School of Computing and Information Systems
COMP90038 Algorithms and Complexity Tutorial Week 12

1. Recall the definition of the knapsack problem. Given a set of items $S = \{i_1, i_2, \ldots i_n\}$ with

   - weights: $w(i_1), w(i_2), \ldots, w(i_n)$
   - values: $v(i_1), v(i_2), \ldots, v(i_n)$

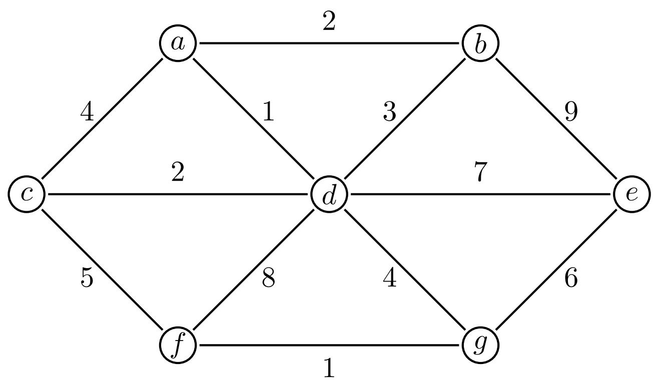
   and a knapsack of capacity W, find the most valuable selection of items that will fit in the knapsack. That is, find a set $I \subseteq S$ such that $\sum_{i \in I} w(i) \leq W$ and so that $\sum_{i \in I} v(i)$ is maximised.

   Define the *benefit* of an item i to be the rational number $v(i)/w(i)$. Consider the following greedy approach to the problem:

   Let $A[1]...A[n]$ hold the items from S, in decreasing order of benefit
   $val \leftarrow 0$
   $weight \leftarrow 0$
   $k \leftarrow 1$
   while $k \leq n \wedge weight + w(A[k]) \leq W$ **do**
       select $A[k]$
       $val \leftarrow val + v(A[k])$
       $weight \leftarrow weight + w(A[k])$
       $k \leftarrow k + 1$

   That is, at each step, from the remaining items we simply pick the one that has the greatest benefit. Give a simple example to show that this greedy algorithm does not solve the knapsack problem.

2. Work through Prim's algorithm for the graph below. Assume the algorithm starts by selecting node a. Which edges are selected for the minimum spanning tree, and in which order?



3. Use Dijkstra's algorithm to find the shortest paths for node e in the previous question's graph. That is, run the algorithm to determine the length of the shortest path from e to v, for all seven nodes v. Is the shortest path from e to b part of the graph's minimum spanning tree?

4. Lemuel Gulliver wishes to compress the string "all␣big␣endians␣and␣all␣small␣endians". Help him by building a Huffman tree for the string (there may be several valid trees) and assign a binary code accordingly, to each of the eleven characters involved (we have used ␣ to make each space character visible). The frequencies are:

a	b	d	e	g	i	l	m	n	s	␣
6	1	3	2	1	3	6	1	5	3	6

   How many bits are required for the encoded string?