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# Introduction

In this case study, we are acting as a financial product controller, which is also known as a market maker. We are asked to ascertain the fair value of a long position in a 10–year Euro interest rate swap, both collateralized and uncollateralized, which is now over three years into the contractual arrangement. To estimate the value of the off-market swap contract, we take a step-by-step approach. We will first introduce the general information and features of swaps. Then we will explain the important dates. After that we calculate the value step-by-step, stripped par rate, zero-coupon bond prices, swap prices and audit checks through the forward rate method and principal method. We will explain the procedure of each part one-by-one later. As for the collateralized swap contract valuation, the process follows the above. Furthermore, we adopt the LIBOR-OIS forward rate projection method.  For both uncollateralized and collateralized swap contracts, we will have cash flows discounted to the trade date, allowing us to know our profit and loss.

# Key Concepts

There are three significant computations for interest rate swaps to be covered which are: (1) pricing an at-market (or par) swap, (2) valuing an off-market swap, and (3) inferring the forward curve that is consistent with a sequence of at-market swaps. “Pricing” to mean would be accustomed to determining the fixed rate at inception and “valuing” to mean determining the market value thereafter. To keep the models straightforward, transaction costs are dismissed, a 30/360 day-count convention is utilized, and expect that the pricing and valuation are on a settlement date. However, similar strategies can be utilized to manage bid-ask spreads, more common day-count conventions such as actual/360 and actual/365, and between-settlement-date calculations. The actual/360 method is preferable than the actual/365 method because the yearly rate is divided by 360, the daily rate is greater than the rate obtained by dividing it by 365, resulting in a higher dollar amount of interest payments.

1.1

1.2

where T refers to the day's counter. 30B is the method of counting days assuming the differences of the month and the year stand for the fixed 30- and 360-days periods, and the remaining part of the days counter are calculated as below where D stands for the day number of dates:

1.3

Beginning with the LIBOR forward curve, pricing an at-market swap entails “monetizing” each forward rate by multiplying by the notional principal and day-count 2 fraction and then discounting the cash flows. The thought is to infer a uniform fixed rate to such an extent that when it is monetized utilizing a similar notional principal and day-count fractions, a similar present value is obtained. In this manner, an at-market swap, which is also called a “par” swap, has an underlying worth of zero by development. The swap fixed rate is an “average” of the LIBOR forward curve, not a simple arithmetic or geometric average, but an average in the time-value-of-money sense in that the two legs of the swap have a similar present value.

A conspicuous test is to obtain the LIBOR forward curve needed to price the swap since it for the most part isn't noticed. It helps in the event that there are actively traded futures contracts on the reference rate, such as the Eurodollar contract at the CME Group in Chicago. Its trading results provide data on 3-month LIBOR for quarterly delivery dates out to a decade. In any case, a convexity adjustment in accordance with the observed futures rate is needed to get the forward rate. That requires an interest rate term structure model and presumptions about the future rate volatility and connections across points along the yield curve. The requirement for this change emerges in light of everyday mark-to-market and settlement practices on exchange-traded futures contracts.

Swap valuation involves: (1) contrasting the legally binding fixed rate to that on an at market swap having in any case matching terms, (2) getting an annuity for the distinction in the fixed rates, and (3) computing the present value of the annuity utilizing a sequence of discount factors corresponding to the settlement dates. An elective methodology is to decipher the interest rate swap as a long/short mix of a bond paying the fixed rate on the swap and a floating-rate bond paying the money market reference rate, e.g., 3 month LIBOR. In the conventional technique for swap valuation, the implied floater maintains its par value on rate-reset dates while the fixed-rate bond can be valued at a premium or discount. The distinction in the prices of these two bonds is the value of the interest rate swap. With OIS discounting, the outcome that the verifiable floating-rate bond paying LIBOR is priced at par value does not hold anymore.

It is valuable to construe the LIBOR forward curve from observed fixed rates on at market swaps. This implied forward curve, also called the projected curve, is utilized to price and value non-standard agreement. For example, a “vanilla” interest rate swap has a consistent notional principal and a prompt start date. Non-vanilla assortments can have shifting notional principals and conceded start dates. Pricing such swaps involves getting the “average”, maybe a “weighted average” on the off chance that the notional principal differs, of the applicable fragment of the forward curve. As displayed later in the paper, the implied LIBOR forward curve calculated for OIS discounting is expected to value collateralized interest rate swaps utilizing the combination-of-bonds approach.

# Details of an Interest Rate Swap Contract

An interest rate swap is an exchange of cash flows between two parties where party A pays a fixed rate and receives a floating rate and party B receives a fixed rate and pays the floating rate. An interest rate swap contract has the following characteristics:

* Notional Principal Amount: It is the predetermined amount of money used as the basis for calculating fixed and floating payments. The notional principal never changes hands, which is known as off-balance-sheet financing.
* Trade Date: It is the month, day, and year in which parties’ contract to enter into a swap and it is also known as the first reset date or fixing date.
* Effective Date: It is also known as the value date is the date first fixed/floating payments start to accrue. The Eurodollar swap rate's value date is **two London business days** after the trade date; called the “spot date”.
* Payment Dates: Its payment frequency would be either annually, semi-annually, or quarterly.
* Payment Date and Holidays Conventions:
  1. If the date falls on a weekend or a bank holiday, payment is made on the next “good” business day if in the same calendar month which is “Modified Following” or “Modified” Business Day.
  2. Otherwise, payment must be made on the first preceding business day (End-of-month effect).
  3. Payment made on the next good business day, even if in a different month which is called “Following” Business Day.
  4. Closest previous good business day would be “Preceding” Business Day.
  5. End of Month (EOM) is the Last good business day in each month.
  6. IMM Settlement Dates are days on which IMM futures contracts settle on the third Wednesday in March, June, September, and December.
* Theoretical Maturity Date: Last payment date. If not adjusted for business day convention, for the maturity date that falls on non-business day, then accruals are calculated up to maturity date only, regardless of day count fraction convention.
* Swaps Tenor: Length of swap’s life from effective (value, spot) date to maturity date.
* Front Stub Period: Length of first payment period if different (generally shorter) from subsequent payment periods.
* Back Stub Period: Length of last payment period if different from preceding payment periods.

# Modified Business Dates

As of 14 December 2000, the Governing Council of the European Central Bank has decided that, from 2002 until further notice, the Trans-European Automated Real-time Gross settlement Express Transfer (TARGET system will be closed, in addition to Saturdays and Sundays, on the following days:

|  |  |
| --- | --- |
| Holiday | Dates |
| New Year’s Day | 1st January |
| Good Friday (Catholic/Protestant) | The last Friday before Easter Sunday |
| Easter Monday (Catholic/Protestant) | The first Monday after Easter Sunday |
| Labour Day | 1st May |
| Christmas Day | 25th December |
| Boxing Day | 26th December |

On these closing days, TARGET as a whole, including all the national real-time gross settlement (RTGS) systems, will be closed.

Modified following alters the official business date spelled out in a contract when it falls on a holiday or bank holiday. To rectify such a date, the effective date of the contract is typically rolled forward or backward to the closest business day. Modified following also helps settle transaction dates when national or bank holidays don't coincide with stock exchange holidays. So, if the payment date falls on a holiday, a modified following could push it ahead or back in time so that it does fall on a business day, and the contract completes. For example:

* If the holiday date is on a Saturday, its alternative closing date would push back to Friday.
* If the holiday date is on a Sunday, its alternative closing date would push ahead to Monday.

Modified business dates make it easier to manage contractual payments of derivatives without explicitly listing every payment date and the exceptions within a contract.

# Methodology

**Linear Stripping**

The valuation of swaps relies on discounting future cash flows, both fixed payments and floating returns. In the above part, we have got the detailed days counting and important trading and settlement dates of the swap contract, as well as the Euribor data and the modified following business dates. We have got the Euribor Par rate from the money market rates and the swap market rates, except for year six, eight and nine, which are known as non-standard grid points. Therefore, the next step is to calculate the par rate for these three years through the linear stripping method. The results are called implied swap rates, also known as “stripped par rates”.

In order to execute linear stripping, we must use the number from observed numbers, which can be observed from the product market curve. Linear stripping means to estimate the stripped par rates linearly.

2.1

2.2

where,

.

**Money Market Rates & Zero-Coupon Price**

Next, we work on the money market rates. Money market rate means the rate from overnight until next one year. As mentioned above, the EUR money market (floating) day count basis is “Actual/360”, we have to adjust the number using equation 1.1. With the adjusted spot money market accruals, we are able to calculate the zero-coupon bond price for the first year with the following equation.

2.3

**Bootstrapping & Zero-Coupon Price**

On the other hand, The EUR swap market (fixed) basis is “Annual 30/360”. We have to adjust the number using equation 1.2. Contrast to the money market scenario, the swap market requires another way to calculate the zero-coupon bond price, which is the bootstrapping method. In terms of the bootstrapping method, the calculation of zero-coupon bond price relies on the current year’s EURIBOR stripped par rate, swap accrual and the previous year’s modified following business date’s swap sum term. The formula is as below.

2.4

After getting the zero-coupon bond prices, we are able to calculate the zero-coupon bond yields with the formula below.

2.5

2.6

# Audit Check

The purpose of an audit check is to confirm if the zero-coupon bond price is corresponding to the par rate. To check if the above situation stands, we will use 2 methods, the principal method and the forward rate method. If the zero-coupon bond price corresponds to the par rate, the P&L result will be zero.

At the dealing date, 24th of December, 2020. There are new par rates. 3 years and 5 months have passed, we look at the 7Y par rate, which is 0.76. We would like to check if the 0.88688 zero-coupon bond price corresponds with the 0.76 par rate at 7Y and if it is traded at par.

**Principal Method**

In principal method, every stochastic cash flow is replaced with two non-stochastic cash flows, especially when the interest rate is reset (26th March 2020). We are in a long position, so we are paying fixed and receiving floating. On the 24th December 2020, the principal is 10,000,000. As we are paying out fixed payments each year, the payments are discounted on the par rate and subtracted from the principal from 1Y to 7Y. At last the P&L equals 0, proving that the swap is trading at par. Below are the equations:

3.1

where,

3.2

where,

**Forward Rate Method**

Forward rates can be derived from spot-interest rates that are the yields that we are acquiring on zero-coupon bonds through a cycle called bootstrapping. A forward rate emerges because of the forward agreement. Despite the fact that the responsibility between two parties prompts the effective execution of a forward agreement. What's more, it has been divided into two legs; the main responsibility is to convey, sell, or take a short situation on the asset and on another leg, to convey, purchase, or take a long situation on the asset.

It expresses that it creates a rate so that two sequential one-year maturities offer a similar return as two - year maturity offers. It is the interest rate a financial backer has ensured to get similar returns between the primary investment maturity and second maturity while picking between the more limited or longer-term. Forward rate over interest rate is addressed numerically as below:

3.3

where

Spot-Forward Parity Relationship:

* The accrual factor over interest period:

3.4

where

* The coverage over interest period:

3.5

where is the appropriate money market day count convention of respective currencies (either 360 or 365).

Equivalently, forward rate adjusted for loan period, holding period (de-annualised) forward rate over accrual period:

. 3.6

# Exponential interpolation and Valuation

The current trade date is 20th December 2020, due to T+2, and the value date is 24th December 2020. The 10-year EUR swap par rates as of the reset date in March, but our current trading date is 24th December. There is a time difference. We have to translate the date to keep the data consistent. We will have to run an approach called exponential interpolation.

The first step is to find the reset dates of the first year until the seventh year(Starting from 2020). We have to modify the reset dates. The dates are shown in the following table.

|  |  |
| --- | --- |
| Gridpoint | Contracted Cash Flow Date |
| 1Y | 29-Mar-21 |
| 2Y | 29-Mar-22 |
| 3Y | 29-Mar-23 |
| 4Y | 28-Mar-24 |
| 5Y | 31-Mar-25 |
| 6Y | 30-Mar-26 |
| 7Y | 30-Mar-27 |

Next we have to calculate the new zero-coupon bond price with a new weight measure(t), which is the date interval between this year’s trading date and the next reset date(Dec-March), divided by the interval of modified business dates.

* Zero-coupon bond price is an exponential function

4.1

* Taking natural logarithms on both sides:

4.2

4.3

. 4.4

* Linear interpolation (stripping) on observed rates,

4.5

4.6

* Rewrite Equation

. 4.7

For simplicity, let and be as follows:

, 4.8

. 4.9

Substituting and into equation and by logarithmic identities:

4.10

4.11

. 4.12

Exponentially interpolated zero-coupon bond:

, 4.13

. 4.14

To extrapolate before first known discount factor, or beyond last discount factor observed at :

4.15

4.16

where,

The T weighting in equation 4.8.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Gridpoint | Contracted Cash Flow Date | Zero-Coupon Bond Price | Forward MM Accrual | LIBOR Forward Rate |
| 1Y | 29-Mar-21 | 0.9997732 | 0.0026389 | 0.0859783 |
| 2Y | 29-Mar-22 | 0.9960606 | 0.0101389 | 0.3676199 |
| 3Y | 29-Mar-23 | 0.9953843 | 0.0101389 | 0.0670127 |
| 4Y | 28-Mar-24 | 0.9910728 | 0.0101389 | 0.4290758 |
| 5Y | 31-Mar-25 | 0.9838368 | 0.0102222 | 0.7194985 |
| 6Y | 30-Mar-26 | 0.9735685 | 0.0101111 | 1.0431105 |
| 7Y | 30-Mar-27 | 0.9598039 | 0.0101389 | 1.4144637 |

# Profit & Loss Calculation

As our value date is different from the contracted cash flow date, there is a difference between expected cash flow and realised cash flow. The profit & loss is no longer zero, meaning that the swap is not trading at par. We use the seven remaining cash flows (from March 2021 to March 2027) of the swap to calculate the profit & loss. The zero coupon bond prices derived in the above using exponential interpolation is used to discount the seven cash flows to the value date, 24/12/2020.

On the inception date, 27th March 2017, the 10Y interest rate is 2.33%. Later on, on the reset date of 2020, 26th March 2020, the 7Y interest rate is 0.76%. This means that we will have a loss. To prove this we will be using the principal method and forward rate method.

i) Principal Method

When using the principal method, we will use the formula 3.1. As we are in a long position we pay fixed and receive floating. The last value date is 26 March 2020, the next cash flow is on 29th March 2021 which is 96 days away from the value date 24th December 2020. The LIBOR floating rate has to multiply by 96/360. For the floating leg, the next cash flow is reset, we replace stochastic cash flow with non-stochastic cash flow. By subtracting the discounted cash flow from principal we are able to get the P&L on value date, which is also known as market-to market. Look at table:

2017

March 27

Inception Date

2020

March 26

Last reset date

2020

Dec 22

Trade date

2022

March 29

+A

Value date

2021

March 29

2023

March 29

2024

March 28

2025

March 31

2026

March 30

2027

March 30

2020

Dec 24

-A

Maturity Date

ii) Forward Rate Method

When calculating the swap cash flow with the forward rate method, we use formula 3.3. The formulas are explained above. As the cash flow has already reset, we have to use the 1Y interest rate set on the 26th March 2020, which is 0.55%, to calculate the first cash flow. Same as above, as the first contract cash flow date is not the same as the value date, we have to multiply LIBOR floating rate by 96/360. For the fixed payment, we are paying a fixed payment set on the inception date. The first cash flow is also adjusted with the interval between value date and reset date The cash flows are discounted, the zero-coupon price is multiplied with the cash flows.

2017

March 27

Inception Date

2020

March 26

Last reset date

2020

Dec 22

Trade date

2022

March 29

Lt

Value date

2021

March 29

Lt

2023

March 29

Lt

2024

March 28

Lt

2025

March 31

Lt

2026

March 30

Lt

2027

March 30

Lt

2020

Dec 24

Lf

Maturity Date

Lf

Lf

Lf

Lf

Lf

Lf

The P&L result is the same for both approaches and both of them recorded a loss of 936,029.31.

# Main Concept of Collateralization Valuation

The main implication of collateralization is that the credit risk on the swap transaction becomes minimal, compared with the non-collateralization. Bilateral CSAs usually entail a zero threshold, meaning only the counterparty has the negative market value for the swap posts collateral, so there still is some “tail” risk of default. In any case, minimal credit risk on the swap implies that the discount factors to get the present value of the annuity for the difference between the contractual and MTM fixed rates, which remains unambiguous, should be based on risk-free interest rates or, at least, nearly risk-free from the perspective of the rating agencies.

Therefore, we will use the dual-curve approach to do the OIS discounting which uses the swap rate to calculate the forward cash flow and the OIS rate to discount the cash flow.

# Calculation of the amount of Cash Collateral

As we only have 7 years left until maturity, therefore we have the OIS par swap rate of 0.38. Moreover, the money market day and OIS day count basis is Actual/360 using formula 1.1.

Similar to the way we did in the uncollateralized, we calculate the OIS stripped par rate of year 6, 8 and 9 by using the linear stripping. Then generating the OIS zero coupon bond price which is the same with the formula used in uncollateralized. After that, we do the audit check as to check whether the zero-coupon bond prices are good for the OIS par swap rate and the profit and loss resulted in zero of the principal method and the forward rate method.

# Exponential interpolation

Due to the 10-year EONIA swap par rates’ reset date is on March 26 and the contracted cash flow date is March 29, but our trade date is on December 22. Therefore, we do the same way as question A in which we need to adjust the date to March and find out the new zero-coupon bond prices of next 7 years which is consistent with the par date (reset date).

After generating the OIS zero coupon price, we will use the formula 3.4 and 3.5 to find out the new forward market accrual and the new OIS forward rate.

# Profit & Loss Calculation of Collateralized

Since the valuation is between two reset dates, the Euribor swap market par rate is calculated as the below formula.

5.1

Since the fixed leg is Euribor and the floating leg is OIS, we need to mix them to find out the forward rate by using the below bootstrapping method.

5.2

After calculating the projected Euribor-OIS forward rate, we can calculate the floating leg, and the swap fixed leg is the same with the calculation in question A. Since the valuation is between two valuation dates, which is in March, and our trade date is in December, we have adjusted the discount rate using the exponential interpolation which is the same as question A. Moreover, the first cash flow should be calculated as 96 days as it is not in the full year. Finally, we will get the profit and loss (paying the fixed and receiving the floating) with -950256.01.

# Results

**Question A**

***Table 1*** EURIBOR Linear stripping on 27 March 2017

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Time | Modified Following Business Date |  | Linear Interpolation |  | EURIBOR Stripped Par Rate |
|  |  | Day | t | (1-t) | R(0,t) |
| Value | 29/3/2017 | Wed |  |  |  |
| O/N | 30/3/2017 | Thu |  |  | 0 |
| 1M | 28/4/2017 | Fri |  |  | 0.13 |
| 3M | 29/6/2017 | Thu |  |  | 0.23 |
| 6M | 29/9/2017 | Fri |  |  | 0.34 |
| 12M | 29/3/2018 | Thu |  |  | 0.55 |
| 2Y | 29/3/2019 | Fri |  |  | 0.61 |
| 3Y | 30/3/2020 | Mon |  |  | 0.82 |
| 4Y | 29/3/2021 | Mon |  |  | 1.08 |
| 5Y | 29/3/2022 | Tue |  |  | 1.32 |
| 6Y | 29/3/2023 | Wed | 0.5 | 0.5 | **1.525** |
| 7Y | 28/3/2024 | Thu |  |  | 1.73 |
| 8Y | 31/3/2025 | Mon | 0.6645397 | 0.3354603 | **1.8809572** |
| 9Y | 30/3/2026 | Mon | 0.3327256 | 0.6672744 | **2.0302735** |
| 10Y | 30/3/2027 | Tue |  |  | 2.18 |

By using linear stripping on 27th March 2017, we get the EURIBOR Stripped Par Rate for year six, eight and nine.

***Table 2*** EURIBOR Linear stripping on 12 December 2020

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Time | Modified Following Business Date |  | Linear Interpolation |  | EURIBOR Stripped Par Rate |
|  |  | Day | t | (1-t) | R(0,t) |
| Value | 24/12/2020 | Thu |  |  |  |
| O/N | 29/12/2020 | Tue |  |  | 0 |
| 1M | 25/1/2021 | Mon |  |  | 0.01 |
| 3M | 24/3/2021 | Wed |  |  | 0.08 |
| 6M | 24/6/2021 | Thu |  |  | 0.19 |
| 12M | 28/12/2021 | Tue |  |  | 0.35 |
| 2Y | 28/12/2022 | Wed |  |  | 0.19 |
| 3Y | 27/12/2023 | Wed |  |  | 0.25 |
| 4Y | 24/12/2024 | Tue |  |  | 0.35 |
| 5Y | 24/12/2025 | Wed |  |  | 0.47 |
| 6Y | 24/12/2026 | Thu | 0.502725 | 0.497275 | **0.614209809** |
| 7Y | 28/12/2027 | Tue |  |  | 0.76 |
| 8Y | 27/12/2028 | Wed | 0.665751 | 0.334249 | **0.900384615** |
| 9Y | 24/12/2029 | Mon | 0.334249 | 0.665751 | **1.039615385** |
| 10Y | 24/12/2030 | Tue |  |  | 1.18 |

By using linear stripping on 27th March 2017, we get the EURIBOR Stripped Par Rate for year six, eight and nine.

***Table 3*** Calculation of Zero-Coupon Bond price using money market rates and bootstrapping on 27 March 2017.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Term Sheet: | |  |  |  |  |  |  |
|  |  |  | Spot MM | Forward MM | Swap |  | Euribor |
| Spot | Forward | Euribor | Accrual | Accrual | Accrual |  | Zero-Coupon |
| MM | MM | Swap | (Actual/360) | (Actual/360) | (30/360) |  | Bond Price |
| (days) | (days) | (days) | a | a | a |  | B(0,t) |
| 0 |  |  |  |  |  |  | 1 |
| 1 |  |  | 2.778E-05 |  |  |  | 1 |
| 30 |  |  | 0.000833333 |  |  |  | 0.999891678 |
| 92 |  |  | 0.002555556 |  |  |  | 0.999412568 |
| 184 |  |  | 0.005111111 |  |  |  | 0.998265237 |
| 365 | 365 | 360 | 0.010138889 | 0.010138889 | 0.01 |  | **0.994454535** |
| 730 | 365 | 360 |  | 0.010138889 | 0.01 |  | **0.987907591** |
| 1097 | 367 | 361 |  | 0.010194444 | 0.010028 |  | **0.97572149** |
| 1461 | 364 | 359 |  | 0.010111111 | 0.009972 |  | **0.9577089** |
| 1826 | 365 | 360 |  | 0.010138889 | 0.01 |  | **0.935956256** |
| 2191 | 365 | 360 |  | 0.010138889 | 0.01 |  | **0.912100535** |
| 2556 | 365 | 359 |  | 0.010138889 | 0.009972 |  | **0.88501629** |
| 2924 | 368 | 363 |  | 0.010222222 | 0.010083 |  | **0.858696673** |
| 3288 | 364 | 360 |  | 0.010111111 | 0.01 |  | **0.830615796** |
| 3653 | 365 | 360 |  | 0.010138889 | 0.01 |  | **0.800669626** |

***Table 4*** Calculation of Zero-Coupon Bond price using money market rates and bootstrapping on 24 December 2020.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Term Sheet: | |  |  |  |  |  |  |
|  |  |  | Spot MM | Forward MM | Swap |  | Euribor |
| Spot | Forward | Euribor | Accrual | Accrual | Accrual |  | Zero-Coupon |
| MM | MM | Swap | (Actual/360) | (Actual/360) | (30/360) |  | Bond Price |
| (days) | (days) | (days) | a | a | a |  | B(0,t) |
| 0 |  |  |  |  |  |  | 1 |
| 5 |  |  | 0.000138889 |  |  |  | 1 |
| 32 |  |  | 0.000888889 |  |  |  | 0.999991111 |
| 90 |  |  | 0.0025 |  |  |  | 0.99980004 |
| 182 |  |  | 0.005055556 |  |  |  | 0.999040366 |
| 369 | 369 | 364 | 0.01025 | 0.01025 | 0.010111 |  | **0.996425324** |
| 734 | 365 | 360 |  | 0.010138889 | 0.01 |  | **0.99619299** |
| 1098 | 364 | 359 |  | 0.010111111 | 0.009972 |  | **0.992516377** |
| 1461 | 363 | 357 |  | 0.010083333 | 0.009917 |  | **0.986100338** |
| 1826 | 365 | 360 |  | 0.010138889 | 0.01 |  | **0.976744043** |
| 2191 | 365 | 360 |  | 0.010138889 | 0.01 |  | **0.963689357** |
| 2560 | 369 | 364 |  | 0.01025 | 0.010111 |  | **0.947787364** |
| 2925 | 365 | 359 |  | 0.010138889 | 0.009972 |  | **0.929794353** |
| 3287 | 362 | 357 |  | 0.010055556 | 0.009917 |  | **0.909560983** |
| 3652 | 365 | 360 |  | 0.010138889 | 0.01 |  | **0.886883313** |

***Table 5*** Bootstrapping & Zero-Coupon Bond Yields 27 March 2017

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Time | Discounted Swap Accrual | Swap Sum Term | | Zero-Coupon Bond Yield |
|  | B(0,t) |  |  | Y(0,t) |
| 12M | 0.009944545 | 0.009945 |  | **0.55763889** |
| 2Y | 0.009879076 | 0.019824 |  | **0.61015978** |
| 3Y | 0.009784318 | 0.029608 |  | **0.82112901** |
| 4Y | 0.009550486 | 0.039158 |  | **1.08539393** |
| 5Y | 0.009359563 | 0.048518 |  | **1.33179629** |
| 6Y | 0.009121005 | 0.057639 |  | **1.54452414** |
| 7Y | 0.008825579 | 0.066465 |  | **1.75960813** |
| 8Y | 0.008658525 | 0.075123 |  | **1.91983554** |
| 9Y | 0.008306158 | 0.083429 |  | **2.0815754** |
| 10Y |  |  |  | **2.24609629** |

***Table 6*** Bootstrapping & Zero-Coupon Bond Yields 24 December 2020

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Time | Discounted Swap Accrual | Swap Sum Term | | Zero-Coupon Bond Yield |
|  | B(0,t) |  |  | Y(0,t) |
| 12M | 0.010074967 | 0.010075 |  | **0.35486111** |
| 2Y | 0.00996193 | 0.020037 |  | **0.18985446** |
| 3Y | 0.009897594 | 0.029934 |  | **0.25002008** |
| 4Y | 0.009778828 | 0.039713 |  | **0.35030178** |
| 5Y | 0.00976744 | 0.049481 |  | **0.47146306** |
| 6Y | 0.009636894 | 0.059118 |  | **0.61805879** |
| 7Y | 0.009583183 | 0.068701 |  | **0.76750699** |
| 8Y | 0.009272116 | 0.077973 |  | **0.91248061** |
| 9Y | 0.009019813 | 0.086993 |  | **1.05817678** |
| 10Y |  |  |  | **1.20698717** |

***Table 7*** The crucial information for doing audit check through the 2 approaches on 27 March 2017

|  |  |  |
| --- | --- | --- |
| Currency |  | Euro |
| Index |  | Euribor |
| Contract Position |  | Long |
| Notional Principal |  | 10,000,000 |
| Maturity (years) |  | 10Y |
| Par Swap Rate |  | 2.18 |

When doing the audit check through Principal Method and Forward Rate Method, we need the above par swap rate, maturity years, notional principal and contract position.

***Table 8*** Audit Check through Principal Method for 27 March 2017

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| Cashflow Date | Euribor Swap Fixed | Euribor Floating | Netted | Euribor Discounted |
| Value |  | 10000000 | 10000000 | 10000000 |
| 12M | -119311.1111 |  | -119311 | -118884.6126 |
| 2Y | -118000 |  | -118000 | -117550.7728 |
| 3Y | -117672.2222 |  | -117672 | -116791.6077 |
| 4Y | -117016.6667 |  | -117017 | -115390.1746 |
| 5Y | -118000 |  | -118000 | -115255.7971 |
| 6Y | -118000 |  | -118000 | -113715.3442 |
| 7Y | -119311.1111 |  | -119311 | -113081.5635 |
| 8Y | -117672.2222 |  | -117672 | -109410.9677 |
| 9Y | -117016.6667 |  | -117017 | -106433.7943 |
| 10Y | -118000 | -10000000 | -1E+07 | -8973485.366 |
|  |  |  | P&L | **0** |

***Table 9*** Audit Check through Forward Rate Method for 27 March 2017

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| Cashflow Date | Euribor Forward Rate (%) | Euribor Floating | Euribor Swap Fixed | Netted | Euribor Discounted |
| 12M | 0.35 | 35875 | -119311.1111 | -83436.11111 | -83137.85406 |
| 2Y | 0.023002764 | 2332.224718 | -118000 | -115667.7753 | -115227.4269 |
| 3Y | 0.366362716 | 37043.34131 | -117672.2222 | -80628.88091 | -80025.48478 |
| 4Y | 0.645270431 | 65064.76845 | -117016.6667 | -51951.89822 | -51229.78441 |
| 5Y | 0.944784558 | 95790.65656 | -118000 | -22209.34344 | -21692.8439 |
| 6Y | 1.33610007 | 135465.7015 | -118000 | 17465.70152 | 16831.51068 |
| 7Y | 1.636879788 | 167780.1783 | -119311.1111 | 48469.06719 | 45938.36943 |
| 8Y | 1.908651239 | 193516.0284 | -117672.2222 | 75843.8062 | 70519.14272 |
| 9Y | 2.212230341 | 222452.051 | -117016.6667 | 105435.3843 | 95899.91173 |
| 10Y | 2.521979578 | 255700.7072 | -118000 | 137700.7072 | 122124.4594 |
|  |  |  |  | P&L | **0** |

***Table 10*** The crucial information for doing audit check through the 2 approaches on 27 March 2017

|  |  |  |
| --- | --- | --- |
| Currency |  | EUR |
| Index |  | LIBOR |
| Contract Position |  | Long |
| Notional Principal |  | 10,000,000 |
| Maturity (years) |  | 7Y |
| Par Swap Rate |  | 0.76 |

When doing the audit check through Principal Method and Forward Rate Method, we need the above par swap rate, maturity years, notional principal and contract position.

***Table 11*** Audit Check through Principal Method for 24 December 2020

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Cashflow Date | Euribor Swap Fixed | Euribor Floating | Netted | Euribor Discounted |
| Value |  | 10000000 | 10000000 | 10000000 |
| 12M | -218000 |  | -218000 | -216791.0886 |
| 2Y | -218000 |  | -218000 | -215363.8548 |
| 3Y | -218605.5556 |  | -218606 | -213298.1383 |
| 4Y | -217394.4444 |  | -217394 | -208200.5944 |
| 5Y | -218000 |  | -218000 | -204038.4638 |
| 6Y | -218000 |  | -218000 | -198837.9166 |
| 7Y | -217394.4444 |  | -217394 | -192397.6246 |
| 8Y | -219816.6667 |  | -219817 | -188755.8402 |
| 9Y | -218000 |  | -218000 | -181074.2436 |
| 10Y | -218000 | -10000000 | -1E+07 | -8181242.235 |
|  |  |  | P&L | **0** |

***Table 12*** Audit Check through Forward Rate Method for 24 December 2020

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Cashflow Date | Euribor Forward Rate (%) | Euribor Floating | Euribor Swap Fixed | Netted | Euribor Discounted |
| 12M | 0.55 | 55763.88889 | -218000 | -162236.1111 | -161336.4364 |
| 2Y | 0.653629919 | 66270.81124 | -218000 | -151729.1888 | -149894.4174 |
| 3Y | 1.225110764 | 124893.2362 | -218605.5556 | -93712.31936 | -91437.12384 |
| 4Y | 1.860131882 | 188080.0014 | -217394.4444 | -29314.44303 | -28074.703 |
| 5Y | 2.292272009 | 232410.912 | -218000 | 14410.91199 | 13487.98322 |
| 6Y | 2.579642168 | 261547.0532 | -218000 | 43547.05316 | 39719.29049 |
| 7Y | 3.018388335 | 306031.0395 | -217394.4444 | 88636.5951 | 78444.83051 |
| 8Y | 2.998433815 | 306506.5677 | -219816.6667 | 86689.90106 | 74440.32959 |
| 9Y | 3.343579001 | 338072.9879 | -218000 | 120072.9879 | 99734.52045 |
| 10Y | 3.688905887 | 374014.0691 | -218000 | 156014.0691 | 124915.7263 |
|  |  |  |  | P&L | **0** |

We used equation 3.1, 3.2 and 3.3 to run the principal method and forward rate method. We have proved that the swap is trading at par.

***Table 13*** Exponential Interpolation on 24 December 2020

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Time | t | (1-t) | t1 | t2 | Zero- Coupon Bond Price, B(0,t) |
| 1M |  |  |  |  | **0.999991111** |
| 3M |  |  |  |  |  |
| 3M5D |  |  |  |  | **0.999773164** |
| 6M |  |  |  |  |  |
| 12M |  |  |  |  |  |
| 1Y3M5D | **0.750684932** | **0.249315068** | **0.935813194** | **0.156246501** | **0.996060591** |
| 2Y |  |  |  |  |  |
| 2Y3M5D | **0.75** | **0.25** | **0.842983651** | **0.18784153** | **0.995384292** |
| 3Y |  |  |  |  |  |
| 3Y3M5D | **0.746556474** | **0.253443526** | **0.809109475** | **0.206432441** | **0.991072776** |
| 4Y |  |  |  |  |  |
| 4Y3M5D | **0.734246575** | **0.265753425** | **0.782995321** | **0.226749089** | **0.983836782** |
| 5Y |  |  |  |  |  |
| 5Y3M5D | **0.736986301** | **0.263013699** | **0.775732569** | **0.230722195** | **0.973568548** |
| 6Y |  |  |  |  |  |
| 6Y3M5D | **0.739837398** | **0.260162602** | **0.772253825** | **0.232418699** | **0.959803913** |

The above table shows the Zero-Coupon Bond Price derived from exponential Interpolation.

***Table 14*** The crucial information for doing profit and loss through the 2 approaches on 24 December 2020



***Table 15*** P&L calculated through Principal Method on 24 December 2020 (market-to-market)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Cashflow Date | Euribor Swap Fixed | Euribor Floating (Reset) | Euribor Floating | Netted | Euribor Discounted |
| 29/3/2021 | -57971.8519 | 9566.666667 | 10000000 | 9951595 | 9949337.44 |
| 29/3/2022 | -218000 |  |  | -218000 | -217141.21 |
| 29/3/2023 | -218000 |  |  | -218000 | -216993.78 |
| 28/3/2024 | -217394.444 |  |  | -217394 | -215453.72 |
| 31/3/2025 | -219816.667 |  |  | -219817 | -216263.72 |
| 30/3/2026 | -218000 |  |  | -218000 | -212237.94 |
| 30/3/2027 | -218000 |  | -1E+07 | -1E+07 | -9807276.4 |
|  |  |  |  | P&L | -**936029.31** |

***Table 16*** P&L calculated through Forward Rate Method on 24 December 2020 (market-to-market)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Cashflow Date | Euribor Forward Rate (%) | | Euribor Floating (Reset) | Euribor Floating | Euribor Swap Fixed | Netted | Euribor Discounted |
| 29/3/2021 | | 0.085978 | 9566.667 |  | -57971.9 | -48405.18519 | -48394.205 |
| 29/3/2022 | | 0.36762 |  | 37272.57 | -218000 | -180727.4296 | -180015.47 |
| 29/3/2023 | 0.067013 | |  | 6794.348 | -218000 | -211205.652 | -210230.79 |
| 28/3/2024 | 0.429076 | |  | 43503.52 | -217394 | -173890.9254 | -172338.56 |
| 31/3/2025 | 0.719498 | |  | 73548.73 | -219817 | -146267.9337 | -143903.77 |
| 30/3/2026 | 1.04311 | |  | 105470.1 | -218000 | -112529.9391 | -109555.61 |
| 30/3/2027 | 1.414464 | |  | 143410.9 | -218000 | -74589.09564 | -71590.906 |
|  |  | |  |  |  | P&L | -**936029.31** |

At 24 December 2020, we discount the future 7 cash flows and got a PV of -936029.31. We suffer loss.

**Principal Method**

The fixed cashflows was set on 27th March 2017. The next floating cashflow was reset on 27th March 2020. The remaining floating cashflows are stochastic, we add the 10,000,000 to turn the cashflows into non-stochastic. After that, we subtract the discounted fixed payments from the principal. The P&L is -936029.31, meaning we have loss EUR 936029.31.

**Forward Rate Method**

We need to pay fixed cashflows which were set on the 27th March 2017. We will receive floating cashflows. The cash flows that we will be receiving is determined by the forward rate of next cashflow on 29th March 2021, which was already reset on 27th March 2020, which is 0.55%. After adjust for the remaining days between value date and next cashflow date, we would need to pay EUR 57971.85 and receive EUR 9566.667 on the next cashflow date. The P&L is -936029.31, meaning we have loss EUR 936029.31.

**Results**

**Question B**

***Table 17***. Linear Interpolation for EONIA Swap Rate on 24/12/2020

|  |  |  |  |
| --- | --- | --- | --- |
|  | Linear Interpolation | | Stripped Par Rate |
| Year | t | (1-t) | R(0,t) |
| 5Y |  |  | 0.24 |
| 6Y | 0.502725 | 0.497275 | **0.30962** |
| 7Y |  |  | 0.38 |
| 8Y | 0.665751 | 0.334249 | **0.46356** |
| 9Y | 0.334249 | 0.665751 | **0.54644** |
| 10Y |  |  | 0.63 |

The above table shows the result of executing linear stripping on the EONIA swap rate, which aim to find the missing stripped par rate of year six, seven and eight.

***Table 18***. Bootstrapping for EONIA SWAP Rate on 24/12/2020

|  |  |  |  |
| --- | --- | --- | --- |
| Year | Swap Accrual (Actual/360) | Zero-Coupon Bond Price | Swap Sum Term |
|  | a | B(0,t) |  |
| 1Y | 0.01025 | **0.998434206** | 0.010233951 |
| 2Y | 0.010138889 | **0.997151845** | 0.020343962 |
| 3Y | 0.010111111 | **0.995135059** | 0.030405884 |
| 4Y | 0.010083333 | **0.992321759** | 0.040411795 |
| 5Y | 0.010138889 | **0.987897286** | 0.050427975 |
| 6Y | 0.010138889 | **0.98130606** | 0.060377329 |
| 7Y | 0.01025 | **0.973265745** | 0.070353302 |
| 8Y | 0.010138889 | **0.962861409** | 0.080115647 |
| 9Y | 0.010055556 | **0.950996315** | 0.089678444 |
| 10Y | 0.010138889 | **0.937514209** |  |

The above table shows the result of zero-coupon bond price, which was produced using bootstrapping method.

***Table 19***. Exponential Interpolation for OIS Zero coupon Rate on 24/12/2020

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | t | (1-t) | t1 | t2 | Zero Coupon Bond Price, B(0,t) |
| **3M5D** |  |  |  |  | **0.99997** |
| 6M |  |  |  |  |  |
| 12M |  |  |  |  |  |
| **1Y3M5D** | **0.750684932** | **0.249315** | **0.935813** | **0.156247** | **0.99809** |
| 2Y |  |  |  |  |  |
| **2Y3M5D** | **0.75** | **0.25** | **0.842984** | **0.187842** | **0.996685** |
| 3Y |  |  |  |  |  |
| **3Y3M5D** | **0.746556474** | **0.253444** | **0.809109** | **0.206432** | **0.994478** |
| 4Y |  |  |  |  |  |
| **4Y3M5D** | **0.734246575** | **0.265753** | **0.782995** | **0.226749** | **0.991242** |
| 5Y |  |  |  |  |  |
| **5Y3M5D** | **0.736986301** | **0.263014** | **0.775733** | **0.230722** | **0.986295** |
| 6Y |  |  |  |  |  |
| **6Y3M5D** | **0.739837398** | **0.260163** | **0.772254** | **0.232419** | **0.979345** |

Same as part a, the valuation date is 24 December 2020, which is different from the contractual cash flow date. As a result, we are not allowed to use the zero-coupon price directly. We will have to adopt the exponential interpolation method. Table 15 shows the adjusted zero-coupon bond price.

***Table 20***. Adjustment for Euribor Swap Rate on Cash Flow Date

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Cashflow Date | Zero Coupon Bond Price | Forward Accrual (Actual/360) | Swap Sum Term | Swap Market Par Rate |
|  | B(0,t) | a |  | R(0,t) |
| 29-Mar-21 | 0.999773164 | 0.002638889 | 0.002638 | **0.085978261** |
| 29-Mar-22 | 0.996060591 | 0.010138889 | 0.012737 | **0.309282866** |
| 29-Mar-23 | 0.995384292 | 0.010138889 | 0.022829 | **0.202183264** |
| 28-Mar-24 | 0.991072776 | 0.010138889 | 0.032878 | **0.27152818** |
| 31-Mar-25 | 0.983836782 | 0.010222222 | 0.042935 | **0.376460463** |
| 30-Mar-26 | 0.973568548 | 0.010111111 | 0.052779 | **0.500799** |
| 30-Mar-27 | 0.959803913 | 0.010138889 | 0.06251 | **0.643035439** |

The above table shows the working for the adjusted swap rate, which turns zero-coupon bond rate into par rate. This process is important as EURIBOR is an off-market data.

***Table 21***. EURIBOR – OIS Forward Rate

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Cashflow Date | Euribor-OIS Forward Rate | Zero Coupon Bond Price | Euribor Swap Sum Term | Euribor-OIS Sum Term |
|  |  |  |  |  |
| 29-Mar-21 | **0.153** | 0.999970084 | 0.002666587 | 0.000403738 |
| 29-Mar-22 | **0.350884807** | 0.998089738 | 0.012786108 | 0.003954524 |
| 29-Mar-23 | **0.066671217** | 0.996685065 | 0.022891387 | 0.004628255 |
| 28-Mar-24 | **0.428963102** | 0.99447827 | 0.032974292 | 0.008953449 |
| 31-Mar-25 | **0.717935875** | 0.991242326 | 0.043106991 | 0.016228078 |
| 30-Mar-26 | **1.038260928** | 0.986295103 | 0.05307953 | 0.026582176 |
| 30-Mar-27 | **1.403382429** | 0.979345079 | 0.063009001 | 0.040517021 |

The above table shows the dual-curve forward rate produced using the EURIBOR and OIS mixed rate.

***Table 22***. Collateralized Swap P&L

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Cashflow Date | EURIBOR-OIS Floating | EURIBOR Swap Fixed | Netted | Discount Factor | OIS Discounted |
|  |  |  |  |  |  |
| 29-Mar-21 | 4037.5 | -57971.85185 | -53934.4 | 0.999970084 | -53932.73835 |
| 29-Mar-22 | 35575.82075 | -218000 | -182424 | 0.998089738 | -182075.7013 |
| 29-Mar-23 | 6759.720565 | -218000 | -211240 | 0.996685065 | -210540.0316 |
| 28-Mar-24 | 43492.0923 | -217394.4444 | -173902 | 0.99447827 | -172942.1104 |
| 31-Mar-25 | 73389.00053 | -219816.6667 | -146428 | 0.991242326 | -145145.3004 |
| 30-Mar-26 | 104979.716 | -218000 | -113020 | 0.986295103 | -111471.3526 |
| 30-Mar-27 | 142287.3851 | -218000 | -75712.6 | 0.979345079 | -74148.77677 |
|  |  |  |  |  |  |
|  |  |  |  | P&L | **-950256.0114** |

The above table is to find the market-to-market value of the collateralized swap. We sum up the OIS discounted cash flow. We have recorded a P&L of -950256.0114, which means that we have loss EUR 950256. This is because the latest EONIA swap rate is lower than the former EONIA swap rate (inception date). The loss of collateralized swap is greater than uncollateralized swap.

# Conclusion

The mark to market profit or loss is calculated for the uncollateralized and collateralized swap in the report. Zero coupon bond prices for LIBOR SWAP were computed at different dates which are in March 2017, March 2020 and December 2020 whereas dates in December 2020 were used for the OIS Swap. Uncollateralized swap uses the Single-curve LIBOR discounting method. Exponential interpolation is used to adjust the discount factors. Subsequently, an audit check was set up using the Principal method and the Forward Rate method to compute the uncollateralized profit or loss results of the long position n in the existing off-market 10–year swap worth which is a loss of 936029.31. On observation, it is proven that both methods had the same results. In the collateralized swap section that utilizes a dual curve method, Projected LIBOR-OIS rates are used to determine the results by using the Forward Rate Method and EONIA is utilized as the discount factor rather than EURIBOR which is considered more of a risky discount factor. Therefore,  the collateralized swap cash results is a loss of 950256.0114 and is to be deposited by the counterparty with the mark-to-market loss of an out-of-the-money swap as of the current trade (dealing) date, 22 December 2020. Theoretically, mark to market value for uncollateralized swap is supposedly lower than the collateralized swap and the report has proven it.

# Appendix (Formula)

1.1

1.2

1.3

2.1

2.2

2.3

2.4

2.5

2.6

3.1

3.2

3.3

3.4

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4.15

4.16

5.1

5.2