EOQ Revisited with Sustainability Considerations

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Abstract

The Economic Order Quantity (EOQ) model is a pure economic model in classical inventory control theory. The model is designed to find the order quantity so as to minimize total cost under a deterministic setting. In this study, we revise the standard EOQ model to incorporate sustainability considerations that include environmental and social criteria in addition to the conventional economics. We propose models for a number of different settings and analyze these revised models. Based on our analysis, we show how these additional criteria can be appended to traditional cost accounting in order to address sustainability in supply chain management. We propose a number of useful and practical insights for managers and policy makers.

Keywords: Economic order quantity (EOQ) model; Inventory control; Sustainability; Supply chain management; Operations Management; Environmental and social criteria

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1. Introduction

The Economic Order Quantity (EOQ) model is a pure economic model in the classical inventory control theory. The model is designed to find the order quantity so as to minimize the total average cost of replenishment under deterministic demand and some simplifying assumptions. These assumptions (listed in section 2.1) are unrealistic; however, simplicity and robustness of the model makes it practical in most cases.

In this paper, we revise this practical model to encompass a wider perspective of sustainability. We modify the standard model to further account for environmental and social criteria. These added criteria stems from the emerging requirement of sustainability in supply chain management. To achieve sustainability, the decision makers should incorporate these added dimensions into the decision making process as well as the conventional economics. This approach is referred to as the “triple bottom line accounting” (Elkington, 1997).

We apply the triple bottom line accounting approach in a methodical manner. Our method is based on revising the standard EOQ model with additional objectives and/or constraints regarding those added criteria. We propose models for a number of different settings and analyze these revised models to characterize the optimal policy analytically and numerically. Based on our analysis, we show how these additional criteria can be appended to traditional cost accounting in order to achieve sustainability in supply chain management. We propose a number of useful and practical results for managers and policy makers.

1.1. Related Literature

Incorporating sustainability into the traditional supply chain and operations management problems has attracted limited attention in the open literature. Many review papers in this context are published during the last two decades including Corbett (2009), Corbett and Klassen (2006), Corbett and Kleindorfer (2001a, 2001b), Gupta (1995), Gupta and Lambert (2009), Kleindorfer et al. (2005), Linton et al. (2007), Sasikumar and Kannan (2009), Seuring and Müller (2008), and Srivastava (2007).

A significant part of the research in this area focuses only on the environmental aspects along with the economical objectives. Social aspects are rarely included or completely ignored (Seuring and Müller, 2008). Furthermore, a methodical approach with a theoretical background in analyzing sustainable supply chain systems is also missing (Seuring and Müller, 2008). Our methodical approach proposed in this paper is one of the first attempts in this direction that we are aware of in the open literature.

There are three recent studies closely related to our work. The first one is a recent paper by Benjaafar et al. (2009) where carbon emission concerns are integrated into simple lot sizing models. The authors point out that existing literature in supply chain management lacks such carbon-enhanced operational models and provide insights based on their models and numerical experiments to show the effect of carbon emissions on optimal operating policies. The authors argue that instead of costly investments, simple operational modifications can reduce the carbon footprint.

Hoen et al. (2010) develop models for transport mode selection problem with emission costs and constraints. Their model is based on the classical newsboy model. The authors argue that emission cost accounting, emission tax or emission trade mechanisms all fail if the aim is to curb the emissions. On the other hand, a direct cap on emissions works. They also calculate emissions for different transport mode choices to estimate the parameters of the proposed models.

Hua et al. (2009) investigate the effect of carbon emissions in inventory control. The authors extend the standard EOQ model to further account for the carbon emissions under cap and trade mechanism. The authors propose a number of insights based on their analytical and numerical analysis and provide conditions of buying and selling carbon credit while reducing costs, emissions and in some cases, both of them.

To our knowledge, there is no paper available in the open literature that provides revised models for alternative environmental management approaches as well as the social dimension of sustainability based on the EOQ model. This paper aims to fill this gap. We revise the classical EOQ model to account for added environmental and social dimensions of sustainability as well as the economic one. In Section §2, we develop and analyze such models. In Section §3, we propose results based on our analysis. Finally in Section §4, we discuss the contributions and implications of our study and propose directions for further research.

2. Model Formulations & Analysis

2.1. The Standard Economic Order Quantity (EOQ) Model

The EOQ model arises from the simplest form of economies of scale (Zipkin, 2000). The model assumes a single item at a single location with a continuous demand. The demand is known and has a constant rate, λ, over time. A supply of the required item is needed in order to satisfy the demand. Therefore, either the items are produced or an order to the supplier is placed. The model assumes constant lead times with an infinite supply capacity where stock-outs are not allowed. Under these assumptions, the EOQ problem is to decide on the ordered quantity Q (or the quantity to be produced) which minimizes the total average cost of replenishment; i.e. we have a single decision variable, Q, which satisfies $Q^* = \arg\min_Q C(Q)$ optimally; where $C(Q)$ denotes the total average cost of replenishment. $C(Q)$ has two components; replenishment cost and the inventory holding cost. The replenishment cost consists of purchasing (or production) cost and the ordering cost (Nahmias, 1993).

Let $I(t)$ denote the inventory level at time t. The inventory level in the EOQ model is assumed to be cyclic, each cycle starting from the inventory level $I(t) = Q$ (assuming zero inventory at $t = 0$) and gradually depleted until the end of the cycle with the constant demand rate. Therefore, we obtain an average inventory of $\frac{Q}{2}$ at each cycle and since the cycles are identical (each time we order Q units), this result holds for any time period of many cycles (Nahmias, 1993). Let h denote the cost per unit item held per unit time. Then the inventory holding part of $C(Q)$ becomes $h\frac{Q}{2}$. Let K denote the fixed/setup cost of ordering/production and c denote the variable cost per unit ordered/produced. Then the sum of purchasing and ordering costs become $K + cQ$ at each cycle. Since each cycle is of length $T = Q/\lambda$, $C(Q)$ is given by,

$$C(Q) = \frac{K + cQ}{Q/\lambda} + h\frac{Q}{2} = \frac{K\lambda}{Q} + \lambda c + \frac{hQ}{2} \tag{1}$$

The optimal Q is then found by solving $C'(Q) = 0$ and checking if $C''(Q) > 0$ for convexity of the cost function. Since $C(Q)$ is convex, the first order condition ensures optimality. Therefore the economic order quantity or the optimal Q of the above optimization problem is given by,

$$Q^* = \sqrt{\frac{2K\lambda}{h}} \tag{2}$$

which is also known as the Wilson's or Harris' formula (Nahmias, 1993; Harris, 1913).

The EOQ model account solely for the economics of the replenishment and inventory holding activity related to the provision of the required items. Apart from the economic aspects, these activities also have impacts on the environment and the society which should be considered as well, as elaborated in the next sections.

2.2. EOQ Model with Environmental Criterion

In this section, the standard EOQ model is revised by taking carbon footprint into account. A typical environmental effect caused by most industrial operations (including inventory control) is the inevitable release of greenhouse gases (GHG) (Turkay, 2008). As a result, in order to assess the environmental performance of an organization, the amount of GHG emissions is commonly used in the green/environmentally conscious supply chain and operations management literature (see Turkay, 2008 and the references therein). The set of greenhouse gases (including carbon dioxide) released by an organization due to its operations is commonly referred to as the carbon footprint (Wiedmann and Minks, 2008).

In this study, we also consider carbon footprint in modeling the environmental criterion. We observe costs, emissions and the refined order quantities under a number of different settings. In the following sections, five environmental management approaches each with different characteristics are modeled and analyzed.

2.2.1. Direct Accounting

The first approach to model carbon footprint is to treat it as an additional source of economic cost. Let f be the fixed cost of environmental impact for each replenishment cycle due to setups, order processing or transportation; v be the variable cost of environmental impact due to the production and related activities, and finally g be the cost of environmental impact due to the inventory holding as a result of material handling and warehousing activities. Environmental cost components as mentioned above (f , v , g) are in the form of monetary units as other EOQ parameters K , c , and h . These cost parameters can be extracted from the cost accounting of environmental management activities of the organization or from the cost of energy used. More specifically, these parameters would be estimated through life cycle assessment data, production data and inventory management data. Although this is not an easy task, the organizations should estimate these parameters in order to comply with the emerging regulatory policies. One can refer to GHG Protocol (GHG Protocol), ISO (ISO), WRI (WRI), EcoTransIT (EcoTransIT) and Carbontrust (Carbontrust) for carbon footprint measurement standards and methodologies.

With these additional parameters, we can rewrite the total average replenishment cost $C(Q)$ as

$$C(Q) = \frac{K + f + (c+v)Q}{Q/\lambda} + (h+g)\frac{Q}{2} = \frac{(K+f)\lambda}{Q} + \lambda(c+v) + \frac{(h+g)Q}{2} \tag{3}$$

and by a similar analysis as in the EOQ model, the optimal order quantity is found as

$$Q_{ee}^{*} = \sqrt{\frac{2(K+f)\lambda}{(h+g)}} \tag{4}$$

along with the optimal cost

$$\begin{aligned} C(Q_{ee}^{*}) &= \frac{(K+f)\lambda}{\sqrt{2(K+f)\lambda \big/ (h+g)}} + \lambda(c+v) + \frac{(h+g)\sqrt{2(K+f)\lambda \big/ (h+g)}}{2} \\ &= \lambda(c+v) + \sqrt{2(K+f)\lambda(h+g)} \end{aligned} \tag{5}$$

where Q_{ee}^{*} denotes the optimal order quantity with economic and environmental criteria. A simple sensitivity of Q_{ee}^{*} with respect to Q^{*} yields

$$\frac{Q_{ee}^{*}}{Q^{*}} = \sqrt{\frac{Kh + fh}{Kh + gK}} \tag{6}$$

where the following set of relationships hold:

(i) $$Q_{ee}^{*} = Q^{*} \Leftrightarrow \frac{K}{h} = \frac{f}{g} \tag{7}$$

(ii) $$Q_{ee}^{*} > Q^{*} \Leftrightarrow \frac{K}{h} < \frac{f}{g} \tag{8}$$

(iii) $\quad Q_{ee}^{*} < Q^{*} \Leftrightarrow \dfrac{K}{h} > \dfrac{f}{g}$ (9)

Hence, the optimal ordering quantity is governed by the trade-off between replenishment and inventory holding costs with the only change of added environmental cost components. Due to this trade-off, the refined optimal ordering quantity may be larger or smaller than the EOQ as it might be equal to it as well depending on the values of the cost components.

Equation (4) resembles the usual EOQ apart from f and g. If these added parameters are incorporated into K and h, then (4) reduces to EOQ exactly. Otherwise, one of the cases in (6), (7) or (8) applies. The unit inventory holding cost, h, incorporates the opportunity cost of the capital committed in addition to the real costs of inventory holding. However, such an argument is clearly not valid for g. Hence, g might be smaller than h in most practical situations. On the other hand, fixed environmental impacts are induced at all stages of the production, transportation, and order processing activities. Hence f might be larger than K. Therefore, (7) is more likely to be realized when the added environmental impact is considered; i.e. $Q_{ee}^{*} > Q^{*}$.

Bonney and Jaber (2010) propose an environmentally enhanced EOQ model similar to the one proposed above. However, they do not consider environmental impacts due to the holding of inventories and conclude that the refined EOQ (referred as the EEOQ) is always larger than the usual EOQ. They also review the current environmental problems with recommendations on reducing environmental impact in inventory management and provide a list of non-cost metrics for incorporating environmental footprint in the inventory context.

The direct accounting approach is an optional one and left to the discretion of the managers of the organization, i.e. organizations may or may not calculate costs by using the environmental cost components as well. However, the above analysis shows that there is value for the organization in investigating the sources of emissions and the related costs.

2.2.2. Carbon Tax

Organizations may be given incentive to account for the environmental costs through an externally applied carbon tax by the regulatory agencies. A simple tax schedule is a linear one; i.e. organizations pay an amount of p money-units for each unit of carbon emitted. However, other tax schedules (convex/progressive, concave/regressive, non-linear, piecewise linear, staircase etc.) may also be applied. We consider only the linear tax schedule in this paper. The refined model with a linear tax schedule is as follows:

$$C(Q) = \frac{K + pf + (c + pv)Q}{Q/\lambda} + (h + pg)\frac{Q}{2} = \frac{(K + pf)\lambda}{Q} + \lambda(c + pv) + \frac{(h + pg)Q}{2} \quad (10)$$

and by a similar analysis as in the EOQ model, the optimal order quantity is found as

$$Q_{ee}^* = \sqrt{\frac{2(K + pf)\lambda}{(h + pg)}} \quad (11)$$

that yield the following optimal cost,

$$C(Q_{ee}^*) = \frac{(K + pf)\lambda}{\sqrt{2(K + pf)\lambda \big/ (h + pg)}} + \lambda(c + pv) + \frac{(h + pg)\sqrt{2(K + pf)\lambda \big/ (h + pg)}}{2}$$
$$= \lambda(c + pv) + \sqrt{2(K + pf)\lambda(h + pg)} \quad (12)$$

Note that f, v, and g now directly denote the amount of emissions in the preceding analysis. Multiplying them with the tax rate p again transforms the emissions into costs in monetary units. Incorporating a linear tax schedule is similar to the direct accounting of the environmental costs except for f and g being replaced by pf and pg. When a sensitivity analysis of Q_{ee}^* with respect to Q^* is conducted, the same conditions may easily be obtained as in the direct accounting (Eq. (5), (6), (7), and (8)) model, independent of the tax rate p. In other words, the tax rate, p, in a linear tax schedule does not have an impact on the optimal policy (the order quantity in this case). However, it affects the total average cost as seen in Eq. (12). Hence, taxing carbon emissions gives incentive to identify emission sources, estimate the emission parameters and curb the emissions to achieve lower operating cost. Therefore, applying a carbon tax schedule suitable with the macroeconomic policy of a country is a useful tool for the regulatory agencies.

2.2.3. Direct cap

Letting f, v, and g directly denote the emission amounts due to their respective activities is another approach, as applied in modeling carbon tax. This approach also facilitates estimating the values of these environmental parameters. We also consider this approach in our analysis. For a more complete discussion on the sources of carbon emissions in inventory control, one can refer to Penman and Stock (1994), Stock (2008), Stock et al. (2010) and Sundarakani et al (2010).

The model in (4) ignores the total impact on environment caused by replenishment and inventory holding, since direct accounting for environmental costs does not give the organization an initiative to curb the emissions. Furthermore, not all organizations consider these costs willingly. This may not be true in case of a carbon tax imposed by the regulatory agencies where tax increases the monetary costs and the organizations are enforced to consider their environmental impact, as a consequence (see discussion on carbon tax above).

An alternative modeling scheme is the one where a direct cap on environmental footprint is imposed either by the regulatory agencies or by the public awareness such that customers are seeking for more environmentally friendly products. In other words, the demand for the products may depend on the emission levels of the organization in supplying those products to the customers.

Assume that there is an upper limit on the amount of GHG emissions, denoted by ζ, as in the case of the countries signed the Kyoto Protocol (UNFCCC, 1997). Assume further that the above environmental cost parameters (f, v, g) now denote the amount of emissions due to their respective activities mentioned before. Since the EOQ model is based on a single cycle and since the cycles are identical, ζ may also be assumed to be an upper bound on the average amount of GHG emissions per cycle (inducing a cap per unit product is another way of modeling which would be used for gathering customer attention by carbon labeling the product; see Brenton et al, 2009; Edwards-Jones et al, 2009; see also Section §2.2.5 for more details in carbon labeling). Therefore, the refined problem becomes

$$\text{Min } C(Q) = \frac{K\lambda}{Q} + \lambda c + \frac{hQ}{2} \tag{13}$$

s.t.

$$\frac{f + vQ}{Q/\lambda} + g\frac{Q}{2} \leq \zeta \tag{14}$$

$$Q \geq 0 \tag{15}$$

This new model resembles the resource constrained EOQ model where a traditional case is to incorporate a linear constraint on the available warehouse space (Nahmias, 1993). However in the above case, the constraint is also nonlinear. Optimal policy is to order the standard optimal of the EOQ model if it satisfies the constraint. In this case, the optimal cost is the usual EOQ optimal given by,

$$C(Q^*) = \frac{K\lambda}{\sqrt{2K\lambda/h}} + \lambda c + \frac{h\sqrt{2K\lambda/h}}{2} = \lambda c + \sqrt{2K\lambda h} \tag{16}$$

and the emission amount becomes

$$EM(Q^*) = \frac{f\lambda}{\sqrt{2K\lambda/h}} + \lambda v + \frac{g\sqrt{2K\lambda/h}}{2} \tag{17}$$

Otherwise, the constraint is binding at optimality and the optimal order quantity for the above problem is found by solving the quadratic equation,

$$\frac{f\lambda}{Q} + v\lambda + g\frac{Q}{2} = \zeta \tag{18}$$

which yields

$$Q_{ee}^* = \frac{\zeta - v\lambda \pm \sqrt{(v\lambda - \zeta)^2 - 2g\lambda f}}{g} \tag{19}$$

and $EM(Q_{ee}^{*}) = \zeta$. Similarly, the optimal total cost may be found by plugging Q_{ee}^{*} into Eq. (11).

Note that any order quantity should be a nonnegative real value to be valid. However, the preceding quadratic equation may not have a real root or have two distinct or identical roots (depending on the parameters in Eq. (14)). If it has real roots, either one or both of the roots may be negative. Hence, the optimal policy is governed by the relationship among environmental and economic parameters of the organization.

If the objective is to minimize purely the emissions, the optimal order quantity would be,

$$Q_e^{*} = \sqrt{\frac{2f\lambda}{g}} \tag{20}$$

and apparently Q_{ee}^{*} is in between Q_e^{*} and Q^{*} provided that $\frac{K}{h} \neq \frac{f}{g}$.

2.2.4. Cap & Trade

Another important mechanism to curb the emissions is the carbon trading markets, simply called as cap and trade. In this setting, companies emitting less than the allowed cap are rewarded whereas those over emitters are penalized. This penalty and reward mechanism is achieved via a carbon trading market. Companies emitting lower than the cap sell their allowances (the amount of emissions they are under the cap) whereas those emitting more than the cap buy such allowances. Therefore, the caps are not strict but encouraging in a cap-and-trade system. Such markets have already been developed in EU and US and the participation of the companies in the system is mandatory (EU, 2010; European Commission, 2010). This market trades significant volumes now, and has a potential to grow up further (EU, 2010).

Let the assumptions of the direct cap system hold along with the model parameters. Since the environmental parameters still denote directly the emission amounts due to their respective activities, we account for emissions directly in the cap and trade system as in the direct cap system. Assume further that p now denotes the price of the carbon which is fixed and externally set by the market mechanism. Let s^{+} denote the amount of allowances sold by the organization and s^{-} denote the amount of allowances bought by the organization; only one of these trading variables may be positive (the organization either buys or sells allowances). The optimal ordering quantity and the amount of allowances either sold or purchased by the organization are found by solving the following mixed integer nonlinear program:

$$\text{Min } C(Q) = \frac{K\lambda}{Q} + \lambda c + \frac{hQ}{2} + p(s^- - s^+) \tag{21}$$

s.t.

$$\frac{f + vQ}{Q/\lambda} + g\frac{Q}{2} - s^- + s^+ \leq \zeta \tag{22}$$

$$s^- \leq y_1 M \tag{23}$$

$$s^+ \leq y_2 \zeta \tag{24}$$

$$y_1 + y_2 = 1 \tag{25}$$

$$Q, s^+, s^- \geq 0 \tag{26}$$

$$y_1, y_2 \in \{0,1\} \tag{27}$$

where M is a large positive number. The optimal cost and the emissions may also be obtained by solving the above model. Note that under the above model with a nonnegative carbon price, there are three options for the organization: (i) organization buys allowances if there is not a feasible Q satisfying Eq. (14), (ii) neither buys nor sells allowances if there is a feasible Q satisfying Eq. (14) at equality, (iii) sells allowances if the constraint is satisfied but is not tight.

In this system, one important parameter is the carbon price, which appears to vary between 0 and 30 euro-cents per ton in the EU ETS (European Carbon Exchange, 2010). This price is an exogenous system parameter determined by the market mechanism and assumed to be fixed in the above model. An alternative and intuitive scenario is the case where carbon price is dependent on the carbon cap. If the cap is tighter, the price of the allowance should obviously be higher. Therefore, one can assume an inversely proportional relation between p and ζ as such:

$$b = p + a\zeta \tag{28}$$

where a and b are assumed to be nonnegative scalars without loss of generality. We can readjust the above model to incorporate such a relationship by plugging in $p = b - a\zeta$.

If regulatory agencies regulate the trading market by setting the carbon price, a macroeconomic view is needed as in the carbon tax model.

2.2.5. Carbon Offsets

The final environmental management mechanism we discuss is carbon offsets which stand for emission reducing investments. These investments may be in the form of energy efficient equipment and facilities, renewable energy resources, energy saving programs, carbon capturing and sequestration (CCS) systems, to name a few. The organization pays a price for the offset in return for less carbon footprint due to increased technology and environmentally friendly resources.

Let the assumptions of the cap and trade system hold except for p now denoting the unit price of the offset purchased by the organization and s^- denote the amount of offset. We assume that the offset directly relax the carbon emission constraint and does not reduce the values of emission parameters although this might be the case and modeled as well. Then, the optimal order quantity and the amount of offset to purchase may be found by solving the following nonlinear program:

$$\text{Min } C(Q) = \frac{K\lambda}{Q} + \lambda c + \frac{hQ}{2} + ps^- \tag{29}$$

s.t.

$$\frac{f + vQ}{Q/\lambda} + g\frac{Q}{2} - s^- \leq \zeta \tag{30}$$

$$Q, s^- \geq 0 \tag{31}$$

The optimal cost and emissions may also be obtained by solving the above nonlinear programming problem. However, it is important to note that buying offsets is reasonable only in the case that there is no feasible Q satisfying Eq. (14). In such a case, it is mandatory (due to the cap exercised) to buy offsets to be able to maintain the operations.

The model for carbon offsets is similar to the model for cap and trade mechanism except for the allowances sold (s^+) in the cap and trade system. As in the cap and trade system, the emission amounts are directly accounted without converting them into monetary units. Moreover, a similar relationship between the offset price, p, and the carbon cap, ζ, may also be considered as in the cap and trade system. Instead of a fixed price, p may be inversely proportional to the carbon cap ζ.

Purchasing offsets is optional for the organization although it is mandatory to participate in the cap and trade system. It may be the case that the market demand requires cleaner products with cleaner technology and energy. In such a case, carbon offsetting becomes obligatory in a sense for the organization for competitive advantage. Nevertheless, this situation enables the organization to carbon-label its products and charge relatively larger prices to those environmentally sensitive customers. Let r_1 denote the price charged to usual customers and r_2 denote the price charged to environmentally sensitive customers when the organization purchase some offsets ($r_2 \geq r_1 \geq c$). Then, the joint pricing and ordering model for the standard case becomes,

$$\text{Min } C(Q) = \frac{K\lambda}{Q} + \lambda(c - r_1) + \frac{hQ}{2} \tag{32}$$

s.t.

$$\frac{f + vQ}{Q/\lambda} + g\frac{Q}{2} \leq M \tag{33}$$

$$Q \geq 0 \tag{34}$$

where M is a sufficiently large nonnegative carbon cap enabling a feasible Q satisfying Eq. (33). The model with offsets can be represented as follows:

$$\text{Min } C(Q) = \frac{K\lambda}{Q} + \lambda(c - r_2) + \frac{hQ}{2} + ps^- \tag{35}$$

s.t.

$$\frac{f+vQ}{Q/\lambda}+g\frac{Q}{2}-s^{-}\leq\zeta \tag{36}$$

$$Q,s^{-}\geq 0 \tag{37}$$

where ζ is a cap which does not allow a feasible Q satisfying Eq. (33) when no offset is purchased. Although an analytical comparison is not straight forward, a numerical comparison of the above two models reveals the extent to which the organization may charge a relatively higher price for more environmentally friendly supply of products (via carbon offsets and tighter carbon caps) to those environmentally sensitive customers. This is not valid, of course, in markets where customers are non-sensitive to environmental friendliness.

2.3. EOQ Model with Social Criterion

The corporate social responsibility (CSR) literature has been the primary area of investigation in terms of incorporating social criteria of sustainability. There is a vast body of literature on this subject; however, a literature review conducted reveals that there are no studies concerned directly with the modeling of CSR aspects in supply chain and operations management problems in the open literature. When seeking appropriate supply chain metrics, SCOR model is the classical reference (Supply Chain Council, 2006). However, it does not provide any social metrics for supply chain modeling. The sustainable supply chain management literature also lacks social aspects as mentioned previously in the literature review section (Seuring and Müller, 2008). As a result, there is no straight forward metric available to use in modeling the social criteria. Therefore, we rely on labor standards put forth by ILO (ILO).

According to ILO; there must be a legal upper limit on working hours for the employees (ILO; NZBCSD, 2003). Moreover, inventory control is among the most time consuming tasks in the operations of organizations (Turner et al, 1993). As a result, a typical social effect caused by inventory control operations is the inevitable exhaustion of available man-hours due to the replenishment and the inventory holding activities. Therefore, in order to assess the social performance of an organization, the amount of man-hours required to perform the inventory control operations can be interpreted as a valid metric. Hence, we account for man-hours in modeling the social criterion. We employ direct accounting and direct cap approaches.

2.3.1. Direct Accounting

The following parameters are used: m the fixed amount of man-hours required, n the variable amount of man-hours required and l the man-hours required for the holding of inventories. By using the labor cost accounting, the cost of labor per man-hour can be easily obtained. Multiplication of the man-hour parameters with this cost factor yields the corresponding labor cost parameters of each activity. Let m, n, and l also denote their respective cost correspondents. Assume further that the total available man-hours during a cycle is denoted by W.

Using a similar aggregation of costs as in deriving Q_{ee}^*, we can derive Q_{se}^*, the optimal order quantity with social and economic criteria, as

$$Q_{se}^* = \sqrt{\frac{2(K+m)\lambda}{(h+l)}} \tag{38}$$

and the corresponding optimal cost as

$$\begin{aligned} C(Q_{se}^*) &= \frac{(K+m)\lambda}{\sqrt{2(K+m)\lambda/(h+l)}} + \lambda(c+n) + \frac{(h+l)\sqrt{2(K+m)\lambda/(h+l)}}{2} \\ &= \lambda(c+n) + \sqrt{2(K+m)\lambda(h+l)} \end{aligned} \tag{39}$$

Furthermore, the following relationships hold:

(i) $$Q_{se}^* = Q^* \Leftrightarrow \frac{K}{h} = \frac{m}{l} \tag{40}$$

(ii) $$Q_{se}^* > Q^* \Leftrightarrow \frac{K}{h} < \frac{m}{l} \tag{41}$$

(iii) $$Q_{se}^* < Q^* \Leftrightarrow \frac{K}{h} > \frac{m}{l} \tag{42}$$

Apparently, the values of m , n , and l rely on the abilities of the employees and the design of the work environment. A discussion on improving these social cost parameters is presented in Section §3. One can argue that n and l are getting smaller due to automation in production environments whereas m is most likely to be stable as a global trend. However, when they are assumed to represent the corresponding costs, they differ significantly between the developed and developing/under-developed countries. Hence, the above equations (40-42) explain why the under-developed countries exercise low quality mass production whereas the developed countries produce quality products in relatively higher lots. This also lays the foundation for mass customization in the developed countries where customization is achieved in a mass production setting.

2.3.2. Direct Cap

Alternatively, we may also employ the constrained-EOQ logic (since there is a legal upper limit on working hours and not all companies account for social costs willingly) and formulate the problem as,

$$\text{Min } C(Q) = \frac{K\lambda}{Q} + \lambda c + \frac{hQ}{2} \tag{43}$$

s.t.

$$\frac{m + nQ}{Q/\lambda} + l\frac{Q}{2} \leq W \tag{44}$$

$$Q \geq 0 \tag{45}$$

If the standard EOQ optimal satisfies the above constraint, it is still optimal for the above model. Then, the optimal cost is again the usual EOQ optimal given by $C(Q^*) = \lambda c + \sqrt{2K\lambda h}$ whereas the required man-hours is obtained by,

$$RM(Q^*) = \frac{m\lambda}{\sqrt{\frac{2K\lambda}{h}}} + \lambda n + l\frac{\sqrt{\frac{2K\lambda}{h}}}{2} \tag{46}$$

When the direct cap constraint is binding at optimality, Q_{se}^* is found as,

$$Q_{se}^{*} = \frac{W - n\lambda \pm \sqrt{(n\lambda - W)^2 - 2l\lambda m}}{l} \tag{47}$$

by solving the quadratic equation

$$\frac{m + nQ}{Q/\lambda} + l\frac{Q}{2} = W \tag{48}$$

with $RM(Q_{se}^{*}) = W$. Similarly, the optimal total cost may be found by plugging Q_{se}^{*} into Eq. (11). The existence of a feasible Q_{se}^{*} depends on the values of the parameters of the above model as discussed previously for Q_{ee}^{*}.

If the objective is to minimize purely the man-hours, the optimal order quantity would be

$$Q_{s}^{*} = \sqrt{\frac{2m\lambda}{l}} \tag{49}$$

and Q_{se}^{*} is in between Q_{s}^{*} and Q^{*} provided that $\frac{K}{h} \neq \frac{m}{l}$.

2.4. Triple Bottom Line Accounting

In this part, we consider the case where the three pillars of sustainability are analyzed simultaneously. We model the three pillars using the direct accounting and direct cap modeling approaches, since these approaches are common for both the environmental and the social criterion. However, different modeling approaches may also be picked for environmental and social pillars (e.g. using cap and trade model for carbon footprint and direct cap modeling for man-hours) as well. We assume that the economic, environmental and social parameters are the same as in the previous sections.

2.4.1. Direct Accounting

By using the direct accounting approach, one can easily find out Q_{ees}^{*} , the optimal order quantity with economic, environmental, and social criteria as

$$Q_{ees}^{*} = \sqrt{\frac{2(K + f + m)\lambda}{(h + l + g)}} \tag{50}$$

with $C(Q_{ees}^{*})$ as

$$C(Q_{ees}^{*}) = \frac{(K+f+m)\lambda}{\sqrt{2(K+f+m)\lambda \big/ (h+g+l)}} + \lambda(c+v+n)$$

$$+\frac{(h+g+l)\sqrt{2(K+f+m)\lambda \big/ (h+g+l)}}{2} \tag{51}$$

$$= \lambda(c+v+n) + \sqrt{2(K+f+m)\lambda(h+g+l)}$$

and deduce the following:

(i) $Q_{ees}^{*} = Q^{*} \Leftrightarrow \frac{K}{h} = \frac{f+m}{g+l}$ (52)

(ii) $Q_{ees}^{*} > Q^{*} \Leftrightarrow \frac{K}{h} < \frac{f+m}{g+l}$ (53)

(iii) $Q_{ees}^{*} < Q^{*} \Leftrightarrow \frac{K}{h} > \frac{f+m}{g+l}$ (54)

2.4.2. **Direct Cap**

The optimal order quantity may also be found using the direct cap modeling approach as the solution of the following quadratic-constrained quadratic program,

$$\text{Min } C(Q) = \frac{K\lambda}{Q} + \lambda c + \frac{hQ}{2} \tag{55}$$

s.t.

$$\frac{f+vQ}{Q/\lambda} + g\frac{Q}{2} \leq \zeta \tag{56}$$

$$\frac{m+nQ}{Q/\lambda} + l\frac{Q}{2} \leq W \tag{57}$$

$$Q \geq 0 \tag{58}$$

If the standard EOQ optimal satisfies both constraints, then it is still optimal for the above quadratic program as well. Otherwise one of the constraints is binding at optimality and either $Q^*_{ees} = Q^*_{ee}$ or $Q^*_{ees} = Q^*_{se}$ provided that there is a feasible solution of the quadratic program. The required man-hours and emission amounts may also be obtained by plugging Q^*_{ees} into Eq. (56) and (57).

2.5. Numerical Analysis

In this section, we present the analysis of a numerical study conducted for the sensitivity of the cap and trade and carbon offset models (presented in §2.2.4 and §2.2.5) with respect to changes in some model parameters. The model parameters used in the numerical experiments are presented in the Appendix B in Table 2. The solutions are obtained using default MINLP and NLP solvers of GAMS using default solver options (GAMS).

The following values (Table 1) are obtained by solving Eq. (2), (16), (17) and (20) using the data given in Table 2 and presented as a benchmark for §2.5.1 and §2.5.2:

Table 1

Pure economic and environmental minimizers along with corresponding costs and emissions

$Q^* = 44.721$	$Q_e^* = 77.459$
$C(Q^*) = 689.443$	$C(Q_e^*) = 703.279$
$EM(Q^*) = 339.442$	$EM(Q_e^*) = 327.459$

Table 1 suggests that one should order/produce in the amount of 44.721 units if the objective is purely to minimize the economic costs. As a result, a cost in the amount of 689.443 monetary-units would be incurred. Furthermore, ordering 44.721 units would lead to emissions in the amount of 339.442 units. On the other hand, the ordering quantity would be in the amount of 77.459 units with a total cost of 703.279 monetary-units if the objective is to minimize purely the emissions. With this environmental policy, the emission amount is reduced by %3.5; to an amount of 327.459 units.

2.5.1. Experiments for the Cap & Trade Model

Figure 1 shows that organizations buy allowances whenever $p \geq 0$ and $\zeta < EM(Q_e^*)$ ($\zeta = 300$). For $p = 0$, the policy is reduced to ordering Q^*, since buying allowances becomes cost-free in this extreme case. In other words, $p = 0$ is equal to carbon cap constraint being inactive whatever the values of ζ is. As p increases, the policy tends to move towards ordering Q_e^* with higher total cost while the amount of allowances bought is reduced gradually to the value of $EM(Q_e^*) - \zeta$. We assume that p is externally set by the regulatory agencies or the market mechanism. In this case, all organizations decide and operate under the given price.

Figure 2 suggests that varying the carbon cap while carbon price is fixed ($p = 5$) does not have an impact on the optimal order quantity (conversely, varying the price affects the policy, as seen in Figure 1). Intuitively, one would expect that when $\zeta > EM(Q^*)$; the policy should be reduced to Q^* (as in Figure 5). However, for such values of ζ, the organization makes money and reduces costs further by selling allowances in a cap and trade system, which is the main difference when compared to the carbon offset mechanism.

When the market price of the carbon is dependent on the cap ($20 = p + 0.04\zeta$) exercised by the regulatory agencies (or by the market mechanism like customer preferences), the effect of varying the carbon cap and price is experienced jointly. Figure 3 is similar to Figure 2 except for the policy being changed as well as higher costs being observed (due to higher carbon prices) for smaller values of ζ. Note how $C(Q)$ is diminished as ζ gets larger. Note also that in a cap and trade system, organizations buy allowances until ζ is relaxed sufficiently (i.e. $\zeta \geq EM(Q_e^*)$) and sells allowances then onwards.

2.5.2. Experiments for the Carbon Offset Model

Figure 4 is an exact equivalent of Figure 1 except for the s^+ series, which is not valid for carbon offset model (since nothing is sold). This shows that when $\zeta < EM(Q_e^*)$ ($\zeta = 300$), both systems respond to changes in p in the same manner (although p denotes carbon price in the cap and trade system whereas it denotes the offset price in the carbon offset mechanism). As price increases, the amount of offset purchased, s^-, decreases whereas order quantity increases (similar to the cap and trade system except for s^- denoting carbon allowances purchased in a cap and trade system).

Figure 5 is also similar to Figure 2 except for the s^+ column, which is not valid for carbon offset model. This shows that when $\zeta < EM(Q_e^*)$, both systems respond to changes in ζ in the same manner given a fixed offset price, p ($p = 5$). However, when $\zeta > EM(Q_e^*)$, no offsets are being purchased and the policy is gradually reduced to Q^* as $\zeta \geq EM(Q^*)$. Furthermore, $C(Q)$ becomes stable as $\zeta \geq EM(Q^*)$. Therefore, the offset system gives incentive to curb the emissions only if $\zeta < EM(Q_e^*)$.

When the offset price is dependent on the cap ($20 = p + 0.04\zeta$) exercised by the regulatory agencies (or by the market mechanism like customer preferences), the effects of varying the carbon cap and offset price changes are experienced jointly. Figure 6 is exactly equivalent to Figure 3 except for the s^+ series in the offset system. As a result, the same policy with same cost structure is observed as in Figure 5 for $\zeta \geq EM(Q_e^*)$.

Finally, the extent to which organizations may carbon-label their products is tested. For $\zeta \geq EM(Q^*)$, there is no need for offsetting and hence carbon labeling is not valid for such caps. However, when the cap is tighter, then the price of the product should increase to achieve the cost value attained when $\zeta \geq EM(Q^*)$ (compared with the case where $r_1 = 15$ and $p = 5$). In order to lower the emissions 10 units (i.e. $\zeta = 329$); for example, the carbon labeling price r_2 should equal to 15.11.

3. Results

In this section, we summarize the results based on the analysis given in previous sections.

I – Incorporating sustainability into standard operational decision making practices has an impact on the operating policies of organizations.

The analysis performed in this paper demonstrates how environmental and social concerns can be incorporated into a standard operational decision-making model for inventory control. Since the optimal order quantity becomes a function of all the economic, social and environmental parameters (see §2.4.1 Eq. (50) and §2.4.2), the cost structure is affected by this revised model (see §2.4.1 Eq.(51)). Therefore, there are important differences in the policies and the resulting costs when the triple bottom line approach of sustainability is considered.

II – Organizations should estimate environmental and social parameters in order to achieve sustainability.

Estimating environmental and social parameters is a necessity for the organizations to comply with the emerging legislative restrictions and market requirements. In order to operate sustainably, organizations should consider their environmental and social impacts in addition to economic factors. For this reason, managers need to identify sources of these impacts and estimate the parameters of these additional criteria in order to be able to model and assess environmental and social impacts of the organizations (see §2.2.1 and §2.2.3).

III – Regulatory agencies' intervention is the key to achieve sustainability until market awareness is established.

Regulatory agencies should work to increase market awareness on decision making strategies that considers sustainability (see §2.2.4, §2.2.5, §2.5.1 and §2.5.1). Such a practice does not only benefit the customers but also the organizations and the whole economy in the long term. However, regulatory agencies should put in place tax schedules and caps until the natural market mechanism is established (see §2.2.2, §2.2.3, §2.3.2 and §2.4.2).

IV – Exercising caps (as in direct cap, cap and trade, and carbon offset models) on emissions (working hours) is the key to strictly curb emissions (working hours).

The analysis show that mechanisms involving direct accounting and tax (see §2.2.1, §2.2.1, §2.3.1 and §2.4.1) give organizations incentive to investigate sources of environmental and social costs; however do not provide a rigid obligation to consider these added dimensions of sustainability. On the other hand, mechanisms involving a cap (direct cap, cap and trade, and carbon offset models; see §2.2.3, §2.2.4, §2.2.5, §2.3.2, §2.4.2, §2.5.1 and §2.5.2) steer the organizations to adjust their policies accordingly. Therefore, a strict control of emissions (working hours) is possible only when caps are exercised by the regulatory agencies.

V – Organizations should find ways to improve not only the economic cost parameters but also the environmental and social cost parameters.

The analysis with the direct accounting approach shows that the revised optimal cost is always larger than the standard EOQ optimal cost even when the policy remains as the EOQ optimal order quantity (see Eq. (5), (39), and (51)). This suggests that organizations should improve their environmental and social impact parameters in addition to the economic ones. However, discussing these improvement opportunities is in the scope of another study, hence left aside.

4. Conclusions, Discussion and Future Work

Sustainability in supply chain management is an emerging requirement for a better business practice. In achieving sustainability, decision makers should adopt environmental and social considerations as well as the traditional economic objectives. This paper attempts to address the issue from an operations management perspective. We utilize a simple and widely used inventory control model; namely the EOQ model, and revise the standard model with additional environmental and social criteria. We provide alternative model formulations for a number of different settings and derive useful and practical results analytically and numerically for decision and policy makers.

This paper also serves as a reference point illustrating alternative environmental and social management approaches and their modeling apart from revising a classical model of an operational issue in supply chain management. As depicted, alternative modeling schemes are available for different approaches.

Further research directions are numerous. In our study, we consider the standard EOQ setting with a single item at a single location with no backlogging, constant lead times, and an unlimited supply. These assumptions may be relaxed to account for multiple items at multiple locations with planned backorders, variable lead times, and a finite production rate; for example.

It is of course possible to include traditional extensions of EOQ model in the above models such as quantity discounts, imperfect quality, and resource constraints such as warehouse space. Furthermore, other forms of cost-accounting (such as NPV accounting instead of considering the average total cost) may also be utilized.

We consider a single organization in our study. Considering multiple organizations and echelons in the chain and analyzing the terms of coordination among supply chain members with these added environmental and social criteria will possibly reveal new insights for sustainable supply chain management. New types of contracts may also be designed which coordinates the chain *sustainably*. An interesting and possibly more complex problem in the cap and trade system; for instance, is the one where organizations decide on the price of the allowances. A proper treatment of such a problem would require a game theoretic analysis.

Finally, there are other environmental and social criteria which may be considered and included in the models. We, however, picked carbon footprint and working hours to illustrate sustainability concerns in the EOQ problem context. This choice is optional, of course, and other environmental and social metrics may be used to assess sustainability in supply chain management.

Acknowledgements

The Scientific and Technological Research Council of Turkey (TUBITAK-BIDEB) and Vehbi Koç Foundation are gratefully acknowledged for providing financial support.

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Appendix A. Notation Used in Model Formulations (in the order of appearance)

λ	*demand rate*
Q	*order/production quantity*
Q^*	*optimal order/production quantity in the standard EOQ model*
$C(Q)$	*total average cost of replenishment*
$I(t)$	*inventory level at time t*
t	*time index*
h	*holding cost*
K	*fixed/setup cost*
c	*variable cost*
T	*cycle length*
$C'(Q)$	*first derivative of* $C(Q)$ *with respect to Q*
$C''(Q)$	*second derivative of* $C(Q)$ *with respect to Q*
f	*fixed amount of GHG emissions*
v	*variable amount of GHG emissions*
g	*amount of GHG emissions due to inventory holding*
Q^*_{ee}	*optimal order quantity with economic and environmental criteria*
$C(Q^*_{ee})$	*optimal cost with economic and environmental criteria*
p	*tax rate / carbon price / offset price*
ζ	*GHG emissions cap*
$C(Q^*)$	*optimal cost in the standard EOQ model*
$EM(Q^*)$	*optimal total average emissions with standard EOQ optimal order quantity*
$EM(Q^*_{ee})$	*optimal total average emissions with economic and environmental criteria*
Q^*_e	*optimal order quantity with pure environmental criterion*
s^+	*amount of allowances sold*
s^-	*amount of allowances / offsets purchased*
y_1	*a binary decision variable*
y_2	*a binary decision variable*
M	*a large nonnegative number*
a	*a nonnegative scalar*
b	*a nonnegative scalar*
r_1	*product price charged to regular customers*
r_2	*product price charged to environmentally sensitive customers*
m	*fixed amount of man-hours required*

n	*variable amount of man-hours required*
l	*man-hours required for inventory holding*
W	*available man-hours*
Q_{se}^*	*optimal order quantity with social and economic criteria*
$C(Q_{se}^*)$	*optimal cost with social and economic criteria*
$RM(Q^*)$	*optimal total average required man-hours with standard EOQ optimal order quantity*
$RM(Q_{se}^*)$	*optimal total average required man-hours with social and economic criteria*
Q_s^*	*optimal order quantity with pure social criterion*
Q_{ees}^*	*optimal order quantity with economic, environmental and social criteria*
$C(Q_{ees}^*)$	*optimal cost with economic, environmental and social criteria*
$EM(Q_e^*)$	*optimal total average emissions with pure environmental criterion*
$C(Q_e^*)$	*optimal cost with pure environmental criterion*

Appendix B. Data Used in Numerical Analysis

The following table (Table 2) gives the base values of the parameters used in the numerical analysis:

Table 2
Base parameter values

Parameter	Cap and Trade	Carbon Offset
λ	50	50
K	40	40
c	12	12
h	2	2
f	60	60
v	5	5
g	1	1
ζ	300	300
p	5	5
M	1000000	*N/A*
a	0,04	0,04
b	20	20
r_1	N/A	*15*
r_2	N/A	*No base value*

Appendix C. Figures for the Numerical Analysis

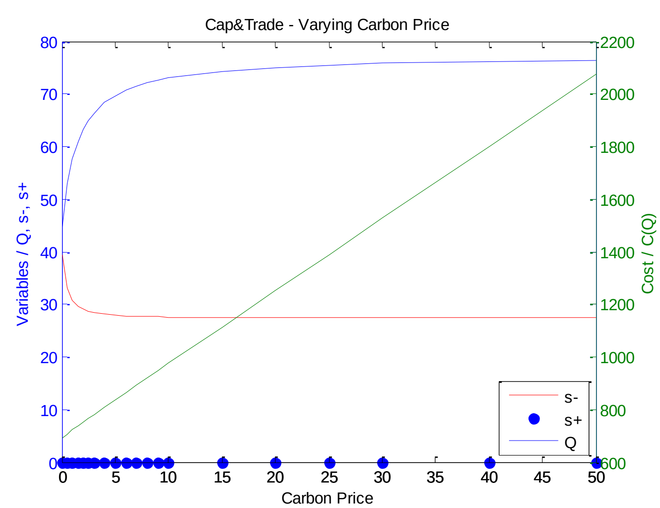

Figure 1. Varying carbon price

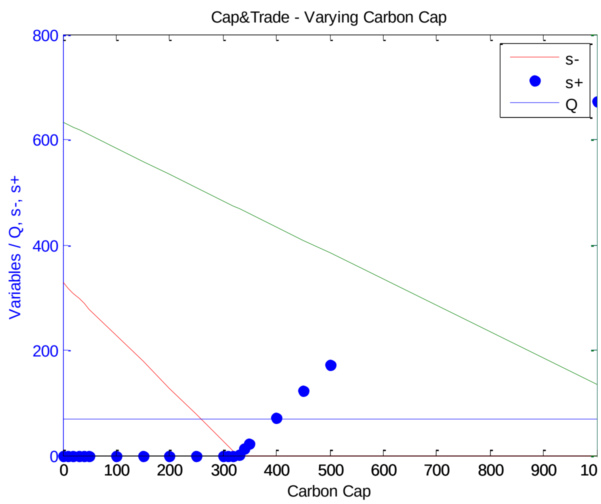

Figure 2. Varying carbon cap

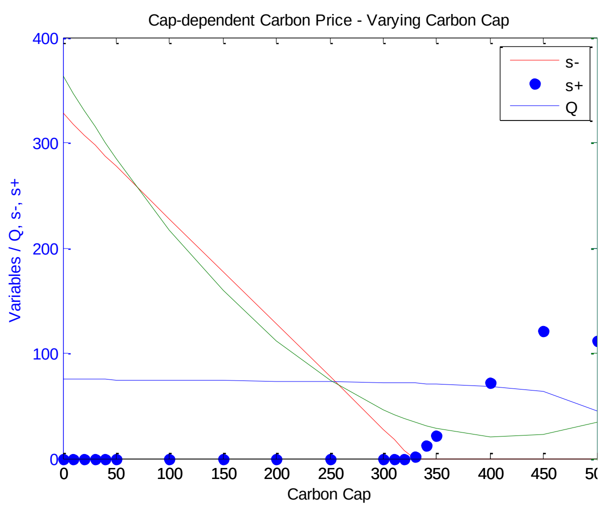

Figure 3. Cap-dependent carbon price

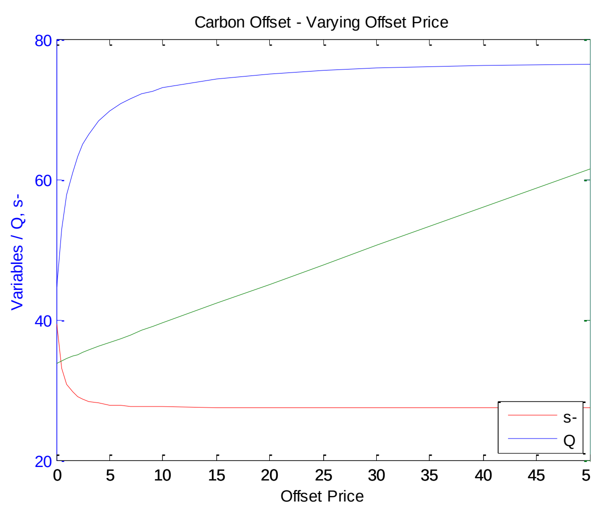

Figure 4. Varying offset price

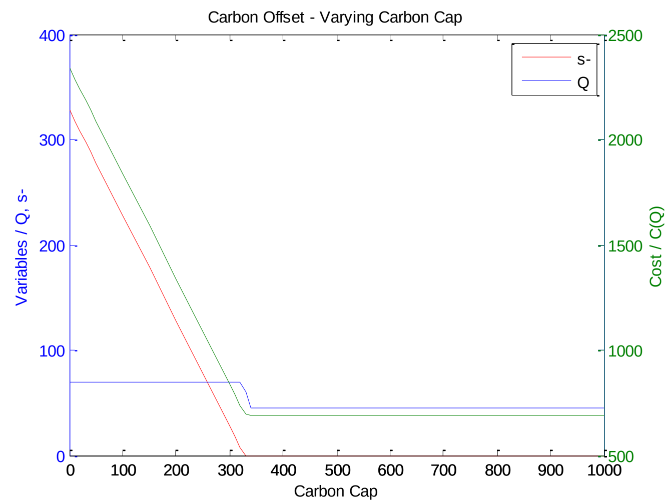

Figure 5. Varying carbon cap

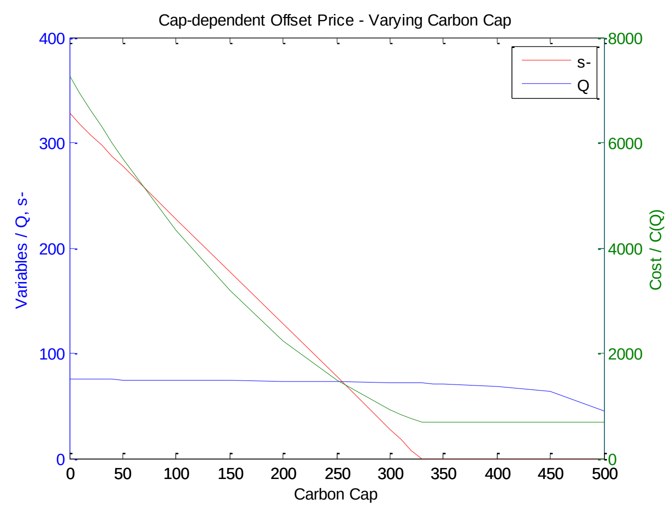

Figure 6. Cap-dependent offset price

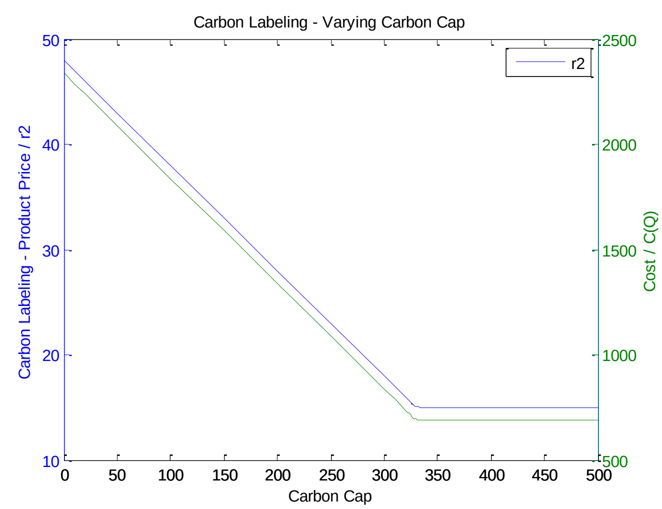

Figure 7. Carbon Labeling