BASIC PRINCIPLES OF STEERING

Agenda

1. Vehicle Cornering
2. Steering Assistance Torque
3. Motor Torque Characteristics
4. Basic Steering Functions
5. Lateral Vehicle Dynamics

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Steering Handbook, Editors: Manfred Harrer, Peter Pfeffer
Springer International Publishing Switzerland 2017
D. Schramm et al., Vehicle Dynamics, DOI: 10.1007/978-3-540-36045-2_10,
Springer-Verlag Berlin Heidelberg 2014

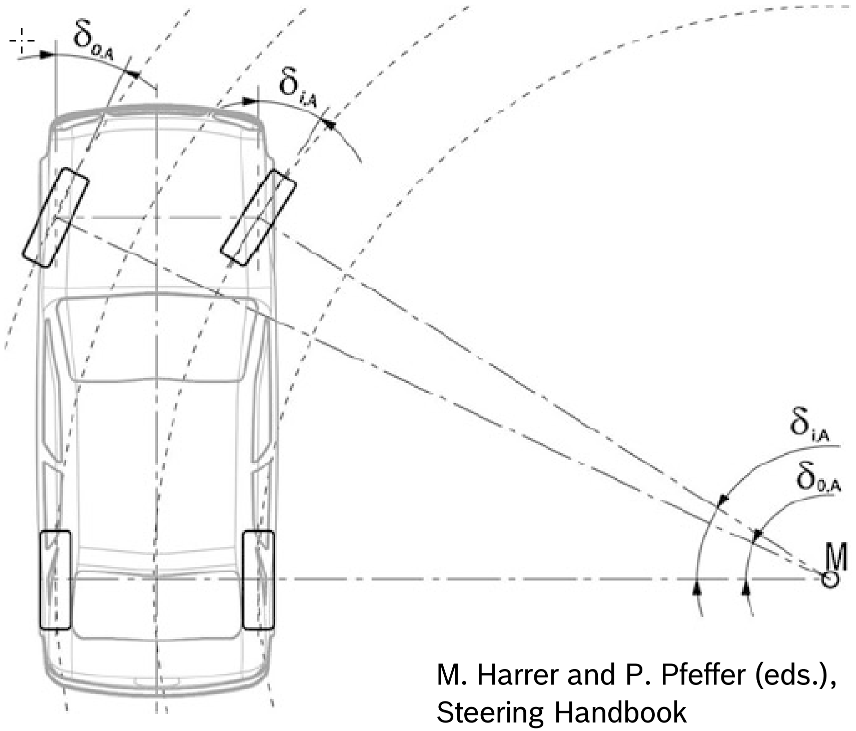
VEHICLE CORNERING

Vehicle Cornering
Slow Cornering – vehicle turning without any lateral force

- all tires have to be oriented tangentially to concentric arcs (centre plane of the wheel)
- the instantaneous center of the car M will be located on the rear axle

$\delta_{i,A}$ - the steering wheel angles inside

$\delta_{o,A}$ - the steering wheel angles outside



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Vehicle Cornering
Slip angle

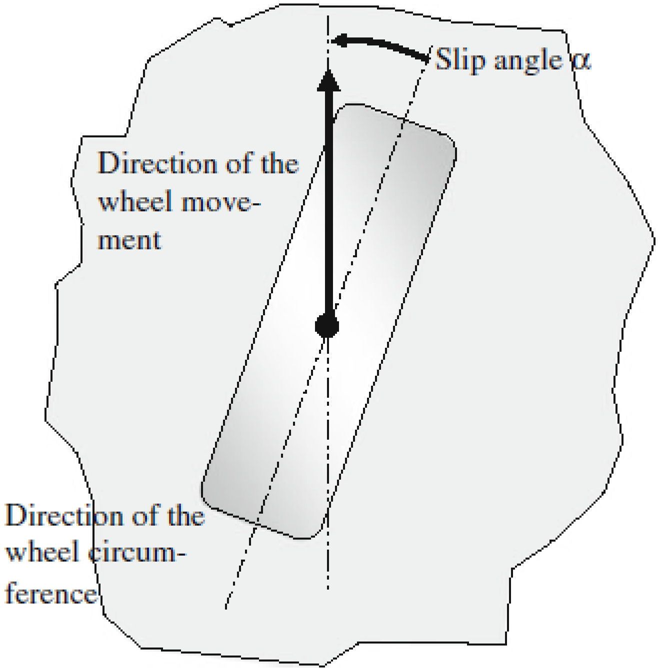
- the angle between the direction of the wheel circumference and the direction of the movement of the wheel

$\alpha_{F,i}$ - front slip angle inside

$\alpha_{F,o}$ - front slip angle outside

$\alpha_{R,i}$ - rear slip angle inside

$\alpha_{R,o}$ - rear slip angle outside



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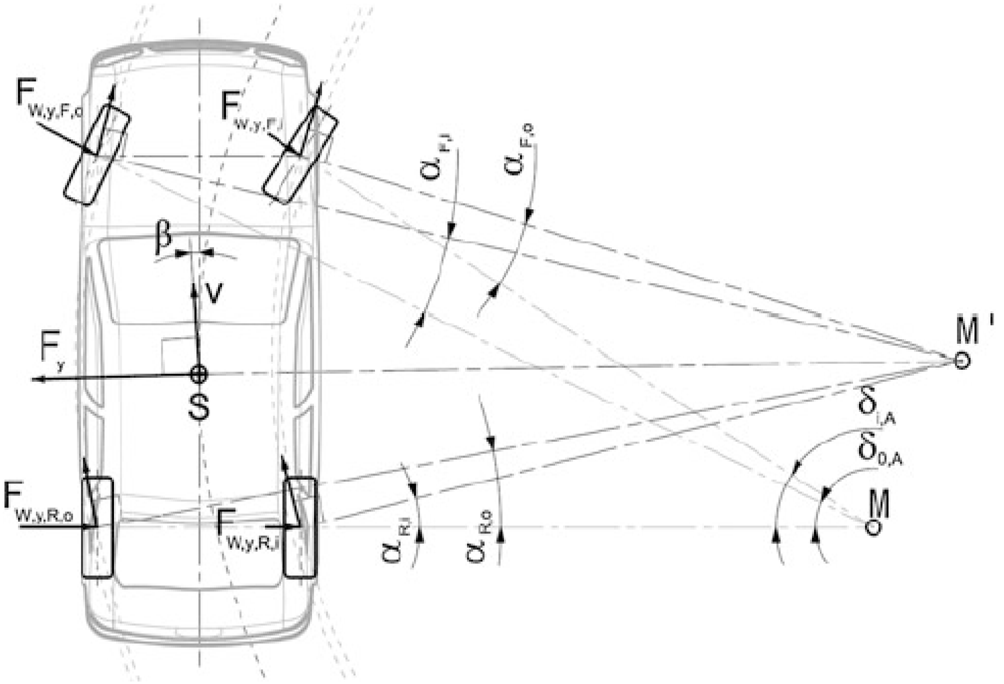
Vehicle Cornering

Fast Cornering – vehicle turning with lateral acceleration

- lateral forces occurs at front and rear wheels
- the center on which the car is cornering results from the intersection of the perpendicular line to the actual path of the moving wheels
- the actual instantaneous center of the car M' moves towards the front axle



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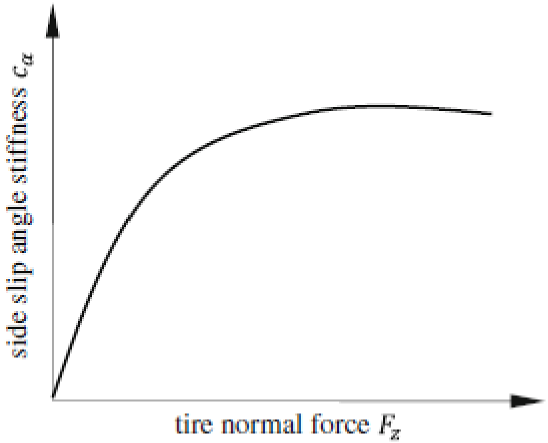
Vehicle Cornering
Lateral force of the tire (F_Y)

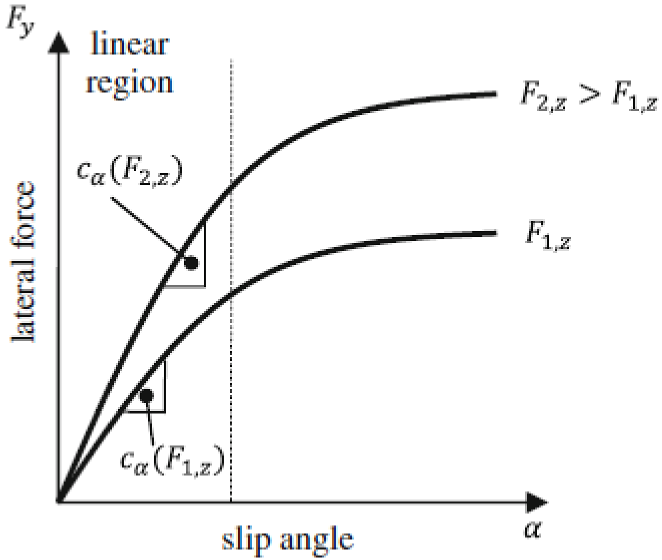
- lateral forces of the tire are produced by the lateral deformation of the rubber

$$F_Y = C_\alpha \alpha$$

C_α - cornering stiffness

α - slip angle





- linear operating range of the tire - lateral acceleration up to 3-4 m/s^2 (on dry road)
- constant wheel load

D. Schramm, M. Hiller, R. Bardini
Vehicle Dynamics Modeling and Simulation

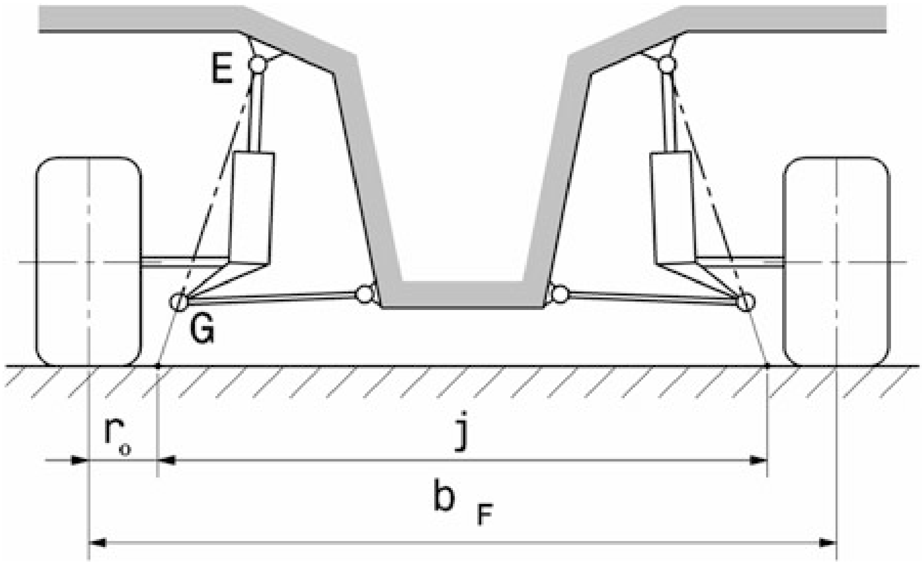
Vehicle Cornering
Characteristics of the steering geometry

EG – steering axis (kingpin axis)

j – distance of the steering axes on the road

b_F – front track

r_0 – scrub radius



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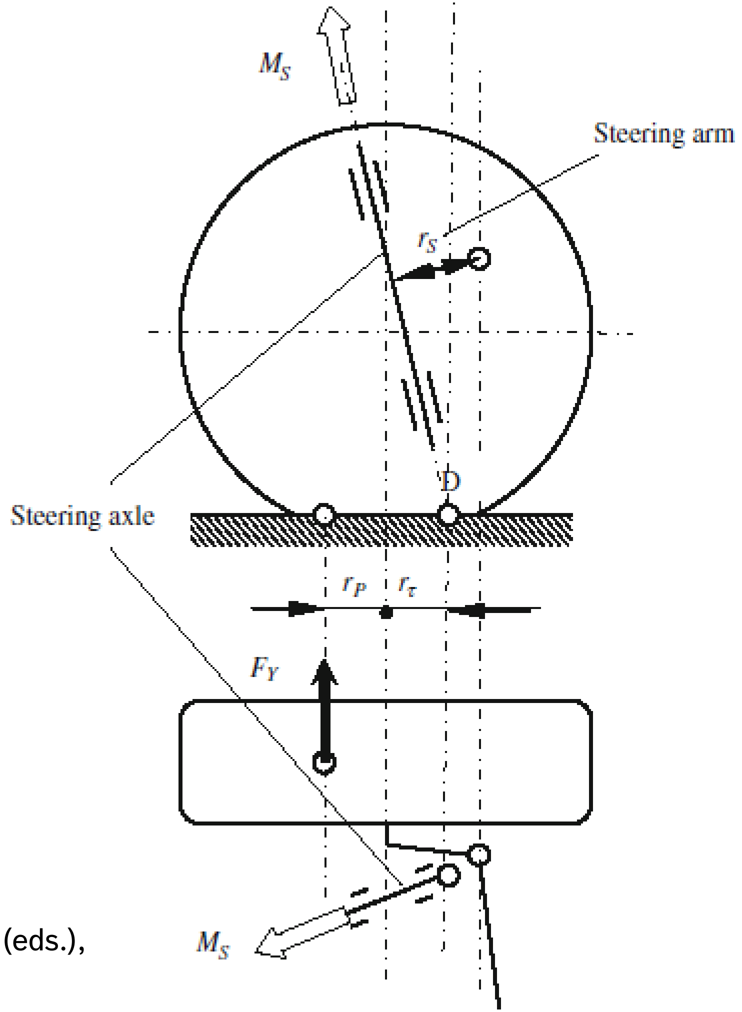
Vehicle Cornering
Steering torque (M_S)

- total steering torque around of the steering axle of the front wheels

$$M_S = F_Y(r_\tau + r_P)$$

r_τ - mechanical trail (distance between the centre of contact patch and the point where the steering axis intersect the ground)

r_P - pneumatic trail (offset between the centre of contact patch and the effective acting point of lateral force)



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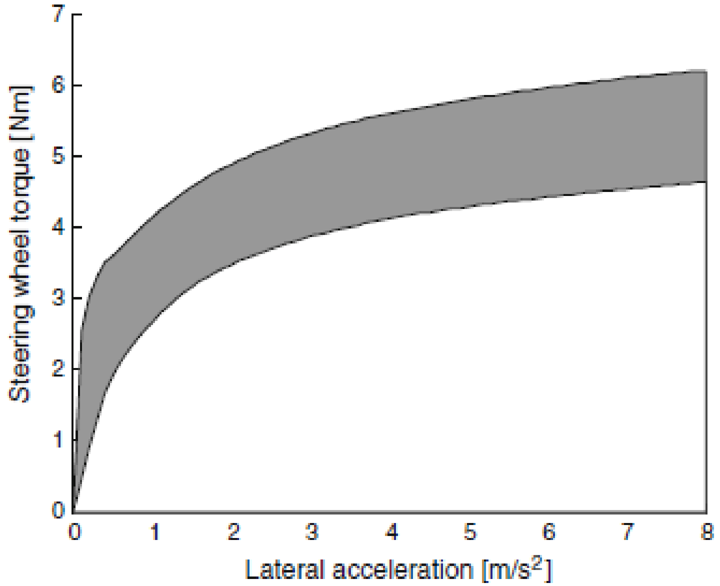
STEERING ASSISTANCE TORQUE

Steering Assistance Torque
Requirements on steering wheel torque

- range of measured steering wheel torques for various sports cars (figure)

- Input
  - steering wheel torque curve in relation to the lateral acceleration of the vehicle

- Output
  - calculation of the vehicle torque for stationary cornering



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Steering Assistance Torque
Steering assitance ratio

$$A_S = \frac{M_S}{i_S M_H}$$

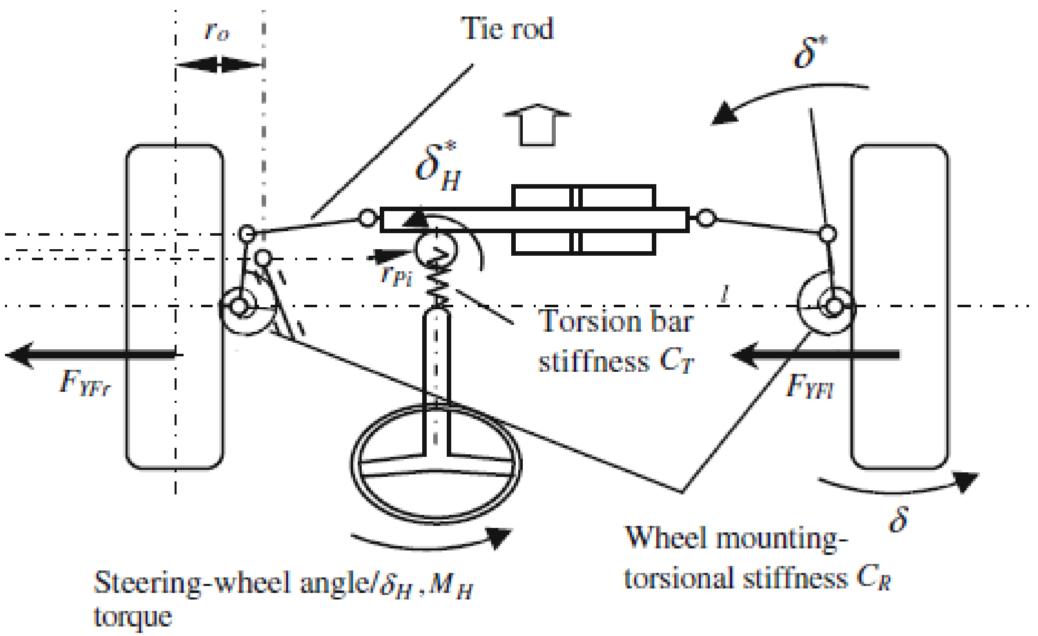
$$M_H = C_T(\delta_H - \delta_H^*)$$

$$M_S = C_R(\delta^* - \delta)$$

- no elasticity between pinion and steering arm

$$\delta_H^* = \delta^* i_S$$

$$i_S = \frac{\delta_H^*}{\delta^*} = \frac{\delta_H - \frac{M_H}{C_T}}{\delta + \frac{M_S}{C_R}}$$



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i_S	- steering ratio
δ_H	- steering wheel angle
δ_H^*	- pinion angle
δ	- steering angle
δ^*	- steering arm angle
M_H	- steering wheel torque
C_T	- torsion bar stiffness
C_R	- axle support stiffness (elasticity of the tie road and the axle mounting)

Steering Assistance Torque
Steering wheel angle and steering angle

$$\delta_H = \delta i_S + M_S i_S \left(\frac{1}{C_R} + \frac{1}{C_T i_S^2 A_S} \right)$$

$$\delta_H = \delta i_S + \frac{F_Y r i_S}{C_S}$$

- for steering without distortion of the torsion bar or for infinite total steering stiffness

$$\delta_H = \delta i_S$$

- effective cornering stiffness - steering stiffness output on cornering stiffness

$$\frac{1}{C_{\alpha,eff}} = \frac{1}{C_\alpha} + \frac{r}{C_S}$$

C_S - total steering stiffness

$C_{\alpha,eff}$ - effective cornering stiffness

$r = r_\tau + r_P$ - total trail

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Steering Assistance Torque
Steering wheel torque (M_H)

- optimum steering reinforcement increase linearly to the lateral acceleration

$$A_S = C_A(D_A + K_A a_Y)$$

- constant total trail
- constant steering ratio

- steering wheel torque

$$M_H = \frac{M_S}{i_S A_S} = \frac{F_Y(r_\tau + r_P)}{i_S A_S} = \frac{m_F r}{i_S A_S} a_Y$$

$$M_H = \frac{C_A}{A_S} a_Y = \frac{1}{D_A + K_A a_Y} a_Y$$

$$C_A = \frac{m_F r}{i_S}$$

$F_Y = m_F a_Y$ - lateral force at the front axle

m_F - mas of the vehicle at the front axle

a_Y - lateral acceleration

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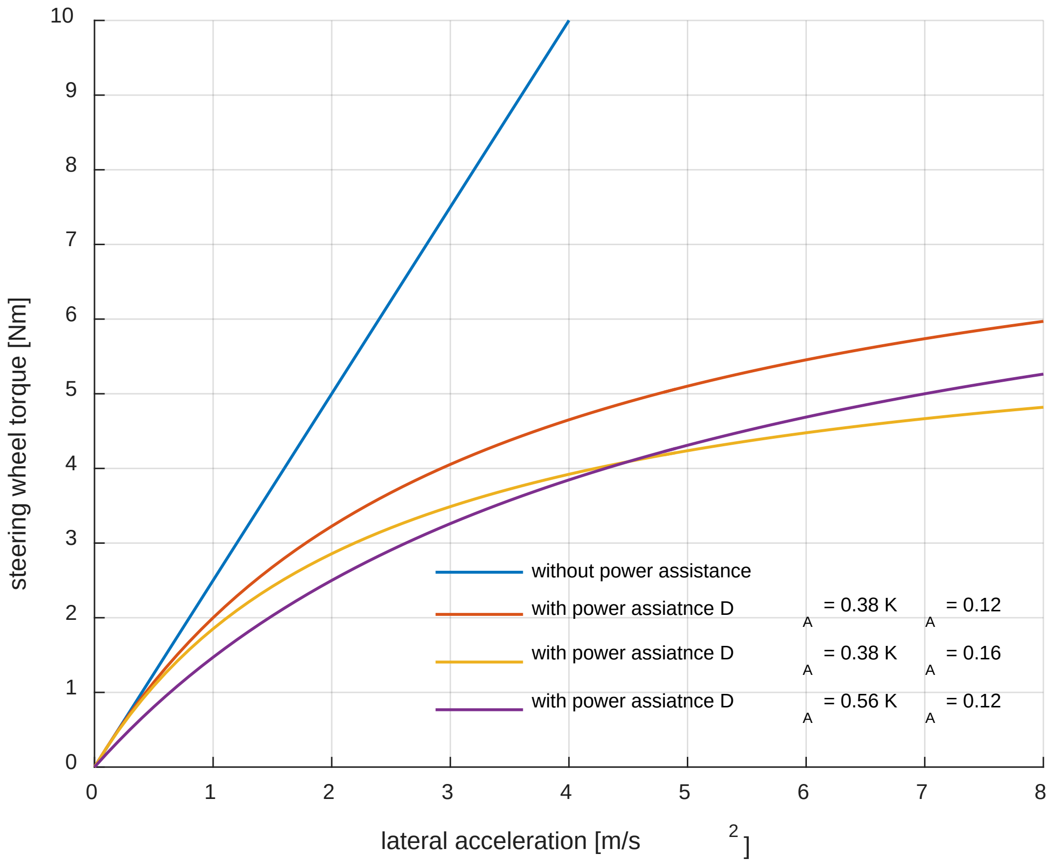
Steering Assistance Torque
Steering wheel torque (M_H)

- the steering wheel torque rises degressively

$$M_H = \frac{1}{\frac{D_A}{a_Y} + K_A}$$

- for vehicles without power steering $A_S = 1$
  - constant lateral acceleration gradient of the steering wheel torque

$$M_H = C_A a_Y$$



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Steering Assistance Torque

Steering assistance torque (M_A)

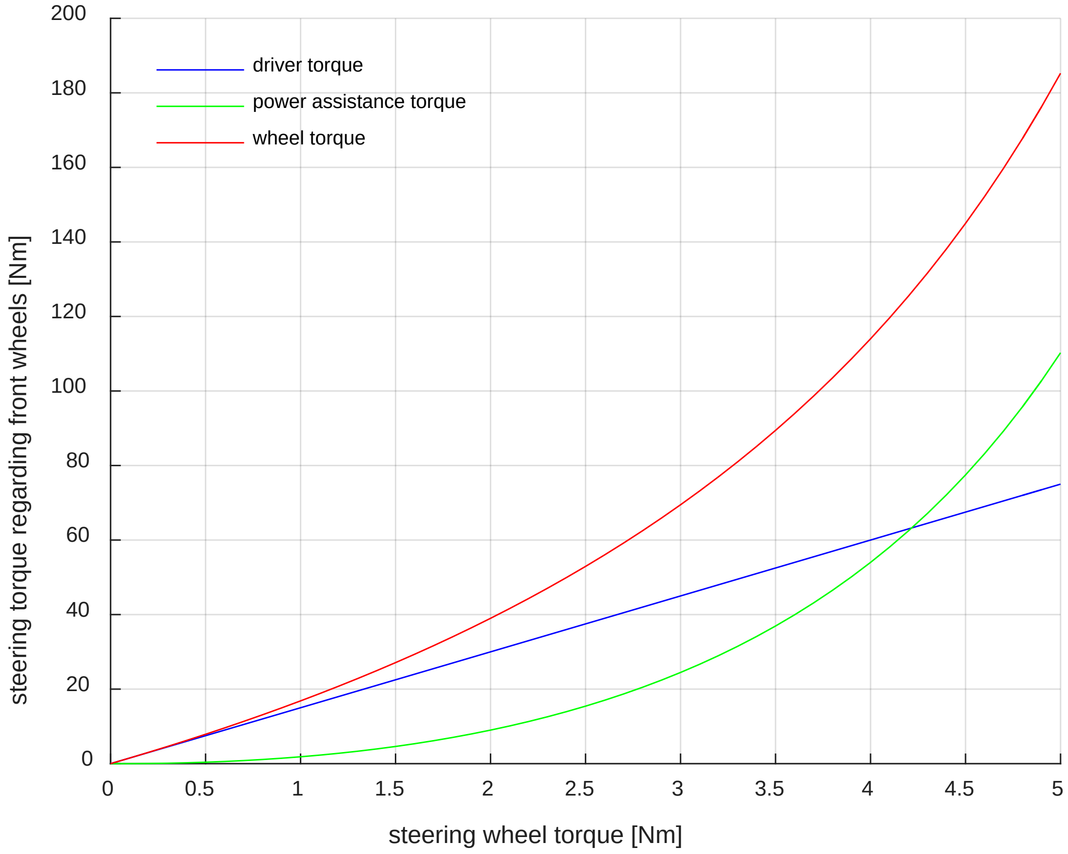
- steering assistance torque is the difference between the steering torque at the wheels and the torque applied by the driver

$$M_A = M_S - M_H i_S$$

$$\begin{aligned} M_A &= M_H i_S A_S - M_H i_S \\ &= M_H \left(m_F r (D_A + K_A a_Y) - i_S\right) \\ &= \frac{a_Y (m_F r D_A + m_F r K_A a_Y - i_S)}{D_A + K_A a_Y} \\ &= \frac{M_H (m_F r D_A + i_S K_A M_H - i_S)}{1 - K_A M_H} \end{aligned}$$

- parameterization of the steering assistance torque based on gradient factor D_A and degressivity factor K_A in relation to the steering wheel torque or lateral acceleration

Steering Assistance Torque
Steering torque regarding front wheels



MOTOR TORQUE CHARACTERISTICS

Motor Torque Characteristics
Steering rack force on EPS

- EPSc; EPSp
  - power assist unit is placed on the steering column
  - power assist torque is transmitted to the steering column
  - steering wheel torque and power assist torque is transferred to rack force by a steering gear

- EPSdp; EPSapa; EPSrc
  - power assist unit is placed on the rack
  - power assist torque is transmitted to the rack by a second pinion, belt, ball screw

- Steering gear ratio
  - the ratio between rack path and steering wheel angle
  - displacement of the rack during one turn of the pinion

Motor Torque Characteristics
Steering rack force on EPS

- steering rack force on EPSc and EPSp

$$F_r v_r = \left(M_H + \frac{\omega_P}{\omega_H} M_P \right) \omega_H$$

$$\omega_H = \frac{2\pi}{i_G} v_r$$

$$F_r = \frac{2\pi}{i_G} (M_H + i_P M_P)$$

$$\omega_P = i_P \omega_H$$

- steering rack force on EPSdp, EPSapa, EPScr

$$F_r \cdot v_r = M_H \cdot \omega_H + M_{dP} \cdot \omega_{dP}$$

$$F_r = \frac{2\pi}{i_G} M_H + i_{dP} M_{dP}$$

$$\omega_{dP} = i_{dP} v_r$$

i_G - steering gear ratio [m/rev]

i_P - motor pinion ratio [-]

i_{dP} - rack pinion ratio [rad/m]

ω_H - steering wheel velocity [rad/s]

ω_P - motor velocity (column pinion) [rad/s]

ω_{dP} - motor velocity (rack pinion) [rad/s]

F_r - rack force [N]

v_r - rack velocity [m/s]

Motor Torque Characteristics
Steering assistance / motor torque

- EPSdp, EPSapa, EPScr

$$M_s\omega_S = M_H\omega_H + M_{dP}\omega_{dP}$$

$$M_S\omega_S = \left(M_H + \frac{\omega_{dP}}{\omega_H}M_{dP}\right)\omega_H$$

$$M_S = \left(M_H + \frac{\omega_{dP}}{\omega_H}M_{dP}\right)i_S$$

$$M_A = M_S - M_H i_S$$

$$= \frac{\omega_{dP}}{\omega_H}\, i_S M_{dP}$$

- EPSc, EPSp

$$M_S = \left(M_H + \frac{\omega_P}{\omega_H}M_P\right)i_S$$

$$M_A = \frac{\omega_P}{\omega_H}\, i_S M_P$$

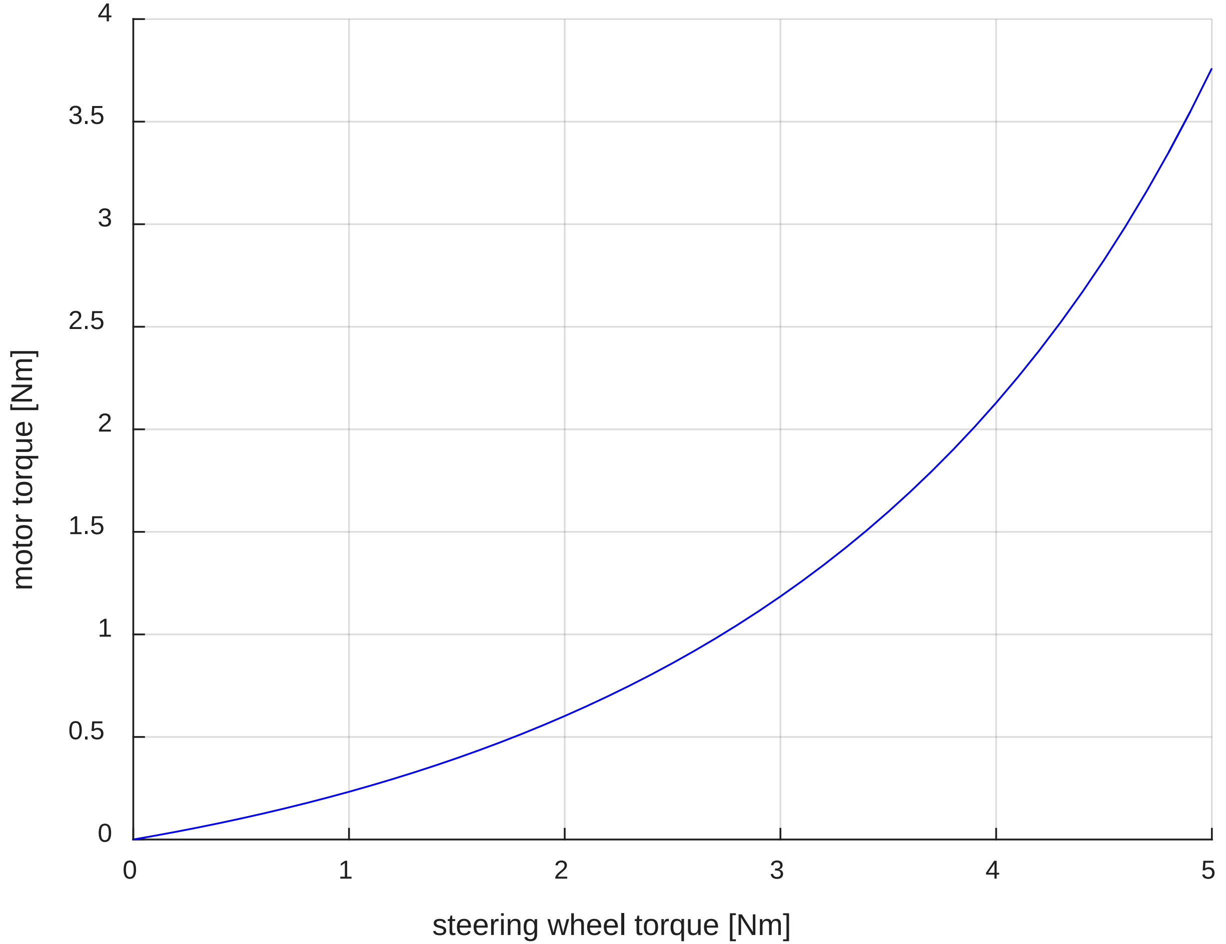
Motor Torque Characteristics
Motor torque characteristics

- Motor torque on EPSc and EPSp

$$M_P = \frac{1}{i_P i_S} M_A$$

- Motor torque on EPSdp, EPSapa, EPScr

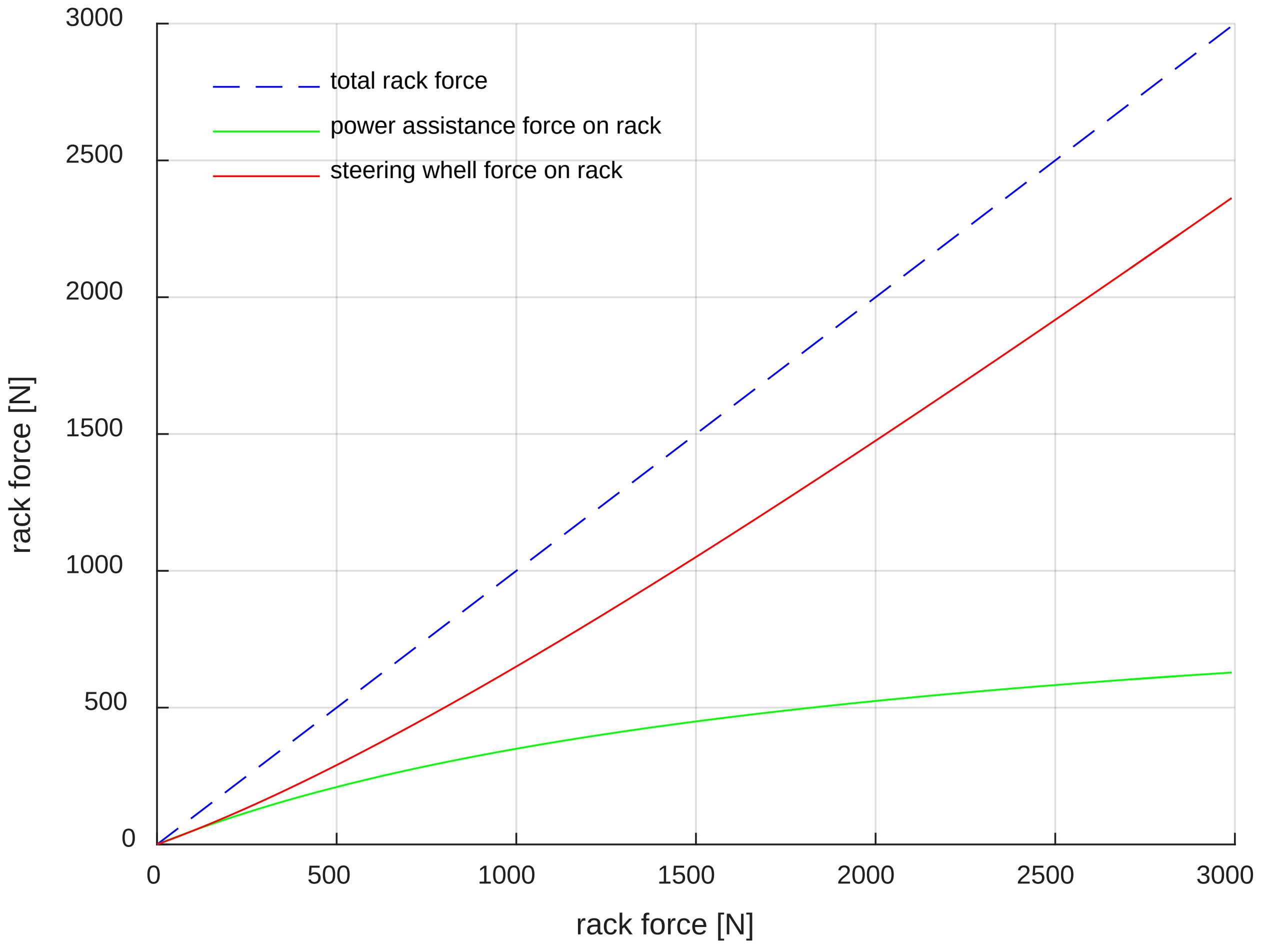
$$M_{dP} = \frac{2\pi}{i_{dP} i_G i_S} M_A$$



Motor Torque Characteristics
Steering rack force on EPS

- steering wheel torque and power assistance torque distribution on forces acting on the rack

$$F_r = \frac{2\pi}{i_G}(M_H + i_P M_P)$$



BASIC STEERING FUNCTIONS

Basic Steering Functions
Friction compensation

- power steering system generates a system friction which is higher than other steering systems
- the feedback of the steering system is affected by higher friction
- useful information about the current driving situation and road condition is accordingly reduced by friction
- a high friction coefficient in the steering system will support the suppression of interferences
- any disturbances in the steering wheel (wheel imbalances, fluctuations of the braking forces) can be reduced by higher friction;
- in steady cornering steering friction will generate a higher torque when the steering angle increase and a lower torque when the steering angle decrease

- reduce the effect of the friction in the steering with regard the torque requested by the power assistance

Basic Steering Functions
Friction compensation

- steady rack displacement with no load and motor off

$$b_H \omega_H = M_H - K_{TB}(\delta_H - i_H x_r)$$

$$b_{dP} \omega_{dP} = -K_{dP}(\delta_{dP} - i_{dP} x_r)$$

$$b_r v_r = i_H K_{TB}(\delta_H - i_H x_r) + i_{dP} K_{dP}(\delta_{dP} - i_{dP} x_r)$$

$$b_r v_r = i_H(M_H - b_H \omega_H) - \frac{i_{dP}^2}{i_H} b_{dP} \omega_H$$

$$M_H = \left(b_H + \frac{1}{i_H^2} b_r + \frac{i_{dP}^2}{i_H^2} b_{dp} \right) \omega_H$$

$$\omega_{dp} = \frac{i_{dP}}{i_H} \omega_H$$

$$\delta_{dP} = \frac{i_{dP}}{i_H} \delta_H$$

$$i_H = \frac{2\pi}{i_G}$$

$$v_r = \frac{\omega_H}{i_H}$$

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Basic Steering Functions
Inertia compensation

- EPS systems have high inertia, the steering movements initiated by the driver have to act against the torque generated by inertia
- a function for inertia compensation has to reduce the inertia effect on the steering torque characteristics; additional torque must be requested from the EPS motor

$$m_r \frac{dv_r}{dt} = i_H K_{TB}(\delta_H - i_H x_r) + i_{dP} K_{dP}(\delta_{dP} - i_{dP} x_r)$$

$$\frac{dv_r}{dt} = \frac{1}{i_{dP}} \frac{d\omega_{dP}}{dt}$$

$$J_H \frac{d\omega_H}{dt} = M_H - K_{TB}(\delta_H - i_H x_r)$$

$$\frac{d\omega_H}{dt} = \frac{i_{dP}}{i_H} \frac{d\omega_{dP}}{dt}$$

$$\frac{m_r}{i_{dP}} \frac{d\omega_{dP}}{dt} = i_H \left(M_H - J_H \frac{i_{dP}}{i_H} \frac{d\omega_{dP}}{dt} \right) + i_{dP} \left(M_{dP} - J_{dP} \frac{d\omega_{dP}}{dt} \right)$$

$$M_{dP} = \left(\frac{1}{i_{dP}^2} m_r + J_{dP} \right) \frac{d\omega_m}{dt}$$

$$J_{dP} \frac{d\omega_{dP}}{dt} = M_{dP} - (\delta_{dP} - i_{dP} x_r)$$

Basic Steering Functions
Active damping

- a friction- and inertia-compensated steering system responds very sensitively to disturbances in the force balance
- bumpy road could lead to high acceleration of the steering system which is perceived by the driver as a kickback
- small change on applied steering wheel torque lead to powerful system movements
- a damping function must be introduced to compensate for these undesirable characteristics

- this function has to request an torque from EPS motor that is oriented against the steering direction, proportional to the current steering speed and parameterised as a function of the vehicle speed

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Basic Steering Functions
Active return

- EPS basic steering functions: power assistance, friction compensation, inertia compensation and active damping displays a steering response that is comparable to that of an hydraulic power steering

- the active return function is design to improve the runback response of the front axle
- EPS motor adds torque to guide the free wheel and driver controlled wheel into the straight ahead position
- it is a function of steering wheel angle, applied steering torque and vehicle speed

LATERAL VEHICLE DYNAMICS

Lateral Vehicle Dynamics
Linear single track model

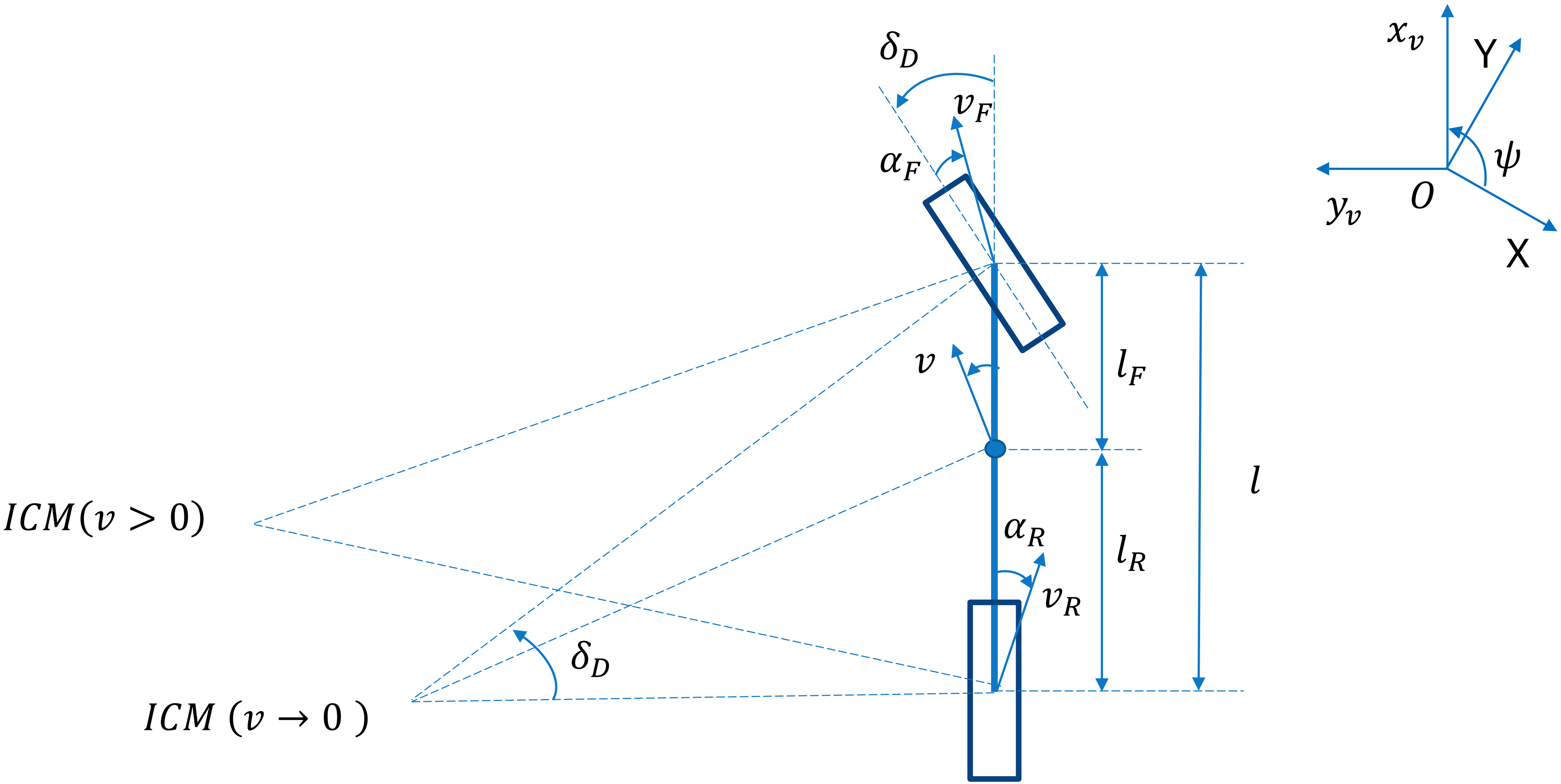
- the linear single track allows to approximate the lateral vehicle dynamics
- following simplification where assumed:
  - all the forces act on a plane flat road; the left an right tyre of each axle is exposed at the same load
  - the equations of the system are linearized; the tyre force is assumed proportional to the slip angle; the trigonometric functions are linearized
  - constant longitudinal velocity

- the model is suitable for lateral accelerations up to 4m/s^2 on dry roads

$$a_y \leq 0.4g \approx 4\,m/s^2$$

- the vehicle is described by a moving coordinate system located at the vehicle center of gravity $(O_v x_v y_v z_v)$ and an inertial coordinate system $(O_{XYZ}XYZ)$

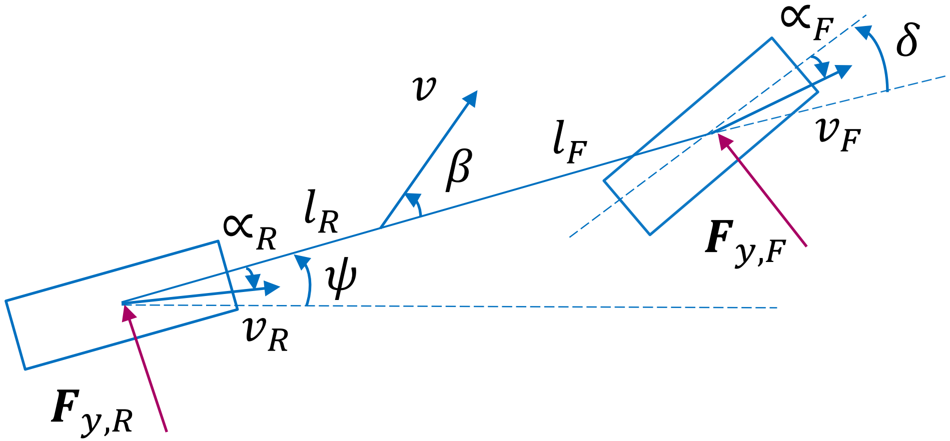
Lateral Vehicle Dynamics
Linear single track model



Lateral Vehicle Dynamics
Vehicle velocity

- the vehicle velocity

$$\boldsymbol{v} = \begin{bmatrix} v\cos\beta \\ v\sin\beta \\ 0 \end{bmatrix}$$



- the front/rear axle vehicle velocity

$$\boldsymbol{v}_F = \boldsymbol{v} + \boldsymbol{\omega} \times \boldsymbol{r}_F$$

$$\boldsymbol{v}_R = \boldsymbol{v} + \boldsymbol{\omega} \times \boldsymbol{r}_R$$

$$\boldsymbol{\omega} = \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} \qquad \boldsymbol{r}_F = \begin{bmatrix} l_F \\ 0 \\ 0 \end{bmatrix} \qquad \boldsymbol{r}_R = \begin{bmatrix} -l_R \\ 0 \\ 0 \end{bmatrix}$$

Lateral Vehicle Dynamics
Vehicle velocity

- the longitudinal component of the speed is equal for every point of the vehicle
- the lateral component of the speed changes by the rotating part of the yaw velocity multiplied by the distance to the front/rear axle

$$\boldsymbol{v}_F = \begin{bmatrix} v\cos\beta \\ v\sin\beta + \dot{\psi} l_F \\ 0 \end{bmatrix} = \begin{bmatrix} v_F\cos(\delta - \alpha_F) \\ v_F\sin(\delta - \alpha_F) \\ 0 \end{bmatrix}$$

$$\boldsymbol{v}_R = \begin{bmatrix} v\cos\beta \\ v\sin\beta - \dot{\psi} l_R \\ 0 \end{bmatrix} = \begin{bmatrix} v_R\cos(-\alpha_R) \\ v_R\sin(-\alpha_R) \\ 0 \end{bmatrix}$$

Lateral Vehicle Dynamics
Front / rear slip angles representations

- from the front/rear vehicle velocity:

$$\tan(-\propto_R) = \frac{v\sin\beta - \dot{\psi} l_R}{v\cos\beta}$$

$$\tan(\delta - \propto_F) = \frac{v\sin\beta + \dot{\psi} l_F}{v\cos\beta}$$

- for small angles, front and rear slip angles have following representation:

$$\alpha_F = \delta - \beta - \frac{\dot{\psi} l_F}{v}$$

$$\sin\beta = \beta;\ \cos\beta = 1$$

$$\alpha_R = -\beta + \frac{\dot{\psi} l_R}{v}$$

Lateral Vehicle Dynamics
Vehicle acceleration

- the vehicle acceleration

$$\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt} + \boldsymbol{\omega} \times \boldsymbol{v}$$

- with the assumption of a constant longitudinal velocity

$$\boldsymbol{a} = \begin{bmatrix} -v\dot{\beta}\sin\beta \\ v\dot{\beta}\cos\beta \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} \times \begin{bmatrix} v\cos\beta \\ v\sin\beta \\ 0 \end{bmatrix} = \begin{bmatrix} -v(\dot{\beta}+\dot{\psi})\sin\beta \\ v(\dot{\beta}+\dot{\psi})\cos\beta \\ 0 \end{bmatrix}$$

- acceleration is perpendicular on velocity: $\boldsymbol{a} \cdot \boldsymbol{v} = 0$
- acceleration magnitude

$$a_n = v(\dot{\beta} + \dot{\psi})$$

Lateral Vehicle Dynamics
Vehicle acceleration

- the acceleration projection on Oy_v axis

$$a_y = a_n \cos\beta$$

- with the assumption of small slip angle, $\cos\beta = 1$

$$a_y = v(\dot{\beta} + \dot{\psi})$$

- the vehicle center of gravity describe a path given by a function of yaw angle, slip angle and radius of curvature

$$v = R(\dot{\psi} + \dot{\beta})$$

- the acceleration projection on Oy_v axis is described by velocity and radius of curvature of the path of the centre of gravity

$$a_y = \frac{v^2}{R}$$

Lateral Vehicle Dynamics
Dynamic equations

- the principle of linear momentum in the lateral direction

$$m a_y = C_{\alpha F}\alpha_F \cos\delta + C_{\alpha R}\alpha_R$$

- the principle of angular momentum around the car axis Oy_v

$$I_z\ddot{\psi} = C_{\alpha F}\alpha_F \cos\delta\, l_F - C_{\alpha R}\alpha_R l_R$$

- for small steering angle, $\cos\delta = 1$

$$mv(\dot{\beta}+\dot{\psi}) = C_{\alpha F}\left(\delta - \beta - \frac{\dot{\psi} l_F}{v}\right) + C_{\alpha R}\left(-\beta + \frac{\dot{\psi} l_R}{v}\right)$$

$$I_z\ddot{\psi} = C_{\alpha F}\left(\delta - \beta - \frac{\dot{\psi} l_F}{v}\right) l_F - C_{\alpha R}\left(-\beta + \frac{\dot{\psi} l_R}{v}\right) l_R$$

Lateral Vehicle Dynamics
Dynamic equations in state space representation

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + Bu$$

- input vector
  - steering angle $u = [\delta]$
- output vector
  - yaw velocity
  - slip angle

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \dot{\psi} \\ \beta \end{bmatrix}$$

- system matrix

$$A = \begin{bmatrix} -\frac{1}{v} a_{11} & -a_{12} \\ -1 - \frac{1}{v^2} a_{21} & -\frac{1}{v} a_{22} \end{bmatrix}$$

- control matrix

$$B = \begin{bmatrix} \frac{C_{\alpha,F} l_F}{\psi} \\ \frac{C_{\alpha,F}}{mv} \end{bmatrix}$$

$$a_{11} = \frac{C_{\alpha,F} l_F^2 + C_{\alpha,R} l_R^2}{\psi}$$

$$a_{12} = \frac{C_{\alpha,F} l_F - C_{\alpha,R} l_R}{\psi}$$

$$a_{21} = \frac{C_{\alpha,F} l_F - C_{\alpha,R} l_R}{m}$$

$$a_{22} = \frac{C_{\alpha,F} + C_{\alpha,R}}{m}$$

Lateral Vehicle Dynamics
Vehicle stability during straight line driving

- assume steering angle equal to zero

$$\delta \ = 0$$

- the linear state space system becomes

$$\dot{\boldsymbol{x}} = A\boldsymbol{x}$$

- the linear system is stable when the polynomial for the characteristic equation of the matrix system has positive coefficients

$$\det(\lambda I - A) = \lambda^2 + a_1\lambda + a_2$$

$$a_1 = \frac{1}{v}(a_{11} + a_{22})$$

$$a_2 = -a_{12} + \frac{1}{v^2}(a_{11}a_{22} - a_{12}a_{21})$$

$$= \frac{C_{\alpha,F}C_{\alpha,R}l^2}{m\psi v^2}\left(1 + \frac{C_{\alpha,R}l_R - C_{\alpha,F}l_F}{C_{\alpha,F}C_{\alpha,R}l^2}mv^2\right)$$

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Lateral Vehicle Dynamics
Vehicle stability during straight line driving

- $a_1 > 0$ for any velocity
- $a_2 > 0$ for any velocity if $C_{\alpha,R} l_R > C_{\alpha,F} l_F$

- the vehicle becomes unstable if $C_{\alpha,R} l_R < C_{\alpha,F} l_F$ and

$$v^2 > \frac{l^2}{m} \frac{C_{\alpha,F} C_{\alpha,R}}{C_{\alpha,F} l_F - C_{\alpha,R} l_R}$$

Lateral Vehicle Dynamics
Stationary steering

- for stationary steering, the steering angle, the yaw rate and slip angle are constant

  $\delta, \dot{\psi}, \beta$ - constant

- the linear/angular momentum principle

$$m\frac{v^2}{R} = C_{\alpha F}\alpha_F + C_{\alpha R}\alpha_R$$

$$C_{\alpha F}\alpha_F l_F = C_{\alpha R}\alpha_R l_R$$

- the driving behaviour for a specific vehicle could be established based on the difference between front and rear slip angle obtained from the linear/angular momentum principle

$$\alpha_F - \alpha_R = \frac{mv^2}{Rl}\left(\frac{l_R}{C_{\alpha F}} - \frac{l_F}{C_{\alpha R}}\right)$$

Lateral Vehicle Dynamics
Stationary steering

- the driving behaviour for a specific vehicle and for a specific steering motion is characterized be the self-steering gradient

$$EG = \frac{m}{l}\left(\frac{l_R C_{\alpha R} - l_F C_{\alpha F}}{C_{\alpha F} C_{\alpha R}}\right)$$

- the difference between front and rea slip angle is described by the self-steering gradient, vehicle velocity and the circle radius the vehicle is driving on

$$\alpha_F - \alpha_R = EG\,\frac{v^2}{R}$$

Lateral Vehicle Dynamics
Stationary steering

- which is the steering angle for a vehicle with a velocity v to follow a circle with radius R?

- the steering angle is obtained from front and rear slip angle (geometrical and velocity representation)

$$\delta = \frac{\dot{\psi} l}{v} + \alpha_F - \alpha_R$$

- recalling the yaw rate $\dot{\psi} = \frac{v}{R}$, lateral acceleration $a_y = \frac{v^2}{R}$ and self-steering gradient EG

$$\delta = \frac{l}{R} + EG a_y$$

Lateral Vehicle Dynamics
Stationary steering

- the ratio between wheel base and the radius of the vehicle path is called Ackerman steering angle

$$\delta_D = \frac{l}{R}$$

- the steering angle is the Ackerman steering angle and dynamic component depending on vehicle velocity

$$\delta = \delta_D + EG\frac{v^2}{R}$$

- the steering angle increase or decrease depending on velocity and the sign of the self steering gradient

Lateral Vehicle Dynamics
Steering driving behaviour

- neutral steering $EG = 0$; steering angle is equal with the Ackerman angle

$$\delta = \delta_D$$

- understeering $EG > 0$; steering angle is greater than the Ackerman angle

$$\delta > \delta_D$$

- oversteering $EG < 0$; steering angle is smaller then the Ackerman angle

$$\delta < \delta_D$$

Lateral Vehicle Dynamics
Yaw amplification factor

- for constant steering angle, yaw rate takes on different values depending on steering gradient EG

$$\dot{\psi} = \frac{v}{l + EGv^2}\delta_{st}$$

$\delta = \delta_{st}$ - constant

- yaw amplification factor for a given velocity

$$\frac{\dot{\psi}}{\delta} = \frac{v}{l + EGv^2}$$

- the yaw amplification factor is small for understeering vehicles $EG > 0$ and large for oversteering vehicles $EG < 0$

Lateral Vehicle Dynamics
Critical velocity

- If the self steering gradient $EG = -\frac{l}{v^2}$ the vehicle becomes instable, small steering inputs lead to infinite yaw rotation

$$\frac{\dot{\psi}}{\delta} \rightarrow \infty$$

- critical velocity v_{cr} is the velocity at witch the yaw amplification factor strives towards an infinite values; it is defined for $EG < 0$

$$v_{cr} = \sqrt{-\frac{l}{EG}} \qquad v_{cr} = \sqrt{\frac{l^2}{m}\frac{C_{\alpha F}C_{\alpha R}}{l_F C_{\alpha F} - l_R C_{\alpha R}}}$$

BOSCH

Lateral Vehicle Dynamics
Characteristics velocity

- characteristic velocity v_{ch} is velocity at witch the yaw amplification factor reaches its maximum

$$\frac{d}{dv}\left(\frac{\dot{\psi}}{\delta}\right) = \frac{l - EGv^2}{(l + EGv^2)^2} = 0$$

$$v_{ch}^2 = \frac{l}{EG}$$

$$v_{ch} = \sqrt{\frac{l^2}{m}\frac{C_{\alpha F}C_{\alpha R}}{l_R C_{\alpha R} - l_F C_{\alpha F}}}$$

- typical values for the characteristic velocity are between 65 and 100km/h

Lateral Vehicle Dynamics
Steering driving behaviour depending on steering gradient