Technical University of Cluj - Napoca
Computer Science Department

Image Processing

(Year III, 2-nd semester)

Lecture 6:

Grayscale Image processing (I)

Statistical image features and applications. Image enhancement

Basic Statistical Properties

Mean and Variance

For the digital image the definition of mean and variance are given by:

$$\mu = \frac{1}{N^2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} f(i, j)$$

$$\sigma^2 = \frac{1}{N^2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} (f(i, j) - \mu)^2$$

Notation

For a 1-D digital signal the mean or average is defined as:

$$\mu = \frac{1}{N} \sum_{i=0}^{N-1} f(i) = \langle f(i) \rangle$$

Similarly in 2-D we have:

$$\mu = \frac{1}{N^2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} f(i, j) = \langle f(i, j) \rangle$$

The variance is then written as:

$$\sigma^2 = \langle \left| f(i, j) - \mu \right|^2 \rangle$$

Basic Statistical Properties

Calculation of Mean and Variance

Looks like a "double scan" through the image

1. Calculate

$$\mu = \langle f(i,j) \rangle$$

2. Calculate

$$\sigma^2 = \langle |f(i,j) - \mu|^2 \rangle$$

But we can expand

$$\sigma^2 = \langle |f(i,j) - \mu|^2 \rangle = \langle |f(i,j)|^2 \rangle - 2\langle f(i,j) \rangle \mu + \mu^2 = \langle |f(i,j)|^2 \rangle - \langle f(i,j) \rangle^2$$

both of which can be formed in a single pass through the image.

We are able to calculate **both** mean and variance by calculating:

$$\langle |f(i,j)|^2 \rangle \quad \& \quad \langle f(i,j) \rangle$$

Histogram

Take the digital image $f(i, j)$ as a random function f with $0 \leq f \leq 255$.

We can define:

The **Probability Distribution Function** as:

$$P(f) = \textbf{Prob. Pixel value} \leq f$$

so that $\quad 0 \leq P(f) \leq 1$

and $\quad P(f_{max})=1$

The **Probability Density Function (PDF)** as:

$$p(f)=dP(f)/df$$

For a digital image if there are M_0 pixels with values $f_0 \rightarrow f_0 + \Delta f$ then PDF can be estimated by:

$$p(f_0)=M_0/N^2 \, \Delta f$$

So if $\Delta f = 1$ then the PDF is the **normalized histogram**

$$p(f)=h(f)/N^2$$

where $h(f)$ is the gray level **histogram of the image f** which shows the distribution of gray-levels over the range of values.

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Histogram

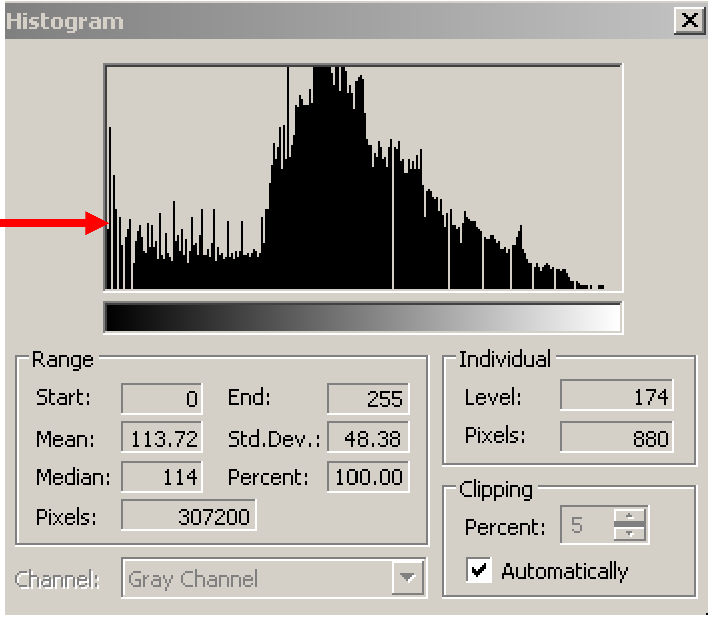
for i=0 to N-1

for j=0 to N-1

increment h(f(i, j))

endfor

endfor



Basic Statistical Properties

Mean and Variance

The *mean* and *variance* can be expressed in terms of the Probability Density Function, (PDF), being given by:

$$\mu = \int_{-\infty}^{\infty} f p(f) df$$

$$\sigma^2 = \int_{-\infty}^{\infty} (f-\mu)^2 p(f) df$$

So in the discrete case of the histogram *h(f):*

$$\mu = \frac{1}{N^2} \sum_{f=0}^{f_{\max}} f h(f)$$

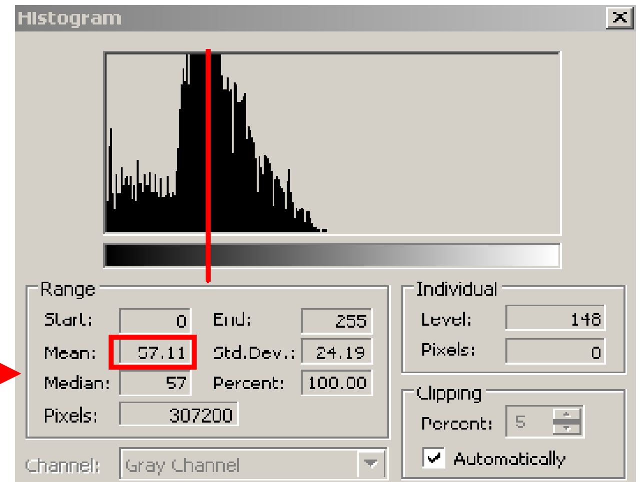
$$\sigma^2 = \frac{1}{N^2} \sum_{f=0}^{f_{\max}} (f-\mu)^2 h(f)$$

Statistical features

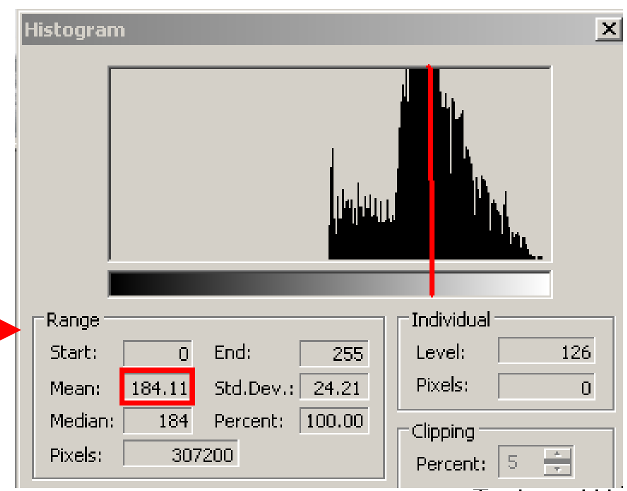
Mean (μ)

⇒ measure of the average brightness of the image/ROI

Dark image



Bright image

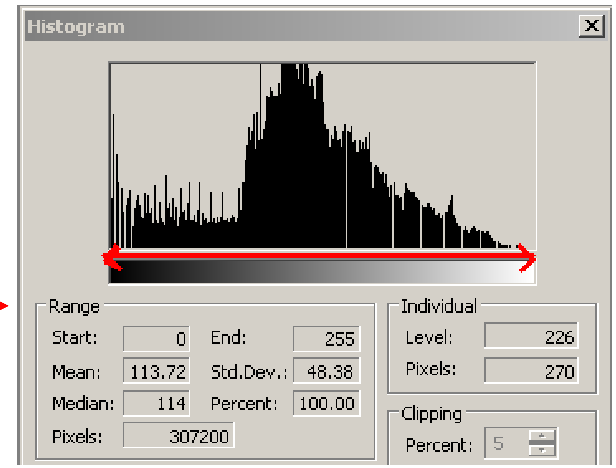


Statistical features

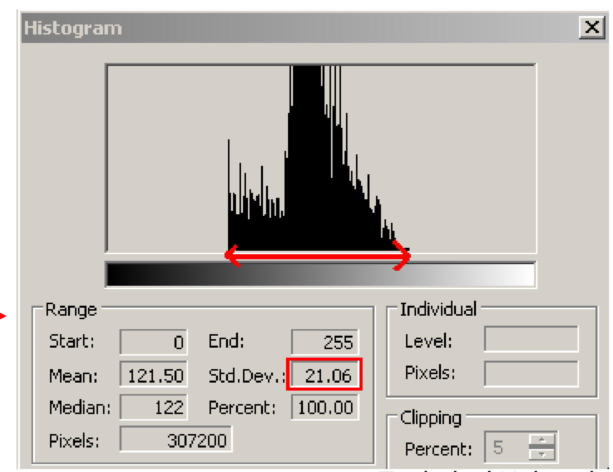
Standard deviation (σ), Variance (σ^2)

$\Rightarrow$ measure of the contrast of the image/ROI

High contrast



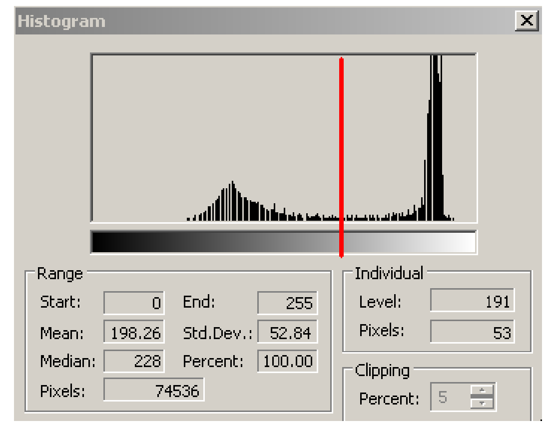
Low contrast



Application: grayscale image segmentation

Basic global thresholding algorithm

- Computes automatically the threshold (T)
- Can be applied on images with **bimodal** histograms



The algorithm

1. Take an initial value for T:

   $T_0 = \mu$ (object area = background area)

   $T_0 = (f_{MAX} + f_{MIN})/2$

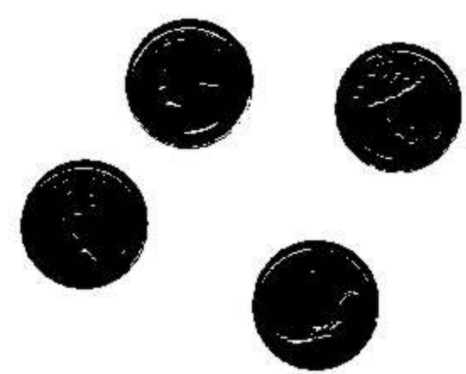
2. Segment the image after T by dividing the image pixels in 2 groups:

$$G1: f[i,j] \leq T \Rightarrow \mu_{G1}$$

$$G2: f[i,j] > T \Rightarrow \mu_{G2}$$

3. $T = (\mu_{G1} + \mu_{G2})/2$

4. Repeat 2-3 until $T_k - T_{k-1} < e$



Efficient implementation $\Rightarrow$ compute the image histogram first then perform all computations on the histogram !!!

Application: grayscale image segmentation

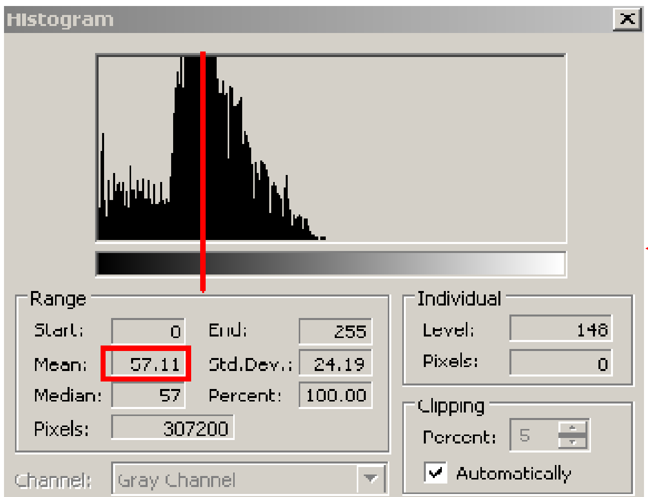
$$\mu_{G_1} = \frac{1}{N^2} \sum_{f=0}^{f_t} f h(f) = \sum_{f=0}^{f_t} f p(f)$$

$$\mu_{G_2} = \frac{1}{N^2} \sum_{f=f_{t+1}}^{f_{\max}} f h(f) = \sum_{f_{t+1}}^{f_{\max}} f p(f)$$

Image enhancement: histogram slide

$$Slide(f[i,j]) = f[i,j] + offset$$

ofset > 0 ⇒ brighter image
ofset < 0 ⇒ darker image



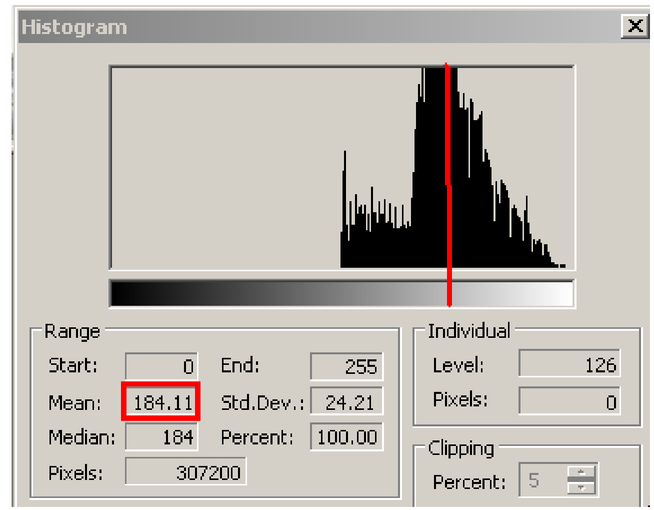
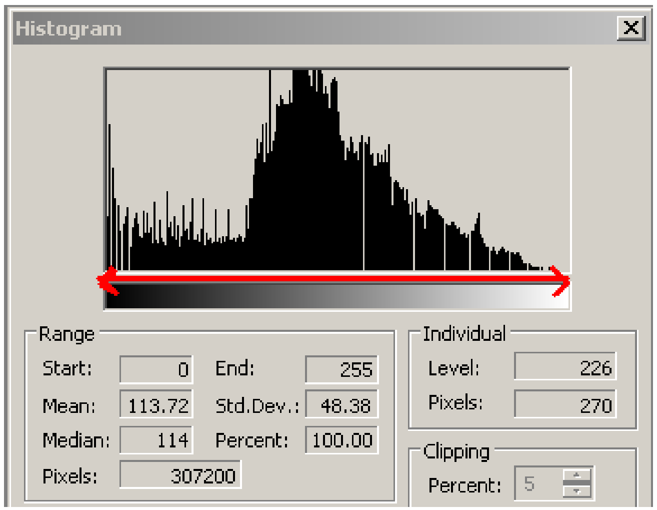


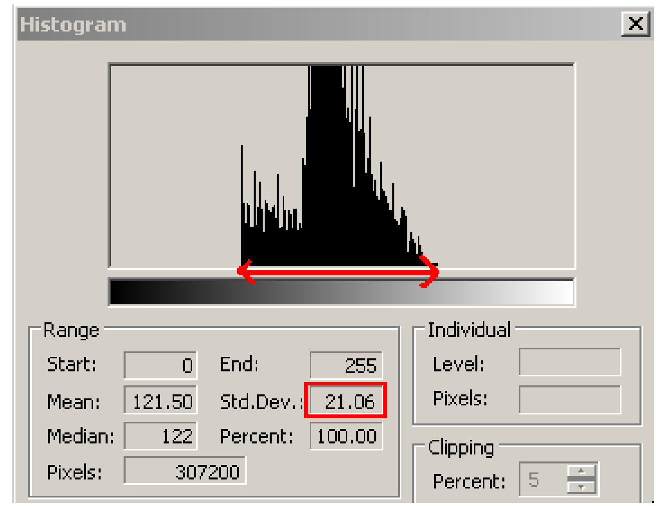
Image enhancement:histogram stretch/shrink

$$\textit{Strecth/Shrink}(f[i,j]) = Final_{MIN} + (Final_{MAX} - Final_{MIN})*(f[i,j]-f_{MIN})/(f_{MAX} - f_{MIN})$$



shrink →

← stretch



shrink →

← stretch

Image enhancement

Gray level mapping using a transformation function

$g_{output} = T\ (f_{input})$

Ex. - gamma correction:

$$g_{out} = c \cdot f_{in}^{\gamma}$$

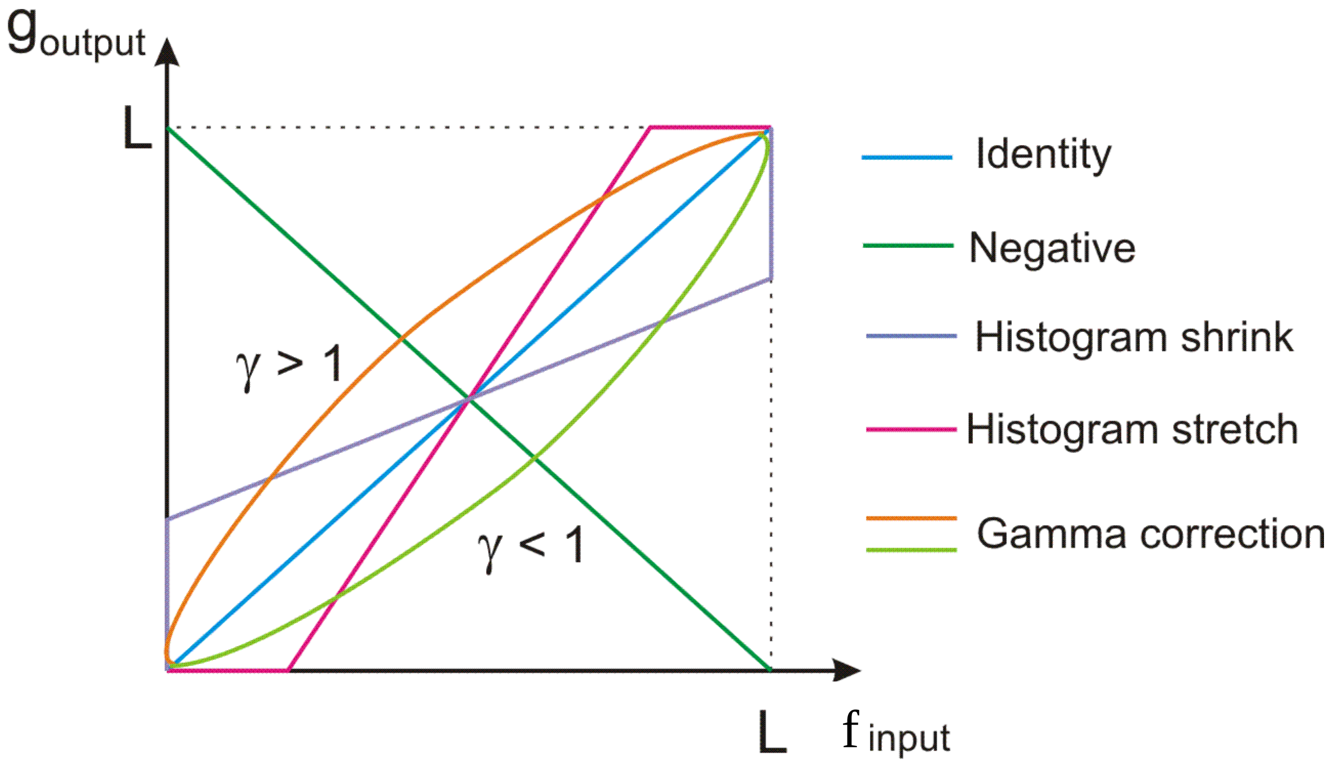
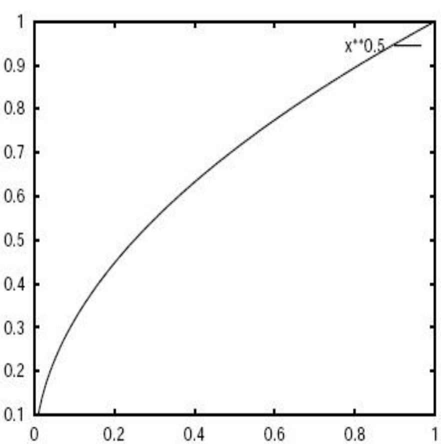
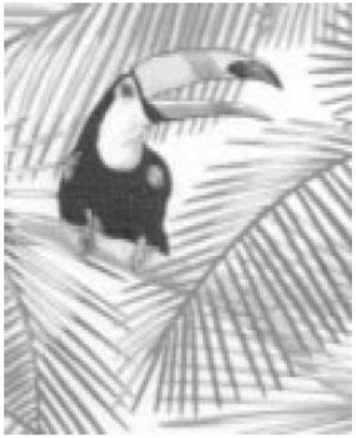


Image enhancement: gamma correction

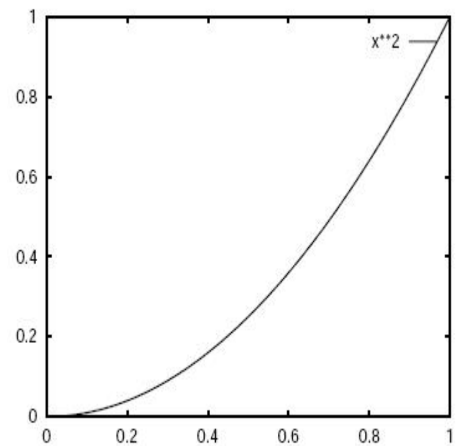
The photographic process in practice contains non-linearities of the type:

- $g(x,y) = f(x,y)^{\gamma}$ where f(x,y) is the real intensity, g(x,y) is the recorded intensity and γ is a constant.
- We digitize and display *g(x,y)*. To correct this we need a transformation of the form: $T(g)=g^{1/\gamma}$. Really $T(g) = g_{max}\,(g/g_{max})^{1/\gamma}$.



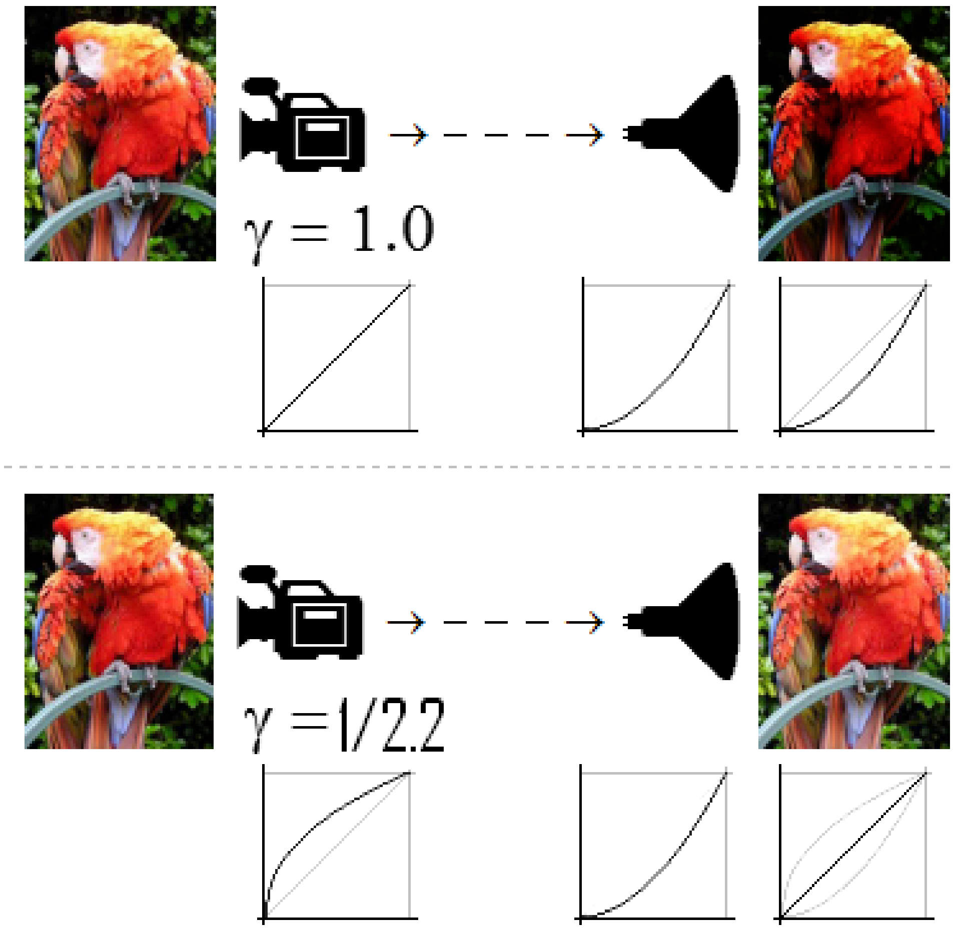


Correction of $\gamma = 2$



Correction of $\gamma = 0.5$

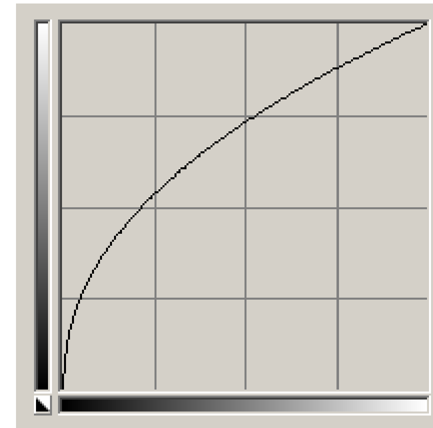
Image enhancement: gamma correction

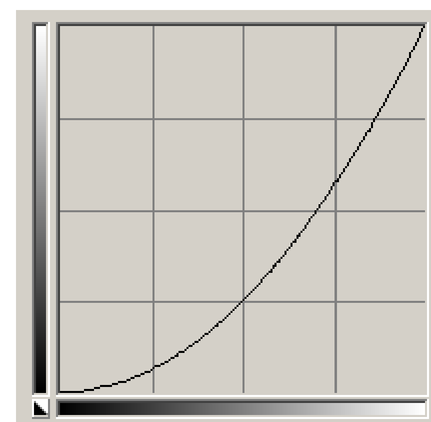


- The three curves represent input–output functions of the camera, the display, and the overall system, respectively.

Image enhancement

Ex. - gamma correction:





More statistical features

- Shannon derived a measure of information content called the **self-information** or **"surprisal"** of a message *m*:

$$I(m) = \log(1/p(m)) = -\log(p(m))$$

  - where **p(m) = Pr(M=m)** is the probability that message *m* is chosen from all possible choices in the message space. The base of the logarithm only affects a scaling factor and, consequently, the units in which the measured information content is expressed. **If the logarithm is base 2, the measure of information is expressed in units of bits**.

- Information is transferred from a source to a recipient only if the recipient of the information did not already have the information to begin with. Messages that convey information that is certain to happen and already known by the recipient contain no real information.

- Infrequently occurring messages contain more information than more frequently occurring messages. This fact is reflected in the above equation - **a certain message, i.e. of probability 1, has an information measure of zero**.

- A **compound message of two (or more) unrelated (or mutually independent) messages would have a quantity of information that is the sum of the measures of information of each message individually**.

More statistical features

Information - the information associated to the gray-level f:

$$I_g = -\log_2 p(f) \quad [bits]$$

$\Rightarrow$ information is large when an unlikely gray-level is generated

Entropy – average information of the image:

$$H = -\sum_{f=0}^{L} p(f) \cdot \log_2 p(f) \quad [bits]$$

$\Rightarrow$ how many bits we need to code the image data:

H is high – pixel values are distributed among many gray levels

$H_{max} = -\sum_{f=o}^{L} \frac{1}{L}\log_2\frac{1}{L} = \sum_{f=o}^{L} \frac{1}{L}\log_2 L = \log_2 L$ [bits] (uniform PDF)

Energy – how the gray-levels are distributed:

$$E = \sum_{f=0}^{L} \left[p(f)\right]^2$$

E (low) – number of gray-levels of the image is high

$E_{max} = 1$ (only one gray-level in the image)

Histogram processing

Histogram equalization

Aim is to distribute pixels equally across available grey level range.

Normalized grayscale levels:

$f \in [0 \ldots L\text{-}1] \Rightarrow r \in [0 \ldots 1]$

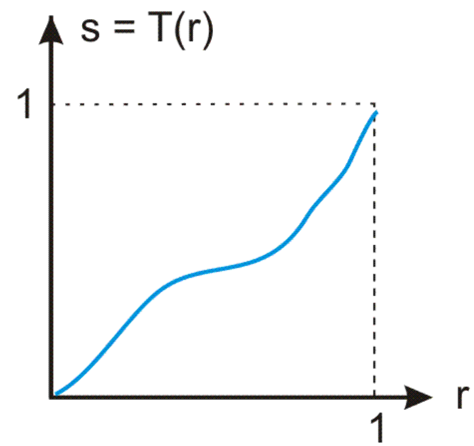
Transformation function:

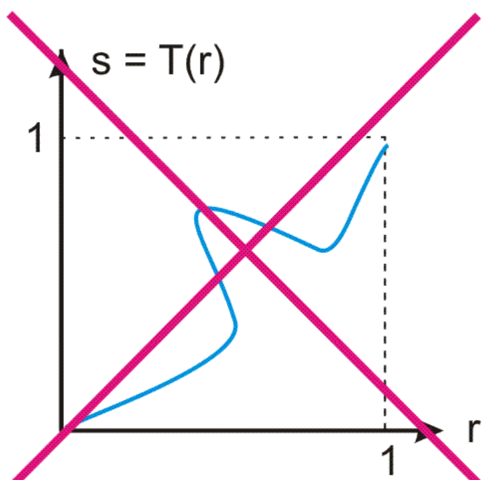
$s = T(r) \in [0 \ldots 1] \Rightarrow g \in [0 \ldots L\text{-}1]$

T features:

(a). single valued and monotonically increasing $\Rightarrow \exists\, r = T^{-1}(s)$

(b). $0 <= T(r) <= 1$





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Histogram equalization

- $p_r(r)$, $p_s(s)$ – probability density functions of the input and output
- $p_r(r)$, $s=T(r)$ are known and T^{-1} satisfies condition *(a)* → $r=T^{-1}(s)$

$$P(s) = Prob(pixel\ value\ < s) = \int_{-\infty}^{s} p_s(s)ds$$

$$P(s) = Prob(T(r) < s)$$

$$T(r) < s \,|T^{-1} \qquad T^{-1}(T(r)) < T^{-1}(s) \qquad r < T^{-1}(s)$$

$$P(s) = Prob\left(r < T^{-1}(s)\right) = \int_{-\infty}^{T^{-1}(s)} p_r(r)dr$$

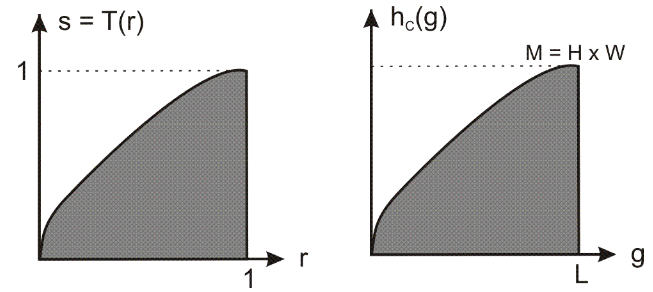
$$p(s) = \frac{dP(s)}{ds} = \frac{d\left(T^{-1}(s)\right)}{ds} p_r\left(T^{-1}(s)\right) = \frac{dr}{ds} p_r(r)$$

$$p_s(s) = \frac{dP(s)}{ds} = p_r(r)\frac{dr}{ds} \qquad (1)$$

Histogram processing

Cumulative histogram / cumulative density function (CDF)

$$s = T(r) = \int_0^r p_r(w)dw \qquad (2)$$



T satisfies (a) & (b)

Leibniz rule: the derivative of a definite integral with respect to its upper limit is simply the integrand evaluated at that limit

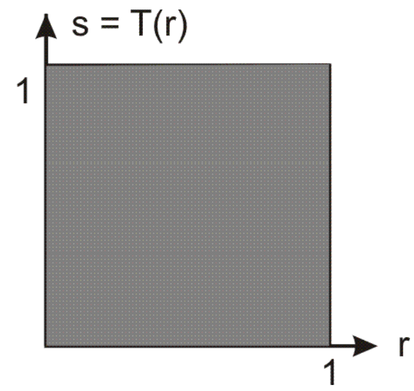
$$\frac{ds}{dr} = \frac{dT(r)}{dr} = \frac{d}{dr}\left[\int_0^r p_r(w)dw\right] = p_r(r) \qquad (3)$$

Histogram processing

Histogram equalization

(1) + (3) $\Rightarrow$

$$p_s(s) = p_r(r)\frac{dr}{ds} = p_r(r)\frac{1}{p_r(r)} = 1 \quad , \quad 0 <= s <= 1$$



$p_s(s)$:

- uniform PDF
- independent from $p_r(r)$

Histogram equalization algorithm

$$p_r(r_k) = \frac{n_k}{n} \quad , \quad k = 0..L$$

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n} \quad , \quad k = 0..L-1$$

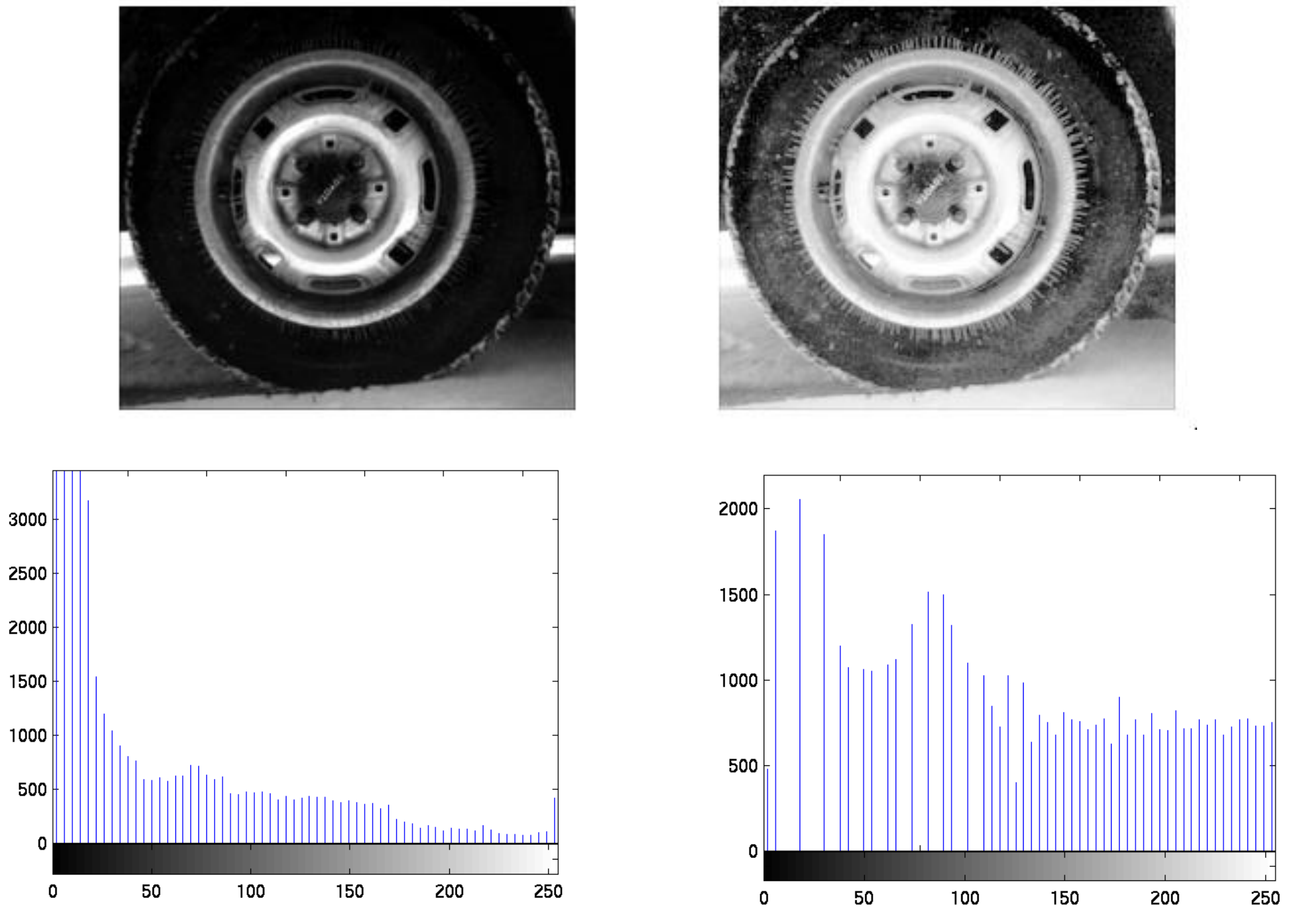
$\Rightarrow$ re-map the gray-scale values of the output image: r_k -> s_k

$$g_k = round(s_k(L-1))$$

$\Rightarrow$ re-scale the gray-scale values of the output image: s_k -> g_k

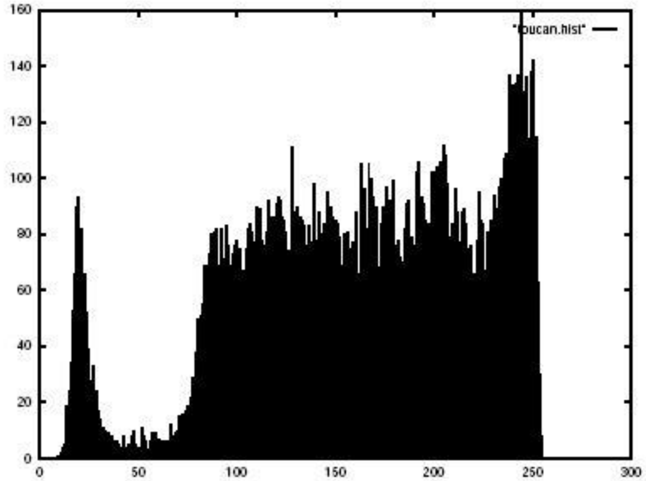
Histogram processing

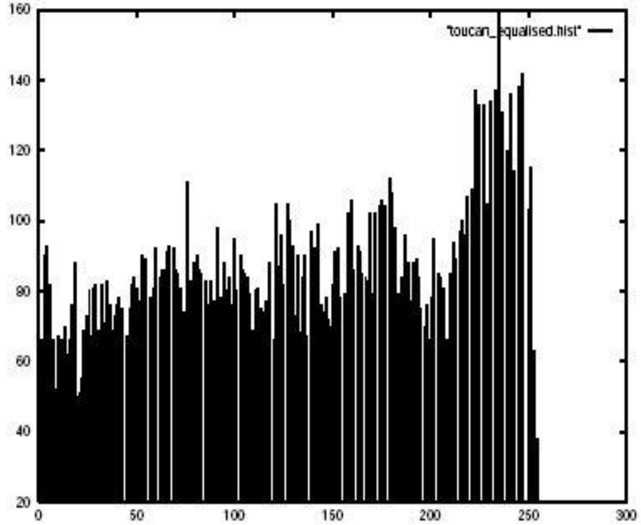
Histogram equalization (results)



Histogram processing

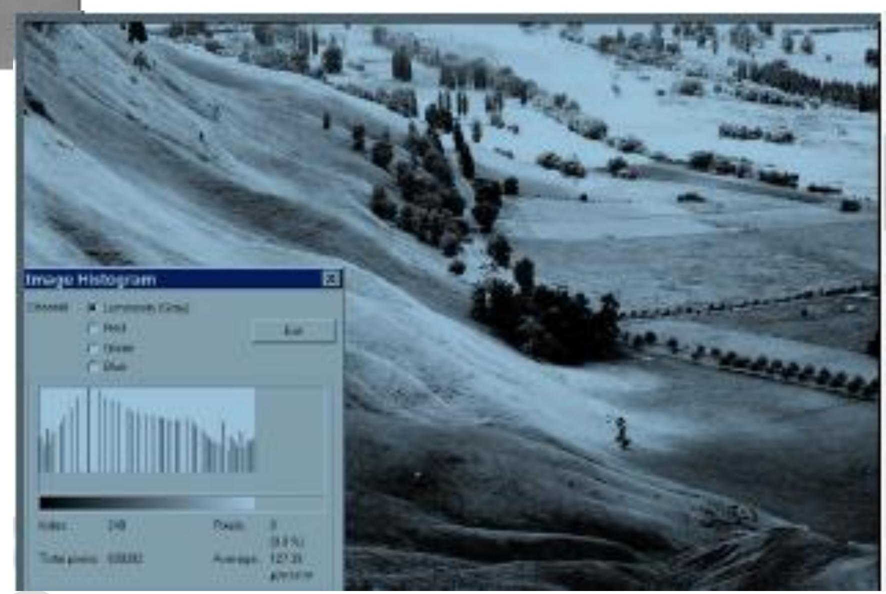
Histogram equalization (results)





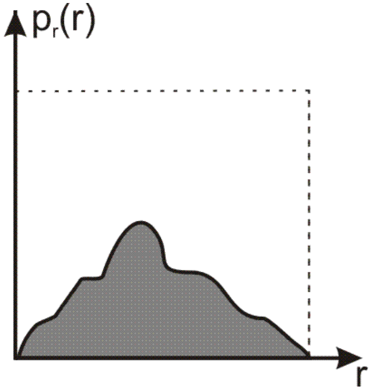
Histogram processing

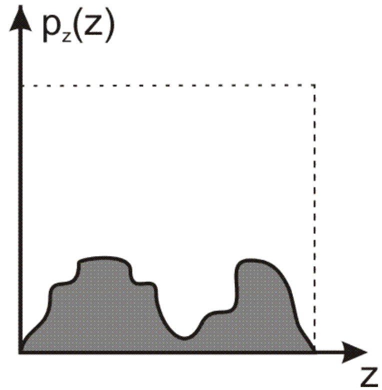
Histogram equalization (results)



Histogram processing

Histogram specification / matching





input image $\Rightarrow$ output image

For the input image we have: $$s = T(r) = \int_0^r p_r(w)dw \qquad (1)$$

We define a random variable z with the property:

$$G(z) = \int_0^z p_z(t)dt = s \qquad (2)$$

(1) + (2) $\Rightarrow$ $$G(z) = T(r)$$

$$z = G^{-1}(s) = G^{-1}[T(r)]$$

Histogram processing

Histogram specification / matching

$s_k \leftrightarrow z_k$

No analytical expressions for T(r) and G^{-1} !?

Algorithm:

1. $r_k \leftrightarrow s_k$:

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n} \quad , \quad k = 0..L$$

2. $s_k \leftrightarrow v_k$:

$$v_k = G(z_k) = \sum_{j=0}^{k} p_z(z_j) = s_k \quad , \quad k = 0..L$$

3. $s_k \leftrightarrow z_k$:

Let z' = z_k, k = 0, ... L

z_k will be the smallest z' satisfying the condition:

(G(z')-s_k) >= 0