

Problem 4-6 Formulation

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Sets

- N Set of nodes $N \subseteq \mathbb{N}$
- E Set of pipelines (edges) between two nodes
- G Set of generator/supplier nodes, $G \subseteq N$

Data

- \mathcal{G} the set of index of nodes that is a supplier node
- \mathcal{NG} the set of index of nodes that is not supplier nodes
- demand* The demand of each nodes
the *demand*¹ represents the demand in the 5th year
the *demand*² represents the demand in the 10th year
- distances_i* Length of an edge of pipeline i
- init_capacity_i* The initial capacity of *generator_i* $\subseteq G$
- add_capacity_{i,j}* The additional capacity of *generator_i* $\subseteq G$
with choosing the upgrade option j, $i \in \{1, 2, 3, 4\}, j \in \{1, 2, 3\}$
- upgrade_cost_{i,j}* The cost of upgrade of *generator_i* $\subseteq G$
with choosing the upgrade option j, $i \in \{1, 2, 3, 4\}, j \in \{1, 2, 3\}$
- end(i) index of pipelines that ends with node i
- start(i) index of pipelines that starts with node i

Variables

X_i^1	The amount of flow in pipeline $_i \in E$ in 5th year
X_i^{20}	The amount of flow in pipeline $_i \in E$ in 10th year and the demand is 80% of original
X_i^{21}	The amount of flow in pipeline $_i \in E$ in 10th year and the demand is original
X_i^{22}	The amount of flow in pipeline $_i \in E$ in 10th year and the demand is 120% of original
Y_i^1	The amount of generation of supplier $_i \in G$ in 5th year, $i \in \{1, 2, 3, 4\}$
Y_i^{20}	The amount of generation of supplier $_i \in G$ in 10th year and the demand is 80% of original
Y_i^{21}	The amount of generation of supplier $_i \in G$ in 10th year and the demand is original
Y_i^{22}	The amount of generation of supplier $_i \in G$ in 10th year and the demand is 120% of original
$b_{i,j}^1 \in \{0,1\}$	A binary variable represents the choice of upgrade option j for generator i, (1: upgrade, 0: no upgrade) $i \in \{1, 2, 3, 4\}$ number index of the generator $j \in \{1, 2, 3\}$ index of option to upgrade
$b_{i,j}^{20} \in \{0,1\}$	option choose in second 5 years period(10th) year and the demand is 80% of original
$b_{i,j}^{21} \in \{0,1\}$	option choose in 10th year and the demand is original
$b_{i,j}^{22} \in \{0,1\}$	option choose in 10th year and the demand is 120% of original
$p_i^1 \in \{0,1\}$	A binary variable indicates upgrade of pipelines i in the 0-5 year (1: upgrade, 0: no upgrade)
$p_i^{20} \in \{0,1\}$	A binary variable indicates upgrade pipelines in the 5-10 year and demand is 80% of original
$p_i^{21} \in \{0,1\}$	A binary variable indicates upgrade pipelines in the 5-10 year and demand is original
$p_i^{22} \in \{0,1\}$	A binary variable indicates upgrade pipelines in the 5-10 year and demand is 120% of original

Objective

$$\begin{aligned}
& \min \left(\sum_{i=1}^4 \sum_{j=1}^3 (b_{i,j}^1 \cdot \text{upgrade_cost}_{i,j}) + \sum_{i \in E} (p_i^1 \cdot \text{distances}_i \cdot 200000) \right. \\
& + 1/3 \cdot \sum_{i=1}^4 \sum_{j=1}^3 (b_{i,j}^{20} \cdot \text{upgrade_cost}_{i,j}) \cdot 0.7 + \sum_{i \in E} (p_i^{20} \cdot \text{distances}_i \cdot 200000 \cdot 0.7) \\
& + 1/3 \cdot \sum_{i=1}^4 \sum_{j=1}^3 (b_{i,j}^{21} \cdot \text{upgrade_cost}_{i,j}) \cdot 0.7 + \sum_{i \in E} (p_i^{21} \cdot \text{distances}_i \cdot 200000 \cdot 0.7) \\
& \left. + 1/3 \cdot \sum_{i=1}^4 \sum_{j=1}^3 (b_{i,j}^{22} \cdot \text{upgrade_cost}_{i,j}) \cdot 0.7 + \sum_{i \in E} (p_i^{22} \cdot \text{distances}_i \cdot 200000 \cdot 0.7) \right)
\end{aligned}$$

Constraints

Choice constraints:

$$\begin{aligned}
p_i^1 + p_i^{20} &\leq 1 & \forall i \in E \\
p_i^1 + p_i^{21} &\leq 1 & \forall i \in E \\
p_i^1 + p_i^{22} &\leq 1 & \forall i \in E \\
\sum_{j=1}^3 b_{i,j}^1 + \sum_{j=1}^3 b_{i,j}^{20} &\leq 1 & \forall i \in \{1, 2, 3, 4\} \\
\sum_{j=1}^3 b_{i,j}^1 + \sum_{j=1}^3 b_{i,j}^{21} &\leq 1 & \forall i \in \{1, 2, 3, 4\} \\
\sum_{j=1}^3 b_{i,j}^1 + \sum_{j=1}^3 b_{i,j}^{22} &\leq 1 & \forall i \in \{1, 2, 3, 4\}
\end{aligned}$$

Capacity constraints:

$$\begin{aligned}
X_i^1 &\leq (p_i^1 + 1) \cdot 280 & \forall i \in E \\
X_i^{20} &\leq (p_i^{20} + 1) \cdot 280 & \forall i \in E \\
X_i^{21} &\leq (p_i^{21} + 1) \cdot 280 & \forall i \in E \\
X_i^{22} &\leq (p_i^{22} + 1) \cdot 280 & \forall i \in E \\
Y_k^1 &\leq \sum_{j=1}^3 b_{i,j}^1 \cdot add_capacity_{i,j} + init_capacity_i & \forall (k, i) \in \mathcal{G} \times \{1, 2, 3, 4\} \\
Y_k^{20} &\leq \sum_{j=1}^3 b_{i,j}^{20} \cdot add_capacity_{i,j} + \sum_{j=1}^3 b_{i,j}^1 \cdot add_capacity_{i,j} + init_capacity_i & \forall (k, i) \in \mathcal{G} \times \{1, 2, 3, 4\} \\
Y_k^{21} &\leq \sum_{j=1}^3 b_{i,j}^{21} \cdot add_capacity_{i,j} + \sum_{j=1}^3 b_{i,j}^1 \cdot add_capacity_{i,j} + init_capacity_i & \forall (k, i) \in \mathcal{G} \times \{1, 2, 3, 4\} \\
Y_k^{22} &\leq \sum_{j=1}^3 b_{i,j}^{22} \cdot add_capacity_{i,j} + \sum_{j=1}^3 b_{i,j}^1 \cdot add_capacity_{i,j} + init_capacity_i & \forall (k, i) \in \mathcal{G} \times \{1, 2, 3, 4\}
\end{aligned}$$

Flow Constraints on supplier nodes:

$$\begin{aligned}
Y_i^1 + \sum_{k \in \text{end}(i)} X_k^1 &= \sum_{k \in \text{start}(i)} X_k^1 + \text{demand}_i^1 & \forall i \in \mathcal{G} \\
Y_i^{20} + \sum_{k \in \text{end}(i)} X_k^{20} &= \sum_{k \in \text{start}(i)} X_k^{20} + 0.8 \cdot \text{demand}_i^2 & \forall i \in \mathcal{G} \\
Y_i^{21} + \sum_{k \in \text{end}(i)} X_k^{21} &= \sum_{k \in \text{start}(i)} X_k^{21} + \text{demand}_i^2 & \forall i \in \mathcal{G} \\
Y_i^{22} + \sum_{k \in \text{end}(i)} X_k^{22} &= \sum_{k \in \text{start}(i)} X_k^{22} + 1.2 \cdot \text{demand}_i^2 & \forall i \in \mathcal{G}
\end{aligned}$$

Flow Constraints on non-supplier nodes:

$$\begin{aligned}
\sum_{k \in \text{end}(i)} X_k^1 &= \sum_{k \in \text{start}(i)} X_k^1 + \text{demand}_i^1 & \forall i \in \mathcal{NG} \\
\sum_{k \in \text{end}(i)} X_k^{20} &= \sum_{k \in \text{start}(i)} X_k^{20} + 0.8 \cdot \text{demand}_i^2 & \forall i \in \mathcal{NG} \\
\sum_{k \in \text{end}(i)} X_k^{21} &= \sum_{k \in \text{start}(i)} X_k^{21} + \text{demand}_i^2 & \forall i \in \mathcal{NG} \\
\sum_{k \in \text{end}(i)} X_k^{22} &= \sum_{k \in \text{start}(i)} X_k^{22} + 1.2 \cdot \text{demand}_i^2 & \forall i \in \mathcal{NG}
\end{aligned}$$

Communication 10 constraints:

$$\begin{aligned}
& \sum_{i=1}^4 \sum_{j=1}^3 (b_{i,j}^{20} \cdot \text{upgrade_cost}_{i,j}) \cdot 0.7 + \sum_{i \in E} (p_i^{20} \cdot \text{distances}_i \cdot 200000 \cdot 0.7) \\
& \leq 2 \left(\sum_{i=1}^4 \sum_{j=1}^3 (b_{i,j}^1 \cdot \text{upgrade_cost}_{i,j}) + \sum_{i \in E} (p_i^1 \cdot \text{distances}_i \cdot 200000) \right)
\end{aligned} \tag{1}$$

$$\begin{aligned}
& \sum_{i=1}^4 \sum_{j=1}^3 (b_{i,j}^{21} \cdot \text{upgrade_cost}_{i,j}) \cdot 0.7 + \sum_{i \in E} (p_i^{21} \cdot \text{distances}_{i,j} \cdot 200000 \cdot 0.7) \\
& \leq 2 \left(\sum_{i=1}^4 \sum_{j=1}^3 (b_{i,j}^1 \cdot \text{upgrade_cost}_{i,j}) + \sum_{i \in E} (p_i^1 \cdot \text{distances}_i \cdot 200000) \right)
\end{aligned} \tag{2}$$

$$\begin{aligned}
& \sum_{i=1}^4 \sum_{j=1}^3 (b_{i,j}^{22} \cdot \text{upgrade_cost}_{i,j}) \cdot 0.7 + \sum_{i \in E} (p_i^{22} \cdot \text{distances}_{i,j} \cdot 200000 \cdot 0.7) \\
& \leq 2 \left(\sum_{i=1}^4 \sum_{j=1}^3 (b_{i,j}^1 \cdot \text{upgrade_cost}_{i,j}) + \sum_{i \in E} (p_i^1 \cdot \text{distances}_i \cdot 200000) \right)
\end{aligned} \tag{3}$$