VIDEO SENSORS II

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CONTENTS
Course structure

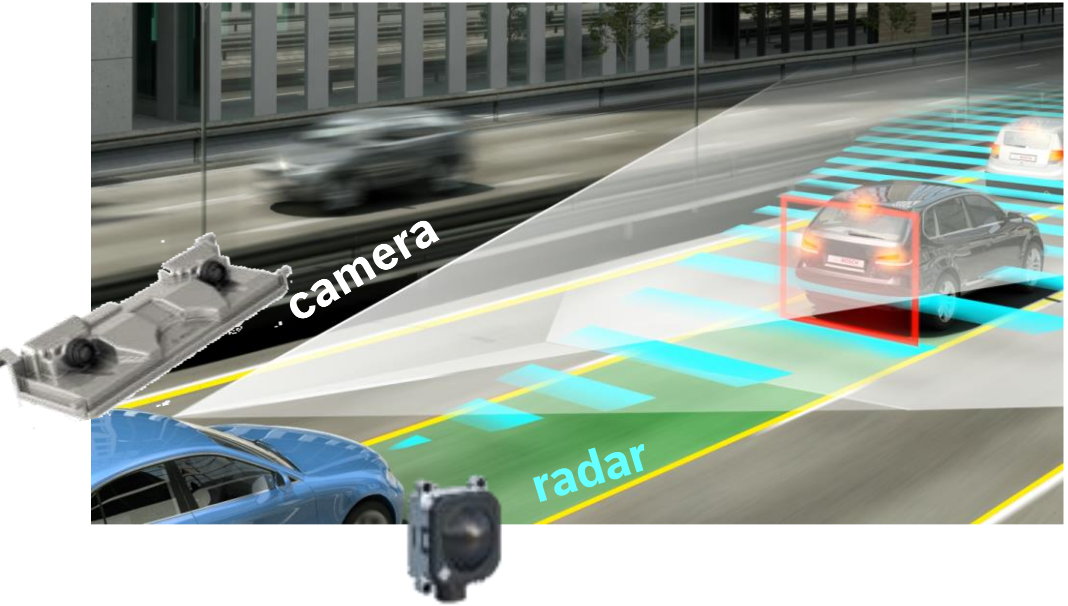
1. Remember from last course and goals for today
2. Theoretical refresh: vectors, dot product, cross product, coordinate systems, transformations
3. Essential matrix: scope, proof
4. Fundamental matrix
5. Fundamental matrix estimation
6. Conclusions

BOSCH

REVIEW OF THE LAST COURSE AND GOALS FOR TODAY

Bosch Engineering Center Cluj
Automated driving activities

SOFTWARE ENGINEERING



Radar Systems

Video Systems

Connectivity

Central processing unit

Ultrasonic Systems

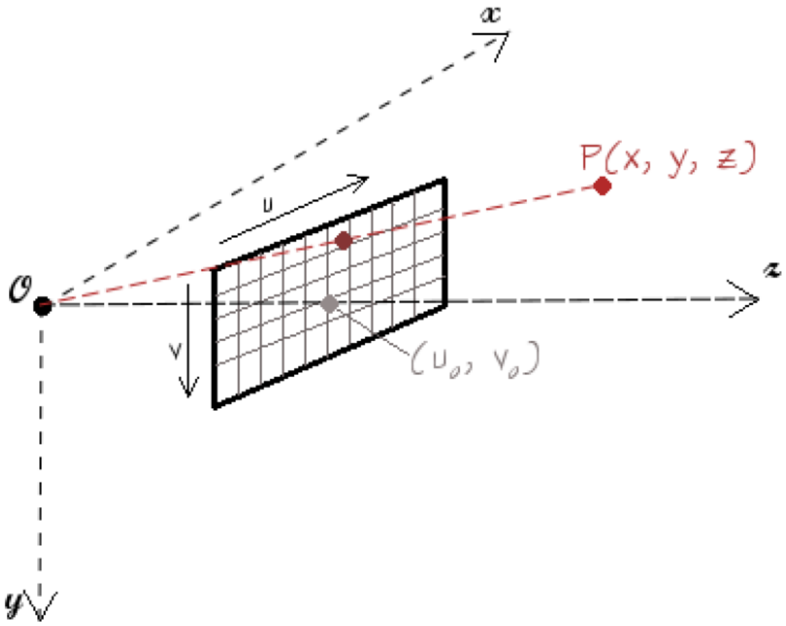
Electric Power Steering

Review of the last course and goals for today

Review - intrinsic parameters of the camera

- O – camera center
- u_0, v_0 - principal point
- u, v – image coordinates
- f – focal length – distance from O to principal point
- dpx, dpy – size of pixels
- f_x, f_y - focal length in pixels units

$$f_x = \frac{f}{dpx} \qquad f_y = \frac{f}{dpy}$$



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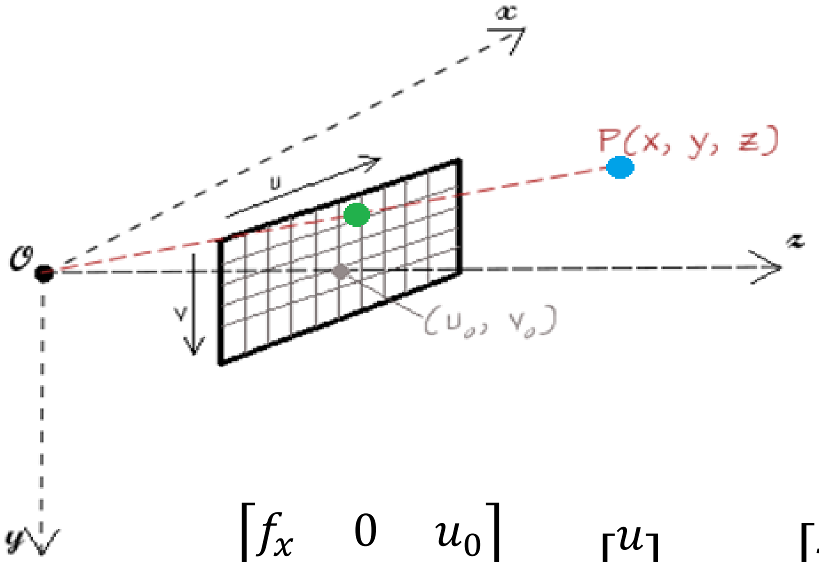
Review of the last course and goals for today

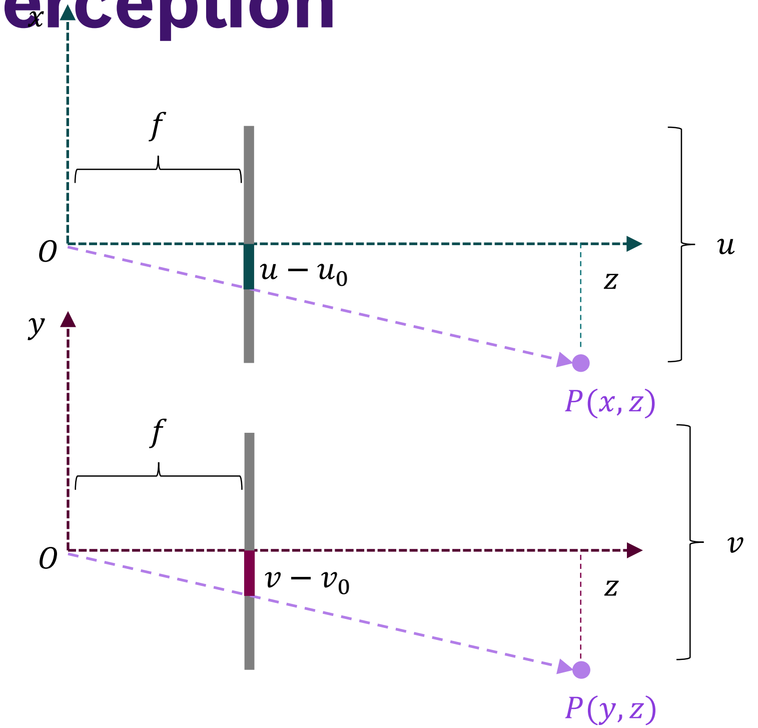
Review - computer vision and 3D perception

1

2



$$K = \begin{bmatrix} f_x & 0 & u_0 \\ 0 & f_y & v_0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K \cdot \begin{bmatrix} x/z \\ y/z \\ 1 \end{bmatrix}$$



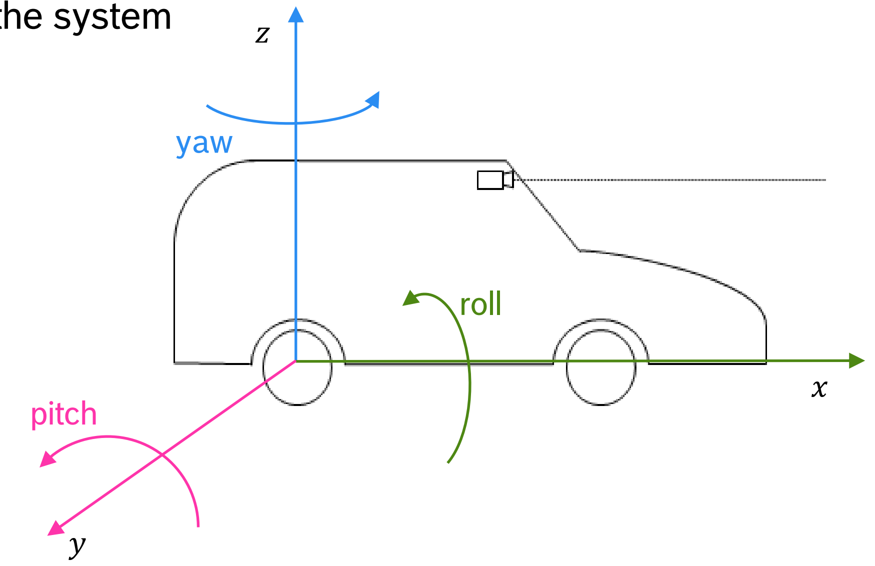
$$\frac{(v - v_0)}{f_y} = \frac{y}{z} \qquad \frac{(u - u_0)}{f_x} = \frac{x}{z}$$

Review of the last course and goals for today
Review - parameters of the camera

Extrinsic parameters

- represent position and orientation of the camera in the car (world) coordinate system
  - Convention – use the center of the rear axis
- R matrix for each axis and T represents the translation vector applied on the system
- they can be written in the following form $[R|T] = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix}$
- there are three rotations applied in 3D space – pitch, roll and yaw

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \sim K \cdot \left(R \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} + T \right) = K \cdot [R|T] \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

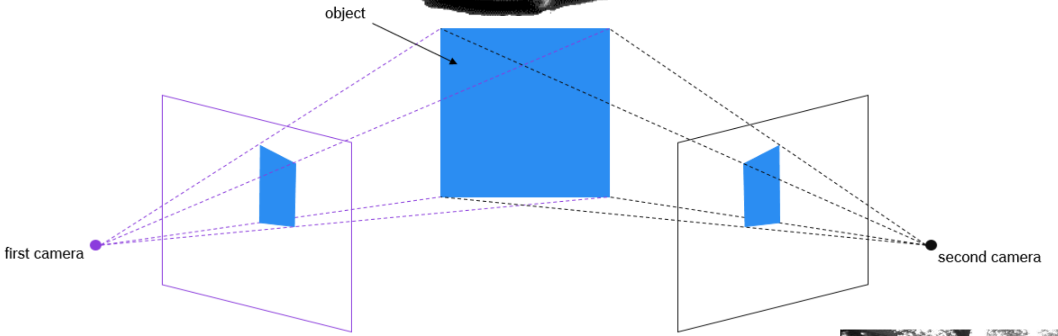


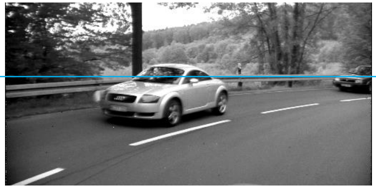
General concepts - review

- 3D reconstruction – the process of creating the 3D shape and position of real objects from images
- in computer vision for automated driving
  - using the stereo system
  - using the mono system in time

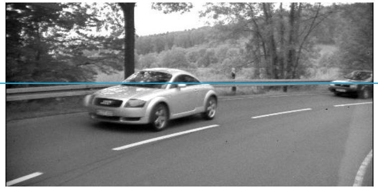
stereo camera

mono camera





Rectification for 3D reconstruction => corresponding image points are on the same scan-line

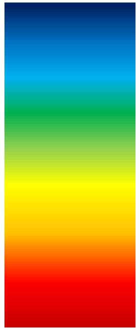


Review of the last course and goals for today

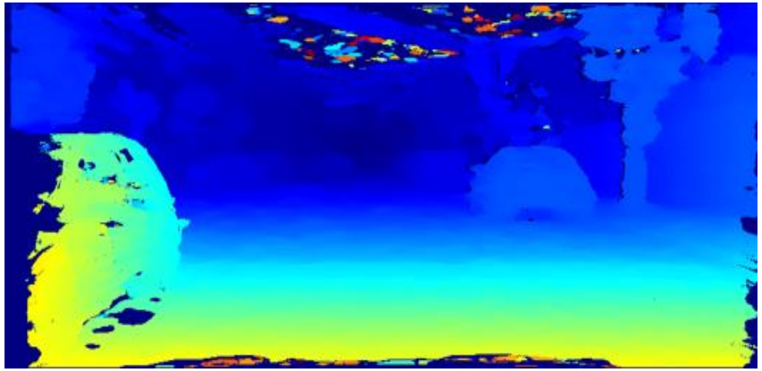
Review – disparity and 3D

Legend

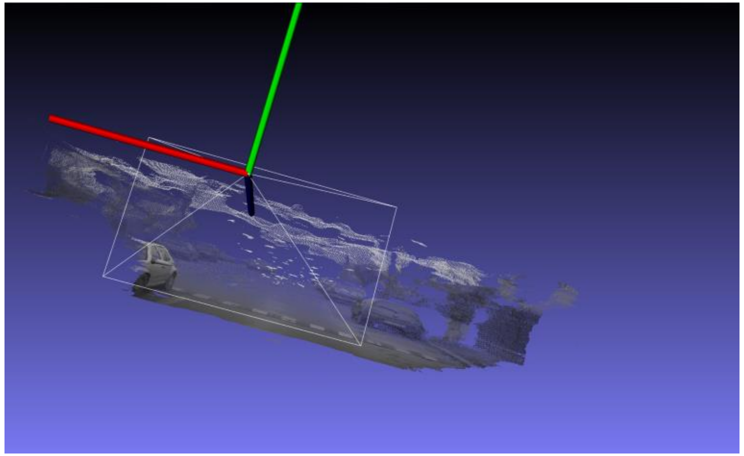
Far objects



Close objects



3D reconstruction



Disparity is small in far range
and big in close range

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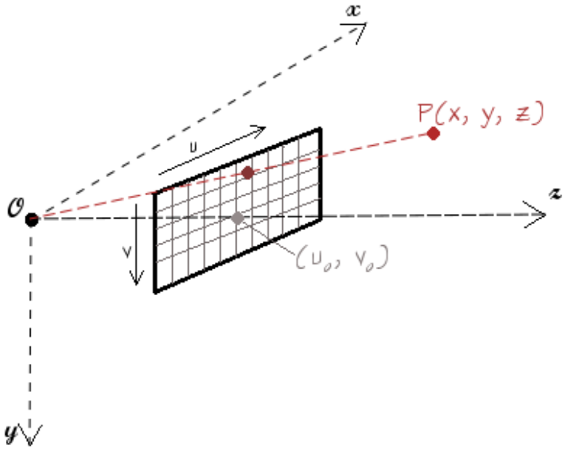
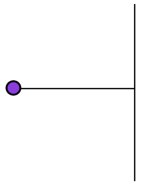
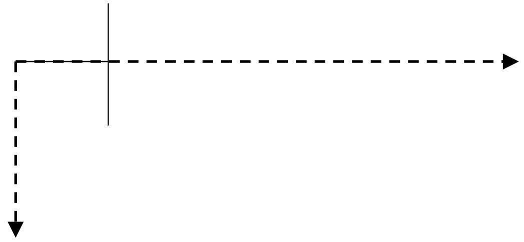
Review of the last course and goals for today
Simplified camera representations

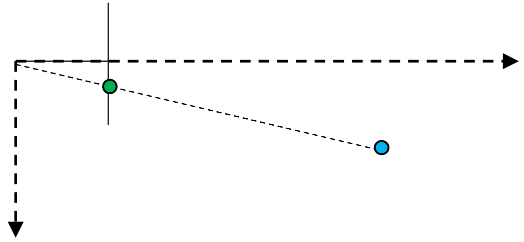




Camera, magenta
is the optical center



Camera with coordinate
system attached



Camera with coordinate
system attached
Blue dot is projected
in the green dot
in the image plane

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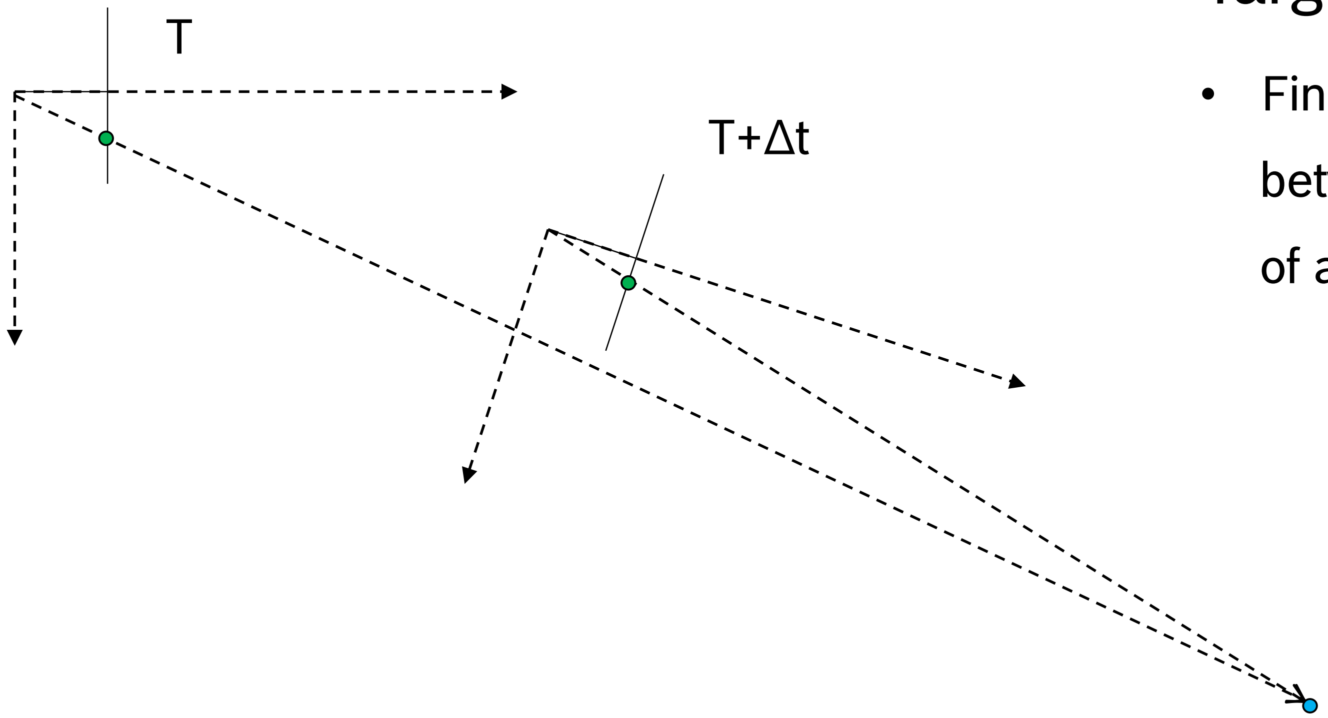
Review of the last course and goals for today

Goal for today – geometry of a mono system

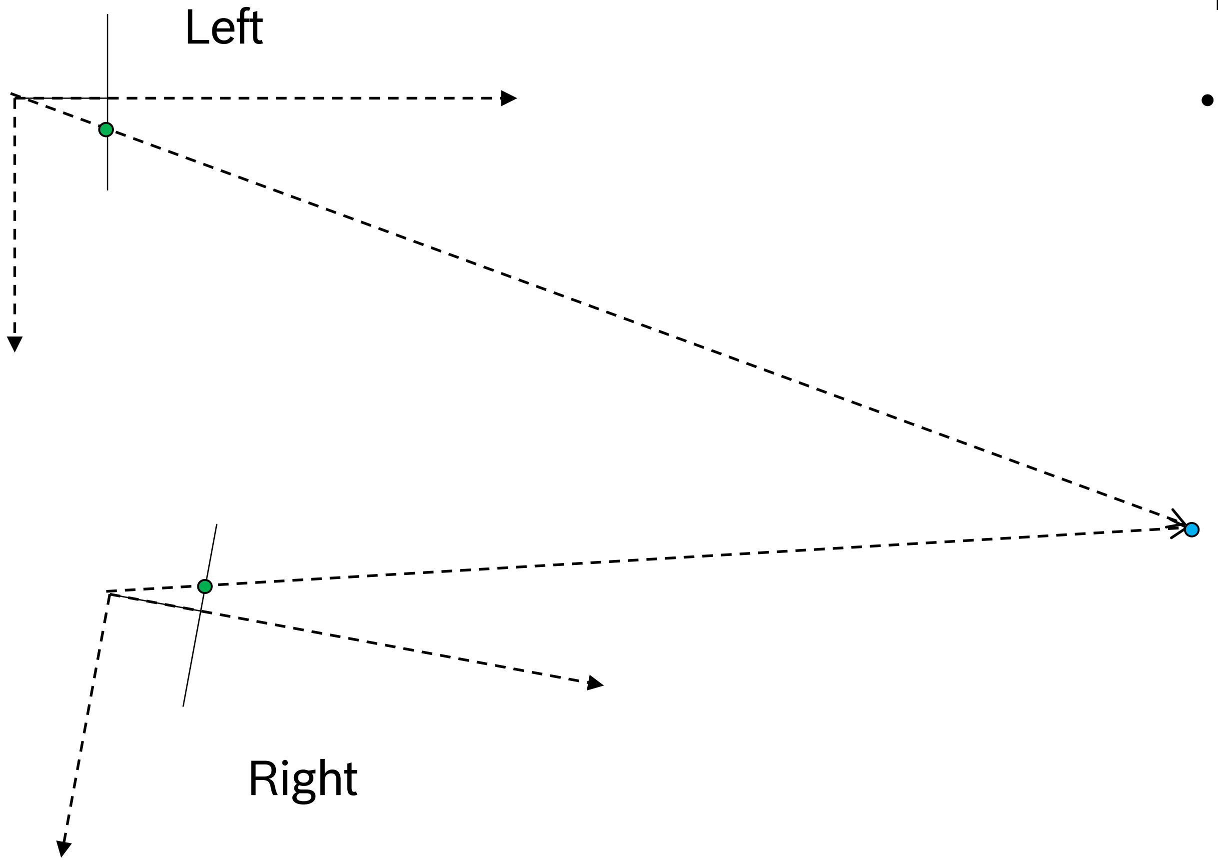


Target:

- Find the mathematical relationship between the two projections (green) of a given point in space (blue)?

Review of the last course and goals for today

Goal for today - geometry of a stereo system



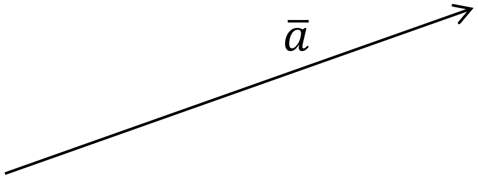
Target:

- Find the mathematical relationship between the two projections (green) of a given point in space (blue)?

BOSCH

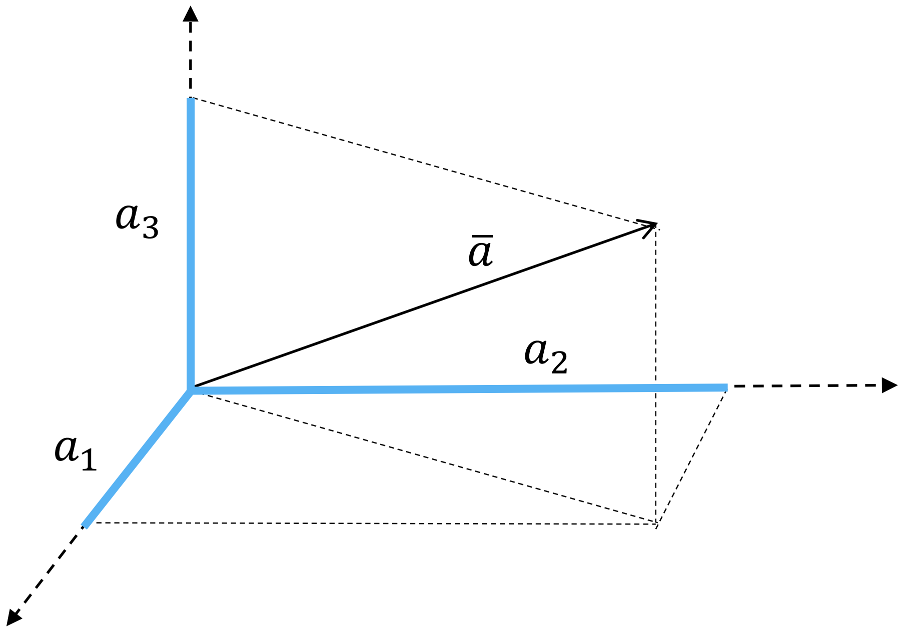
THEORETICAL REFRESH

Theoretical refresh
Vectors



- A quantity that has direction and magnitude
  - General theory about vectors and vector spaces you get from your linear algebra course
  - For computer vision we need 3-dimensional vector spaces, in the linear algebra course you studied n-dimensional vector spaces
  - Magnitude will be noted as $|\bar{a}|$

Theoretical refresh
Vector’s coordinates



- A quantity that has direction and magnitude
- Has coordinates when represented in a certain coordinate system
  - Coordinates are vector’s projection on some orthogonal axes
  - $\bar{a} = (a_1 \quad a_2 \quad a_3)$ for 3-dimensional case
  - Based on axis unit vectors: $\bar{a} = a_1\bar{\imath} + a_2\bar{\jmath} + a_3\bar{k}$
  - $\bar{a} = (a_1 \quad a_2 \quad \dots \quad a_n)$ for n-dimensional case

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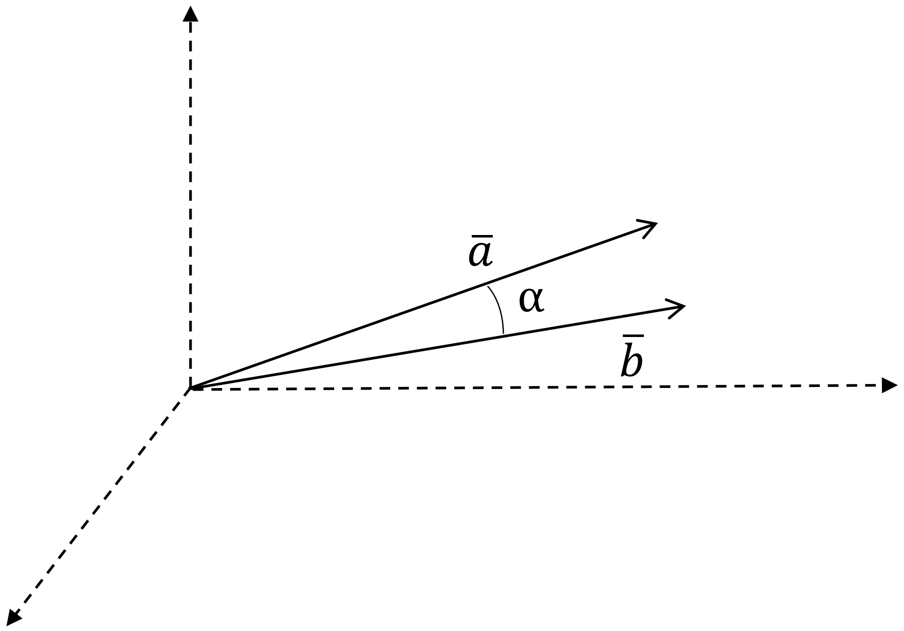
Theoretical refresh
Dot product

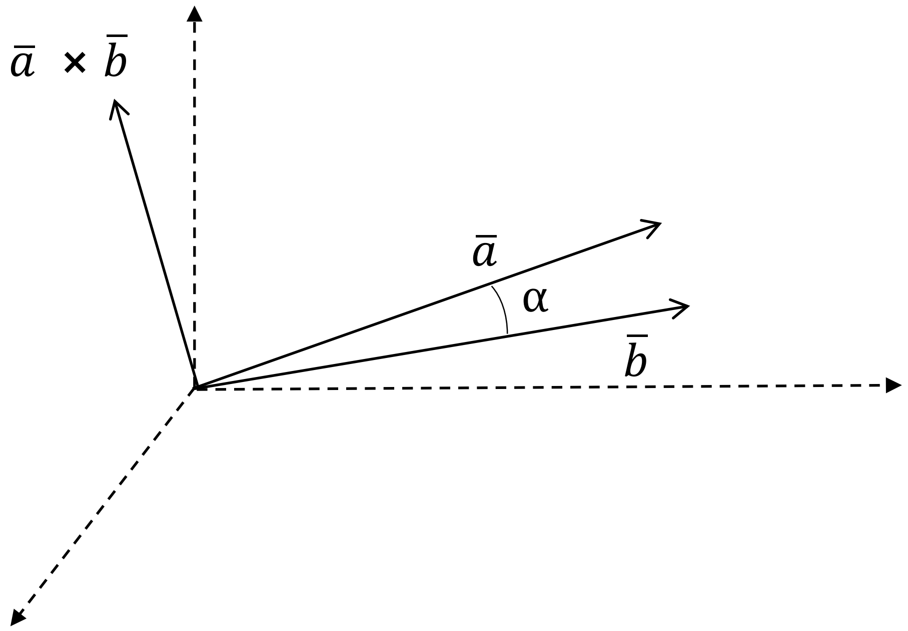


- A quantity that has direction and magnitude
- Has coordinates when represented in a certain coordinate system
- Dot product
  - $\bar{a} \cdot \bar{b} = |\bar{a}||\bar{b}|\cos(\alpha)$
  - If $\bar{a} = (a_1 \quad a_2 \quad a_3)$ and $\bar{b} = (b_1 \quad b_2 \quad b_3)$ then

$$\bar{a} \cdot \bar{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}' \cdot \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = [a_1 \quad a_2 \quad a_3] \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

  - Remark
    - If vectors are perpendicular then the dot product is zero

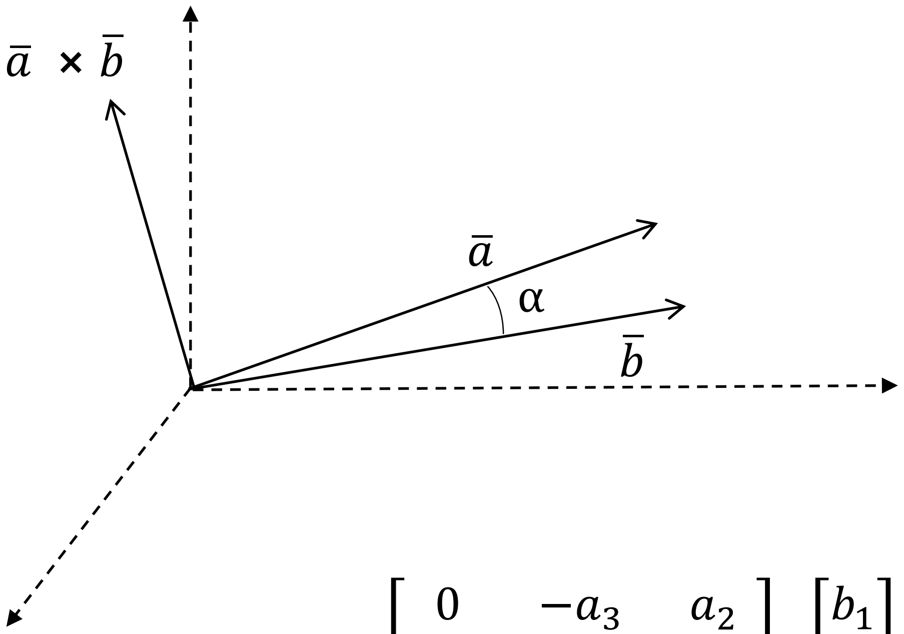
Theoretical refresh
Cross product



- A quantity that has direction and magnitude
- Has coordinates when represented in a certain coordinate system
- Dot product
- Cross product
  - Direction perpendicular to the two vectors
  - Magnitude $\left|\bar{a} \times \bar{b}\right| = |\bar{a}|\left|\bar{b}\right|\sin(\alpha)$

$$\left|\bar{a} \times \bar{b}\right| = \begin{vmatrix} \bar{\imath} & \bar{\jmath} & \bar{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

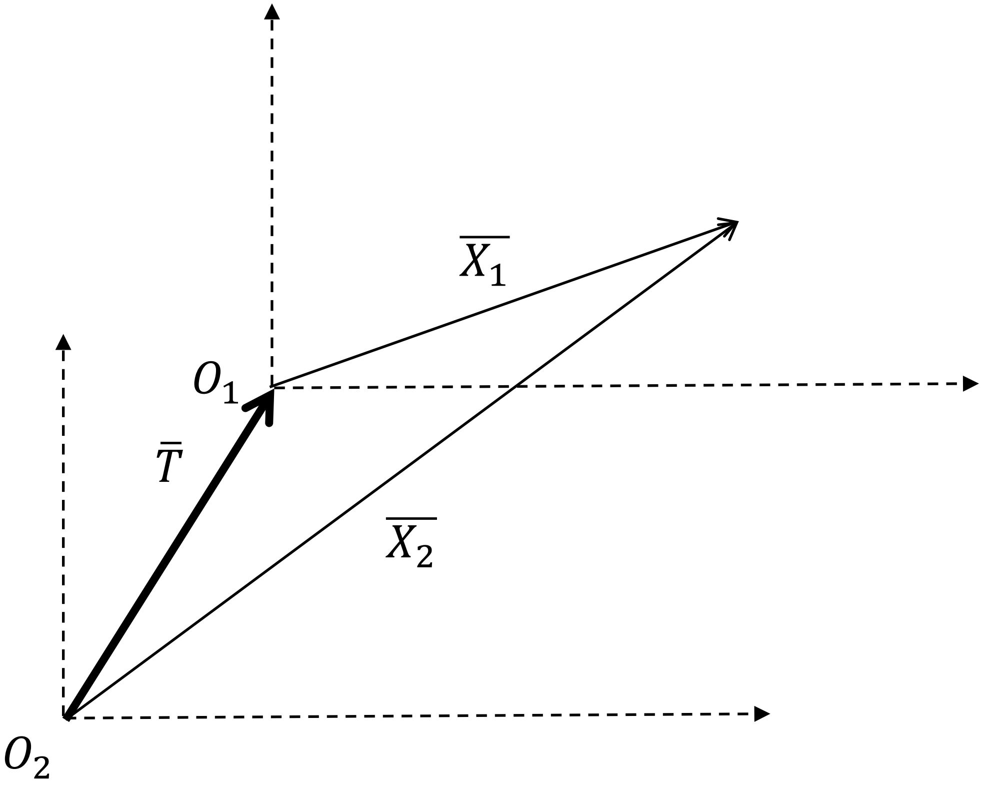
Theoretical refresh
Cross product matrix form



- A quantity that has direction and magnitude
- Has coordinates when represented in a certain coordinate system
- Dot product
- Cross product
- Cross product matrix form

$$\bar{a} \times \bar{b} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \bar{a}_\times \cdot \bar{b} \quad \Longleftrightarrow \quad \left|\bar{a} \times \bar{b}\right| = \begin{vmatrix} \bar{\imath} & \bar{\jmath} & \bar{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

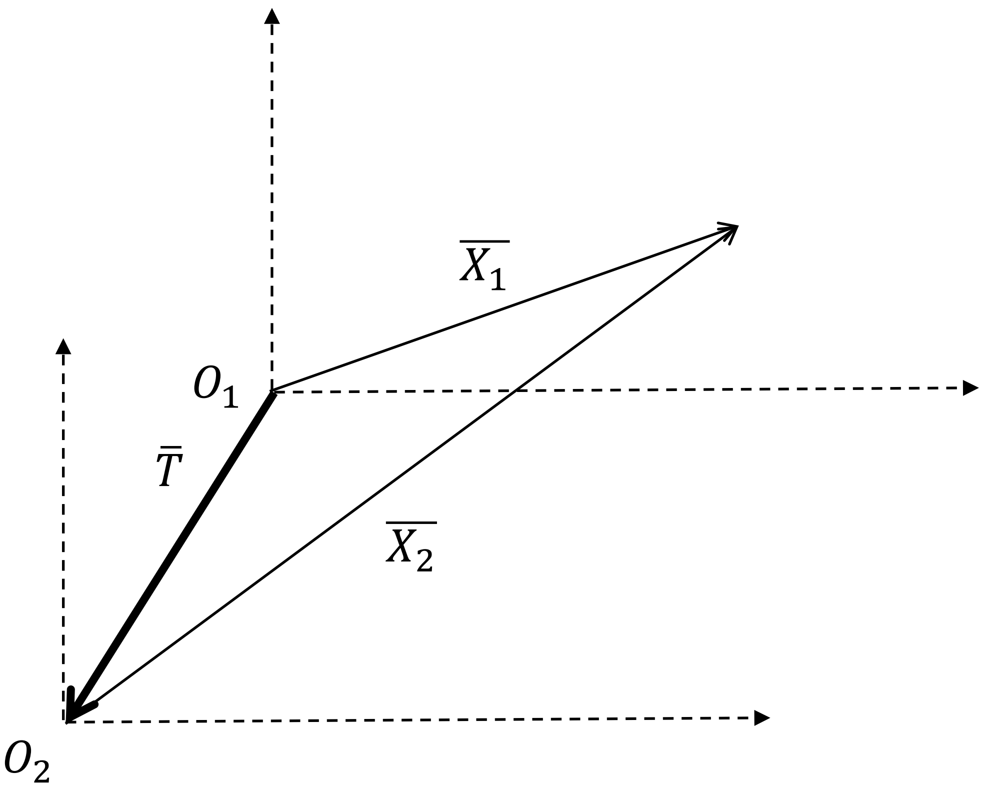
Theoretical refresh
Translation



$$\overline{X_1} = \overline{X_2} - \overline{T} \qquad \overline{X_2} = \overline{X_1} + \overline{T}$$

- A quantity that has direction and magnitude
- Has coordinates when represented in a certain coordinate system
- Dot product
- Cross product
- Cross product matrix form
- Translation
  - $\overline{T}$ represents the coordinates of the **first** coordinate system in the **second** coordinate system

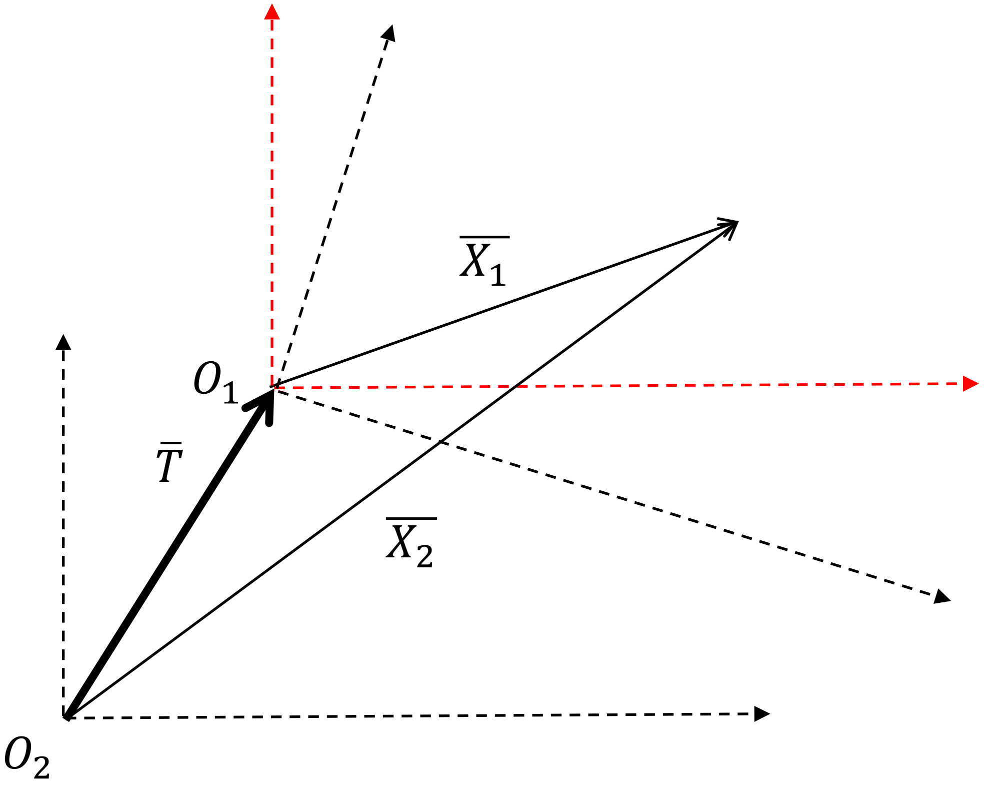
Theoretical refresh
Translation



$$\overline{X_1} = \overline{X_2} + \bar{T} \qquad \overline{X_2} = \overline{X_1} - \bar{T}$$

- A quantity that has direction and magnitude
- Has coordinates when represented in a certain coordinate system
- Dot product
- Cross product
- Cross product matrix form
- Translation
  - $\bar{T}$ represents the coordinates of the **second** coordinate system in the **first** coordinate system

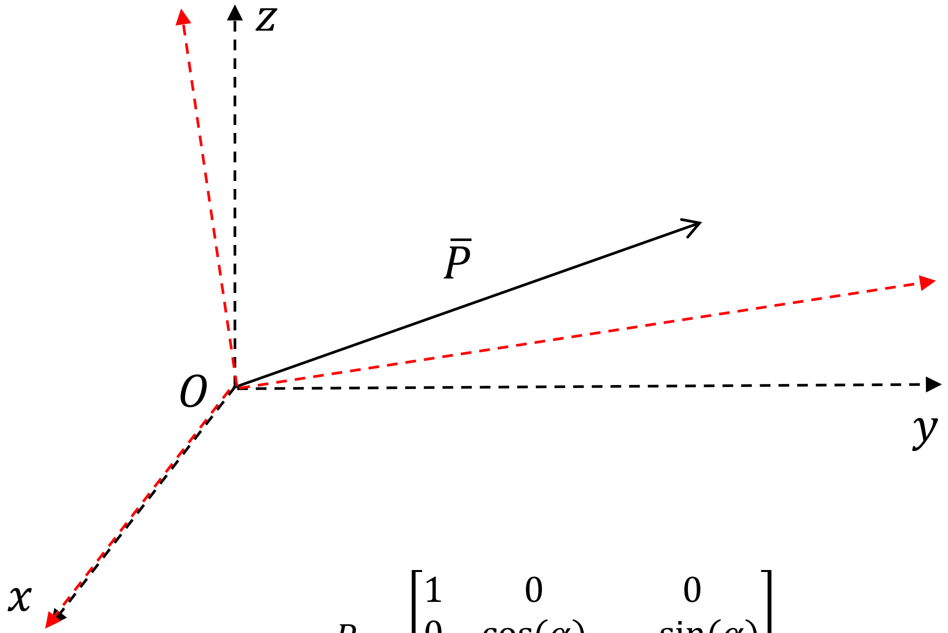
Theoretical refresh
Rotation matrix



$$R \cdot \overline{X_1} = \overline{X_2} - \overline{T} \qquad \overline{X_2} = R \cdot \overline{X_1} + \overline{T}$$

- A quantity that has direction and magnitude
- Has coordinates when represented in a certain coordinate system
- Dot product
- Cross product
- Cross product matrix form
- Translation
- Rotation
  - Rotation matrix $R \cdot R^t = I$
  - $R \cdot \overline{X_1}$ are the coordinates of $\overline{X_1}$ in the coordinate system with red axes

Theoretical refresh
Rotation around single axis

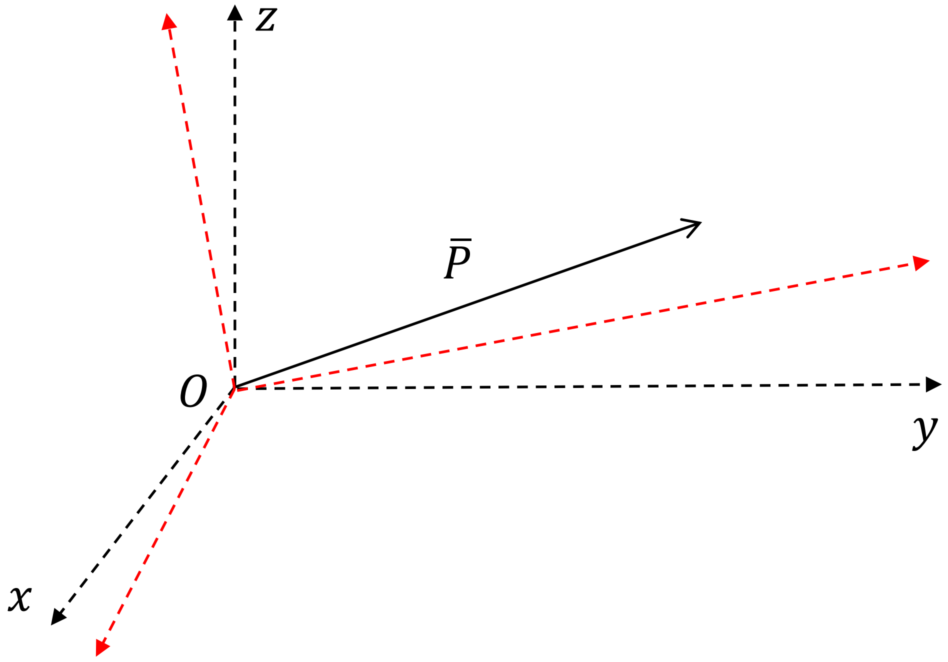


$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix}$$

$$R \cdot \bar{P} = R \cdot \begin{bmatrix} x_P \\ y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \boldsymbol{x_P} \\ \cos(\alpha)\, y_P - \sin(\alpha) z_P \\ \sin(\alpha)\, y_P + cos(\alpha) z_P \end{bmatrix}$$

- A quantity that has direction and magnitude
- Has coordinates when represented in a certain coordinate system
- Dot product
- Cross product
- Cross product matrix form
- Translation
- Rotation
  - Rotation matrix $R \cdot R^t = I$
  - $R \cdot \bar{P}$ are the coordinates of the vector $\bar{P}$ in the coordinate system with red axes

Theoretical refresh
General 3D rotation

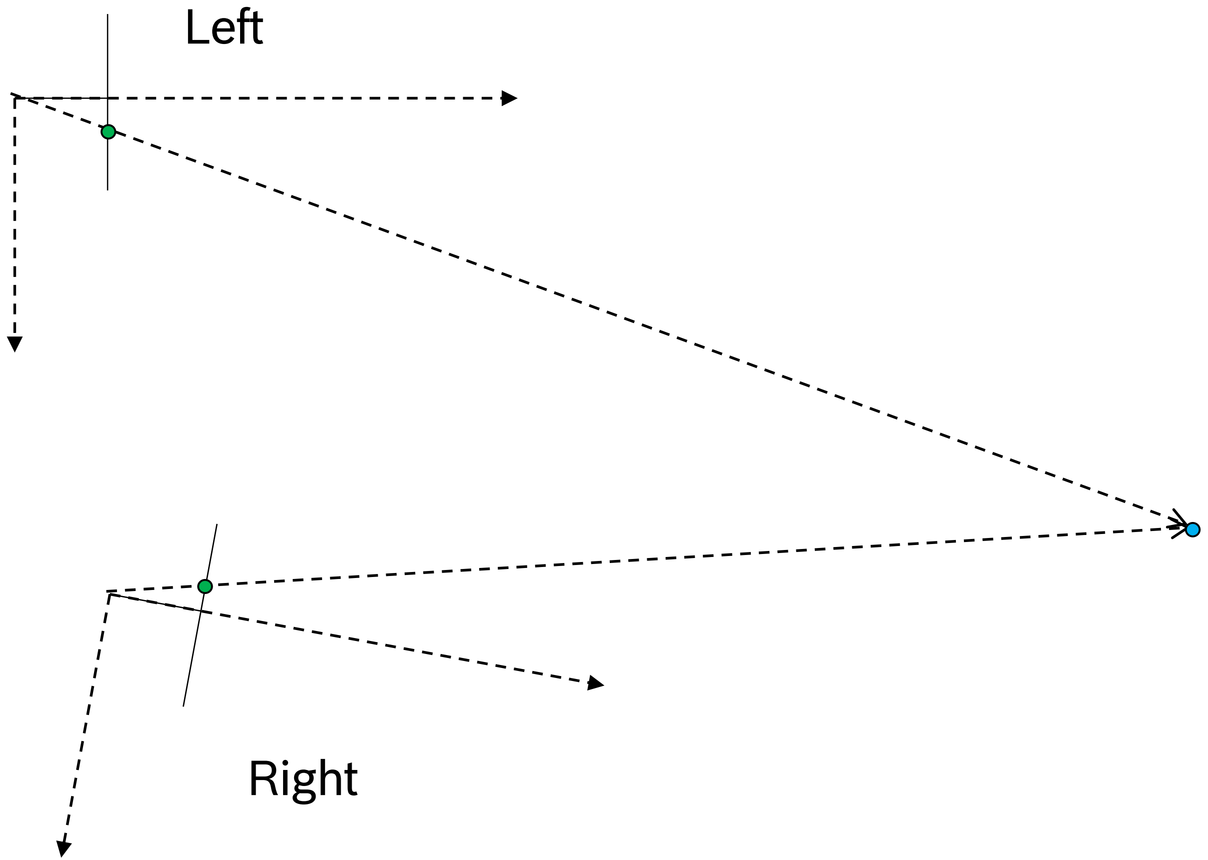


$$R = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix}$$

- A quantity that has direction and magnitude
- Has coordinates when represented in a certain coordinate system
- Dot product
- Cross product
- Cross product matrix form
- Translation
- Rotation
  - Rotation matrix $R \cdot R^t = I$
  - $R \cdot \bar{P}$ are the coordinates of the vector $\bar{P}$ in the coordinate system with red axes

ESSENTIAL MATRIX

Essential matrix
Geometry of two cameras



Target:

- Find the mathematical relationship between the two projections (green) of a given point in space (blue)?

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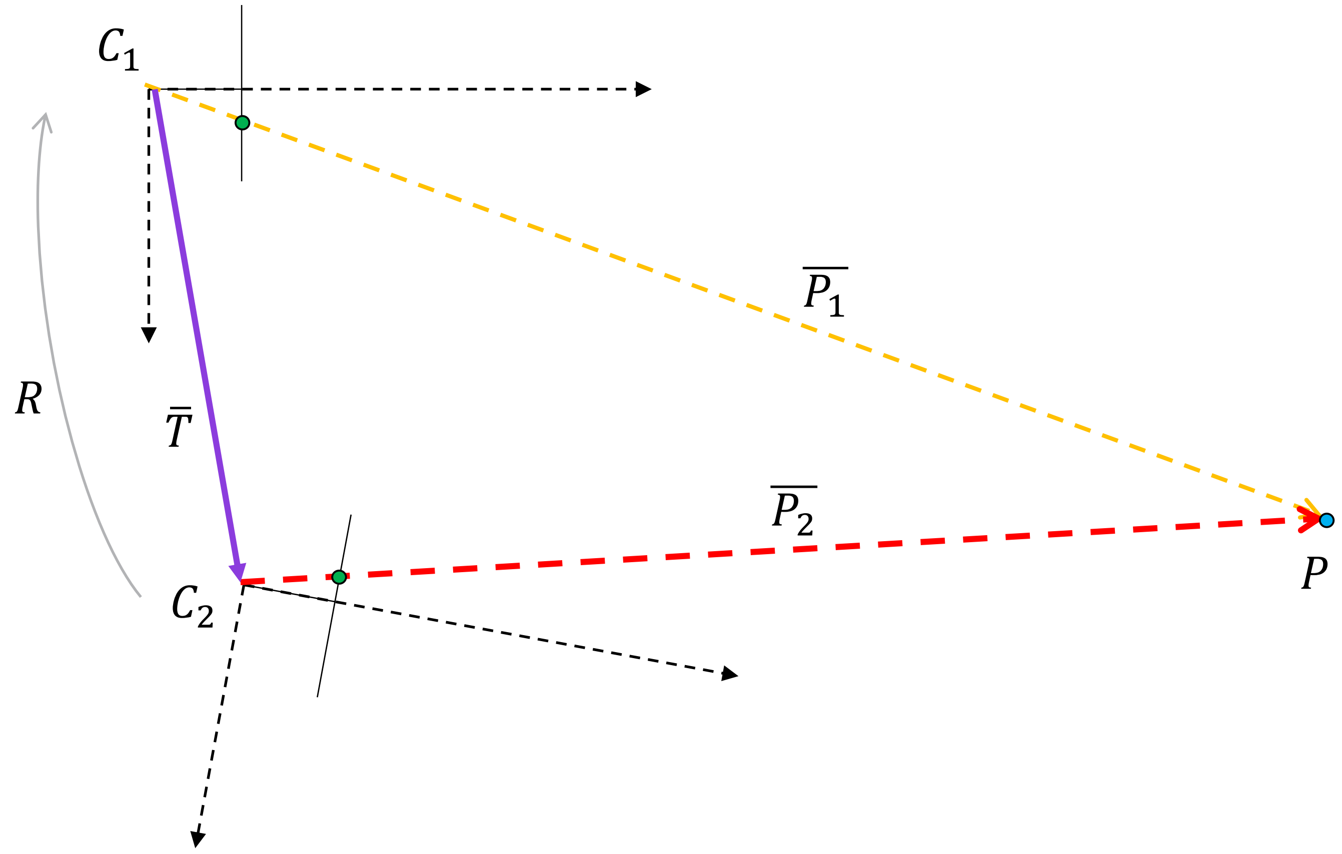
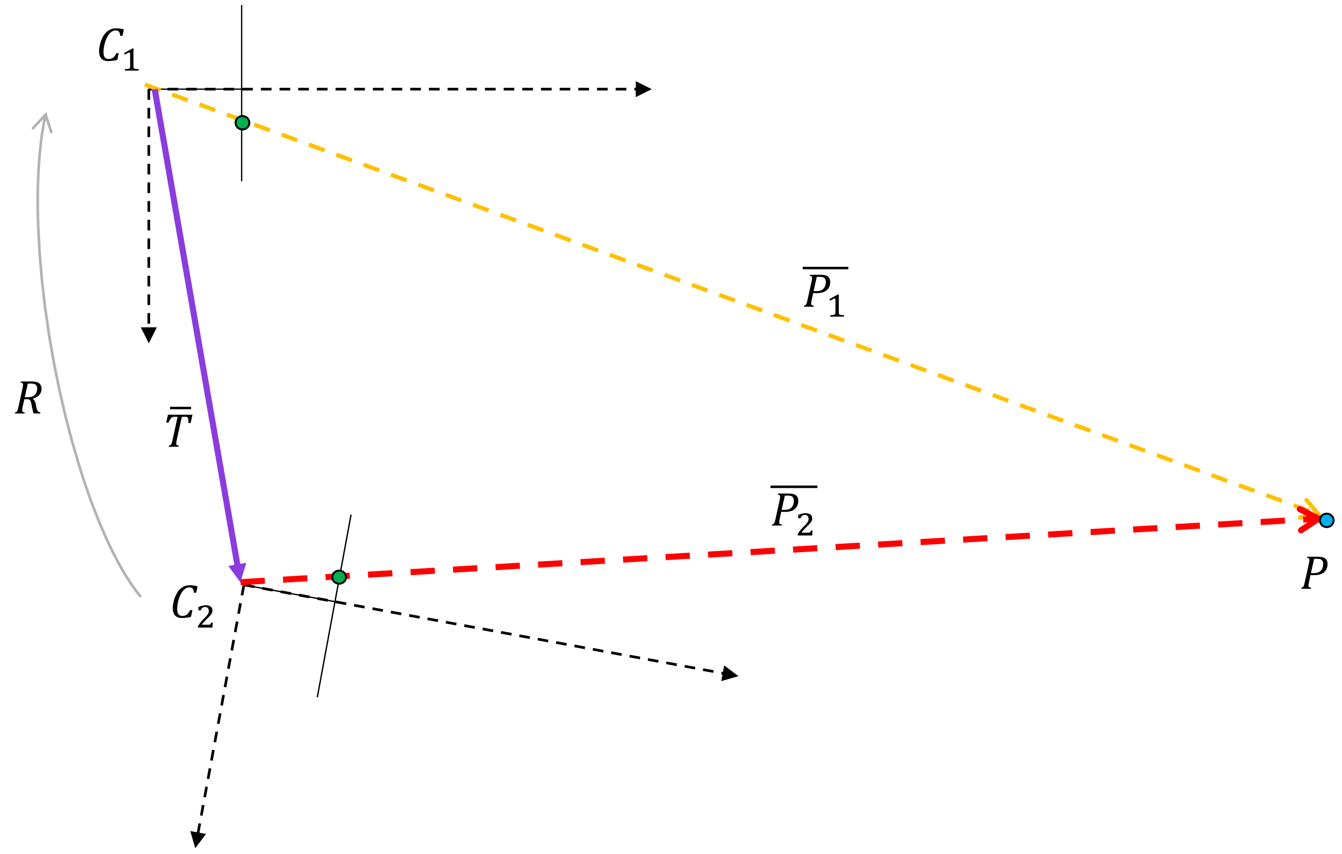
Essential matrix

Geometry of two cameras



- Consider two projections (green) of a given point in space (blue)
- $\bar{T}$ translation vector between the cameras (magenta)
- R rotation matrix from the C_2 coordinate system to C_1
- $\overline{P_1}$ column vector with coordinates of the point P in C_1 (orange)
- $\overline{P_2}$ column vector with coordinates of the point P in C_2 (red)

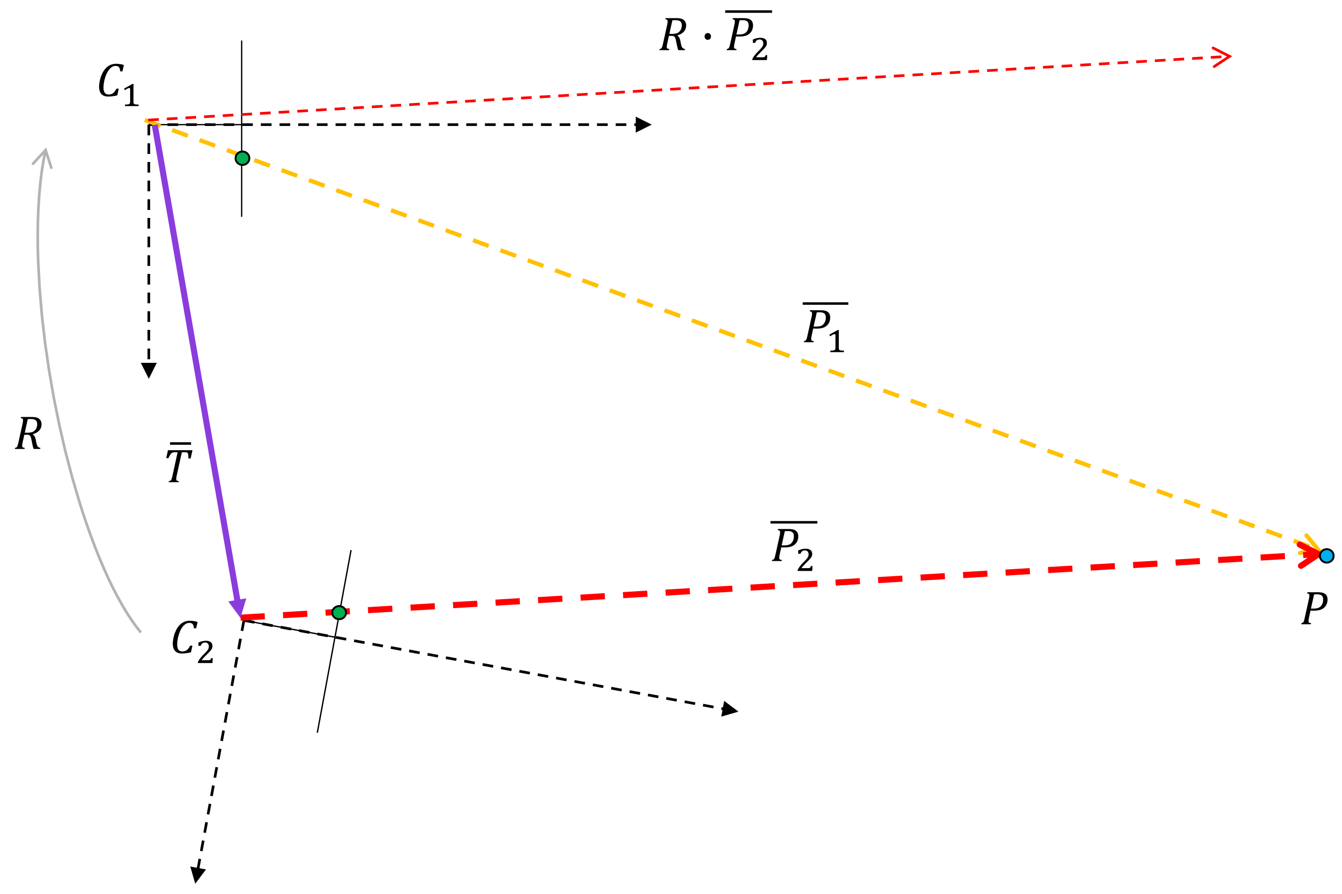
Essential matrix
Geometry of two cameras



- Constraint – the magenta, orange and red vectors are co-planar
  - **Cross product of any two of them, dot product with the third is zero!**
- Expressed in the C_1 the values of the three column vectors are:

$$\bar{T},\ \overline{P_1},\ R \cdot \overline{P_2}$$

Essential matrix
Geometry of two cameras



- Constraint – the magenta, orange and red vectors are co-planar
  - **Cross product of any two of them, dot product with the third is zero!**
- Expressed in the C_1 the values of the three column vectors are:

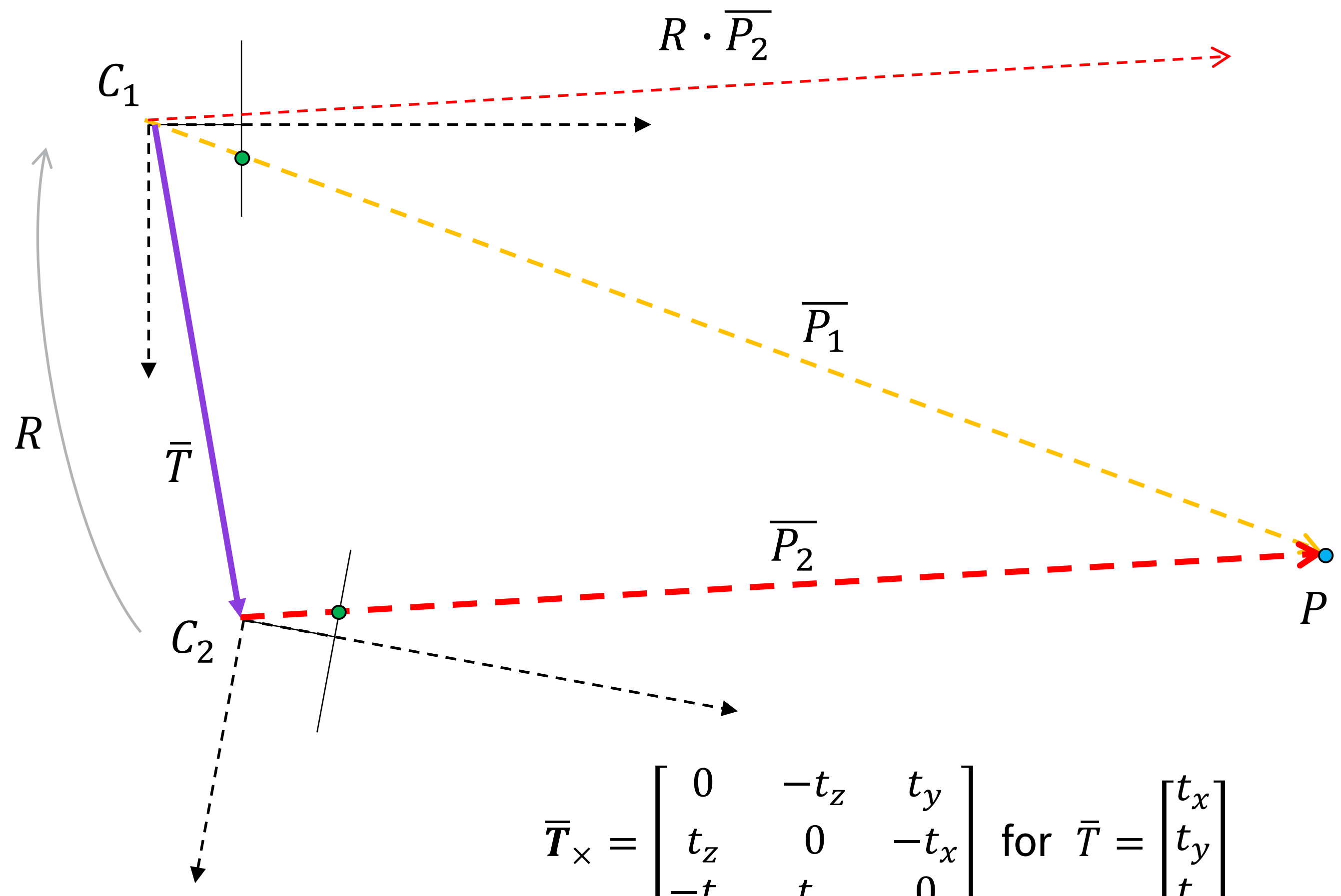
$$\bar{T},\ \overline{P_1},\ R \cdot \overline{P_2}$$

$$\overline{P_1} \cdot (\bar{T} \times R \cdot \overline{P_2}) = 0$$

  - In matrix form (‘ means transpose):

$$\overline{P_1}' \cdot (\bar{T}_x \cdot R \cdot \overline{P_2}) = \overline{P_1}' \cdot (\bar{\boldsymbol{T}}_\times \cdot \boldsymbol{R}) \cdot \overline{P_2} = 0$$

Essential matrix
Geometry of two cameras



$$\overline{\boldsymbol{T}}_\times = \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} \text{ for } \overline{T} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

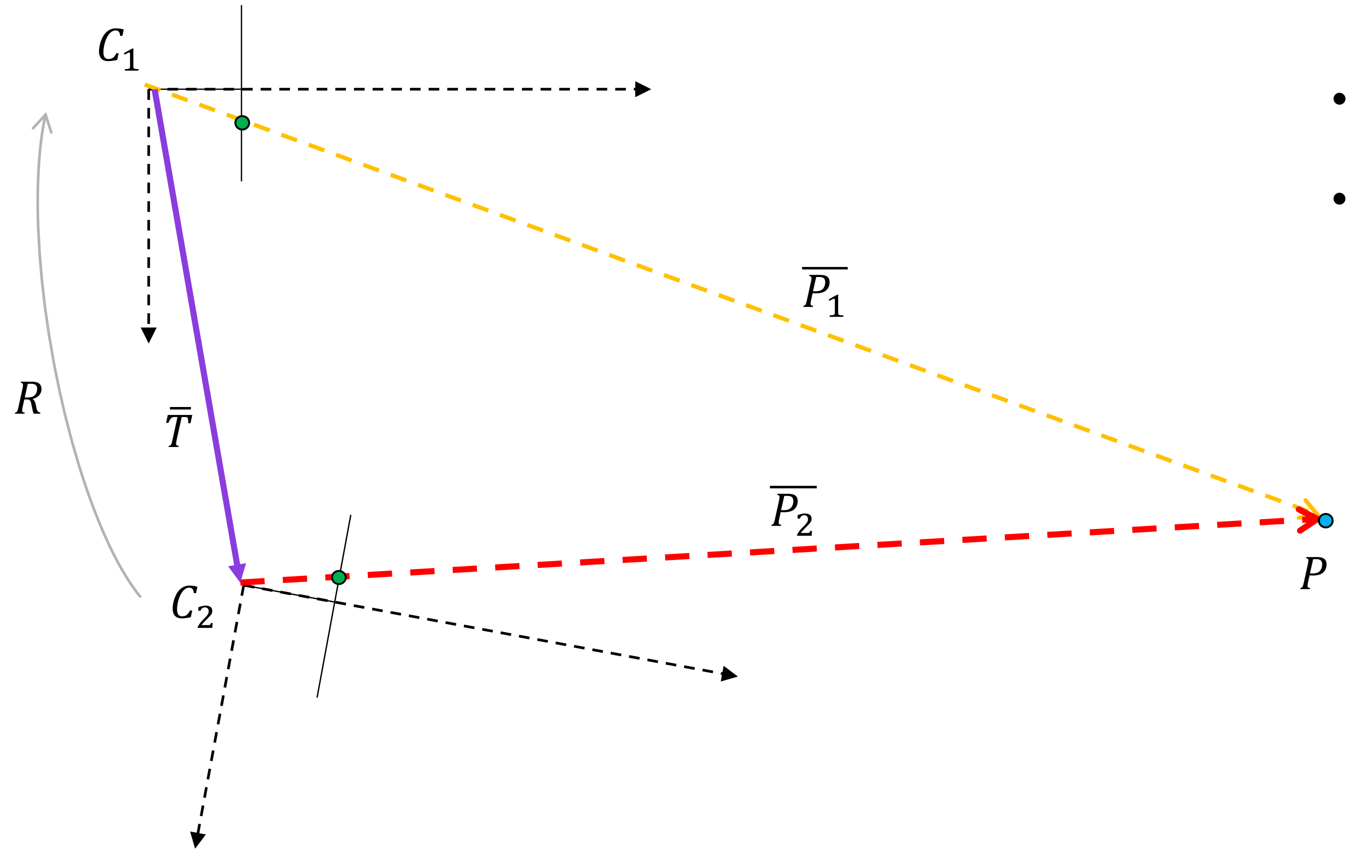
- Constraint – the magenta, orange and red vectors are co-planar
  - **Cross product of any two of them, dot product with the third is zero!**
  - In matrix form:

    $$\overline{P_1}' \cdot (\overline{\boldsymbol{T}}_\times \cdot \boldsymbol{R}) \cdot \overline{P_2} = 0$$

  - $\boldsymbol{E} = \overline{\boldsymbol{T}}_\times \cdot \boldsymbol{R}$ is the **essential matrix** relating the two cameras

Essential matrix

Geometry of two cameras

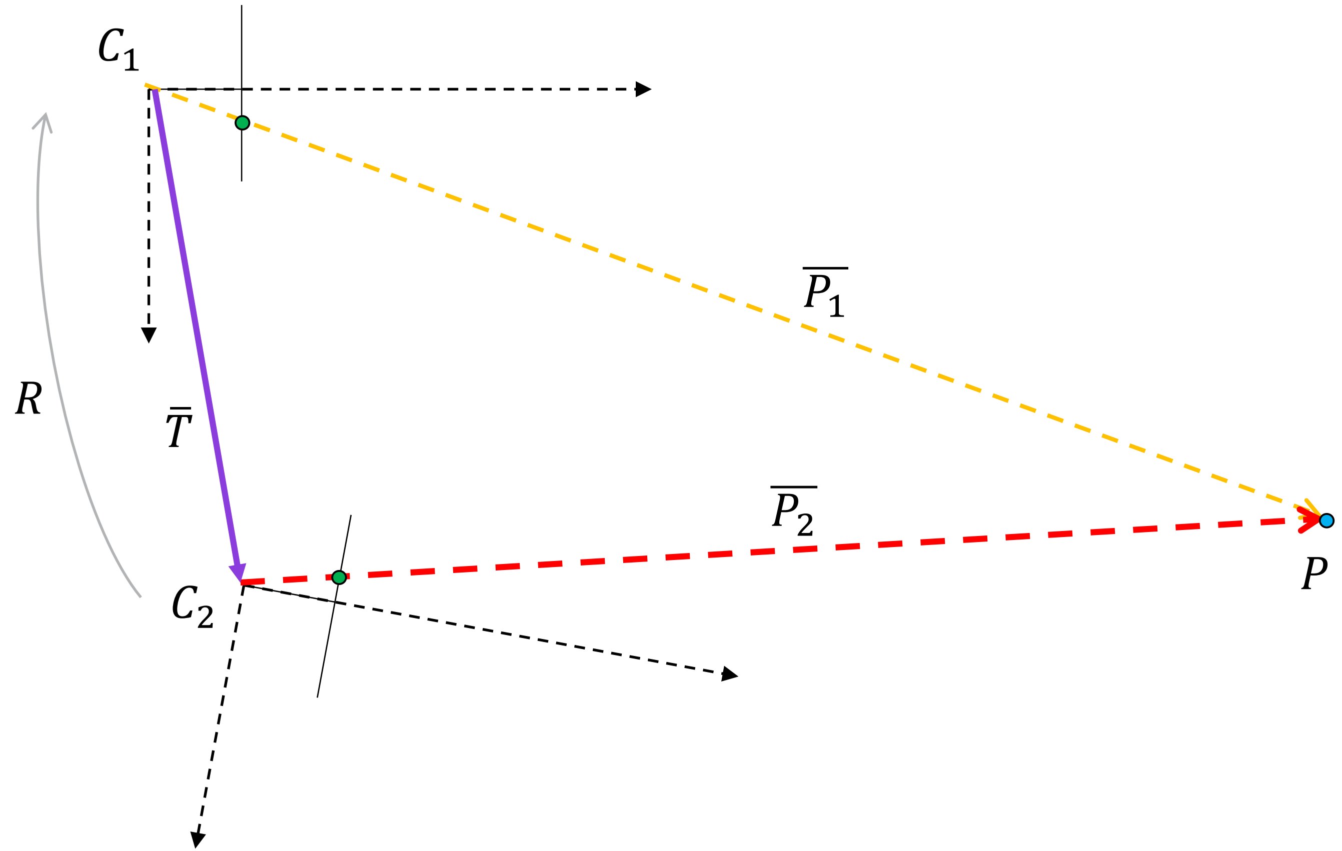


$$\overline{P_1}' \cdot (\overline{\boldsymbol{T}}_\times \cdot \boldsymbol{R}) \cdot \overline{P_2} = 0$$

- $\boldsymbol{E} = \overline{\boldsymbol{T}}_\times \cdot \boldsymbol{R}$ essential matrix
- Remarks:
  - Equation still holds by multiplying $\overline{P_1}$, $\overline{P_2}$ or $\overline{\boldsymbol{T}}_\times$ with a scalar factor
    - Example: $\boldsymbol{k_1}\overline{P_1}' \cdot (\boldsymbol{k_2}\bar{T}_\times \cdot R) \cdot \boldsymbol{k_3}\overline{P_2} = 0$
  - Thus $\overline{\boldsymbol{T}}_\times$ has only 2 degrees of freedom (DoF). Let them be t_1 and t_2
  - Essential matrix has 5 DoF:
    - $\mathrm{E}(\alpha, \beta, \gamma, t_1, t_2)$
    - $\overline{P_1}' \cdot \mathrm{E}(\alpha, \beta, \gamma, t_1, t_2) \cdot \overline{P_2} = 0$

FUNDAMENTAL MATRIX

Fundamental matrix
Geometry of two cameras



$$\overline{P_1}' \cdot (\overline{\boldsymbol{T}}_\times \cdot \boldsymbol{R}) \cdot \overline{P_2} = 0$$

$\boldsymbol{E} = \overline{\boldsymbol{T}}_\times \cdot \boldsymbol{R}$ essential matrix

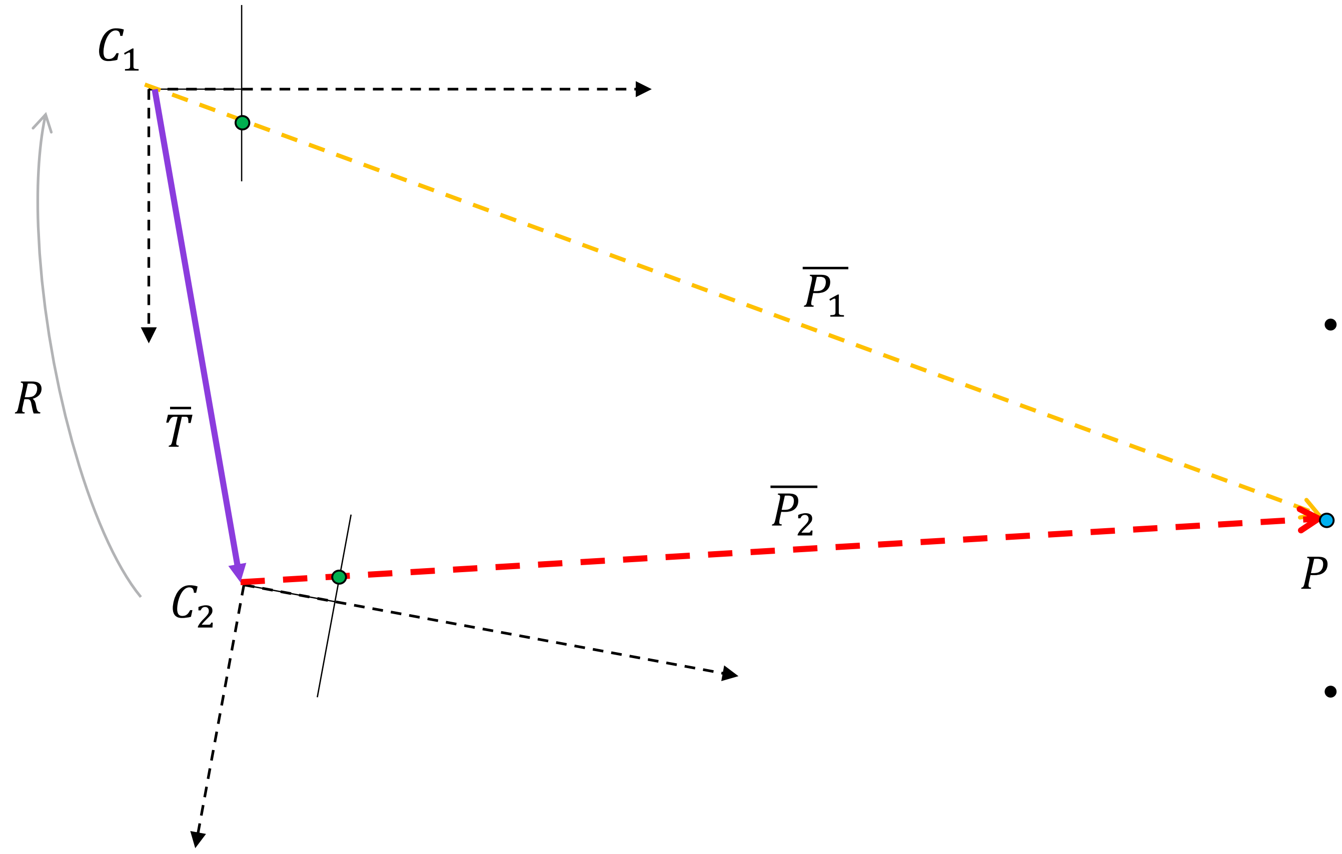
$$\overline{P_1} = \begin{bmatrix} x_{P_1} \\ y_{P_1} \\ z_{P_1} \end{bmatrix} \quad \overline{P_2} = \begin{bmatrix} x_{P_2} \\ y_{P_2} \\ z_{P_2} \end{bmatrix}$$

- Essential matrix constraint becomes:

$$\begin{bmatrix} x_{P_1} \\ y_{P_1} \\ z_{P_1} \end{bmatrix}' \cdot (\overline{\boldsymbol{T}}_\times \cdot \boldsymbol{R}) \cdot \begin{bmatrix} x_{P_2} \\ y_{P_2} \\ z_{P_2} \end{bmatrix} = 0$$

- It can be divided with z_{P_1} and z_{P_2}

Fundamental matrix
Geometry of two cameras



$$\overline{P_1}' \cdot (\overline{\boldsymbol{T}}_\times \cdot \boldsymbol{R}) \cdot \overline{P_2} = 0$$

$$\overline{P_1} = \begin{bmatrix} x_{P_1} \\ y_{P_1} \\ z_{P_1} \end{bmatrix} \quad \overline{P_2} = \begin{bmatrix} x_{P_2} \\ y_{P_2} \\ z_{P_2} \end{bmatrix}$$

- Essential matrix constraint becomes:

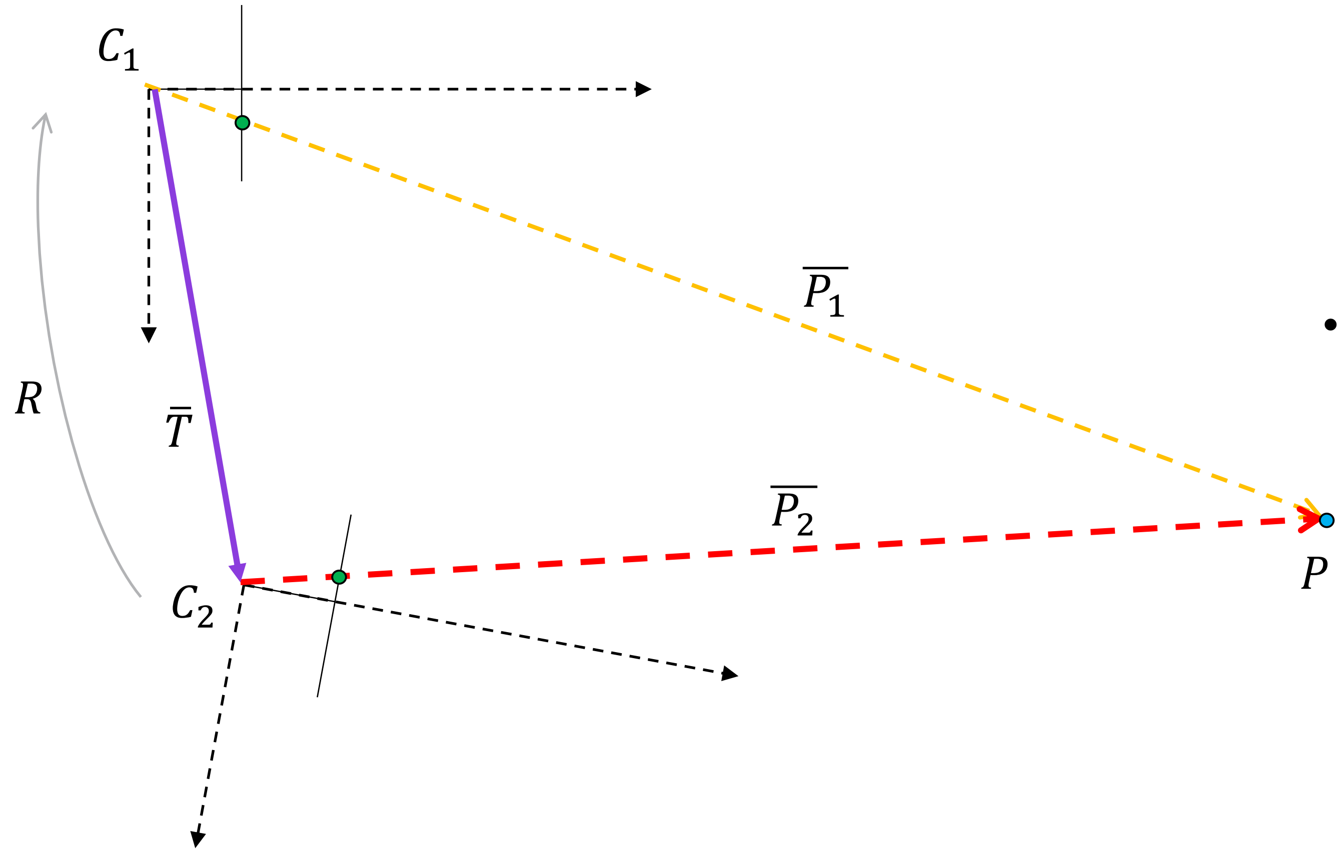
$$\begin{bmatrix} x_{P_1}/z_{P_1} \\ y_{P_1}/z_{P_1} \\ 1 \end{bmatrix}' \cdot (\overline{\boldsymbol{T}}_\times \cdot \boldsymbol{R}) \cdot \begin{bmatrix} x_{P_2}/z_{P_2} \\ y_{P_2}/z_{P_2} \\ 1 \end{bmatrix} = 0$$

- But remember the projection equation:

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K \cdot \begin{bmatrix} x/z \\ y/z \\ 1 \end{bmatrix} \qquad K^{-1} \cdot \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} x/z \\ y/z \\ 1 \end{bmatrix}$$

BOSCH

Fundamental matrix
Geometry of two cameras



$$\overline{P_1}' \cdot (\overline{\boldsymbol{T}}_\times \cdot \boldsymbol{R}) \cdot \overline{P_2} = 0$$

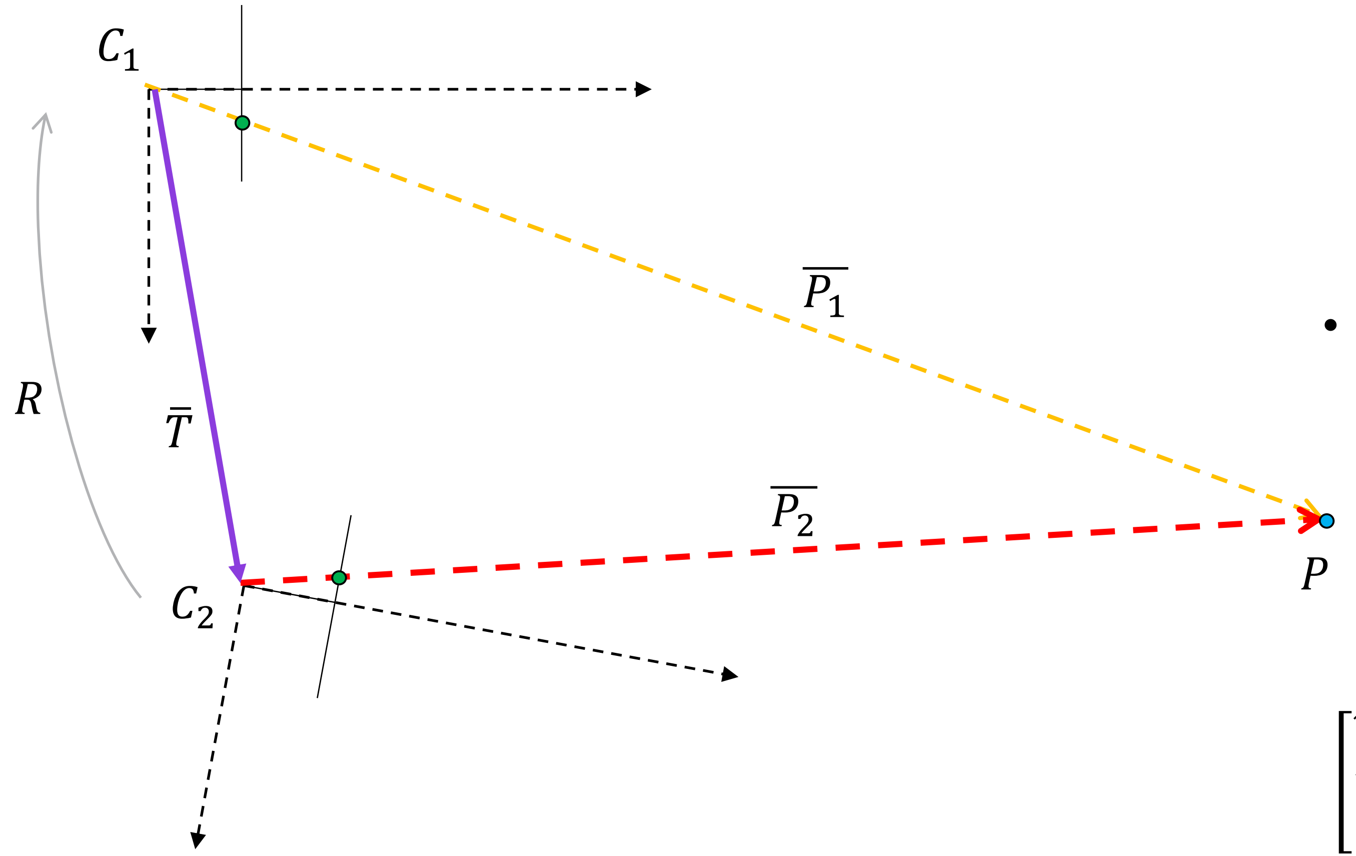
$$\overline{P_1} = \begin{bmatrix} x_{P_1} \\ y_{P_1} \\ z_{P_1} \end{bmatrix} \quad \overline{P_2} = \begin{bmatrix} x_{P_2} \\ y_{P_2} \\ z_{P_2} \end{bmatrix}$$

- Essential matrix constraint becomes:

$$\begin{bmatrix} x_{P_1}/z_{P_1} \\ y_{P_1}/z_{P_1} \\ 1 \end{bmatrix}' \cdot (\overline{\boldsymbol{T}}_\times \cdot \boldsymbol{R}) \cdot \begin{bmatrix} x_{P_2}/z_{P_2} \\ y_{P_2}/z_{P_2} \\ 1 \end{bmatrix} = 0$$

$$\left(K_1^{-1} \cdot \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}\right)' \cdot (\overline{\boldsymbol{T}}_\times \cdot \boldsymbol{R}) \cdot K_2^{-1} \cdot \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = 0$$

Fundamental matrix
Geometry of two cameras



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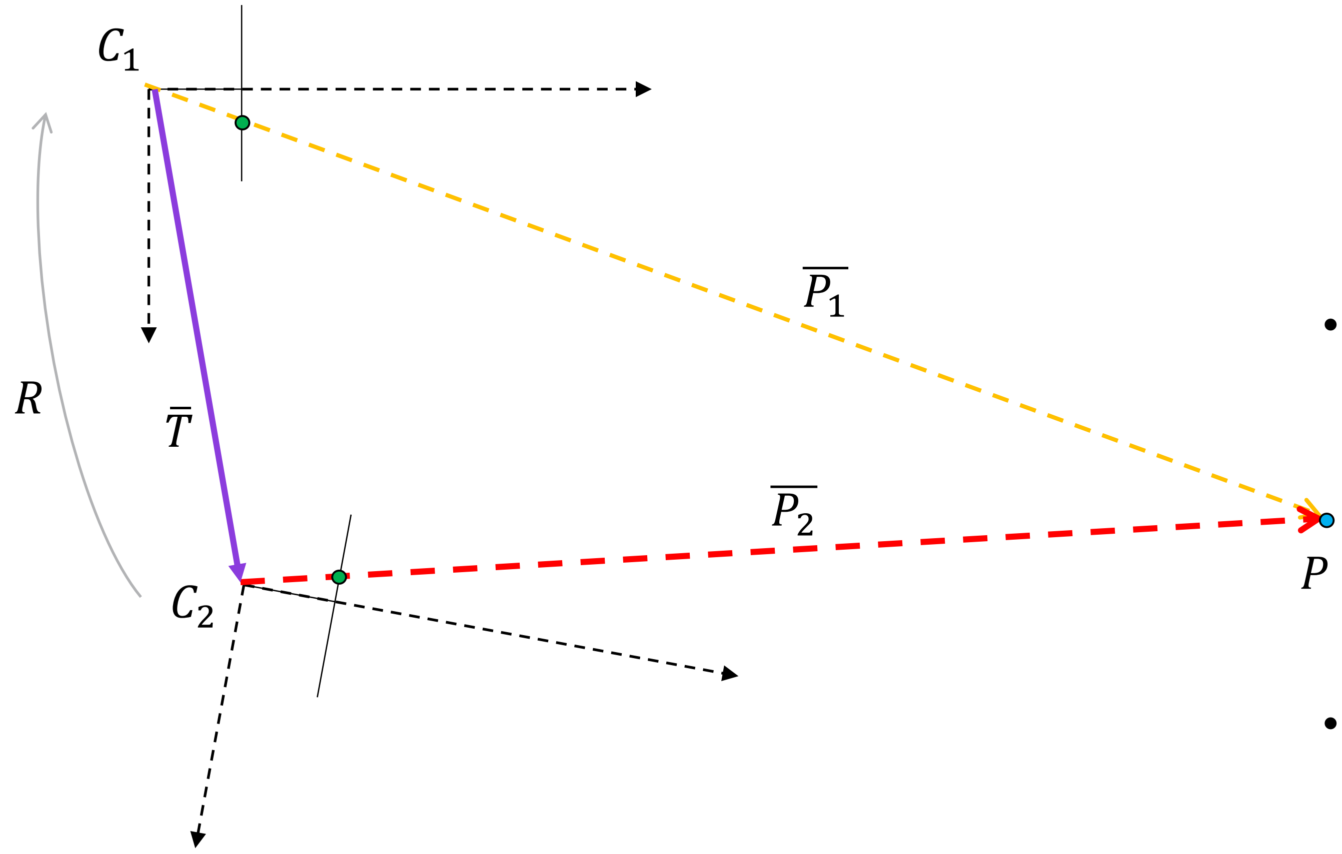
$$\begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}' \cdot K_1^{-T} \cdot (\overline{\boldsymbol{T}}_\times \cdot \boldsymbol{R}) \cdot K_2^{-1} \cdot \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = 0$$

Fundamental matrix
Geometry of two cameras



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- Fundamental matrix constraint:

$$\begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}' \boxed{\cdot K_1^{-T} \cdot (\overline{\boldsymbol{T}}_\times \cdot \boldsymbol{R}) \cdot K_2^{-1} \cdot} \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = 0$$

- Fundamental relates pixel coordinates from different views of the same scene

Fundamental matrix
Intuition

$$\begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}' \cdot K_1^{-T} \cdot (\overline{\boldsymbol{T}}_\times \cdot \boldsymbol{R}) \cdot K_2^{-1} \cdot \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = 0$$

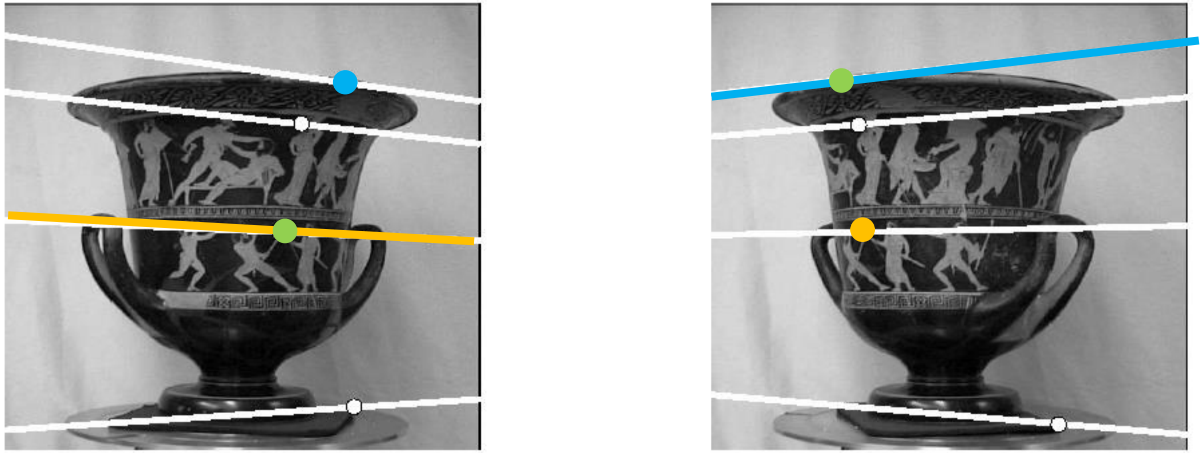
$$\begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}' \cdot F \cdot \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = 0$$

If $\begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}' \cdot F = [a \quad b \quad c]$ then the constraint

becomes $[a \quad b \quad c] \cdot \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = 0$

If $F \cdot \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix}$ then the

constraint becomes $[u_1 \quad v_1 \quad 1] \cdot \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix} = 0$



- https://www.robots.ox.ac.uk/~vgg/hzbook/hzbook2/HZepipolar.pdf

Refresh line equation: *ax+by+c=0*

$$[a \quad b \quad c] \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

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Fundamental matrix
Intuition

$$\begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}' \cdot K_1^{-T} \cdot (\overline{\boldsymbol{T}}_\times \cdot \boldsymbol{R}) \cdot K_2^{-1} \cdot \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = 0$$

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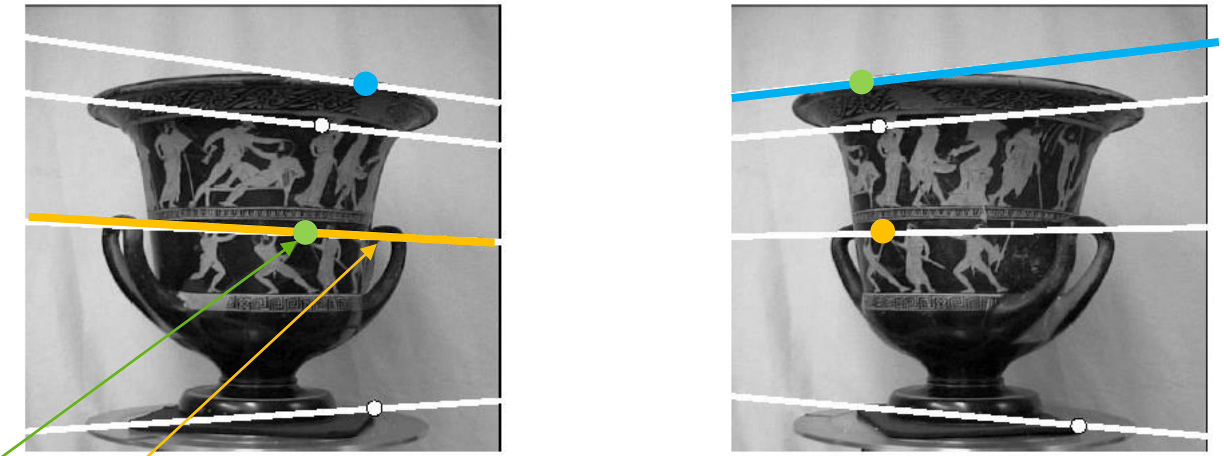


- https://www.robots.ox.ac.uk/~vgg/hzbook/hzbook2/HZepipolar.pdf

Refresh line equation: *ax+by+c=0*

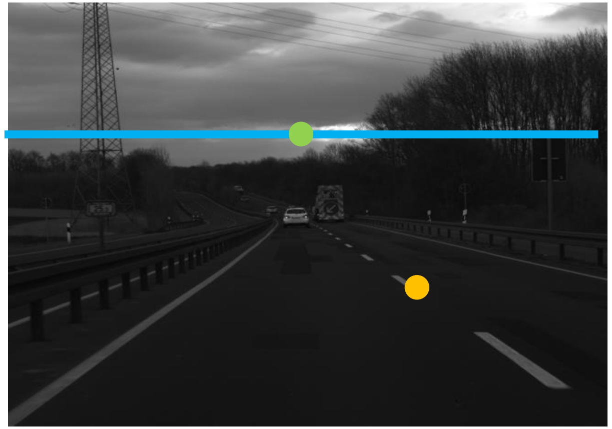
$$[a \quad b \quad c] \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

Fundamental matrix

Intuition on rectified images

$$\begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}' \cdot K_1^{-T} \cdot (\overline{\boldsymbol{T}}_\times \cdot \boldsymbol{R}) \cdot K_2^{-1} \cdot \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}' \cdot F \cdot \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = 0$$



If $\begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}' \cdot F = \begin{bmatrix} a & b & c \end{bmatrix}$ then the constraint

becomes $\begin{bmatrix} a & b & c \end{bmatrix} \cdot \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = 0$

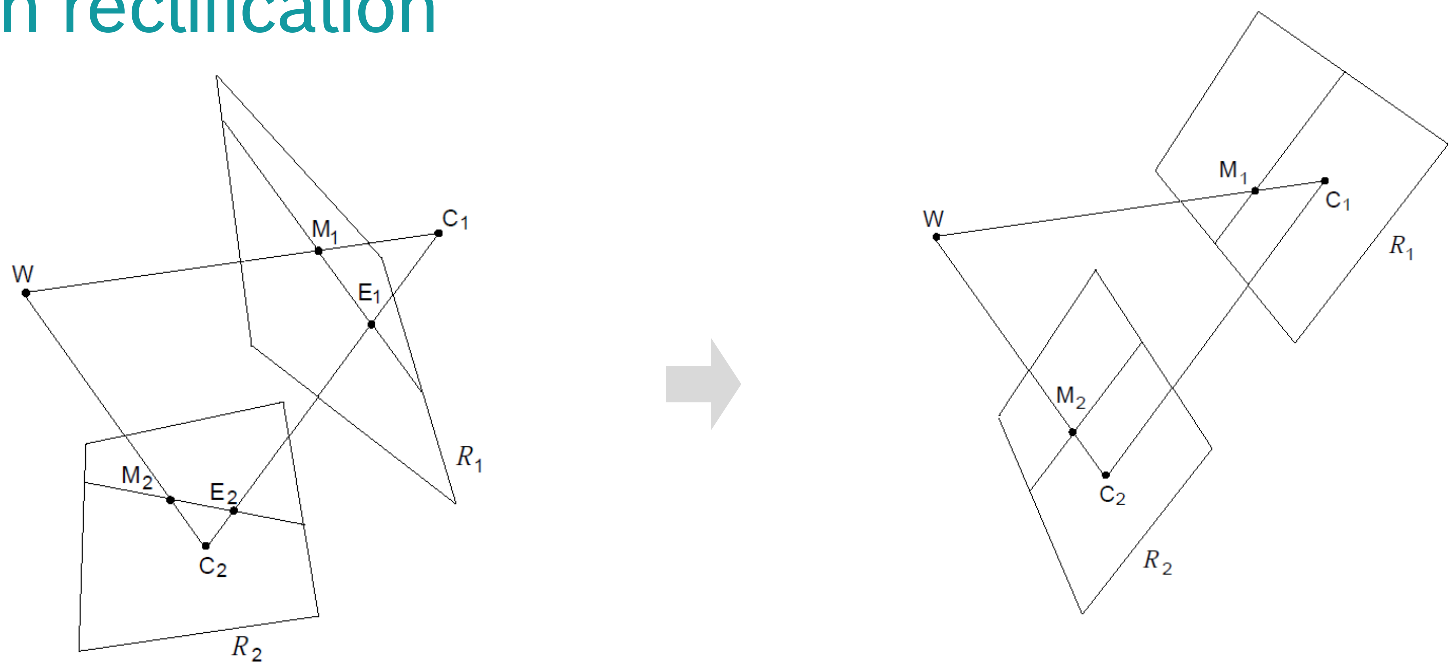
If $F \cdot \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix}$ then the

constraint becomes $\begin{bmatrix} u_1 & v_1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix} = 0$

Refresh line equation: *ax+by+c=0*

$$\begin{bmatrix} a & b & c \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

Fundamental matrix
Intuition on rectification



A compact algorithm for rectification of stereo pairs

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Received: 25 February 1999 / Accepted: 2 March 2000

ESSENTIAL MATRIX ESTIMATION

Essential matrix estimation
Mathematica model

$$\begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}' \cdot K_1^{-T} \cdot (\overline{\boldsymbol{T}}_\times \cdot \boldsymbol{R}) \cdot K_2^{-1} \cdot \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = 0$$

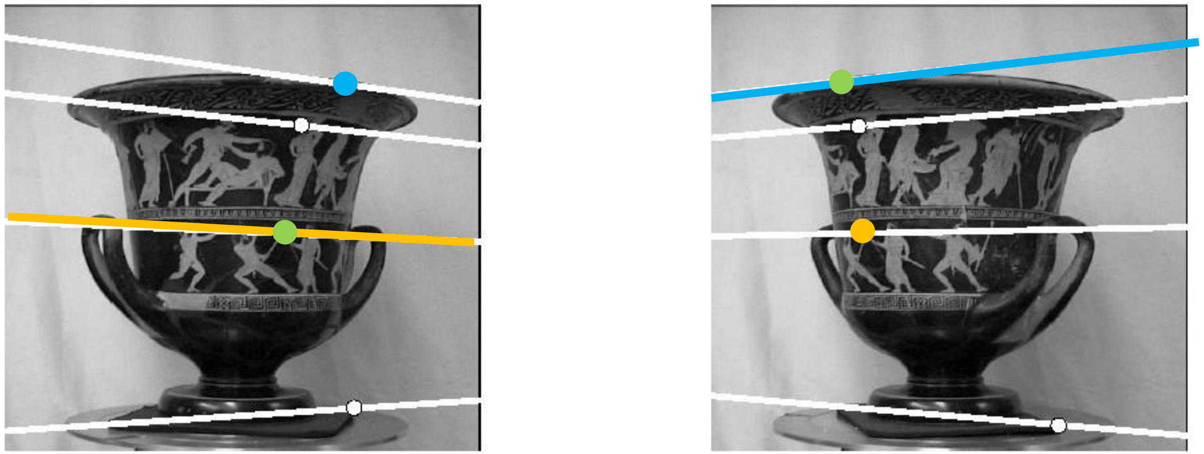
For **more points** we have:

$$\begin{bmatrix} u_{1,i} \\ v_{1,i} \\ 1 \end{bmatrix}' \cdot K_1^{-T} \cdot \mathrm{E}(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{t_1}, \boldsymbol{t_2}) \cdot K_2^{-1} \cdot \begin{bmatrix} u_{2,i} \\ v_{2,i} \\ 1 \end{bmatrix} = 0$$

Let:

$$r_i(\alpha, \beta, \gamma, t_1, t_2) = \begin{bmatrix} u_{1,i} \\ v_{1,i} \\ 1 \end{bmatrix}' \cdot K_1^{-T} \cdot \mathrm{E}(\alpha, \beta, \gamma, t_1, t_2) \cdot K_2^{-1} \cdot \begin{bmatrix} u_{2,i} \\ v_{2,i} \\ 1 \end{bmatrix}$$

$r_i(\alpha, \beta, \gamma, t_1, t_2)$ - means how good the essential matrix constraint is respected given that point correspondences have noise (i.e. how close is green point to orange line)



- https://www.robots.ox.ac.uk/~vgg/hzbook/hzbook2/HZepipolar.pdf

If $F \cdot \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix}$ then the constraint becomes $\begin{bmatrix} u_1 & v_1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix} = 0$

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Essential matrix estimation
Mathematica model

For **more points** we have:

$$\begin{bmatrix} u_{1,i} \\ v_{1,i} \\ 1 \end{bmatrix}' \cdot K_1^{-T} \cdot \mathrm{E}(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{t_1}, \boldsymbol{t_2}) \cdot K_2^{-1} \cdot \begin{bmatrix} u_{2,i} \\ v_{2,i} \\ 1 \end{bmatrix} = 0$$

Let:

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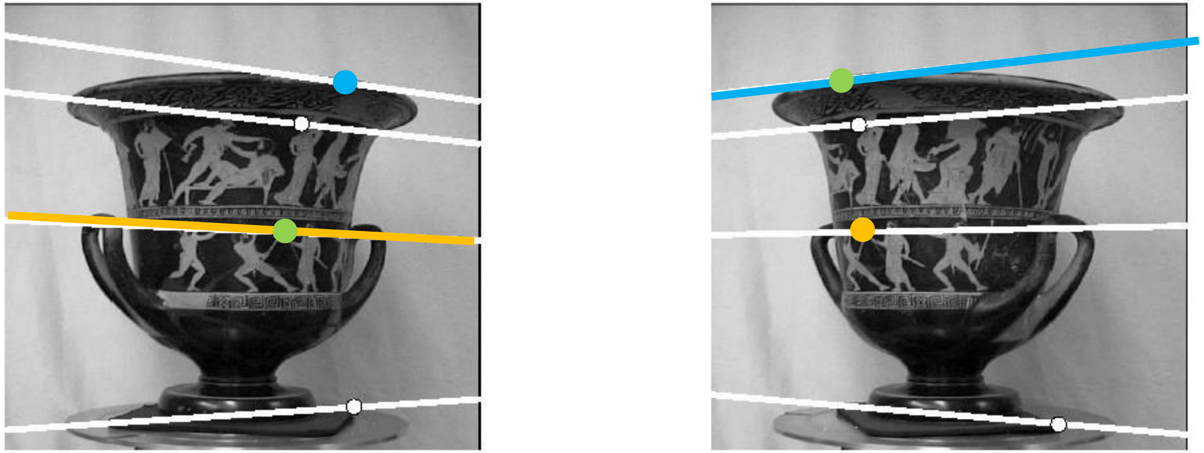
$$= \begin{bmatrix} u_{1,i} \\ v_{1,i} \\ 1 \end{bmatrix}' \cdot K_1^{-T} \cdot \mathrm{E}(\alpha, \beta, \gamma, t_1, t_2) \cdot K_2^{-1} \cdot \begin{bmatrix} u_{2,i} \\ v_{2,i} \\ 1 \end{bmatrix}$$



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Find $\alpha, \beta, \gamma, t_1, t_2$ by doing:

$$argmin \sum_i \left(r_i(\alpha, \beta, \gamma, t_1, t_2)\right)^2$$

If $F \cdot \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix}$ then the constraint becomes $\begin{bmatrix} u_1 & v_1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix} = 0$

Essential matrix estimation
Mathematica model

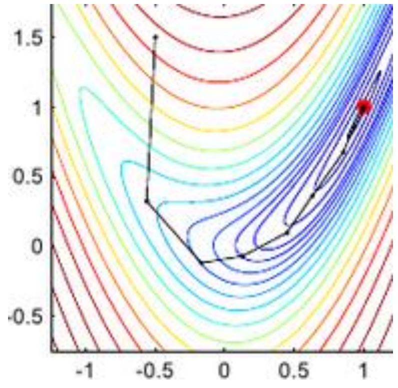
Find $\alpha, \beta, \gamma, t_1, t_2$ by doing:

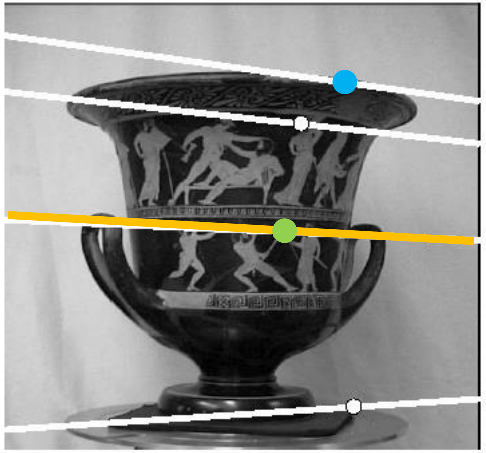
$$argmin \sum_i \left(r_i(\alpha, \beta, \gamma, t_1, t_2)\right)^2$$

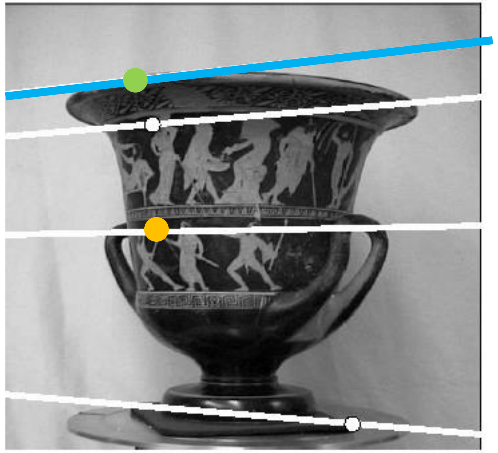
Gauss-Newton (see your numerical calculus course)

https://en.wikipedia.org/wiki/Gauss%E2%80%93Newton_algorithm

Basic idea: find solution for the minimum iteratively







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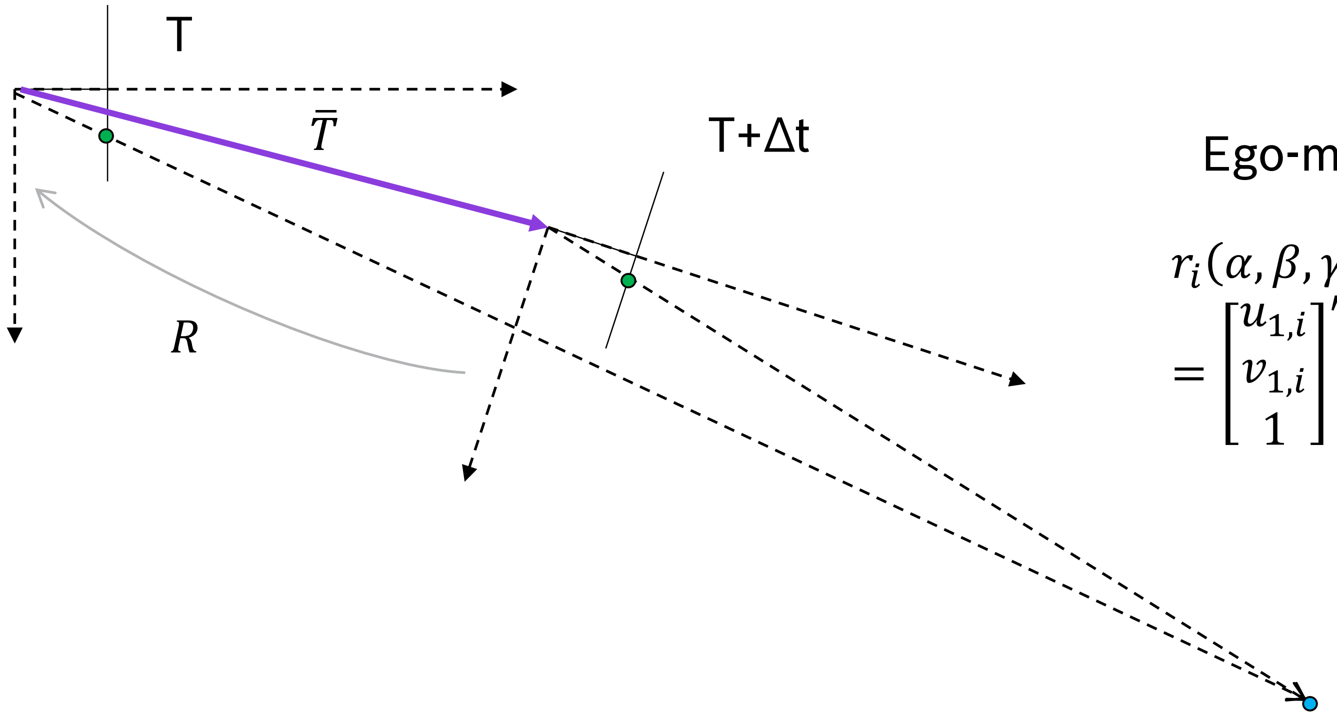
Review of the last course and goals for today
Goal for today – geometry of a mono system



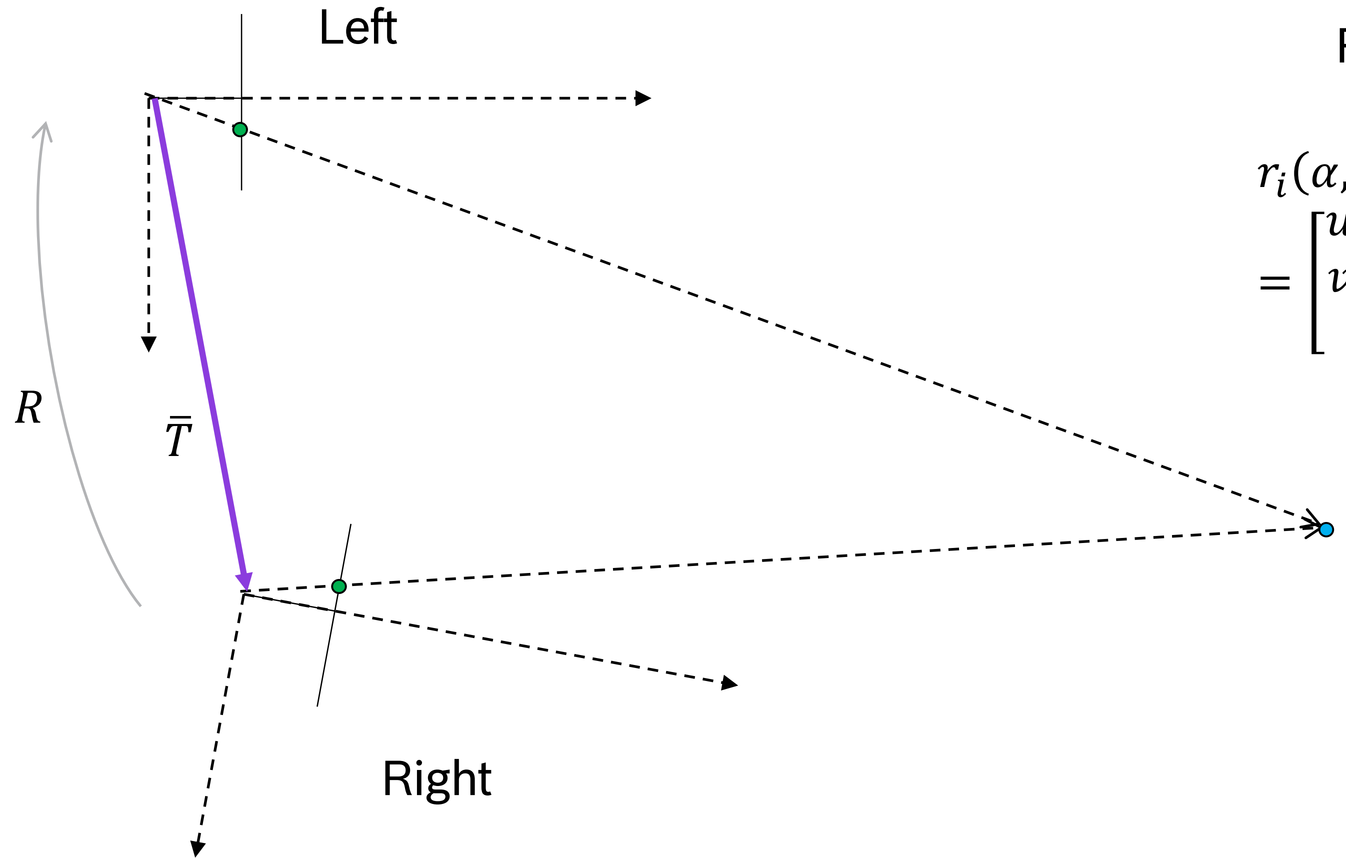
Ego-motion estimation for mono

$$r_i(\alpha,\beta,\gamma,t_1,t_2) = \begin{bmatrix} u_{1,i} \\ v_{1,i} \\ 1 \end{bmatrix}' \cdot K_1^{-T} \cdot \mathrm{E}(\alpha,\beta,\gamma,t_1,t_2) \cdot K_2^{-1} \cdot \begin{bmatrix} u_{2,i} \\ v_{2,i} \\ 1 \end{bmatrix}$$

$$argmin \sum_i \left(r_i(\alpha,\beta,\gamma,t_1,t_2)\right)^2$$

Review of the last course and goals for today

Goal for today - geometry of a stereo system



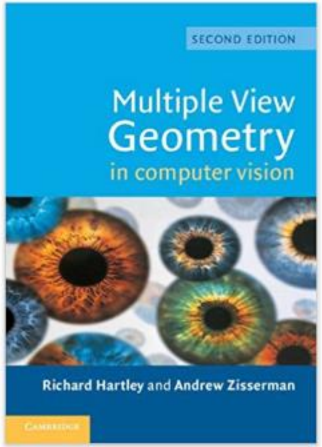
Relative pose estimation for stereo

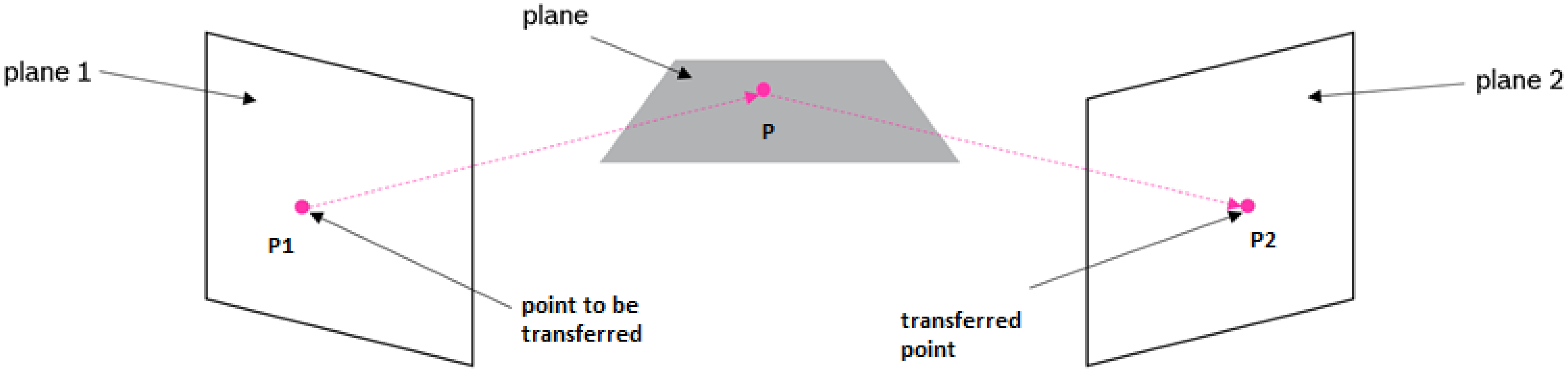
$$r_i(\alpha,\beta,\gamma,t_1,t_2) = \begin{bmatrix} u_{1,i} \\ v_{1,i} \\ 1 \end{bmatrix}' \cdot K_1^{-T} \cdot \mathrm{E}(\alpha,\beta,\gamma,t_1,t_2) \cdot K_2^{-1} \cdot \begin{bmatrix} u_{2,i} \\ v_{2,i} \\ 1 \end{bmatrix}$$

$$argmin \sum_i \left(r_i(\alpha,\beta,\gamma,t_1,t_2)\right)^2$$

PROJECTIVE GEOMETRY
References

- https://www.amazon.com/Multiple-View-Geometry-Computer-Vision/dp/0521540518
- http://robotics.stanford.edu/~birch/projective/
- http://robotics.stanford.edu/~birch/projective/projective.pdf
- https://www.robots.ox.ac.uk/~vgg/hzbook/hzbook2/HZepipolar.pdf
- http://mathworld.wolfram.com/ProjectiveGeometry.html





$$P_1 = H_1 \cdot P$$

$$P_2 = H_2 \cdot P$$

$$H_{image} = H_2 \cdot H_1^{-1}$$

$$P_2 = H_{image} \cdot P_1$$

H_{image}, H_1, H_2 **– homography matrices**

Transfer via plane

For details see the references

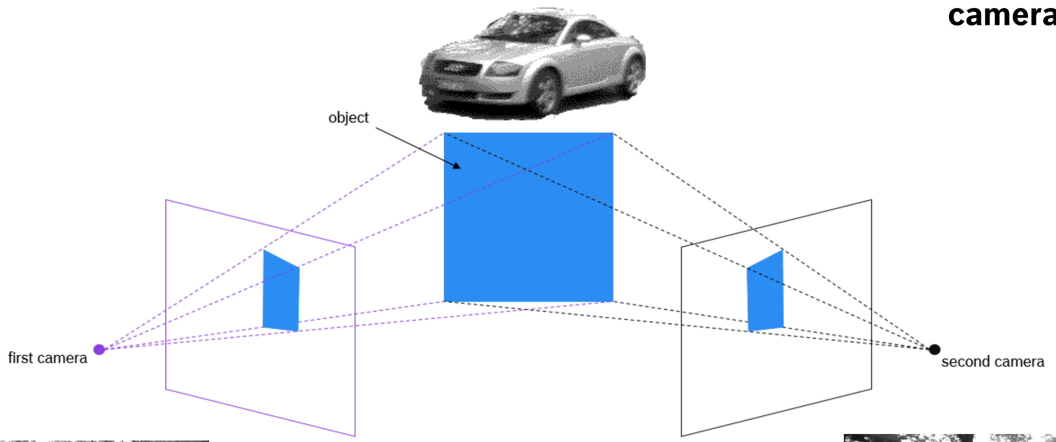
CONCLUSIONS

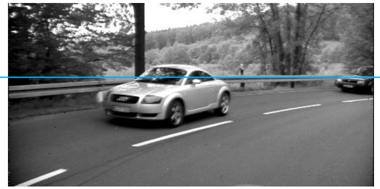
Conclusions
Conclusions

- Essential matrix as key part for
  - Rectification
  - Ego-motion estimation
- Same theory applicable for mono & stereo

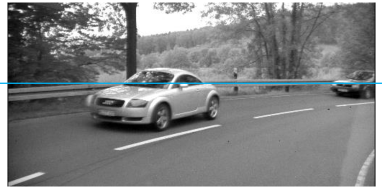
stereo camera

mono camera





Rectification for 3D reconstruction => corresponding image points are on the same scan-line



Thank you for your attention!