Math 263 : Discrete Mathematics | Towson University

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HW-5.2-5.3

1. (40 *points*) Consider the following statement denoted by $P(n)$ where $n \geq 2$ is an integer.

$$P(n): \quad 5 \mid (7^n - 2^n)$$

a) (8 *points*) State and prove the basis step.

Step 1: Basis step

Show that $P(2)$ is true

$P(2)$: $5 \mid (7^2 - 2^2)$

$5 \mid 49 - 4$

$5 \mid 45$

$45 = 5(k)$ where $k = 9$ & $9 \in \mathbb{Z}$

By definition of "$\mid$", $5 \mid (7^n - 2^n)$

$P(2)$ is true

b) (7 *points*) Write the statements that correspond to $P(k)$ & $P(k+1)$.

$P(k)$: $5 \mid (7^k - 2^k)$

$P(k+1)$: $5 \mid (7^{k+1} - 2^{k+1})$

c) (25 *points*) Prove that $P(n)$ is true for all integers $n \geq 2$ using the Principle of Mathematical Induction by writing and completing the inductive step.

Step 2: Inductive Step

To show that

$\forall k \in \mathbb{Z} \geq 2$, if $P(k)$ is true then $P(k+1)$ is true.

Let $k \geq 2$ be any integer such that $P(k)$ is true,

Prove that $P(k+1)$ is true.

$7^k - 2^k = 5r$ by definition of "$\mid$" for some $r \in \mathbb{Z}$.

$$7^{k+1} - 2^{k+1} = 7^k \cdot 7 - 2^k \cdot 2$$
$$= (5r + 2^k) \cdot 7 - 2^k \cdot 2$$
$$= 35r + 2^k \cdot 7 - 2^k \cdot 2$$
$$= 35r + 2^k(7-2)$$
$$= 5(7r + 2^k)$$

$7^{k+1} - 2^{k+1} = 5s$, where $s = 7r + 2^k$

$s \in \mathbb{Z}$ because $7, r, 2 \in \mathbb{Z}$ & sums & products of integers are integers.

note $k \geq 2$ & $k \in \mathbb{Z}$.

Thus, $P(k+1)$ is true.

This completes the inductive step.

Thus $P(n)$ is true $\forall n \geq 2$ & $n \in \mathbb{Z}$.

2. (30 *points*) Consider the following statement denoted by $P(n)$ where $n \geq 2$ is an integer.

$$P(n): \quad 5 + 8 + 11 + \cdots + (3n+2) = \frac{n(3n+7)}{2}$$

a) (8 *points*) State and prove the basis step.

Step 1: Basis Step

Show that $P(2)$ is true,

$P(2)$: $5+8+11+\ldots+3(2)+2 = \frac{(2)(3(2)+7)}{2}$

$P(2)$: $5+8 \;|\; \frac{2\cdot(6+7)}{2}$

LHS $13 \;|\; 13$ RHS

Since LHS & RHS of $P(2)$ are equal, $P(2)$ is true.

b) (7 *points*) Write the equations that correspond to $P(k)$ & $P(k+1)$.

$P(k)$: $5+8+11+\ldots+(3k+2) = \frac{k(3k+7)}{2}$

$P(k+1)$: $5+8+11+\ldots+(3(k+1)+2) = \frac{(k+1)(3(k+1)+7)}{2}$

c) (15 *points*) Prove that $P(n)$ is true for all integers $n \geq 2$ using the Principle of Mathematical Induction by writing and completing the inductive step.

Step 2: Inductive Step

To show that

$\forall k \in \mathbb{Z} \geq 2$, if $P(k)$ is true then $P(k+1)$ is true.

Let $k \geq 2$ be any integer such that $P(k)$ is true.

Prove that $P(k+1)$ is true

LHS of $P(k+1)$ $= 5+8+11+\ldots+(3(k+1)+2)$

$= 5+8+11+\ldots+(3k+5)$

$= \frac{k(3k+7)}{2} + (3k+5)$

$= \frac{3k^2+13k+10}{2}$

RHS of $P(k+1)$ $= \frac{(k+1)(3(k+1)+7)}{2}$

$= \frac{(k+1)(3k+10)}{2}$

$= \frac{3k^2+10k+3k+10}{2}$

$= \frac{3k^2+13k+10}{2}$

Since LHS of $P(k+1)$ = RHS of $P(k+1)$,

$P(k+1)$ is true.

This completes the inductive step.

Thus $P(n)$ is true $\forall n \geq 2$ $n \in \mathbb{Z}$.