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Including sustainability criteria into inventory models

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ABSTRACT

Research on sustainability performance has considerably enriched operations management literature in recent years. However, work with quantitative models is still scarce. This paper contributes by revisiting classical inventory methods taking sustainability concerns into account. We believe that reducing all aspects of sustainable development to a single objective is not desirable. We thus reformulate the classical economic order quantity model as a multiobjective problem. We refer to this model as the sustainable order quantity model. Then, a multi-echelon extension of the sustainable order quantity model is studied. For both models, the set of efficient solutions (Pareto optimal solutions) is analytically characterized. These results are used to provide some insights about the effectiveness of different regulatory policies to control carbon emissions. We also use an interactive procedure that allows the decision maker to quickly identify the best option among these solutions. The proposed interactive procedure is a new combination of multi-criteria decision analysis techniques.

1. Introduction

Sustainable development (SD) is becoming a key issue for companies worldwide. Facing governments, customers and other stakeholders' pressures, the firms are undertaking initiatives to reduce their environmental and social impacts while continuing to be profitable. Following this trend, the literature dealing with SD and operations is becoming abundant (Linton et al., 2007; Carter and Rogers, 2008; Kleindorfer et al., 2005; Seuring and Müller, 2008). However, the papers dealing with quantitative models have up to now mainly focused on reverse and closed-loop logistics or on waste management (Sbihi and Eglese, 2007). We are interested here in investigating the potential for optimizing operations with SD concerns. Operations management decisions can be classified into three levels, i.e. strategic, tactical and operational. Although sustainable supply chains have to be considered globally, researchers and practitioners lack clear guidelines on how to allocate efforts between these three decision levels (Carter and Rogers, 2008). This paper focuses on the operational level for two reasons. First, operational adjustments are effective to improve the sustainability of supply chains (Hua et al., 2011). Second, operational decisions can be easily adjusted in connection with the other decision levels if needed.

This paper aims to include SD criteria into inventory models. The related literature is quite limited and has mainly focused on carbon footprint. Venkat (2007) considers a two-echelon serial system and studies the impact of batch size in terms of carbon emissions. Two main conclusions are presented. First, frequent deliveries of small batches can increase the carbon footprint of the supply chain if the distances are important. Second, carbon emissions associated with the storage of products that require refrigeration can counterbalance the advantages of full truck-load deliveries. Benjaafar et al. (2010) incorporate carbon emission constraints on single and multi-stage lot-sizing models with a cost minimization objective. Four regulatory policy settings are considered, based respectively on a strict carbon cap, a tax on the amount of emissions, the cap-and-trade system and the possibility to invest in carbon offsets to mitigate carbon caps. Insights are derived from an extensive numerical study. In a paper proposing a research agenda for designing environmentally responsible inventory systems, Bonney and Jaber (2011) briefly present an illustrative model that includes vehicle emissions cost into the economic order quantity (EOQ) model. The authors refer to this model as the environmental economic order quantity. Emissions associated with the storage of products are not taken into account. The order quantity is thus larger than the classical EOQ. Tao et al. (2010) studies the integration of a green cost into the economic production quantity model and the EOQ model. A green cost per product unit and time unit is added to the classical models and optimal solutions are analytically derived. Hua et al. (2011) extend the EOQ model to take carbon emissions into account under the cap and trade system. Analytical and numerical results are presented and managerial insights are derived. Jaber et al. (in press) include emissions from manufacturing processes into a two-echelon supply chain model. Different emissions trading schemes are studied. Analytical and numerical results are used to provide managerial insights. The efficiency of the different emissions trading schemes under study

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is also discussed. Finally, Arslan and Turkay (2010) include carbon emissions and working hours into the EOQ model. The four regulatory policies studied in Benjaafar et al. (2010) are considered. Moreover, the authors study the case where the carbon footprint is treated as an additional economic cost.

Except Venkat (2007) who does not consider the cost, these papers can be classified as a regulation based integration of SD (or its restriction to carbon footprint) into inventory models. The insights drawn are relevant and much more work can be done in this direction. However, the regulation is not the only green pressure for companies. Firms indeed are becoming increasingly proactive with respect to SD. The concept of SD popularized by Brundtland's report (WCED, 1987) was initially seen as an answer to the resource depletion problem. The United Nations and national governments have been the driving force behind SD, and a lot of companies were at first reluctant to include SD concerns into their business model. Firms were mainly convinced that SD issues would erode their competitiveness. The situation has evolved in the nineties and the link between sustainability and profitability became a true debate (Porter and van der Linde, 1995). Nowadays, even though this question remains open in the literature, more and more consumers are becoming aware of SD issues (Wakeland et al., 2009; Blengini and Shields, 2010). Firms thus seek to get competitive advantage by selling greener products. This trend is reflected in a 2008 survey of 40 chief executive officers from many of the largest third-party logistics industries worldwide (Lieb and Lieb, 2010). In order of importance, the top three reasons to establish sustainability programs were "The corporate desire to do the right thing", "The pressure from customers" and "The corporate desire to enhance company image". Another recent survey of 582 European companies highlights that regulation is no longer considered as the most important reason to establish sustainability programs (BearingPoint, 2011). In order to reflect this trend, other ways to include sustainability criteria into inventory models should be studied.

One possible way is the one followed by El Saadany et al. (2011) who study a two-echelon supply chain model where the demand is assumed to be a function of the price and product's environmental quality. Analytical results and numerical examples are provided. To our knowledge, this is the only published work that directly account for customers' pressure while studying sustainable inventory systems. Another way to include sustainability criteria into inventory models is proposed in this paper. Instead of specifically focusing on customers' pressure, we assume that the firm will decide on economic, environmental and social tradeoffs by taking into account all the green pressures that are faced. This work is thus complementary to the existing literature. Moreover, we believe that reducing all aspects of sustainability to a single objective is not desirable. This paper then adopts the concept of strong sustainability (Neumayer, 2003). We thus study a multiobjective formulation of the EOQ model. We refer to this model as the sustainable order quantity (SOQ) model. A multi-echelon extension of the SOQ model is then studied. For both models, the set of efficient solutions (Pareto optimal solutions) is analytically identified. However, the study of all these solutions can become too time-consuming in practice, especially in an operational context where decisions may be taken several times a day. In this setting, the decision process should quickly end up with a unique solution. We thus propose an interactive multiobjective optimization procedure that enables the firm to provide preference information about economic, environmental and social tradeoffs in order to quickly identify a satisfactory solution. Our contribution is thus threefold. First we present an innovative way to include SD criteria into inventory models. Second, the multiobjective optimization results are used to highlight some insights about the effectiveness of different regulatory policies to control carbon emissions. Finally, the proposed interactive procedure enables users quickly to identify a satisfactory solution and to implement the model in practice.

The paper is organized as follows. The proposed interactive procedure is presented in Section 2 after a review of the related theoretical background. In Section 3, the multiobjective formulation of the EOQ model is presented. An extension of the SOQ model to the multi-echelon case is studied in Section 4. Finally, a discussion and conclusion are presented in Section 5.

2. A new interactive procedure to identify a satisfactory solution

2.1. Theoretical background

Methods developed for multiobjective optimization problems can be classified into four classes i.e. no-preference methods, *a priori* methods, a posteriori methods and interactive methods, depending on the role of the decision maker (DM) in the solution process (Miettinen, 1999). The method proposed in this paper belongs to the latter class. In interactive methods, the preference information obtained from the DM is used to direct the solution process and only a subset of solutions is generated and evaluated. Solving a multiobjective optimization problem interactively is a constructive process consisting of several iterations where the DM builds a conviction of what is possible and confronts this knowledge with his/her preferences that may also evolve. In this setting, the most important stopping criterion is the DM's conviction that a satisfactory solution has been reached. (Miettinen et al., 2008).

In this paper, a non-empty set of alternatives (operational decisions) A is evaluated on a family of n criteria $C_1; C_2; \ldots; C_n$ with $C_i : A \rightarrow \Re\ \forall i \in [1, n]$ (the symbol $\forall$ corresponds to "for all"). We assume that the criteria represent SD impacts that should be minimized. An alternative $a \in A$ is said to be dominated if $\exists b \in A$ so that $\forall i \in [1, n]$, $C_i(b) \leqslant C_i(a)$ with at least one strict inequality (the symbol$\exists$ corresponds to "there exists"). The non-dominated solutions are called efficient solutions (Pareto optimal solutions) and the set of efficient frontier.

To rank the different alternatives of A, an aggregation model is constructed on the basis of preference information provided by the DM. This aggregation model is called a preference model. The preference model considered in this paper is in the form of an additive value function $V : A \rightarrow \Re$, such that $\forall a \in A$,

$$V(a) = \sum_{i=1}^{n} v_i(C_i(a)), \tag{1}$$

where v_i are monotonic decreasing marginal value functions, $v_i : \Re \rightarrow \Re,\ \forall i \in [1, n]$ (Keeney and Raiffa, 1976). The bigger is $V(a)$, the better is alternative a for the DM. One possible way to elicit such a preference model is to directly ask the DM for some parameters of the targeted value function. Another approach consists of deducing value functions that are compatible with preference information given by the DM. In this second approach know as the preference disaggregation paradigm (Jacquet-Lagreze and Siskos, 1982), a finite subset of A called the learning set A_L is proposed to the DM who is required to compare some alternatives of A_L. This approach allows the DM to gain more insights about his/her own preferences and a better knowledge of the problem. Furthermore, judgments on alternatives are acknowledged as less demanding in terms of cognitive effort. The main difficulty encountered when using preference disaggregation is that several value functions are often compatible with the information obtained from the DM. The available methods can then be classified into two classes, depending on how they handle the multiplicity of compatible value functions. The first one includes UTA-GMS (Greco et al., 2008a) and GRIP (Figueira et al., 2009). These methods deal with all the value functions compatible with the preference information obtained from the DM and seek robust conclusions. For the second class of methods known as meta-UTA techniques, a particular value function is selected by using some predefined rules (Siskos et al., 2005; Jacquet-Lagrèze and Siskos, 2001). There are four main meta-UTA techniques, i.e. UTA* (Siskos and Yannacopoulos, 1985), UTAMP I (Beuthe and Scannella, 1996), UTAMP II (Beuthe and Scannella, 2001) and ACUTA (Bous et al., 2010). Moreover, Kadziński et al. (2012) propose a method for selecting a representative value function in the GRIP framework.

Combining preference disaggregation and interactive methods is not a new idea. Jacquet-Lagrèze et al. (1987) propose a method that optimizes an additive value function, which has been interactively assessed, to focus on a particular alternative of A. However, this method does not allow the DM to learn about the problem as the value function assessment is the unique interactive phase. Stewart (1987) proposes an interactive method for the progressive elimination of elements from a finite set of alternatives. In this method, the set of utility functions compatible with the preference information given by the DM is used to eliminate elements of A. Siskos and Despotis (1989) use UTA to select a value function that is optimized within a feasible region defined at each iteration on the basis of satisfaction levels. Figueira et al. (2008) present an interactive procedure where GRIP is used to build a set of additive value functions compatible with the preference information obtained from the DM. This set is applied to A to deduce necessary and possible rankings that will help the DM to either select a solution or give new preference information.

The proposed interactive procedure combines the idea of Jacquet-Lagrèze et al. (1987) consisting in optimizing a particular additive value function to focus on a "new" solution with the interactive methodology proposed by Figueira et al. (2008). The next section describes the proposed interactive procedure.

2.2. The proposed interactive procedure

This section proposes an interactive procedure aiming at quickly identifying a solution that is satisfactory for the DM. The study of all efficient solutions can indeed be too time-consuming in practice, especially in an operational context where decisions may be taken several times a day. In this context, it can be useful to start with a rather small but representative learning set and to present a "new" interesting solution to the DM. Our interactive procedure is based on this idea and consists of a number of iterations. At each iteration, a value function reflecting the preference information given by the DM is obtained by using the preference disaggregation approach. This value function is then optimized on A to highlight a "new" solution that is proposed to the DM. The procedure stops when a satisfactory solution is found. The proposed interactive procedure is described in Fig. 1.

This interactive procedure allows the DM to learn about the problem and identify what is possible as a "new" solution a^* is presented at each iteration. It also enables the DM to have evolving preferences as he/she can come back to the preference information given in *Step 2*. Moreover, the generated value function is not required to perfectly represent DM's preferences. Indeed, this value function is only used to point out a possibly interesting solution a^*. If a^* is judged unsatisfactory, new preference information can be given and a new value function can be generated.

The proposed procedure is compatible with any meta-UTA techniques. In what follows, we decide to use the ACUTA method (Bous et al., 2010) as an example. In ACUTA, the chosen piecewise linear decreasing value function is generated by computing the analytic center of the feasible value functions polyhedron. This definition is implicit and ensures uniqueness. Being situated "as far as possible" from the boundaries of the feasible value functions' polyhedron, the solution may also be considered as representative. There is however no guarantee that the selected value function perfectly represents DM's preferences. As already explained, the procedure enables the DM to either validate or reject the result. Note that the computation of the analytic center is not a linear problem. However, computations were performed using the Diviz software platform (Consortium Decision Deck, 2006–2010) and computation time remains reasonable in all of our experiments.

2.3. Discussion

Before presenting the SOQ model and applying the proposed procedure in an example, some general comments on the method can be made.

As already mentioned, several value functions are generally compatible with the preference information obtained from the DM. In the proposed interactive procedure, a specific one is chosen without any validation by the DM. We have indeed argued that this value function is only used to point out a possibly interesting solution. Instead of validating the preference model, the DM can either validate or reject the solution found by optimizing a specific value function. Another method is proposed by Stewart (1987) where the optimality of every alternative in A is checked for every utility function compatible with the preference information obtained from the DM. If the optimality of an alternative is inconsistent in every case, this one is eliminated. In this method, a non-eliminated alternative is randomly added to A_L and the DM is asked to indicate some preference information taking this new element into account. However, the work of Stewart (1987) is limited to the case where A is finite. As it will be shown in the following models, operational decision problems are often characterized by an infinite decision space. Moreover, the interactive method of Stewart (1987) does not allow the DM to have evolving preferences. By contrast, our procedure enables the DM to come back to the preference information given in *Step 2*.

It may also happen that the preference information obtained from the DM in *Step 2* leads to an empty set of compatible value functions. In this case, two options can be considered. Either the DM can reduce the number of pairwise comparisons made by focusing on the ones he/she is more comfortable with. Doing so, the problem of finding a compatible value function will be less constrained. Or it can be concluded that the DM's preferences are not compatible with an additive value function model. The proposed algorithm is also compatible with non-additive value function models. For instance, Angilella et al. (2004) propose a preference disaggregation method for non-additive value functions.

In our procedure, the appropriateness of the result is deeply influenced by the selection of the learning set in *Step 1*. The learning set should not contain too many alternatives, yet it should be representative enough of the problem. The problem of selecting the most appropriate learning set may deserve future research. However, the proposed procedure can be easily modified to make the learning set denser in the region of the proposed solution a^*. Instead of presenting only one solution to the DM at each iteration, some solutions in the neighborhood of a^* could also be proposed. We will nevertheless focus on the procedure proposed in Fig. 1 in what follows.

The procedure can also take strict caps on some criteria into account. In this case, it can be assumed that the additive value function generated in *Step 3* represents the DM's preferences under reasonable limits. The learning set can be restricted to alternatives that respect the caps and the limitations can be added to the optimization problem in *Step 4* by using constraints.

The next sections are devoted to the study of the SOQ model and to an extension of this model in the multi-echelon case.

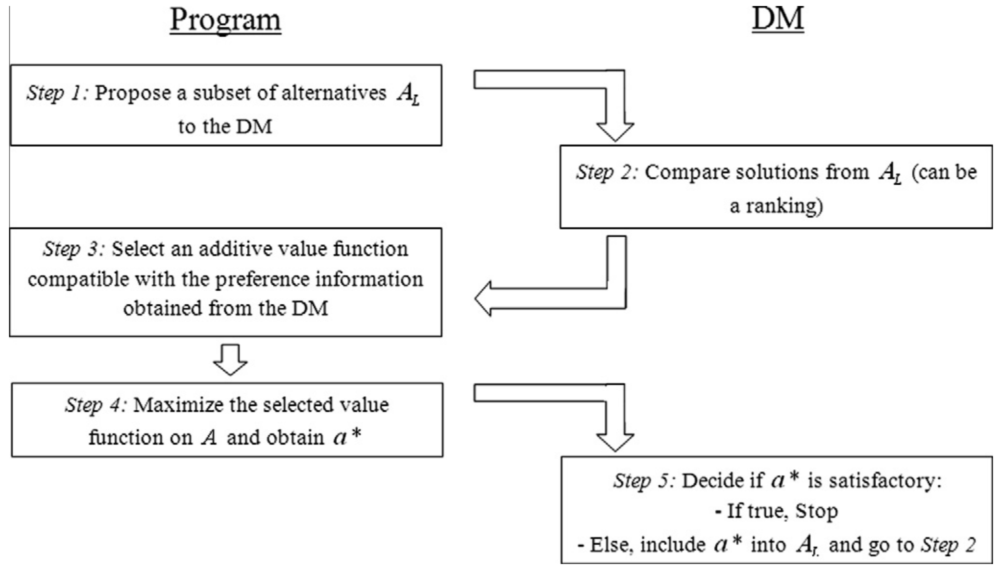

Fig. 1. The proposed interactive procedure.

3. The sustainable order quantity model

3.1. Including SD criteria into the EOQ model

The EOQ model was first derived by Harris (1913), but Wilson (1934) is also recognized in connection with this model. Assuming a constant and continuous demand, a fixed leadtime and no shortage allowed, the average total cost per time unit has the following expression:

$$C(Q) = PD + \frac{Q}{2}h + \frac{D}{Q}O, \tag{2}$$

with Q is the batch quantity (decision variable), P the fixed purchase cost per product unit, D the demand per time unit, h the constant inventory holding cost per product unit and time unit, O is the fixed ordering or setup cost.

As the cost function C is strictly convex for $Q \in \Re_+^*$, the optimal batch quantity has the following expression:

$$Q^* = \sqrt{\frac{2OD}{h}}. \tag{3}$$

It can be noticed that the value P does not affect the optimal order quantity. This parameter will thus be omitted in what follows.

Considering that minimizing the cost may not be the unique company objective, environmental and social objectives will be included into the model. We refer to this multiobjective extension of the EOQ model as the sustainable order quantity (SOQ) model. Determining the set of indicators that appropriately reflects the sustainable performances of operations is beyond the scope of this paper. Note that Bouchery et al. (2010) propose a methodology to build sustainable key performance indicators (KPIs) for distribution supply chains. A set of such KPIs for delivery and warehousing processes is also suggested.

From a general point of view, environmental and social impacts may be associated with any process of the product lifecycle. This paper focuses on operational decision models and specifically aims at including SD criteria into inventory models. In the EOQ model, decision on the order quantity affects both ordering and warehousing operations. A structure similar to Formula 2 will thus be used to quantify SD impacts. This assumption is also used in other papers (Hua et al., 2011; Benjaafar et al., 2010; Arslan and Turkay, 2010). Alternatives structures may be developed in future research.

Let n be the number of criteria ($n \in \aleph^*$). In this paper, each economic, environmental or social impact C_i is thus evaluated by using the following formula:

$$C_i(Q) = \frac{Q}{2}h_i + \frac{D}{Q}O_i, \quad \forall i \in [1, n], \tag{4}$$

with h_i, $i \in [1,n]$ is the constant inventory holding impact per product unit and time unit pertaining to criteria i, O_i, $i \in [1,n]$ is the fixed ordering impact pertaining to criteria i.

In the decision space, the set of possible values for Q is $A = \Re_+^*$. Let $C : A \to \Re^n$, $C(a) = \{C_1(a); \ldots; C_n(a)\}$, $\forall a \in A$, with C_i defined by Formula 4, $\forall i \in [1,n]$. $A^C = C(A) = \{(C_1(Q), \ldots, C_n(Q)) | Q \in A\}$ is the image of A in the criterion space (evaluation space). From a practical point of view, some alternatives of A are not of interest to the DM as there exists other alternatives that have lower impacts in every criterion. We will analytically determine the efficient frontier E of the SOQ model and derive some properties of its image $E^C = C(E)$ in the criterion space. We also introduce the following notations:

$\Re_+^n = \{(x_1, \ldots, x_n) | x_i \in \Re_+, \forall i \in [1,n]\}$ is the nonnegative subset of $\Re^n$,

Let S_1 and S_2 two subsets of $\Re^n : (S_1 + S_2) = \{s_1 + s_2 | s_1 \in S_1, s_2 \in S_2\}$ is the Minkowski sum,

$E_+^C = \left(E^C + \Re_+^n\right)$. For $n = 2$, E_+^C thus includes all the elements of E^C as well as all the elements situated at the top right of E^C (see Fig. 3 for a graphical example).

As $C_i(Q)$ is strictly convex for $Q \in \Re_+^*$, $\forall\ i \in [1,n]$, the single objective minimum is expressed as follows:

$$Q_i^* = \sqrt{\frac{2O_iD}{h_i}}. \tag{5}$$

We can assume without loss of generality that the criteria are arranged so that $Q_1^* \leqslant, \ldots, \leqslant Q_n^*$.

Theorem 1. *Let E be the efficient frontier of the SOQ problem and E^C its image in the criterion space, then:*

$E = [Q_1^*, Q_n^*]$,

E_+^C is convex.

Proofs from here onwards are provided in Appendix A. Note that Theorem 1 is valid as soon as C is a general strictly convex function. We illustrate the results with two criteria ($n = 2$), for instance the

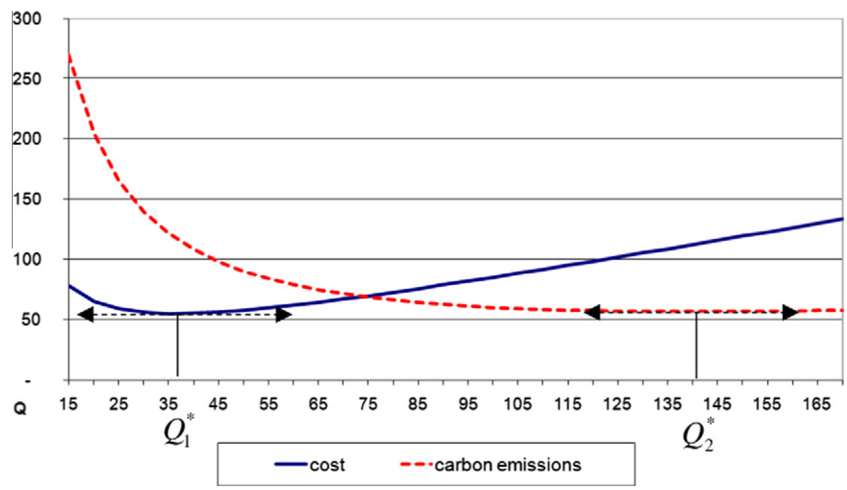

Fig. 2. Cost and carbon emissions in function of the batch size.

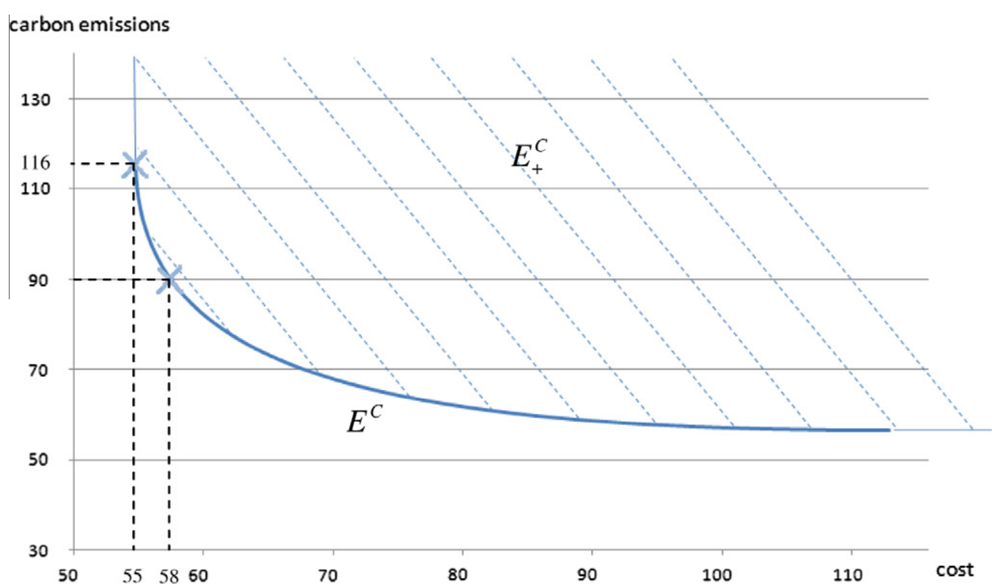

Fig. 3. The image of the efficient frontier in the criterion space.

cost and the carbon footprint. As an example, let $D = 20$ product units per time unit, $O_{cost} = O_1 = 50$, $h_{cost} = h_1 = 1.5$, $O_{emissions} = O_2 = 200$ and $h_{emissions} = h_2 = 0.4$. It can be noticed that the parameters' units are omitted. Indeed, they are not useful as only the ratios O_i/h_i matter. The parameters must nevertheless be expressed with the same unit within a criterion. Applying Formula (5), we get $Q_1^* \approx 37$ and $Q_2^* \approx 141$. Fig. 2 illustrate the results.

By applying Theorem 1, we obtain that $E = [37; 141]$. The image of the efficient frontier is illustrated in Fig. 3.

It can be noticed that a significant carbon emissions reduction can be achieved by an operational adjustment that requires only a small financial effort. In this example, the carbon emissions can be reduced by 22% (from 116 to 90) for a 5% cost increase (from 55 to 58) starting from the minimal cost (see Fig. 3). This highlights that operational adjustments are effective to improve the sustainability of supply chains. On the contrary, the financial effort will increase when getting closer to the minimum amount of emissions. In this case, the firms will tend to invest in carbon-reducing technologies in addition to operational adjustments.

In the next section, a numerical example is used to illustrate the type of interaction and the type of result that the interactive procedure proposed in Section 2.2 can produce for the SOQ model.

3.2. Applying the proposed interactive procedure to the SOQ model

In this example, three criteria are taken into account for the SOQ model. We do not advocate that the proposed criteria are the most relevant ones but they are proposed as an example. As greenhouse gases reduction is nowadays a key issue, we decide to choose the carbon footprint as an environmental criterion. The fixed amount of carbon emissions per order represents the emissions related to order processing and transportation. An amount of carbon emissions can also be associated with the storage of each unit per time unit. The social dimension of SD has received less attention in the literature (White and Lee, 2009). There is a lack of consensus on how to assess the social performance of operations. In this example, the injury rate is used as a social criterion. Injuries are indeed a major social impact of operations and are caused both by ordering and warehousing operations. We focus on a didactic example and we imagine an interaction with a fictitious DM so as to illustrate the type of interaction and the type of result that the proposed method can produce. For the numerical calculation, the chosen values are presented in Table 1.

Applying Formula 5, the three single objective optima can be calculated (see Table 2).

Table 1
Example parameter's values.

		Demand rate (D)	25		
Ordering cost (O_1)	100	Ordering carbon footprint (O_2)	320	Ordering injury rate (O_3)	119
Holding cost (h_1)	1	Holding carbon footprint (h_2)	0.45	Holding injury rate (h_3)	0.27

Table 2
Single objective optima.

	Q_i^*	Cost C_1	Carbon emissions C_2	Injuries C_3
Economic order quantity	71	70.7	128.7	51.5
Environmental order quantity	189	107.7	84.9	41.3
Social order quantity	148	90.9	87.4	40.1

Applying Theorem 1, the efficient frontier consists of any batch sizes between [71; 189]. The range on each criterion also appears in Table 2. The final solution will depend on the relative importance the DM gives to each of the three criteria.

Iteration 1:

Step 1: We decide to include the economic order quantity a_1, the environmental order quantity a_5 and the social order quantity a_3 into the learning set. The corresponding batch sizes are 71, 189 and 148 respectively (see Table 2). Only the images of the alternatives in the criterion space are presented to the DM (see Table 3). We also include two other solutions a_2 and a_4 into the learning set with corresponding batch sizes of 110 (in the middle of [71; 148]) and 169 (in the middle of [148; 189]).

Step 2: Assume that the DM provides the following preference information: $a_2 \succ a_4 \succ a_1$ ($\succ$ corresponds to strict preference).

Step 3: ACUTA is used with the provided preference information to compute a compatible value function:

$$V(Q) = \sum_{i=1}^{3} v_i(C_i(Q)). \tag{6}$$

Step 4: $V(Q)$ can then be maximized. The optimum is found for $Q = 120$, the corresponding alternative is a_6 (80.8; 93.7; 41.0).

Step 5: The DM may consider that a_6 is not satisfactory, this one is added to A_L.

Iteration 2:

Step 2: The DM may provide the following additional information: $a_2 \succ a_6 \succ a_4 \succ a_1$.

Steps 3 and 4: With this new information, a new value function can be generated and optimized. The optimum is found for $Q = 102$, the corresponding alternative is a_7 (75.5; 101.4; 42.9).

Step 5: The DM may consider that a_7 is not satisfactory, this one is added to A_L.

Iteration 3:

Step 2: The following preference information may be given by the DM: $a_2 \succ a_7 \succ a_6 \succ a_4 \succ a_1$.

Step 3 and 4: The optimum of the new value function is found for $Q = 109$, the corresponding alternative is a_8 (77.4; 98.0; 42.0).

Step 5: Assume that the solution a_8 is satisfactory for the DM, the procedure stops.

It can be noticed that the resulting solution is relatively close to alternative a_2 which was randomly generated in *Step 1*. However, the DM feels more confident with alternative a_8 as he/she has learnt about the problem and about his/her own preferences. The proposed procedure enables an effective interaction with the DM as a satisfactory solution is quickly identified.

The next section is devoted to the sensitivity analysis of the results.

3.3. Sensitivity analysis

The previous section has shown that the proposed interactive procedure allows the DM to quickly find a satisfactory solution for the SOQ model. However, this procedure will be used in practice only if it ensures a certain kind of robustness. The following result proves that the procedure is quite insensitive to a slight change or an estimation error for any parameter of the model.

Recall that in the SOQ model, n criteria ($n \in \aleph^*$) are evaluated by using Formula 4, $C_i(Q) = \frac{Q}{2} h_i + \frac{D}{Q} O_i$, $\forall i \in [1, n]$. Assume that the value function generated in the last iteration of the proposed interactive procedure represents DM's preferences. This value function is noted $V^*(Q) = \sum_{i=1}^{n} v_i^*(C_i(Q))$ and is maximal for $Q = Q^*$. By using ACUTA, $\forall i \in [1, n]$, v_i^* is piecewise linear decreasing. The following theorem proves that V^* behaves as a cost function $C_{eq}(Q) = \frac{Q}{2} h_{eq} + \frac{D}{Q} O_{eq}$ in a neighborhood of Q^*, with $h_{eq} = \sum_{i=1}^{n} \alpha_i h_i$ and $O_{eq} = \sum_{i=1}^{n} \alpha_i O_i$. It implies that V^* has the same robustness as the cost function in the EOQ model.

Theorem 2. *There exists $Q_{\min} < Q_{\max} \in \Re_+^*$ such that:*

$$Q^* \in [Q_{\min}, Q_{\max}],$$
$$\forall Q \in [Q_{\min}, Q_{\max}], \quad V^*(Q^*) - V^*(Q) = C_{eq}(Q) - C_{eq}(Q^*).$$

The coefficients α_i can be obtained by using the following formula for $Q \in [Q_{\min}, Q_{\max}]$ such that $Q \neq Q^*$:

$$\alpha_i = \frac{v_i^*(C_i(Q) - C_i(Q^*))}{C_i(Q) - C_i(Q^*)}. \tag{7}$$

For $Q \notin [Q_{\min}, Q_{\max}]$, a deviation appears between $V^*(Q^*) - V^*(Q)$ and $C_{eq}(Q) - C_{eq}(Q^*)$. Fig. 4 illustrates Theorem 2, the chosen value function is the one obtained in iteration 3 of Section 3.2. For this example, recall that $Q^* = 109$. V^* behaves like a cost function in the EOQ model for a wide range of values as the segment $[Q_{\min}, Q_{\max}]$ is equal to [95; 140].

This result strengthens the proposed interactive procedure for two main reasons. First, this ensures robust results even if an error occurs when estimating a parameter of the model. This is a crucial point when dealing with sustainability criteria as companies often face difficulties to get reliable sustainability measures. Second, this implies valid results for a longer period of time. Slight changes in parameter values often occur in operational situations. As the procedure is quite insensitive to these changes, performing the interactive procedure is not required every time. Note that Theorem 2

Table 3
The initial learning set.

	Cost C_1	Carbon emissions C_2	Injuries C_3
a_1	70.7	128.7	51.5
a_2	77.7	97.5	41.9
a_3	90.9	87.4	40.1
a_4	99.3	85.4	40.4
a_5	107.7	84.9	41.3

also implies that V^*can be considered as a weighted sum of the criteria in the neighborhood of Q^*.

4. The two-echelon serial sustainable order quantity model

4.1. Problem presentation and preliminary results

This section presents an extension of the EOQ model in a multi-echelon case. The considered model is a serial system with 2 echelons, where one warehouse supplies a single retailer (see Fig. 5). The model was first studied by Schwarz (1973).

The retailer faces a constant continuous demand. Leadtimes are assumed to be zero for clarity (fixed leadtimes can be easily handled) and no shortage is allowed. Moreover, initial inventories are assumed to be zero. Fixed ordering costs and linear holding costs are supported at each location. Let Q_r and Q_w be the batch quantities ordered by the retailer and by the warehouse respectively. An entire batch is delivered at the same time. The following result is taken from Schwarz (1973).

Preliminary Result. *An optimal policy is stationary-nested and respects the zero-inventory condition i.e.:*

Q_r and Q_w are time invariant,

$Q_w = k.Q_r$, with $k \in \aleph^*$,

The retailer orders only if its inventory level is null,

The warehouse orders when both the retailer and the warehouse have no inventory.

To simplify the notations, let $Q_r = Q$. The total cost can then be expressed as a function of Q and k:

$$C(k,Q) = (h_r + (k-1)h_w)\frac{Q}{2} + \left(O_r + \frac{O_w}{k}\right)\frac{D}{Q}, \tag{8}$$

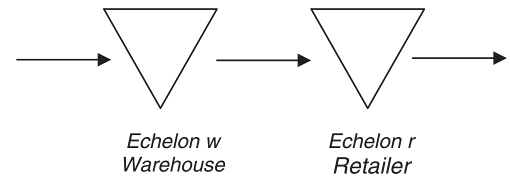

Fig. 5. The two-echelon serial system.

with Q is the batch quantity at the retailer (first decision variable), k the strictly positive integer such that $Q_w = k{\cdot}Q$ (second decision variable), D the demand per time unit, h_r the constant inventory holding cost per product unit and time unit at the retailer, h_w the constant inventory holding cost per product unit and time unit at the warehouse, O_r the fixed ordering cost at the retailer, O_w is the fixed ordering cost at the warehouse.

If $h_r < h_w$, the minimum of Formula 8 is found for $k^* = 1$. Else, let $k^{\inf} = \sqrt{\frac{O_w(h_r - h_w)}{O_r h_w}}$. k^* is a strictly positive integer that can be found by using the following rule. If $k^{\inf} < 1$, it is optimal to choose $k^* = 1$. Else, let $k' \leqslant k_{\inf} \leqslant k' + 1$ with $k' \in \aleph^*$. If $\frac{k^{\inf}}{k'} \leqslant \frac{k'+1}{k^{\inf}}$ then it is optimal to choose $k^* = k'$. Otherwise, $k^* = k' + 1$ (Axsäter, 2006). It follows that,

$$Q^* = \sqrt{\frac{2D(O_r + \frac{O_w}{k^*})}{h_r + (k^* - 1)h_w}}. \tag{9}$$

We will now consider the case where several criteria ($n \geqslant 2$) have to be taken into account and we refer to this problem as the two-echelon serial SOQ problem. Theorem 3 proves that each efficient ordering policy (efficient solution) can be found in the set of "basic" policies.

Theorem 3. *For the two-echelon serial SOQ problem, an ordering policy leading to an efficient solution is basic i.e.:*

The retailer orders only if its inventory level is null,

The warehouse orders when both the retailer and the warehouse have no inventory.

All deliveries made to the retailer between successive deliveries to the warehouse are of equal size.

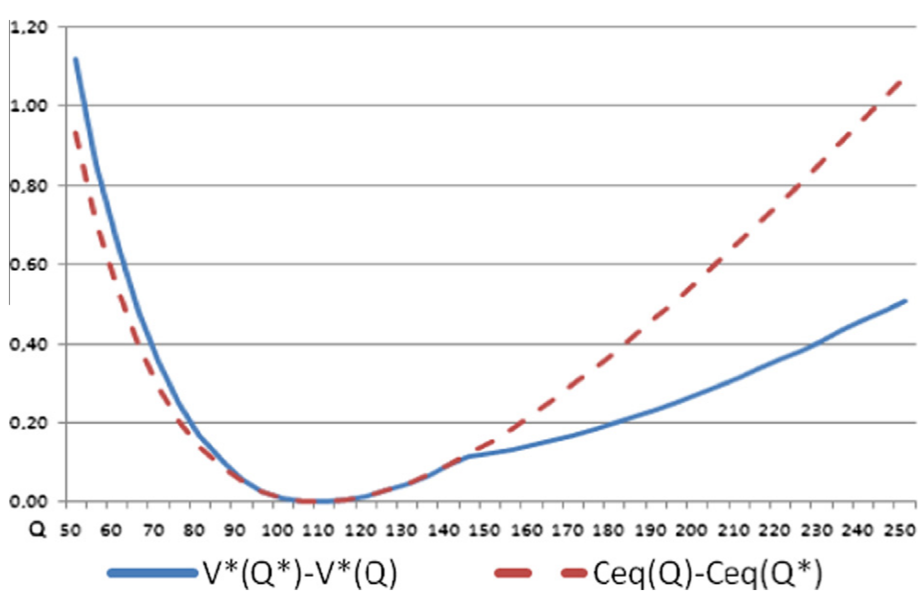

Fig. 4. Illustration of Theorem 2.

In what follows, we will restrict our attention to stationary policies as non-stationary ones are often too complicated to be implemented in practice. Note that the following results can be extended if non-stationary policies are of interest for the DM. When focusing on stationary policies, Theorem 3 can be strengthened. An ordering policy leading to an efficient solution is then stationary nested and respects the zero inventory condition as in Preliminary result. The simplified notations Q and k are kept. Each SD criteria is thus evaluated by using the following formula:

$$C_i(k,Q) = (h_{ir} + (k-1)h_{iw})\frac{Q}{2} + \left(O_{ir} + \frac{O_{iw}}{k}\right)\frac{D}{Q}, \quad \forall i \in [1,n], \tag{10}$$

with h_{ir}, $i \in [1,n]$ is the constant inventory holding impact i per product unit and time unit at the retailer, h_{iw}, $i \in [1,n]$ the constant inventory holding impact i per product unit and time unit at the warehouse, O_{ir}, $i \in [1,n]$ the ordering impact i per order at the retailer, $O_{iw}, i \in [1,n]$ is the ordering impact i per order at the warehouse.

$\forall i \in [1,n]$, if $h_{ir} < h_{iw}$, $k_i^* = 1$. Else, k_i^* is a strictly positive integer that can be found by using the rule described earlier with $k_i^{\inf} = \sqrt{\frac{O_{iw}(h_{ir}-h_{iw})}{O_{ir}h_{iw}}}$, $\forall i \in [1,n]$. The minimum of Formula 10 is found for:

$$Q_i^* = \sqrt{\frac{2D\left(O_{ir} + \frac{O_{iw}}{k_i^*}\right)}{h_{ir} + (k_i^*-1)h_{iw}}}, \quad \text{and } k_i^* \text{ defined above}, \quad \forall i \in [1,n]. \tag{11}$$

The next section is devoted to the identification of the efficient frontier of the two-echelon serial SOQ problem.

4.2. Multiobjective optimization of the two-echelon serial SOQ model

In this section, some theorems that characterize the efficient frontier of the two-echelon serial SOQ problem are presented. Compared with the single-echelon SOQ model, a strictly positive integer k is added as decision variable. Let n be the number of criteria ($n \in \aleph^*$). In the decision space, the set of possible alternatives A is $\left\{(k,Q)|k \in \aleph^*, Q \in \Re_+^*\right\}$. Let $C : A \to \Re^n$, $C(a) = \{C_1(a); \ldots; C_n(a)\}$, $\forall a \in A$, with C_i defined by Formula 10, $\forall i \in [1,n]$. The image of A in the criterion space is $A^C = \{(C_1(k,Q),\ldots,C_n(k,Q))|(k,Q) \in A\}$. Let E be the efficient frontier of the problem and $E^C = C(E)$ its image in the criterion space. Moreover, let $E_+^C = \left(E^C + \Re_+^n\right)$.

We will first consider the case with k fixed. $A_k^C = \{(C_1(k,Q),\ldots,C_n(k,Q))|Q \in \Re_+^*\}$, $\forall k \in \aleph^*$. The efficient frontier of this sub-problem is noted E_k and E_k^C is its image in the criterion space. Let $E_{k+}^C = \left(E_k^C + \Re_+^n\right)$. As Formula 11 is strictly convex in Q, assume that $Q_i^{k^*}$ minimizes $C_i(k,Q)$.

Theorem 4. *Let E_k be the efficient frontier of the two-echelon serial SOQ with k fixed and E_k^C its image in the criterion space, then:*

$$E_k = \left[\min_i\left(Q_i^{k^*}\right), \max_i\left(Q_i^{k^*}\right)\right],$$

E_{k+}^C is convex.

It can be noticed that $E^C \subset \bigcup_{k=1}^{\infty} E_k^C$. We could intuitively expect that $E^C \subset \bigcup_{k=\min(k_i^*)}^{\max(k_i^*)} E_k^C$. However, a counterexample can be found even for $n = 2$ (Table 4).

Applying Formula 11 to the example presented in Table 4, we get $k_1^* = 3$ and $k_2^* = 3$. It could then be tempting to conclude that $E^C = E_3^C$. However, some elements of E_4^C are also efficient. This can be seen in Fig. 6. In this example, $E^C \subset \left\{E_3^C \cup E_4^C\right\}$.

Theorem 5 states that a lower bound $k_{\min}$ and an upper bound $k_{\max}$ exist such that $E^C \subset \bigcup_{k=k_{\min}}^{k_{\max}} E_k^C$.

Theorem 5. *There exists $(k_{\min}, k_{\max}) \in \aleph^{*2}$ such that:*

$$1 \leqslant k_{\min} \leqslant \min_i(k_i^*),$$
$$\max_i(k_i^*) \leqslant k_{\max},$$
$$E^C \subset \bigcup_{k=k_{\min}}^{k_{\max}} E_k^C.$$

It can also be noticed in the above example that E_+^C is non-convex. This result can be generalized as soon as E^C is not included into a single set E_k^C. This condition necessarily holds when $\min_i(k_i^*) \neq \max_i(k_i^*)$. However, the example shows that the converse is not true. This result is stated in Theorem 6.

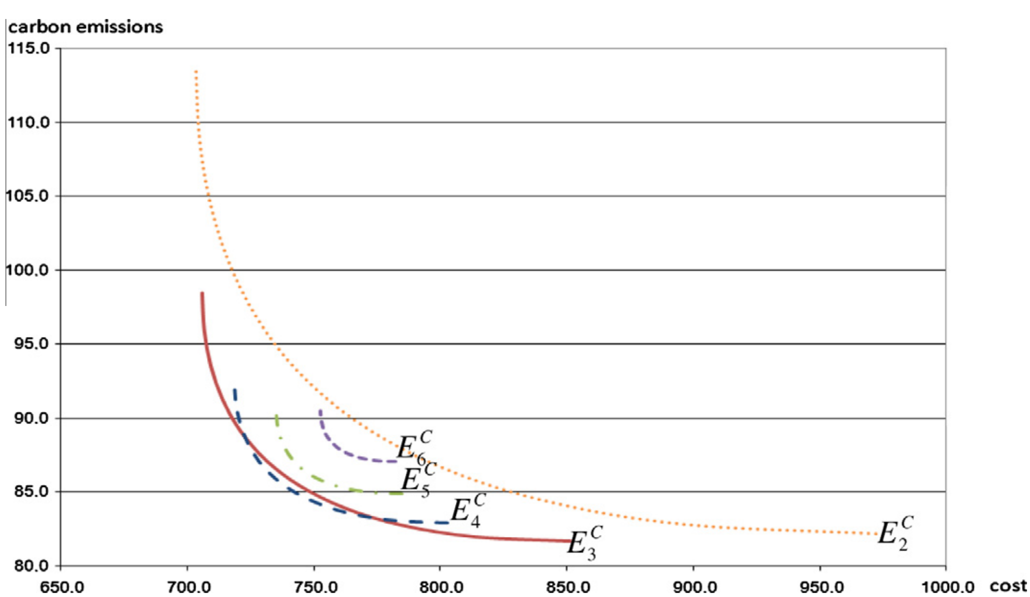

Fig. 6. The example criterion space.

Table 4
Example data set.

Demand rate (D)	50		
Holding impact 1 at the retailer (h_{1r})	10	Holding impact 2 at the retailer (h_{2r})	4
Holding impact 1 at the warehouse (h_{1w})	6	Holding impact 2 at the warehouse (h_{2w})	0.5
Ordering impact 1 at the retailer (O_{1r})	50	Ordering Impact 2 at the retailer (O_{2r})	10
Ordering impact 1 at the warehouse (O_{1w})	500	Ordering Impact 2 at the warehouse (O_{2w})	10

Table 5
Example data set.

Demand rate (D)	20		
Holding impact 1 at the retailer (h_{1r})	8	holding impact 2 at the retailer (h_{2r})	2
Holding impact 1 at the warehouse (h_{1w})	4	holding impact 2 at the warehouse (h_{2w})	0.15
Ordering impact 1 at the retailer (O_{1r})	80	ordering Impact 2 at the retailer (O_{2r})	45
Ordering impact 1 at the warehouse (O_{1w})	350	ordering Impact 2 at the warehouse (O_{2w})	70

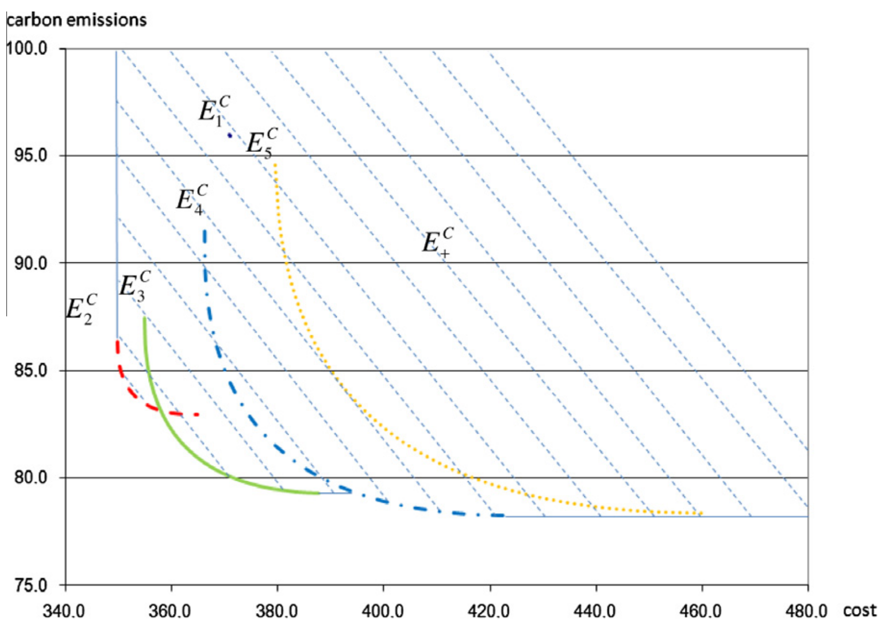

Fig. 7. The image of the efficient frontier in the criterion space.

Theorem 6. *If* $\min_i(k_i^*) < \max_i(k_i^*)$, *then* E_+^C *is non-convex.*

An illustration of the two-echelon serial SOQ problem is given with two criteria (the cost and the carbon footprint). Parameter values can be found in Table 5.

It can be noticed that E_+^C is non-convex in this example (see Fig. 7). In this case, some efficient solutions cannot be generated by using a linear combination of the objectives. For instance, $E_2^C \cap E_3^C$ is an efficient solution that cannot be found by optimizing a linear combination of the two objectives. However, this solution can represent a desirable compromise for the company. The interactive procedure described in Section 2.2 enables proposing such solutions by optimizing an additive value function instead of a simple weighted sum. This strengthens the proposed interactive procedure.

5. Discussion and conclusion

5.1. Regulatory policies and carbon emissions

Even if regulation is not directly modeled in this paper, some insights about the effectiveness of different regulatory policies to control carbon emissions can be highlighted. Existing regulatory policies may be divided into two categories, i.e. policies based on a carbon price and policies based on a carbon cap. Carbon emissions are controlled by a carbon price for the carbon tax policy as well as for the cap and trade system. Hua et al. (2011) have indeed proven that emissions levels depend only on the carbon price in the EOQ model with a fixed carbon price under the cap and trade system.

Our results show that regulatory policies based on a carbon price have some technical drawbacks. First, the minimum amount of emissions cannot be achieved as it would imply an infinite carbon price. Moreover, companies will have two possibilities to optimize their supply chains when facing a given carbon price. Companies can indeed make operational adjustments but they can also invest in carbon-reducing technologies. The different options may give the same overall result with different operational cost and carbon emissions levels. For a given carbon price, the amount of carbon emissions is thus quite unpredictable. It can then be concluded that regulatory policies based on a carbon price are not very effective in controlling carbon emissions, even if establishing these policies encourages reducing emissions.

The two-echelon serial SOQ model enables giving additional insights. As E_+^C can be non-convex, some interesting operational solutions are ruled out whatever the chosen carbon price is. This can be seen as a limitation induced by setting a carbon price. This problem could be overcome by using non-linear tax schemes. Note that the proposed interactive procedure could be used in this case to find a non-linear carbon tax function that seems satisfactory for policy makers. The marginal value functions v_i can be interpreted as the tax function and could thus be presented to policy makers.

Using a carbon cap can be seen as more effective to green supply chains as the previous drawbacks are avoided. Using a carbon cap indeed allow to directly control carbon emissions. Moreover, every efficient solution including the minimum amount of emissions can be achieved. This kind of regulatory policy may nevertheless be harder to implement as a cap has to be set up for each company.

5.2. Conclusion

This paper has presented a novel approach for integrating SD criteria into inventory model. In order to consider the different aspects of sustainability, we formulate inventory models as multiobjective problems. We have thus considered a multiobjective formulation of the EOQ model called the SOQ model. A multi-echelon extension of the SOQ model has been also studied. For both models, the efficient frontier is analytically characterized. In this paper, it is assumed that the firm can decide on economic, environmental and social tradeoffs by taking into account the different green pressures that are faced. An interactive procedure that allows the company to quickly identify the most preferred option is thus proposed. In the existing literature, the firm is usually assumed to face a single green pressure source. The company then aims at minimizing its cost under the pressure constraint. The positioning taken in this paper is thus complementary to the existing literature. It appropriately reflects actual trends as companies are becoming increasingly proactive with respect to SD. This also enables firms to go beyond strict regulatory requirements in terms of sustainability performances.

Two main findings can be highlighted when focusing on multiobjective optimization results. First, operational adjustment is proved to be an effective way to reduce SD impacts. In the SOQ model, the flat region of the cost function corresponds to a steeper region of the other criteria functions. It enables reducing any SD impact by requiring a small increase in cost. This conclusion is also made by Hua et al. (2011) based on another model. Second, we have identified problems with non-convex efficient frontiers. In this case, some efficient solutions cannot be generated by using a weighted sum. We thus propose an interactive procedure that enables to quickly take advantage of operational adjustment. This procedure is proven to be robust and allows focusing on all efficient solutions even if the efficient frontier is non-convex. Even if the method was designed for inventory decisions, it may also be efficiently applied in other operations management contexts as product and packaging design, facility location and distribution optimization for example.

Further research directions are numerous. First of all, other inventory models could be revisited. Even if the popularity of the EOQ model has been recognized by both academics and practitioners, this model has also been criticized by some researchers on the basis that its assumptions are never met (Jaber, 2009). For instance, Jaber et al. (2004) propose to use a thermodynamic approach to model the hidden costs of inventory systems. The SOQ model may also be revisited by using this approach. Imperfect operations may also be considered as they could lead to possible defective items that require rework, recycling or scrap. This could indeed affect the SD performance of the supply chain. The study of more complex models could nevertheless be difficult as optimal solutions are often hard to determine even in the single-objective case (see e.g. Crowston et al., 1973; Roundy, 1985 or Roundy, 1986). If that is the case, close to optimal solutions could be used. Other operations management decisions could also be revisited by considering the problem as multiobjective. For instance, facilities location, supplier selection and transportation mode selection also affect the sustainability of the supply chain. Moreover, the SD criteria could be modeled with more precision. In this paper, a structure similar to the classical cost function of the EOQ model is used as a first attempt. Alternative structures could be used in future work. Note that the presented multiobjective optimization results are valid as soon as the criteria are modeled by using general strictly convex functions.

An interesting question arises when the DM has to explain his/her choices. The additive value functions selected in the proposed procedure can of course be shown to explain the decision. Another solution could consist in adapting the dominance-based rough set approach proposed in Greco et al. (2008b). In this case, the DM could select a rule that is concordant with the given preference information. This rule could then be added to the model and a new learning set could be generated. This approach could simplify the explanation of the decision.

Finally, operational decisions usually involve more than one DM within a company and more than one company along the supply chain. Research aiming at including SD criteria into inventory models with multiple actors should be conducted. For instance, investigating the impact of collaboration among companies with SD concerns is of major interest. The way to handle the multiplicity of the DMs within a company should also be studied.

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Appendix A

Proof of Theorem 1. Identification of the efficient frontier:

If $Q_1^* = Q_n^*$, $E = Q_1^*$ as $C_i(Q_1^*)$ is the unique minimum on each criteria i.

Assume that $Q_1^* < Q_n^*$:

$C_1(Q)$ is strictly increasing on $[Q_1^*, Q_n^*]$,

$C_n(Q)$ is strictly decreasing on $[Q_1^*, Q_n^*]$,

$\forall i \in [1,n]$,$C_i(Q)$ is strictly increasing on $[Q_n^*, \infty)$ and strictly decreasing on $(0, Q_1^*]$ then the solution is dominated if $Q \notin [Q_1^*, Q_n^*]$,

then $E = [Q_1^*, Q_n^*]$.

Convexity:

As $\Re_+^n$ is convex, we only have to prove that $\forall (a,b) \in E^C \times E^C$, the segment $[a,b]$ is included into E_+^C.

Let $(a,b) \in E^C \times E^C$, if $a = b$, $a \in E_+^C$ by definition.

Else, let $a = C(Q_a)$ and $b = C(Q_b)$ with $(Q_a, Q_b) \in [Q_1^*, Q_n^*] \times [Q_1^*, Q_n^*]$.

$a \in E_+^C$ and $b \in E_+^C$.

$\forall \lambda \in]0,1[$, let $x_\lambda = \lambda.\ a + (1-\lambda).b$.

As C is strictly convex, x_λ is dominated by $C(\lambda.\ Q_a + (1-\lambda).Q_b)$.

So, $x_\lambda \in E_+^C$. □

Proof of Theorem 2. v_i^* are piecewise linear decreasing then there exits $(Q_{\min}, Q_{\max}) \in \Re_+^* \times \Re_+^*$ such that:

$$Q^* \in [Q_{\min}, Q_{\max}],$$
$$\forall i \in [1,n],\ \text{it exists}\ \alpha_i \in \Re_+^* | \forall Q \in [Q_{\min}, Q_{\max}],$$
$$v_i^*(C_i(Q)) = v_i(C_i(Q^*)) - \alpha_i(C_i(Q) - C_i(Q^*)).$$

By applying Formula (6), we obtain that $\forall Q \in [Q_{\min}, Q_{\max}]$:

$$V^*(Q) = \sum_{i=1}^{n} v_i^*(C_i(Q))$$
$$= V^*(Q^*) + \left(\frac{h_{eq} \cdot Q^*}{2} + \frac{O_{eq} \cdot D}{Q^*}\right) - \left(\frac{h_{eq} \cdot Q}{2} + \frac{O_{eq} \cdot D}{Q}\right), \text{ with}$$
$$h_{eq} = \sum_{i=1}^{n} \alpha_i \cdot h_i \text{ and } O_{eq} = \sum_{i=1}^{n} \alpha_i \cdot O_i.$$

It follows that:

$$\forall Q \in [Q_{\min}, Q_{\max}], \quad V^*(Q^*) - V^*(Q) = C_{eq}(Q) - C_{eq}(Q^*). \qquad \square$$

Proof of Theorem 3. Similar to that of Schwarz (1973)

(1) The retailer orders only if its inventory level is null:Consider any feasible policy that does not satisfy (1) at some time t. Every holding impacts in the interval $[0,t]$ will be reduced by reducing the amount of the preceding delivery by the inventory on hand at time t (or to zero) and increasing the amount of the delivery at time t by the same amount. This adjustment does not increase the number of deliveries and ordering impacts are thus reduced or kept equal. By repeating this adjustment for every retailer delivery time, a policy satisfying (1) will result.
(2) The warehouse orders when both the retailer and the warehouse have no inventory:The fact that the warehouse orders when its inventory level is null is proven in the same manner as (1). To prove that the warehouse orders when the retailer has no inventory, we remark that on the other case, the warehouse order can be postponed until the retailer orders. This will decrease every holding impact at the warehouse without modifying the ordering impacts. By applying (1), this condition happens when the inventory at the retailer is null.
(3) All deliveries made to the retailer between successive deliveries to the warehouse are of equal size:Assume that there are n deliveries to the retailer of lot sizes $Q_k, k \in [1,n]$ such that $\sum_{k=1}^{n} Q_k = Q$ between any two successive deliveries to the warehouse. The only impacts affected by these lot sizes are the holding impacts at the retailer. As D is constant, the minimum of all holding impacts at the retailer is reached when $\forall k \in [1,n]$, $Q_k = \frac{Q}{n}$. $\square$

Proof of Theorem 4. Similar to that of Theorem 1. $\square$

Proof of Theorem 5. The existence of $k_{\min}$ is trivial.

Moreover, the mono-objective optima defined in Formula 11 are included in E by definition, then $1 \leqslant k_{\min} \leqslant \min_i(k_i^*)$.

It also implies that if $k_{\max}$exists, $\max_i(k_i^*) \leqslant k_{\max}$.

$\forall i \in [1,n]$, $C_i(k,Q)$ tends to infinity as k tends to infinity. Let $e(k_e, Q_e) \in E$.

It exits $t \in \aleph^*$ such that $\forall i \in [1,n]$, $\forall Q \in \Re_+^*$, $\forall n \in \aleph$, $C_i(k_e, Q_e) < C_i(t+n, Q)$.

Then e dominates all elements of $\bigcup_{k=t}^{\infty} E_k$. That proves the existence of $k_{\max}$. $\square$

Proof of Theorem 6. By using Theorem 5, $E^C \subset \bigcup_{k=k_{\min}}^{k_{\max}} E_k^C$.

As $\min_i(k_i^*) < \max_i(k_i^*)$, there exists $e_{k_{\min}} \in E_{k_{\min}}^C | e_{k_{\min}} \in E^C$ and $e_{k_{\max}} \in E_{k_{\max}}^C | e_{k_{\max}} \in E^C . E_{\min_i(k_i^*)+}^C \neq E_{\max_i(k_i^*)+}^C$ and both are convex by using Theorem 4 thus E^C is non-convex. $\square$

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