A Green Cost Based Economic Production/Order Quantity Model

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Abstract
In this paper we investigate the integration of green cost into the economic production quantity (EPQ) model and economic order quantity (EOQ) model. The trade off among set up/order cost, holding cost and green cost is fundamental in managing a modern environment-conscious inventory system. The main contribution of the paper to the literature is the inclusion of green cost into the production/order quantity model. We derive the optimal production/order quantity considering production/order efficiency and environment impact. We further investigate the percentage cost penalty (PCP) when the production quantity deviates from the optimal production/order quantity and the percentage cost difference (PCD) when green cost is not considered in EPQ/EOQ model.

Keywords: Economic production/order quantity; Inventory control; Lot sizing; Green supply chain management

1. INTRODUCTION

Since businesses have been organized to bring products and services to customers, research on supply chain management is growing with practice. A supply chain (Beamon, 1998), at its highest level, is comprised of two basic, integrated processes: (i) the production planning and inventory control process, and (ii) the distribution and logistics process. Inventory control is one of the most important areas in supply chain management and has received much coverage in the management literature. Mathematical models for inventory control can be traced back to Harris, who first introduced economic order quantity (EOQ) model in 1913 and Taft, who presented economic production quantity (EPQ) model in 1918.

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The economic production quantity (EPQ) model is a variation of the economic order quantity (EOQ) model with noninstantaneous receipt hence we cover both models in this paper. Although EPQ/EOQ has been criticized for their restrictive set of assumptions (see for example, Adkins, 1984; Zangwill, 1987; and Woolsey, 1988), both lot size models have been studied extensively. Many researchers acknowledge the restrictive assumptions of the EOQ and EPQ and construct models to overcome these limitations. beyond. For example, Jaber & Bonney (2001), Jaber & Salamek (1995), Muth & Spremann (1983), Keachie & Foutana (1966) varied the production rate by integrating learning and/or forgetting into EPQ model.

As far as we know, there is little, if any research that integrates environment thinking into the EPQ or EOQ models. By introducing an environmental budget term (AQ) in the formulation of the models, our research explores the impact on the environment as well as economic performance and hopefully derives a cost-effective environmentally conscious production/order quantity.

In the past decades, there has been a growing awareness in society of the impact that economic decisions have on the environmental. Cases in point are global warming, carbon emissions and the recent environment disaster of BP oil spill on Gulf of Mexico. Organizations are also conscious about their environmental performance in response of the regulation of ISO 14000, which was launched in 1966 by the international organization for standardization and promoted environmental friendly economic activities. Green concepts such as green strategy, green design, green production, green operation, green partner, etc., affect all stages of the supply chain and can directly impact decisions made in the management of supply chains. As defined by Srivastava (2007), research on green supply chain management can be classified into three groups: (i) importance of green supply chain management, as discussed in Porter (1995 a, b) and Sarkis (2001) who consider the competitive aspects of green, (ii) green design, which focuses on environmental conscious design using less hazardous or non-hazardous materials or process and is discussed by Guide & Srivastava (1997a, 1997b), and (iii) green operations, which has been explored by Johnson (1998) with emphasis on recycling and reverse logistics.

Our green production/order research pioneers to connect the two groups of research: research on economic production/order quantity (EPQ/EOQ) mathematical models, which have been the backbone of inventory management, and research on green, which is an emerging trend for modern management.

1.1 Green Cost

"By considering pollution prevention separately from other manufacturing needs, such as productivity and quality improvements, most pollution prevention programs fail to develop the vital synergies and working relationships with manufacturers that are essential to drive both pollution prevention and manufacturing competitiveness"

-------1994, the US Office of Technology reported to the Congress

We integrate an environmental budget, interchangeably green cost, AQ in our production/order quantity model. For an environment proactive partner in supply chain, say, the manufacturer, the environmental budget can be used to correct detrimental environment impacts (for example, pollution of soil) that may result due to its economic activity. Green cost is determined by the carrying time and extensity of the economic activity. In our model, green cost, AQ, is introduced as a linear function of production/order quantity. The green cost per unit time per unit quantity (A) is fixed although it may be determined by many factors in real life, such as the properties of industries. Electric services are considered the highest pollution cost while the retail trade the least.

1.2 Paper Organization

In this paper we propose a green cost based economic production/order quantity (EPQ/EOQ) model and derive the optimal production/order quantity considering both of production/order economic performance (setup/order and carrying cost) and environment performance (green cost). This paper is organized as follows. In section 2, we present the formulation of the green economic production/order quantity (EPQ/EOQ) model and provide the optimal solution. In section 3, we investigate two types of percentage cost analyses, the percentage cost penalty (PCP) and the percentage cost difference (PCD). The summary and conclusion is present in section 4. Supporting derivations of the PCP are presented in Appendices 1 & 2.

2. MODEL DEVELOPMENT

2.1 Assumptions and Nomenclature

We adopt the traditional EPQ modeling assumptions: (i) a single product is considered, (ii) the demand rate and production rate are constant over time with the production rate greater than the demand rate, (iii) constant annual demand (iv) zero lead time, (v) the unit production cost is independent of the production quantity, and (vi) a perfect product is produced.

The following notation is used in the development of the model:

D: annual demand

d: demand rate

p: production rate (p>d)

Q: production/order quantity

Q*: economic production/order quantity

H: holding cost per unit per unit time

S: set up/order cost per production cycle

A: green cost per unit per unit time

GT(Q): total annual cost with green cost

GT(Q*): minimal cost with green cost

T(Q): total annual cost without green cost

PCP: percentage cost penalty when production quantity deviates from optimal production quantity.

PCD: percentage cost difference when green cost is not considered.

2.2 Model definition and development

2.2. 1 Green economic production quantity (Green EPQ) model

Since green cost is a component of green production, the environmental conscious producer should balance economic production performance and environmental performance when deciding on a cost-effective production quantity. The total annual cost for the EPQ model with green cost is composed of the sum of annual set up cost, annual holding cost and green cost and is defined as

$$\mathrm{GT}(\mathrm{Q}) = \frac{DS}{Q} + \frac{HQ}{2}\frac{(p-d)}{\mathrm{p}} + AQ \qquad (1)$$

Examining (1) we note that set up cost increases when production quantity Q decreases, whereas holding cost and green cost increases with Q. To determine the production quantity Q that minimizes the annual total cost, we differentiate (1) with respect to Q, set the result equal to zero and then solve for Q. This gives:

$$\frac{dGT(Q)}{dQ} = -\frac{DS}{Q^2} + \frac{H}{2}\left(\frac{P-D}{P}\right) + A \tag{2}$$

and

$$Q^* = \sqrt{\frac{2DSp}{H(p-d)+2Ap}}\,. \tag{3}$$

The second derivative of (1) is

$$\frac{d^2\,GT(Q)}{dQ^2} = \frac{2DS}{Q^3} \tag{4}$$

Which for D>0 and S>0, insures that (3) is the optimal production quantity, Q*

Substituting Q* into (1) and simplifying yields the minimal total cost

$$GT(Q^*) = \sqrt{\frac{(H(p-d)+2Ap)(2DS)}{p}} \tag{5}$$

When we don't incorporate green cost in EPQ model, A=0 and our green cost based EPQ model becomes classic EPQ model (see Table 1).

Table 1: Summary of Green EPQ and Classic models

	Green EPQ	Classic EPQ
Total cost	$\frac{DS}{Q} + \frac{HQ}{2}\left(\frac{p-d}{p}\right) + AQ$	$\frac{DS}{Q} + \frac{HQ}{2}\left(\frac{p-d}{p}\right)$
Optimal quantity	$\sqrt{\frac{2DSp}{H(p-d)+2Ap}}$	$\sqrt{\frac{2DSp}{H(p-d)}}$
Minimal cost	$\sqrt{\frac{(H(p-d)+2Ap)(2DS)}{p}}$	$\sqrt{\frac{H(p-d)(2DS)}{p}}$

When green cost is integrated into the EPQ model, the smaller optimal production quantity results in comparison to that of an EPQ without a green cost consideration. A proactive manufacturer not only considers production efficiency but also environment effect, as the result of trade off, smaller production quantity is preferred and environment cost is budgeted.

2.2. 2 Green economic order quantity (Green EOQ) model

When the production rate p is much greater than the demand rate d, p>>d, p-d≈p, and the green economic production quantity model becomes the green economic order quantity model with total annual cost and the optimal order quantity defined as

$$GT(Q) = \frac{DS}{Q} + \frac{HQ}{2} + AQ \tag{6}$$

$$Q^* = \sqrt{\frac{2DS}{H+2A}} \tag{7}$$

The resulting minimum total cost is

$$GT(Q^*) = \sqrt{2DS(H+2A)} \tag{8}$$

When A=0, the green EOQ model becomes classic EOQ model. Table 2 provides a comparison of green EOQ and classic EOQ model.

Table 2: Summary of Green EOQ and classic EOQ models

	Green EOQ	Classic EOQ
Total cost	$\frac{DS}{Q} + \frac{HQ}{2} + AQ$	$\frac{DS}{Q} + \frac{HQ}{2}$
Optimal quantity	$\sqrt{\frac{2DS}{H+2A}}$	$\sqrt{\frac{2DS}{H}}$
Minimal cost	$\sqrt{2DS(H+2A)}$	$\sqrt{2DSH}$

Similar to the comparison of the green and classic EPQ models, the green EOQ results in a smaller order quantity than that of the classic EOQ model. A proactive buyer not only considers order efficiency but also environment effect, as the result of trade off, smaller order is preferred and environment cost is budgeted.

3. SENSITIVITY OF GREEN EPQ/EOQ MODEL: PCP and PCD

We investigate the sensitivity of green EPQ/EOQ under two scenarios: percentage cost penalty (PCP) when production/order quantity deviates from optimal production; percentage cost difference (PCD) when the component of green cost is not considered.

3.1 PCP for Green EPQ and Green EOQ

Percentage cost penalty (PCP) is defined as

$$\mathrm{PCP} = \frac{GT(Q) - GT(Q*)}{GT(Q*)} \tag{9}$$

we have PCP for Green EPQ (see detailed derivation from appendix 1)

$$\mathrm{PCP} = \frac{1}{2}\left(\frac{Q}{Q^*} + \frac{Q^*}{Q}\right) - 1 \tag{10}$$

we have PCP for Green EOQ (see detailed derivation appendix 2)

$$\mathrm{PCP} = \frac{1}{2}\left(\frac{Q}{Q^*} + \frac{Q^*}{Q}\right) - 1 \tag{11}$$

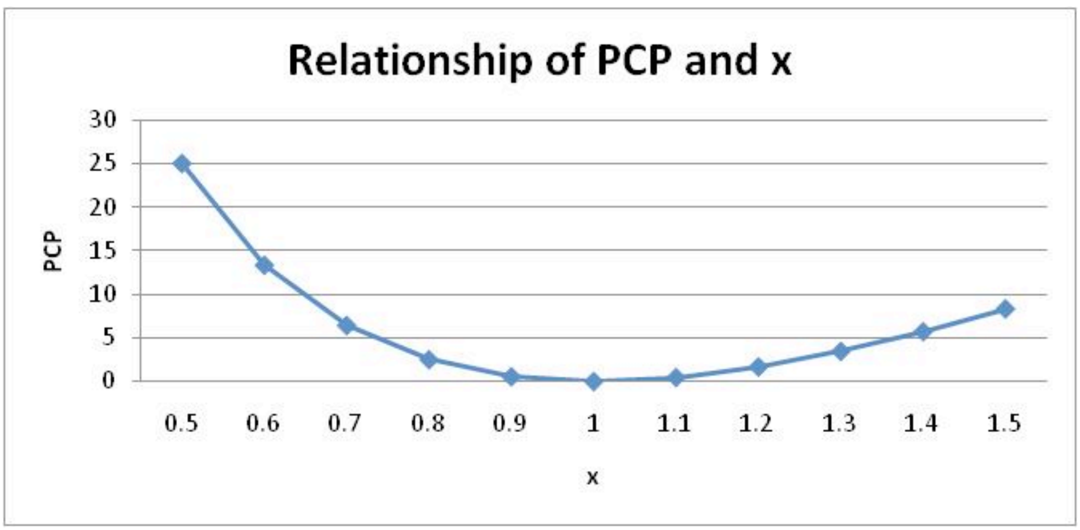
For PCP, green EOQ model and green EPQ model have the same mathematical expression with EOQ from Wagner (1969). Since they have the same expression, we adopt the technique from Guiffrida & Papp (2008) EOQ model to change the form of expression of PCP in term of x, showing how the percentage cost penalty (PCP) reacts with underestimate or overestimate production/order quantity in our green EPQ and green EOQ models. x is the ratio of how much production/order quantity Q deviates from optimal production/order quantity Q*.

we set Q=xQ* whereby

$$X = \begin{Bmatrix} 0 < x < 1 & when\ Q < Q* \\ x = 1 & when\ Q = Q* \\ x > 1 & when\ Q > Q* \end{Bmatrix}$$

Then Eq. (10) and Eq. (11) become

$$\text{PCP} = \frac{1}{2}\left(\frac{1+x^2}{x}\right) - 1 \tag{12}$$



From the above graph, we can see that our green EPQ/EOQ models are robust. In other words, when production/order quantity is over or under optimal quantity, the percentage cost penalty (PCP) varies slowly.

3.2 PCD for Green EPQ and Green EOQ

When a green cost is not considered in EPQ/EOQ model, the percentage cost penalty (PCD) is defined as

$$\text{PCD} = \frac{GT(Q)-T(Q)}{T(Q)} \tag{13}$$

Substituting (1) and $\text{T}(\text{Q}) = \frac{DS}{Q} + \frac{HQ}{2}\left(\frac{p-d}{p}\right)$ from classic EPQ model into (13), we have

$$\text{PCD} = \frac{AQ}{\frac{DS}{Q}+\frac{HQ}{2}(\frac{p-d}{p})} \tag{14}$$

When p>>d, p-d≈p, Eq.13 becomes Eq.14, which is PCD for green EOQ

$$\text{PCD} = \frac{AQ}{\frac{DS}{Q}+\frac{HQ}{2}} \tag{15}$$

For green EPQ/EOQ ,when A, D,S, H, p, d and q are variables, PCD is sensitive to any change of these variables; when A, D,S, H, p and d are constants, PCD is a function of production/order quantity of green EPQ/EOQ.

4. Summary and Future Research

In our research, we have derived economic production quantity and economic order quantity models that incorporate green costs into their formulations. The model solutions demonstrate that for the EPQ model a smaller production quantity results. Similar results were found for the EOQ order quantity. For the environmental conscious producer or buyer, the tradeoff between economic performance and environment performance is facilitated by the models, as well as the allocation of environment cost that is budgeted, suggests that traditional business practices involving economic lot sizing decisions within the supply chain may need to be reexamined.

We have also explored the sensitivity of the green EPQ and green EOQ models when the production/order quantity deviates from optimal production/order quantity, and when a green cost is integrated in the green EPQ and EOQ models. We illustrate both graphically and by formula the robustness of the model solutions. An understanding of the robust nature of the models contributes to a decision maker's ability to further gauge the sensitivity of their lot sizing decisions under varying parameter values. Several opportunities for future research on green EPQ/EOQ lot sizing models exist. First, we could explore modeling the green cost as a non-linear function of production/order quantity. Second, the green cost per unit per unit time parameter, (A), which was constant in the models presented herein could be defined for specific industries thereby enhancing the overall scope of application for our green EPQ/EOQ models.

Appendix 1: PCP for green EPQ model

$$\text{PCP} = \frac{GT(Q) - GT(Q*)}{GT(Q*)}$$

Substituting $\text{GT(Q)} = \frac{DS}{Q} + \frac{HQ}{2}\frac{(p-d)}{\text{p}} + AQ$ and $\text{GT}(\text{Q}^*) = \frac{H(p-d)+2Ap}{p}\sqrt{\frac{2DSp}{H(p-d)+2Ap}}$ into PCP:

$$= \frac{\frac{DS}{Q}+\frac{HQ}{2}(\frac{p-d}{p})+\text{AQ}}{\frac{H(p-d)+2Ap}{p}\sqrt{\frac{2DSp}{H(p-d)+2Ap}}} - 1$$

Substituting $\text{Q}^* = \sqrt{\frac{2DSp}{H(p-d)+2Ap}}$ into PCP,

$$= \frac{Q(\frac{DS}{Q^2}+\frac{H}{2}(\frac{p-d}{p})+\text{A})}{Q*(\frac{H(p-d)+2Ap}{p})} - 1$$

$Q^{*^2} = \frac{2DSp}{H(p-d)+2Ap}$, substituting $DS = \frac{Q*^2}{2P}(H(p-d)+2Ap)$ into PCP

$$= \frac{Q\left(\frac{H(p-d)+2Ap}{2p}\right)\left(\frac{Q*^2+Q^2}{Q^2}\right)}{Q*\left(\frac{H(p-d)+2Ap}{p}\right)} - 1$$

$$= \frac{1}{2}\left(\frac{Q}{Q^*} + \frac{Q^*}{Q}\right) - 1$$

Appendix 2: PCP for green EOQ model

$$\mathrm{PCP} = \frac{GT(Q) - GT(Q*)}{GT(Q*)}$$

Substituting $\mathrm{GT}(Q) = \frac{DS}{Q} + \frac{HQ}{2} + AQ$ and $\mathrm{GT}(Q^*) = (H+2A)\sqrt{\frac{2DS}{H+2A}}$ into PCP :

$$= \frac{\frac{DS}{Q}+\frac{HQ}{2}+AQ}{(H+2A)\sqrt{\frac{2DS}{H+2A}}} - 1$$

Substituting $Q^* = \sqrt{\frac{2DS}{H+2A}}$ into PCP,

$$= \frac{Q\left(\frac{DS}{Q^2}+\frac{H}{2}+A\right)}{Q*(H+2A)} - 1$$

$Q^{*^2} = \frac{2DS}{H+2A}$, substituting $DS = \frac{Q*^2}{2}(H+2A)$ into PCP

$$= \frac{Q\left(\frac{H+2A}{2}\right)\left(\frac{Q*^2+Q^2}{Q^2}\right)}{Q*(H+2A)} - 1$$

$$= \frac{1}{2}\left(\frac{Q}{Q^*} + \frac{Q^*}{Q}\right) - 1$$

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