Faculty of Industrial and Mechanical Engineering

Course: Construction Mechanic

Group: I4-GIM (B)

Homework :03

KUKA LBR iiwa 14 R820 (ix third joint for simplification)

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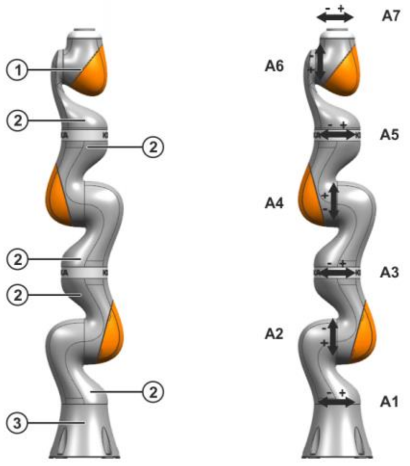
I. Introduction

I.1. Description of LBR iiwa

Kuka LBR iiwa 14 R820

The LBR iiwa is classified as a lightweight robot and is a jointed-arm robot with **7 axes**. All motor units and current-carrying cables are protected beneath cover plates. Each axis is protected by means of axis range sensors and can be adjusted by means of internal sensors. Each joint is equipped with a position sensor on the input side, torque sensors on the output side and temperature sensors. The robot can thus be operated with position and impedance control. The temperature sensors prevent thermal overloading of the robot. The robot is equipped redundantly and consists of the following principal components.



Main assemblies and robot axis

1. In-line wrist
2. Joint module
3. Base frame

In-line wrist: the robot is fitted with a 2-axis in-line wrist. The motors are located in axes A6 and A7.

Joint module: the joint modules consist of an aluminum structure. The drive units are situated inside these modules. In this way, the drive units are linked to one another via the aluminum structures.

Base frame: The base frame is the base of the robot. Interface A1 is located at the rear of the base frame. It constitutes the interface for the connecting cables between the robot, the controller and the energy supply system.

Technical data

I.2.Basic data

	LBR iiwa 14 R820
Number of axes	7
Number of controlled axes	7
Volume of working envelope	1.8 m³
Pose repeatability (ISO 9283)	± 0.15 mm
Weight	approx. 29.9 kg
Rated payload	14 kg
Maximum reach	820 mm
Protection rating	IP 54
Protection rating, in-line wrist	IP 54

	LBR iiwa 14 R820
Sound level	< 75 dB (A)
Mounting position	Floor
Footprint	-
Permissible angle of inclination	-
Default color	-
Controller	KUKA Sunrise Cabinet
Transformation name	-

Ambient temperature during operation	5 °C to 33 °C (278 K to 306 K)
Ambient temperature during storage/transportation	0 °C to 45 °C (273 K to 318 K)
Air humidity	20 % to 80 %

I.3.Axis data, LBR iiw 14 R820

Range of motion	
A1	±170 °
A2	±120 °
A3	±170 °
A4	±120 °
A5	±170 °
A6	±120 °
A7	±175 °
Speed with rated payload	
A1	85 °/s
A2	85 °/s
A3	100 °/s
A4	75 °/s
A5	130 °/s
A6	135 °/s
A7	135 °/s

The shape and size of the working envelop for the robot Kuka LBR iiw R820

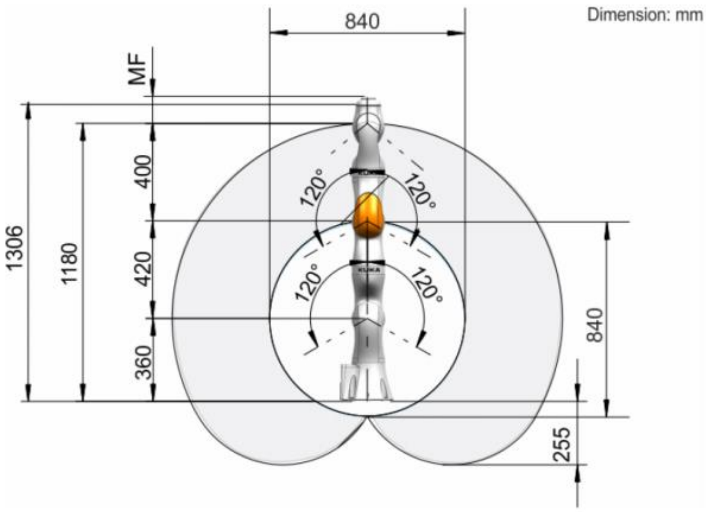


Figure 4: Kinematic Lengths

Working envelop of robot Kuka LBR iiw R820

I.4.Payloads of Kuka LBR iiw R820

The KUKA LBR iiwa 14 R820 robot is a 7-axis robot arm, it offers a 14 kg payload and 820 mm reach. The repeatability of the KUKA LBR iiwa 14 R820 robot is 0.1 mm.

Common applications of the KUKA LBR iiwa 14 R820 robot include: arc welding, dispensing, remote tcp, and spot welding.

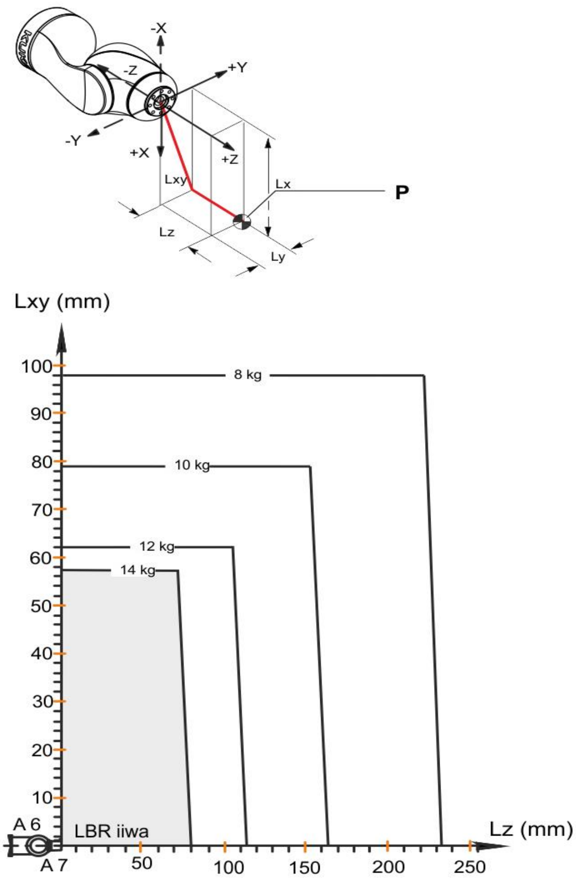
Payloads

Rated payload	14 kg
Rated mass moment of inertia	0.3 kgm²
Rated total load	14 kg
Rated supplementary load, base frame	0 kg
Maximum supplementary load, base frame	-
Rated supplementary load, rotating column	0 kg
Maximum supplementary load, rotating column	-
Rated supplementary load, link arm	0 kg
Maximum supplementary load, link arm	-
Rated supplementary load, arm	0 kg
Maximum supplementary load, arm	-
Nominal distance to load center of gravity	
Lxy	40 mm
Lz	44 mm

Load center of gravity P

For all payloads, the load center of gravity refers to the distance from the face of the mounting flange on axis A7.

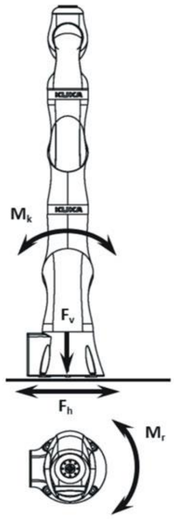
Payload diagram



This loading curve corresponds to the maximum load capacity. Both values (payload and mass moment of inertia) must be checked in all cases. Exceeding this capacity will reduce the service life of the robot and overload the motors and the gears; in any such case KUKA Customer Support must be consulted beforehand. The values determined here are necessary for planning the robot application. For commissioning the robot, additional input data are required in accordance with the operating and programming instructions of the control software.

I.4.Foundation data, Kuka LBR iiw R820

Foundation load



Foundation loads for floor-mounted robots

The specified forces and moments already include the payload and the inertia force (weight) of the robot.

Vertical force	541.2 N
Horizontal force	228.4 N
Tilting moment	281.6 Nm
Torque about axis 1	172.6 Nm

II. Objective

2. Develop kinematic of robot

In order to formulate a mathematical model for forward kinematics and inverse kinematics of the Kuka14
certain assumptions have been considered under which the robot will operate. The list of assumptions
is as below:

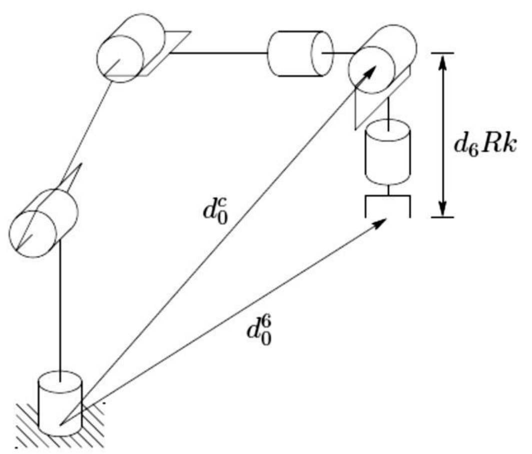
1. All bodies are assumed to be perfectly rigid
2. No slip between mating parts
3. No backlash in the gears of the motors
4. Required power, torque and weld wire feed is always available to the system
5. A finite length straight cylindrical weld torch is mounted at the last joint of the robot arm
6. Robotic arm has a fixed base

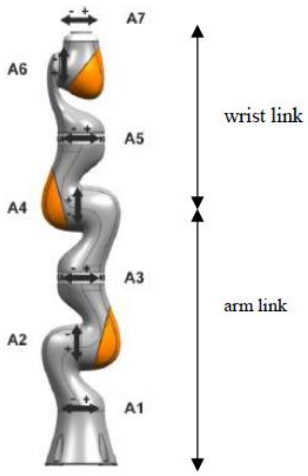
7. No physical obstruction in the workspace of the robot
8. All the revolute joints are at zero degrees in zeroth condition.
9. For developing inverse kinematics model, 6-DOFs are only considered, instead of 7-DOFs, by locking the 3*rd* joint, to avoid infinite number of solutions
10. Robot's base is placed at the origin of the world frame and is oriented in such a way that the axis
directions of the world frame and the base frame coincide with each other.
11. Desired position and orientation of the end-effector lies withing the robot's workspace

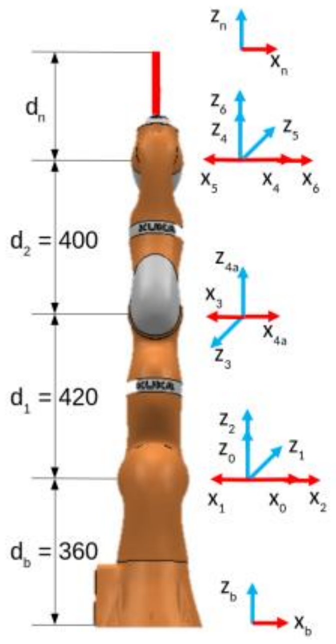
- IK (Inverse Kinematic) of robot Kuka iiwa

IK analytical approach of Kuka

This robot has 6 dof industrial robot:







3. Solve Forward Kinematic Problem

Forward kinematics is used to determine the end-effector's position and the orientation for a given set of joint parameters. For Kuka14 since all the 7 joints are revolute joints, let us denote the joint parameters by θi for i, where θi is the angular position of the respective joint measured in degrees.

Frame	θ_{i-1}	d_{i-1}	a_i	α_i
b-0	0	d_b	0	0
0-1	$180^\circ + \theta_1$	0	0	90°
1-2	$180^\circ + \theta_2$	0	0	90°
2-3	$180^\circ + \theta_3$	d_1	0	-90°
3-4a	$180^\circ + \theta_4$	0	0	-90°
4a-4	0	d_2	0	0
4-5	$180^\circ + \theta_5$	0	0	90°
5-6	$180^\circ + \theta_6$	0	0	90°
6-n	θ_7	d_n	0	0

Table 1: DH table

Using the above DH table, all A and T matrices are calculated from equation (1) and (2).

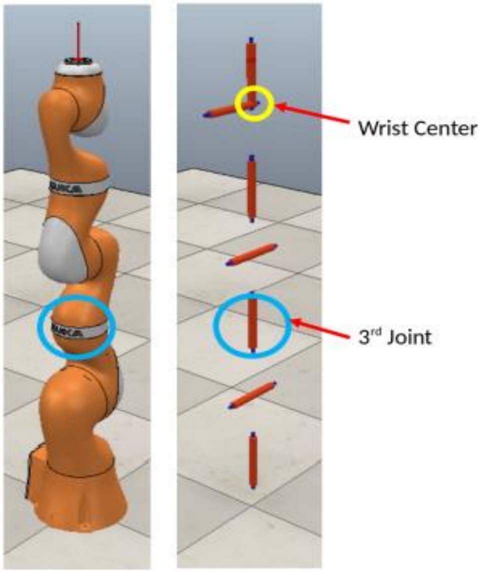
$$A_i = \begin{bmatrix} \cos(\theta_{i-1}) & -\sin(\theta_{i-1})\cos(\alpha_i) & \sin(\theta_{i-1})\sin(\alpha_i) & \cos(\theta_{i-1})a_i \\ \sin\theta_{i-1} & \cos(\theta_{i-1})\cos(\alpha_i) & -\cos(\theta_{i-1})\cos(\alpha_i) & \sin(\theta_{i-1})a_i \\ 0 & \sin(\alpha_i) & -\cos(\alpha_i) & d_{i-1} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(1)

$$T_i^b = A_0 \times A_1 \times A_2 \ldots\ldots A_i \qquad \text{where i=0,1,2,3,4,5,6,7} \qquad (2)$$

4. Solve Inverse Kinematic Problem

Inverse kinematics is used to find the joint parameters for a given desired position and an orientation of the end-effector with respect to the world frame. After formulating the model for the forward kinematics, the next step is to solve the inverse kinematics of Kuka14 robotic arm.



Joints of Kuka LBR iiwa 14 R820

The presence of the wrist center allows the inverse kinematics problem to be decoupled into 2 problems of:

1. Inverse Position depending on bottom 4 joints (joints 1 to 4)
2. Inverse Orientation depending on top 3 joints (joints 5 to 7)

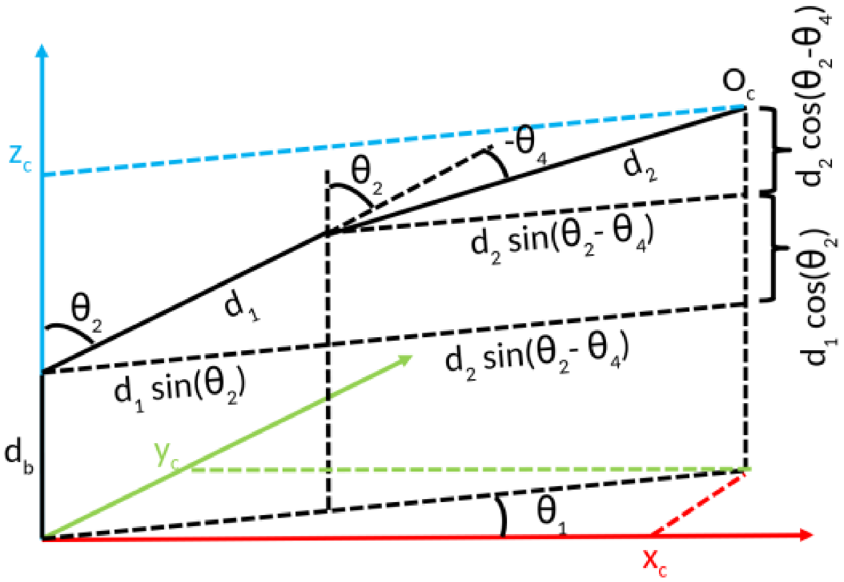
With respect to the base frame, let the desired position of the end-effector be denoted by O_d^b and the desired orientation by R_d^b. Also, the distance of the end-effector frame from the frame $x_6 - z_6$ is known (d_n). Hence, we can determine the position of the wrist center (O_d^b) by traversing a distance of d_n units in the direction of the negative z_n axis. This can be represented mathematically by eq (3).

$$O_d^b = O_d^b - (d_n \times R_d^b) \quad (3)$$

O_d^b can be decomposed into its components and represented as (x_c^b, y_c^b, z_c^b).

4.1. Inverse Position

In our context, inverse position refers to the problem of calculating the 4 joints parameters that will position. The wrist center at O_c^b. Since 4 unknowns (4 joint parameters) are required to be solved with 3 known value i.e. x_c^b, y_c^b and z_c^b, Hence this forms an under-constraint with infinite solution. Because of this reason it was started previously that inverse kinematic will be solved by locking 3^{rd} joints.



Inverse position geometrical representation

Inverse position for joint parameters θ_1, θ_2, and θ_4 can be solved geometrically from the figure. We get,

$$x_c^b = (d_1 \sin(\theta_2) + d_2 \sin(\theta_2 - \theta_4)) \times \cos(\theta_1) \quad (4)$$

$$y_c^b = (d_1 \sin(\theta_2) + d_2 \sin(\theta_2 - \theta_4)) \times \sin(\theta_1) \quad (5)$$

$$z_c^b = d_b + d_1 \cos(\theta_2) + d_2 \cos(\theta_2 - \theta_4) \quad (6)$$

Let $z_{cc}^b = z_c^b - d_b$. We can write equation (6) as

$$z_{cc}^b = d_1 \cos(\theta_2) + d_2 \cos(\theta_2 - \theta_4) \quad (7)$$

Solving for θ_1: By dividing (5) and (4), We get

$$\tan(\theta_1) = \frac{y_c^b}{x_c^b} \quad \text{or } \theta_1 = atan2(\frac{y_c^b}{x_c^b}) \quad (8)$$

Solving for θ_4: By squaring and adding (5) and (4), we get

$$x_c^{b^2} + y_c^{b^2} = (d_1 \sin(\theta_2) + d_2 \sin(\theta_2 - \theta_4))^2 \quad (9)$$

By squaring and adding (7) and (9), we get

$$x_c^{b^2} + y_c^{b^2} + z_{cc}^{b^2} = d_1^2 + d_2^2 + 2d_1 d_2 \cos(\theta_4) \quad (10)$$

$$\cos(\theta_4) = \frac{x_c^{b^2} + y_c^{b^2} + z_{cc}^{b^2} - d_1^2 - d_2^2}{2d_1 d\_2} \quad \text{or } \theta_4 = acos(\frac{x_c^{b^2} + y_c^{b^2} + z_{cc}^{b^2} - d_1^2 - d_2^2}{2d_1 d\_2}) \quad (11)$$

Solving for θ_2: By expanding the term $\cos(\theta_2 - \theta_4)$, we can rewrite (7) as

$$=> z_{cc}^b = d_1 \cos(\theta_2) + d_2 \cos(\theta_2) \cos(\theta_4) - d_2 \sin(\theta_2) \sin(\theta_4)$$

$$=> z_{cc}^b - d_1 \cos(\theta_2) - d_2 \cos(\theta_2) \cos(\theta_4) = -d_2 \sin(\theta_2) \sin(\theta_4)$$

Squaring both sides:

$$\Rightarrow \; (z_{cc}^{b} - d_1\cos(\theta_2) - d_2\cos(\theta_2)\cos(\theta_4))^2 = (-d_2\sin(\theta_2)\sin(\theta_4))^2$$

$$\Rightarrow \; (z_{cc}^{b} - d_1\cos(\theta_2) - d_2\cos(\theta_2)\cos(\theta_4))^2 = d_2^2(1 - cos^2(\theta_2))(1 - cos^2(\theta_4))$$

$$\Rightarrow \; cos^2(\theta_2)(d_2^1 + d_2^2 + 2d_1d_2\cos(\theta_4)) - 2z_{cc}^{b}(d_1 + d_2\cos(\theta_4))\cos(\theta_2) + [z_{cc}^{b^2} - d_2^2(1 - cos^2(\theta_4))] = 0$$

Using (10), we get

$$cos^2(\theta_2)(x_c^{b^2} + y_c^{b^2} + z_{cc}^{b^2}) - \frac{z_{cc}^{b}}{d_1}(x_c^{b^2} + y_c^{b^2} + z_{cc}^{b^2} + d_1^2 - d_2^2)\cos(\theta_2) + [z_{cc}^{b^2} - d_2^2(1 - cos^2(\theta_4))] = 0 \quad (12)$$

The above equation is a quadratic equation in $\cos(\theta_2)$ of the form $Ax^2 + Bx + c = 0$. It can be solved by using the quadratic formula to find the solutions for $\cos(\theta_2)$.

4.2. Inverse Orientation

Now we need to solve for θ_5, θ_6, and θ_7. These are calculated by comparing the orientation matrix for the end-effector obtained from the mathematical model of forward kinematics, with the already known orientation matrix R_d^b.

$$R_d^b = R_7^b \qquad (13)$$

Transformation matric:

$$R_d^b = \begin{bmatrix} R_i^j & O_i^j \\ \vec{0} & 1 \end{bmatrix} \qquad \text{where, } R_d^b = A_j \times A_{j+1} \times A_{j+2} \times \ldots\ldots..\times A_i$$

$$\text{or } R_d^b = A_j \times A_{j-1} \times A_{j-2} \times \ldots\ldots.\times A_i \qquad (14)$$

The orientation of the end-effector with respect to base frame can be represented as:

$$R_7^b = R_4^b R_7^4$$

$$\Rightarrow R_d^b = R_7^b R_7^b$$

Pre-multiplying by $R_4^{b^{-1}}$, we get

$$R_4^{b^{-1}} R_d^b = R_7^4$$

For rotation matric, $R^{-1} = R^T$, we can write

$$R_4^{b^T} R_d^b = R_7^4 \qquad (15)$$

Denote the L.H.S of equation (15) as R_{wrist}, R_7^4 can be express in term of θ_5, θ_6 and θ_7

$$\Rightarrow R_7^4 = \begin{bmatrix} -\sin(\theta_5)\sin(\theta_7) + \cos(\theta_5)\cos(\theta_6)\cos(\theta_7) & -\sin(\theta_5)\cos(\theta_7) - \cos(\theta_5)\cos(\theta_6)\sin(\theta_7) & \cos(\theta_5)\sin(\theta_6) \\ \cos(\theta_6)\sin(\theta_5)\cos(\theta_7) + \cos(\theta_5)\sin(\theta_7) & \cos(\theta_5)\cos(\theta_7) - \cos(\theta_6)\sin(\theta_5)\sin(\theta_7) & \sin(\theta_5)\sin(\theta_6) \\ -\sin(\theta_6)\cos(\theta_7) & \sin(\theta_6)\sin(\theta_7) & \cos(\theta_6) \end{bmatrix} \quad (16)$$

By equating the (16) to R_{wrist}, we can write

$$\tan(\theta_5) = \frac{(R_{wrist})_{23}}{(R_{wrist})_{13}} \quad \text{or} \quad \theta_5 = atan2(\frac{(R_{wrist})_{23}}{(R_{wrist})_{13}})$$

$$\cos(\theta_6) = (R_{wrist})_{33} \quad \text{or} \quad \theta_6 = \text{acos}\,((R_{wrist})_{33})$$

$$\tan(\theta_7) = \frac{(R_{wrist})_{32}}{-(R_{wrist})_{31}} \quad \text{or} \quad \theta_7 = atan2(\frac{(R_{wrist})_{32}}{-(R_{wrist})_{31}})$$