Math 263 : Discrete Mathematics Towson University

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HW-3.3

1. (20 *points*) Let $A = \{-2, 0, 2, 3\}$ & $B = \{-1, 0, 1\}$. Test if the following statement is true or false. Accordingly prove or disprove it.

   a) $\forall\, x \in A, \exists\, y \in B$ such that $xy < y$.

   Hint: First prove using basic algebra that for all (real) values of x & y $xy < y$ is logically equivalent to $y(x-1) < 0$

scratch

$xy < y$

$xy - y < 0$

$y(x-1) < 0$

proof: let $x = -2 \in A, \exists y = 1 \in B$ such that

$y(x-1) = 1((-2)-1)$

$= 1 \cdot -3$

$= -3 < 0$ ✓

let $x = 0 \in A, \exists y = 1 \in B$ such that

$y(x-1) = 1(0-1)$

$= 1(-1)$

$= -1 < 0$ ✓

let $x = 2 \in A, \exists y = -1 \in B$ such that

$y(x-1) = -1(2-1)$

$= -1 \cdot 1$

$= -1 < 0$ ✓

let $x = 3 \in A, \exists y = -1 \in B$ such that

$y(x-1) = -1(3-1)$

$= -1(2)$

$= -2 < 0$ ✓

thus, $\forall x \in A$, $xy < y$ is true $\exists y \in B$; this proves the given statement.

   b) $\exists\, y \in B$ such that $\forall\, x \in A$, $xy < y$.

$y(x-1) < 0$

Scratch

A	B			
x	y	$y(x-1) < 0$	$0(-2-1)$	$1(3-1)$
-2	-1	$-1(-2-1) < 0$	$0 < 0$ X	$2 < 0$ X
0	0	$3 < 0$ X		
2	1	Thus False		
3				

The given statement is false because its negation:

$\forall y \in B, \exists x \in A$ such that $y(x-1) \geq 0$

Proof of negation: let $y = -1 \in B, \exists x = -2 \in A$ such that

$y(x-1) = (-1)((-2)-1)$

$= 3 \geq 0$

let $y = 0 \in B, \exists x = -2 \in A$ such that

$y(x-1) = (0)((-2)-1)$

$= 0 \geq 0$

let $y = 1 \in B, \exists x = 3 \in A$ such that

$y(x-1) = (1)((3)-1)$

$2 \geq 0$

thus, $\forall y \in B$, $\exists x \in A$ such that $y(x-1) \geq 0$

this proves the negation.

2. (30 *points*) Write the formal/informal versions of the following statements (if asked), and then write its negation both formally and informally. In the informal version, do not use variables, quantifiers, if-then, for all, there exists and other technical words.

a) (9 *points*) $\exists$ a book x such that $\forall$ people y, y has not read x.

Informal Version : Some books have not been read by anyone.

Formal Negation : $\forall$ books x, $\exists$ a person y such that y has read x.

Informal Negation: Different people have read every book.

b) (12 *points*) No one loves everyone.

$\equiv$ Everyone is not loved by someone

Formal Version : $\forall$ people x, $\exists$ a person y such that y does not love x.

Formal Negation : $\exists$ a person x such that $\forall$ people y, y loves x.

Informal Negation: Everyone loves someone.

c) (9 *points*) Every action has an equal and opposite reaction. ("opposite" means "negative of")

Formal Version : $\forall$ action x, $\exists$ a reaction y such that $y = -x$.

Formal Negation : $\exists$ an action x such that $\forall$ reactions y, $y \neq -x$.

Informal Negation: Some actions do not have an equal and opposite reaction