Problem 4-6 Formulation

October 18, 2023

Sets

N	Set of nodes $N \subseteq$N
E	Set of pipelines (edges) between two nodes
G	Set of generator/supplier nodes, $G \subseteq N$

Data

$\mathcal{G}$	the set of index of nodes that is a supplier node
$\mathcal{NG}$	the set of index of nodes that is not supplier nodes
$demand$	The demand of each nodes
	the $demand^1$ represents the demand in the 5th year
	the $demand^2$ represents the demand in the 10th year
$distances_i$	Length of an edge of pipeline i
$init\_capacity_i$	The initial capcity of $generator_i \subseteq G$
$add\_capacity_{i,j}$	The additional capcity of $generator_i \subseteq G$
	with choosing the upgrade option j, $i \in \{1, 2, 3, 4\}$,$j \in \{1, 2, 3\}$
$upgrade\_cost_{i,j}$	The cost of upgrade of $generator_i \subseteq G$
	with choosing the upgrade option j, $i \in \{1, 2, 3, 4\}$,$j \in \{1, 2, 3\}$
end(i)	index of pipelines that ends with node i
start(i)	index of pipelines that starts with node i

Variables

X_i^1	The amount of flow in pipeline$_i \in E$ in 5th year
X_i^{20}	The amount of flow in pipeline$_i \in E$ in 10th year and the demand is 80% of original
X_i^{21}	The amount of flow in pipeline$_i \in E$ in 10th year and the demand is original
X_i^{22}	The amount of flow in pipeline$_i \in E$ in 10th year and the demand is 120% of original
Y_i^1	The amount of generation of supplier$_i \in G$ in 5th year, $i \in \{1,2,3,4\}$
Y_i^{20}	The amount of generation of supplier$_i \in G$ in 10th year and the demand is 80% of original
Y_i^{21}	The amount of generation of supplier$_i \in G$ in 10th year and the demand is original
Y_i^{22}	The amount of generation of supplier$_i \in G$ in 10th year and the demand is 120% of original
$b_{i,j}^1 \in \{0{,}1\}$	A binary variable represents the choice of upgrade option j for generator i, (1: upgrade, 0: no upgrade)
	$i \in \{1,2,3,4\}$ number index of the generator
	$j \in \{1,2,3\}$ index of option to upgrade
$b_{i,j}^{20} \in \{0{,}1\}$	option choose in second 5 years period(10th) year and the demand is 80% of original
$b_{i,j}^{21} \in \{0{,}1\}$	option choose in 10th year and the demand is original
$b_{i,j}^{22} \in \{0{,}1\}$	option choose in 10th year and the demand is 120% of original
$p_i^1 \in \{0{,}1\}$	A binary variable indicates upgrade of pipelines i in the 0-5 year (1: upgrade, 0: no upgrade)
$p_i^{20} \in \{0{,}1\}$	A binary variable indicates upgrade pipelines in the 5-10 year and demand is 80% of original
$p_i^{21} \in \{0{,}1\}$	A binary variable indicates upgrade pipelines in the 5-10 year and demand is original
$p_i^{22} \in \{0{,}1\}$	A binary variable indicates upgrade pipelines in the 5-10 year and demand is 120% of original

Objective

(Note: if no discount, just remove the 0.7 in below equation)

$$
\begin{aligned}
&\min\Big(\sum_{i=1}^{4}\sum_{j=1}^{3}(b_{i,j}^1 \cdot \text{upgrade\_cost}_{i,j}) + \sum_{i\in E}(p_i^1 \cdot \text{distances}_i \cdot 200000) \\
&+ 1/3 \cdot \sum_{i=1}^{4}\sum_{j=1}^{3}(b_{i,j}^{20} \cdot \text{upgrade\_cost}_{i,j}) \cdot 0.7 + \sum_{i\in E}(p_i^{20} \cdot \text{distances}_i \cdot 200000 \cdot 0.7) \\
&+ 1/3 \cdot \sum_{i=1}^{4}\sum_{j=1}^{3}(b_{i,j}^{21} \cdot \text{upgrade\_cost}_{i,j}) \cdot 0.7 + \sum_{i\in E}(p_i^{21} \cdot \text{distances}_i \cdot 200000 \cdot 0.7) \\
&+ 1/3 \cdot \sum_{i=1}^{4}\sum_{j=1}^{3}(b_{i,j}^{22} \cdot \text{upgrade\_cost}_{i,j}) \cdot 0.7 + \sum_{i\in E}(p_i^{22} \cdot \text{distances}_i \cdot 200000 \cdot 0.7)\Big)
\end{aligned}
$$

Constraints

Choice constraints:

$$p_i^1 + p_i^{20} \leq 1 \qquad \forall i \in E$$

$$p_i^1 + p_i^{21} \leq 1 \qquad \forall i \in E$$

$$p_i^1 + p_i^{22} \leq 1 \qquad \forall i \in E$$

$$\sum_{j=1}^{3} b_{i,j}^1 + \sum_{j=1}^{3} b_{i,j}^{20} \leq 1 \qquad \forall i \in \{1,2,3,4\}$$

$$\sum_{j=1}^{3} b_{i,j}^1 + \sum_{j=1}^{3} b_{i,j}^{21} \leq 1 \qquad \forall i \in \{1,2,3,4\}$$

$$\sum_{j=1}^{3} b_{i,j}^1 + \sum_{j=1}^{3} b_{i,j}^{22} \leq 1 \qquad \forall i \in \{1,2,3,4\}$$

Capacity constraints:

$$X_i^1 \leq (p_i^1 + 1) \cdot 280 \qquad \forall i \in E$$

$$X_i^{20} \leq (p_i^{20} + 1) \cdot 280 \qquad \forall i \in E$$

$$X_i^{21} \leq (p_i^{21} + 1) \cdot 280 \qquad \forall i \in E$$

$$X_i^{22} \leq (p_i^{22} + 1) \cdot 280 \qquad \forall i \in E$$

$$Y_k^1 \leq \sum_{j=1}^{3} b_{i,j}^1 \cdot add\_capacity_{i,j} + init\_capacity_i \qquad \forall (k,i) \in \mathcal{G} \times \{1,2,3,4\}$$

$$Y_k^{20} \leq \sum_{j=1}^{3} b_{i,j}^{20} \cdot add\_capacity_{i,j} + \sum_{j=1}^{3} b_{i,j}^1 \cdot add\_capacity_{i,j} + init\_capacity_i \qquad \forall (k,i) \in \mathcal{G} \times \{1,2,3,4\}$$

$$Y_k^{21} \leq \sum_{j=1}^{3} b_{i,j}^{21} \cdot add\_capacity_{i,j} + \sum_{j=1}^{3} b_{i,j}^1 \cdot add\_capacity_{i,j} + init\_capacity_i \qquad \forall (k,i) \in \mathcal{G} \times \{1,2,3,4\}$$

$$Y_k^{22} \leq \sum_{j=1}^{3} b_{i,j}^{22} \cdot add\_capacity_{i,j} + \sum_{j=1}^{3} b_{i,j}^1 \cdot add\_capacity_{i,j} + init\_capacity_i \qquad \forall (k,i) \in \mathcal{G} \times \{1,2,3,4\}$$

Flow Constraints on supplier nodes:

$$Y_i^1 + \sum_{k \in end(i)} X_k^1 = \sum_{k \in start(i)} X_k^1 + demand_i^1 \qquad \forall i \in \mathcal{G}$$

$$Y_i^{20} + \sum_{k \in end(i)} X_k^{20} = \sum_{k \in start(i)} X_k^{20} + 0.8 \cdot demand_i^2 \qquad \forall i \in \mathcal{G}$$

$$Y_i^{21} + \sum_{k \in end(i)} X_k^{21} = \sum_{k \in start(i)} X_k^{21} + demand_i^2 \qquad \forall i \in \mathcal{G}$$

$$Y_i^{22} + \sum_{k \in end(i)} X_k^{22} = \sum_{k \in start(i)} X_k^{22} + 1.2 \cdot demand_i^2 \qquad \forall i \in \mathcal{G}$$

Flow Constraints on non-supplier nodes:

$$\sum_{k \in end(i)} X_k^1 = \sum_{k \in start(i)} X_k^1 + demand_i^1 \qquad \forall i \in \mathcal{NG}$$

$$\sum_{k \in end(i)} X_k^{20} = \sum_{k \in start(i)} X_k^{20} + 0.8 \cdot demand_i^2 \qquad \forall i \in \mathcal{NG}$$

$$\sum_{k \in end(i)} X_k^{21} = \sum_{k \in start(i)} X_k^{21} + demand_i^2 \qquad \forall i \in \mathcal{NG}$$

$$\sum_{k \in end(i)} X_k^{22} = \sum_{k \in start(i)} X_k^{22} + 1.2 \cdot demand_i^2 \qquad \forall i \in \mathcal{NG}$$

Communication 10 constraints:

$$\begin{aligned}\sum_{i=1}^{4}\sum_{j=1}^{3}(b_{i,j}^{20}\cdot \text{upgrade\_cost}_{i,j})\cdot 0.7+\sum_{i\in E}(p_i^{20}\cdot \text{distances}_i\cdot 200000\cdot 0.7)\\ \leq 2(\sum_{i=1}^{4}\sum_{j=1}^{3}(b_{i,j}^{1}\cdot \text{upgrade\_cost}_{i,j})+\sum_{i\in E}(p_i^{1}\cdot \text{distances}_i\cdot 200000))\end{aligned} \tag{1}$$

$$\begin{aligned}\sum_{i=1}^{4}\sum_{j=1}^{3}(b_{i,j}^{21}\cdot \text{upgrade\_cost}_{i,j})\cdot 0.7+\sum_{i\in E}(p_i^{21}\cdot \text{distances}_{i,j}\cdot 200000\cdot 0.7)\\ \leq 2(\sum_{i=1}^{4}\sum_{j=1}^{3}(b_{i,j}^{1}\cdot \text{upgrade\_cost}_{i,j})+\sum_{i\in E}(p_i^{1}\cdot \text{distances}_i\cdot 200000))\end{aligned} \tag{2}$$

$$\begin{aligned}\sum_{i=1}^{4}\sum_{j=1}^{3}(b_{i,j}^{22}\cdot \text{upgrade\_cost}_{i,j})\cdot 0.7+\sum_{i\in E}(p_i^{22}\cdot \text{distances}_{i,j}\cdot 200000\cdot 0.7)\\ \leq 2(\sum_{i=1}^{4}\sum_{j=1}^{3}(b_{i,j}^{1}\cdot \text{upgrade\_cost}_{i,j})+\sum_{i\in E}(p_i^{1}\cdot \text{distances}_i\cdot 200000))\end{aligned} \tag{3}$$