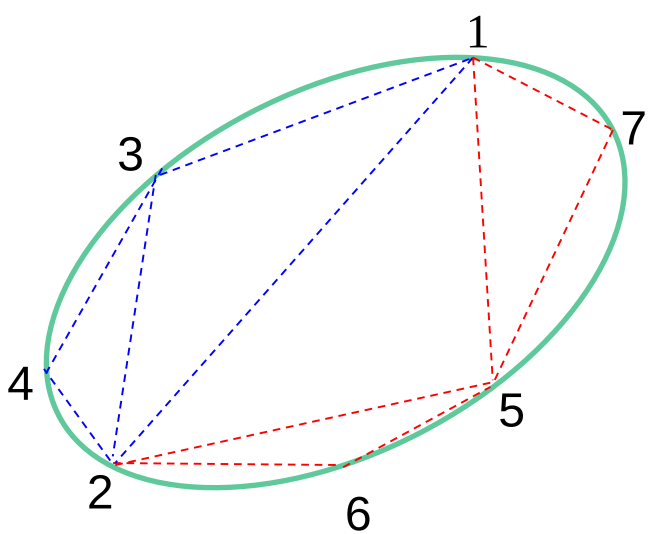
Polygonal Approximation



Procedure POLIGONAL_APROX(Q)
begin
A:=Create_List();
B:=Create_List();
i=Index_of_first_point(Q);
j=Index_of_the_most_far_point(Q);
Insert(j,A); Insert(j,B);
Insert(i,A);
while((A!=NULL)
{
Let k and l the indexes of the last elements of the lists A and B;
Let P_k P_l the segment generated by these two points;
Let m the index of the most far point to P_kP_l segment among the contour points starting with P_k and ending with P_l, when the contour is scanned in counterclock-wise direction.
if ((d=Distance($P_kP_{l,}$, P_m) > ε)
then Insert(m, A)
else { Delete(k, A)
Insert(k, B); }
}
end

$$\text{distance}(P_1, P_2, (x_0, y_0)) = \frac{|(y_2 - y_1)x_0 - (x_2 - x_1)y_0 + x_2y_1 - y_2x_1|}{\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}}.$$

Polygonal Approximation – Example



A	B
2 1 3 4	2
2 1 3	2 4
2 1	2 4 3
2	2 4 3 1
2 5	2 4 3 1

A	B
2 5 7	2 4 3 1
2 5	2 4 3 1 7
2	2 4 3 1 7 5
2 6	2 4 3 1 7 5

A	B
2	2 4 3 1 7 5 6
	2 4 3 1 7 5 6 2

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