

COMP90038 Algorithms and Complexity

Balanced Trees

Remember when I talk about the tree and the tree value, the `.Key` or `.Value`
All I do with the rotation is changing the `.left` and `.right` pointing to, i'm ranging
It in some way.

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Lecture 15

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Approaches to Balanced Binary Search Trees

To optimise the performance of BST search, it is important to keep trees (reasonably) balanced.

Instance simplification approaches: Self-balancing trees

- AVL trees
- Red-black trees
- Splay trees

You are going to be asked for the height of the tree in the EXAM, whether the answer or the algorithm !!!!!!!!!

AVL tree is the type of the balanced tree.

Representational changes:

- 2-3 trees
- 2-3-4 trees
- B-trees

AVL Trees

Named after Adelson-Velsky and Landis.

Recall that we defined the height of the empty tree as -1 .

For a binary (sub-) tree, let the **balance factor** be the difference between the height of its left sub-tree and that of its right sub-tree.

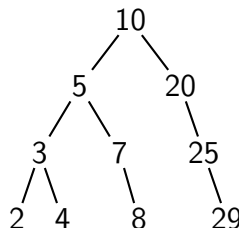
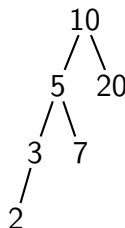
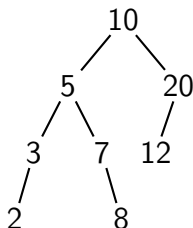
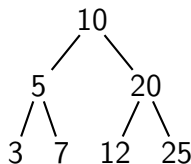
An **AVL tree** is a BST in which the **balance factor** is -1 , 0 , or 1 , for every sub-tree.



AVL Trees: Examples and Counter-Examples

We are going to be asked to build a tree like this, the AVL tree
Giving a stream of nodes, build a AVL tree well insert the tree

Which of these are AVL trees?



Building an AVL Tree

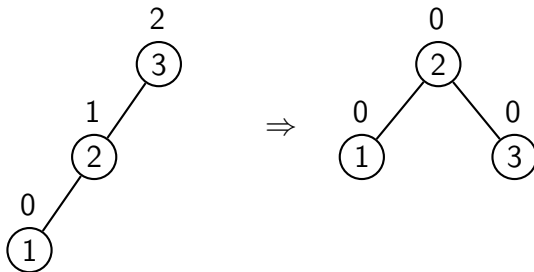
As with standard BSTs, insertion of a new node always takes place at the fringe of the tree.

If insertion of the new node makes the AVL tree unbalanced (some nodes get balance factors of 2 or -2), transform the tree to regain its balance.

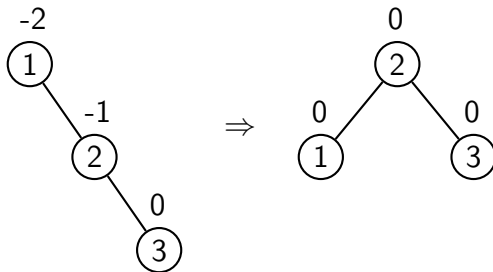
Regaining balance can be achieved with one or two simple, local transformations, so-called **rotations**.

AVL Trees: R-Rotation

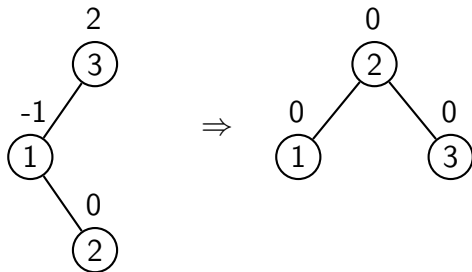
The rotation always start with the lower node



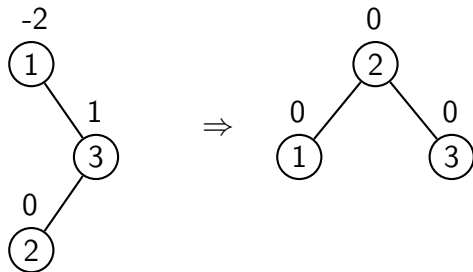
AVL Trees: L-Rotation



AVL Trees: LR-Rotation

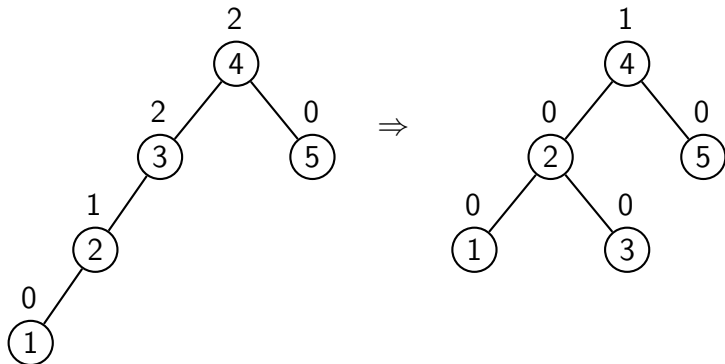


AVL Trees: RL-Rotation



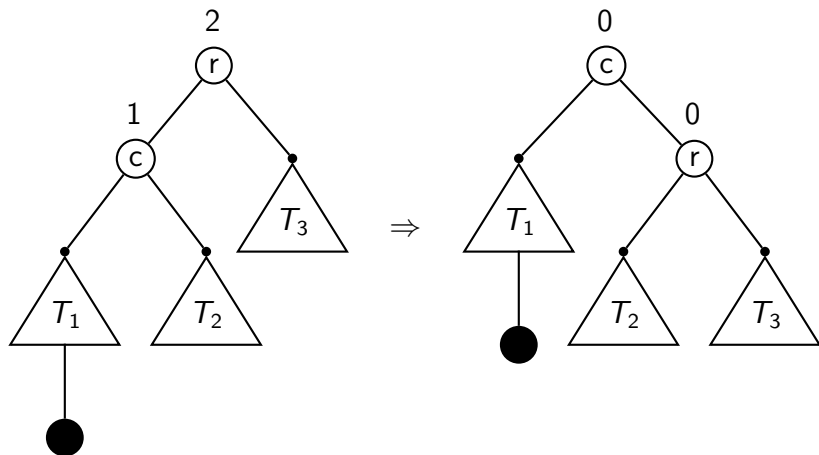
AVL Trees: Where to Perform the Rotation

Along an unbalanced path, we may have several nodes with balance factor 2 (or -2):



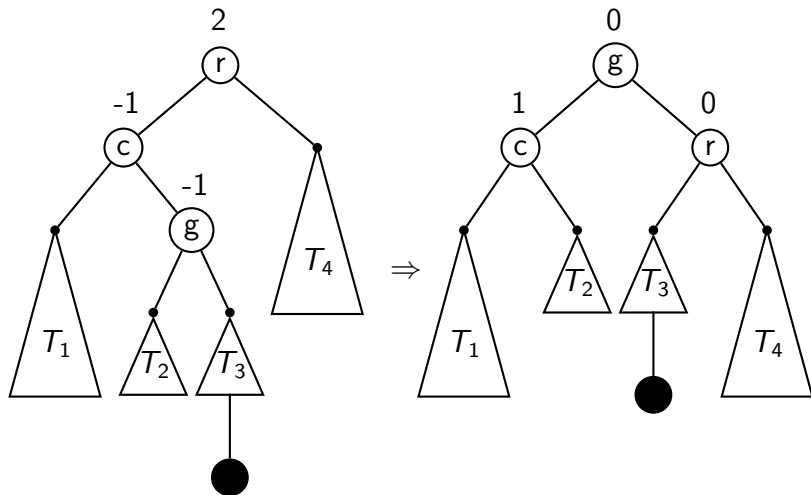
It is always the **lowest** unbalanced subtree that is re-balanced.

AVL Trees: The Single Rotation, Generally



This shows an **R-rotation**; an **L-rotation** is similar.

AVL Trees: The Double Rotation, Generally



This shows an **LR-rotation**; an **RL-rotation** is similar.

Properties of AVL Trees

The notion of “balance” that is implied by the AVL condition is sufficient to guarantee that the depth of an AVL tree with n nodes is $\Theta(\log n)$.

For random data, the depth is very close to $\log_2 n$, the optimum.

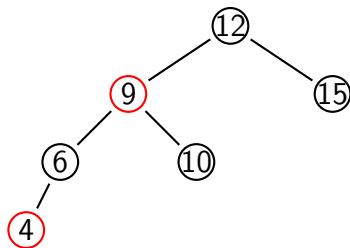
In the worst case, search will need at most 45% more comparisons than with a perfectly balanced BST.

Deletion is harder to implement than insertion, but also $\Theta(\log n)$.

Other Kinds of Balanced Trees

A **red-black tree** is a BSTs with a slightly different concept of “balanced”. Its nodes are coloured red or black, so that

- 1 No red node has a red child.
- 2 Every path from the root to the fringe has the same number of black nodes.



A worst-case red-black tree (the longest path is twice as long as the shortest path).

A **splay tree** is a BST which is not only self-adjusting, but also **adaptive**. Frequently accessed items are brought closer to the root, so their access becomes cheaper.

2-3 Trees

2-3 trees and 2-3-4 trees are search trees that allow more than one item to be stored in a tree node.

As with BSTs, a node that holds a single item has (at most) two children.

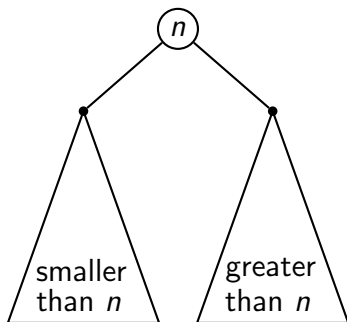
A node that holds two items (a so-called **3-node**) has (at most) three children.

And for 2-3-4 trees, a node that holds three items (a **4-node**) has (at most) four.

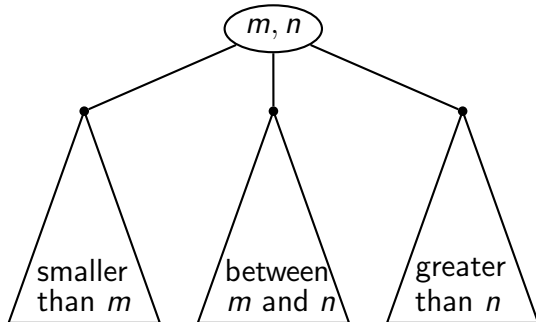
This allows for a simple way of keeping search trees balanced.

2-Nodes and 3-Nodes

2-node



3-node



Insertion in a 2–3 Tree

To insert a key k , pretend that we are searching for k .

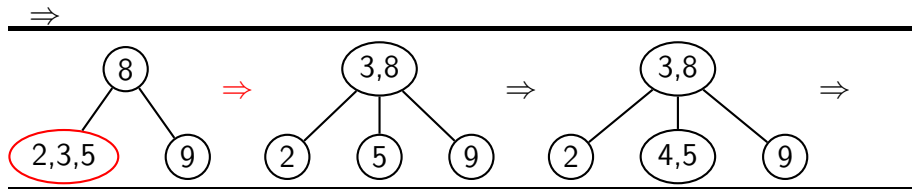
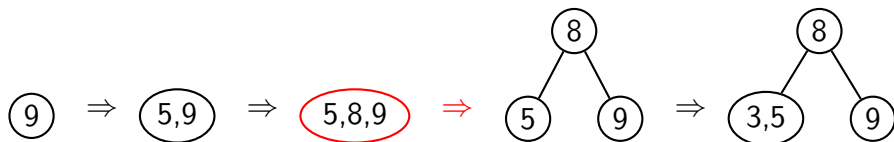
This will take us to a leaf node in the tree, where k should now be inserted; if the node we find there is a 2-node, k can be inserted without further ado.

Otherwise we had a 3-node, and the two inhabitants, together with k , momentarily form a node with three elements; in sorted order, call them k_1 , k_2 , and k_3 .

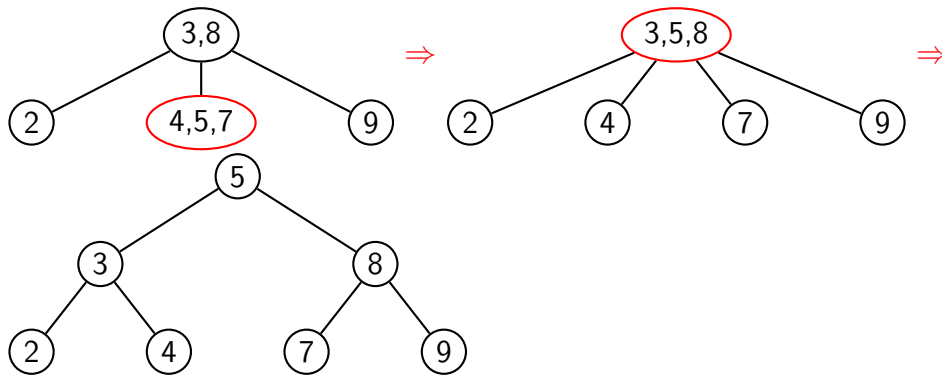
We now **split** the node, so that k_1 and k_3 form their own individual 2-nodes. The middle key, k_2 is **promoted** to the parent node.

The promotion may cause the parent node to overflow, in which case **it** gets split the same way. The only time the tree's height changes is when the root overflows.

Example: Build a 2-3 Tree from 9, 5, 8, 3, 2, 4, 7



Example: Build a 2-3 Tree from 9, 5, 8, 3, 2, 4, 7



Exercise: 2–3 Tree Construction

Build the 2–3 tree that results from inserting these keys, in the given order, into an initially empty tree:

C, O, M, P, U, T, I, N, G



2-3 Tree Analysis

Worst case search time results when all nodes are 2-nodes.

The relation between the number n of nodes and the height h is:

$$n = 1 + 2 + 4 + \dots + 2^h = 2^{h+1} - 1$$

That is, $\log_2(n + 1) = h + 1$.

In the best case, all nodes are 3-nodes:

$$n = 2 + 2 \times 3 + 2 \times 3^2 + \dots + 2 \times 3^h = 3^{h+1} - 1$$

That is, $\log_3(n + 1) = h + 1$.

Hence we have $\log_3(n + 1) - 1 \leq h \leq \log_2(n + 1) - 1$.

~~Useful formula: $\sum_{i=0}^n a^i = \frac{a^{n+1} - 1}{a - 1}$, for $a \neq 1$.~~

Coming Soon to a Theatre Near You

How to buy time, by spending a bit of space.