PHW 7.1-7.3 p2

1 f is an onto function because
$\forall y \in \mathbb{Z}, \exists x \in X$ such that $f(x) = y$.

Proof: let y be any integer

Case 1: $y \in \mathbb{Z}^-$. let $y = -k$ where $k \in \mathbb{Z}^+$

let $x = \underbrace{000\ldots0}_{k \text{ zeros}}$

$f(x) = 2(0) - k = -k = y$. so $f(x) = y$

Case 2: $y = 0$

let $x = 100$

$f(x) = 2(1) - 1(2) = 0$. so $f(x) = y$

Case 3: $y \in \mathbb{Z}^+$ & y is even

so $y = 2k$ for some $k \in \mathbb{Z}^+$

let $x = \underbrace{111\ldots1}_{\frac{y}{2}}$

$f(x) = 2(\frac{y}{2}) = y$. so $f(x) = y$

Case 4: $y \in \mathbb{Z}^+$ & y is odd

$y = 2k+1$ for $k \in \mathbb{Z} \geq 0$

let $x = \underbrace{111\ldots1}_{k+1}0$

$f(x) = 2(k+1) - 1 = 2k+1 = y$. so $f(x) = y$.

So, in each case, $\exists x \in X$ such that $f(x) = y$.
This proves f is onto

2a f is onto because
$\forall y \in \mathbb{R}, \exists x \in \mathbb{R}$ such that $f(x) = y$

proof: let y be any real number. let $x = \frac{7-y}{-2}$. Note $x \in \mathbb{R}$ because $7, y, -2 \in \mathbb{R}$ & differences & quotients (other than by 0 ($\because -2 \neq 0$)) of real #'s are real numbers

$f(x) = f(\frac{7-y}{-2}) = y$

so, $f(x) = y$.

2b f is not onto because

$\exists\, y \in \mathbb{Z}$ such that $\forall x \in \mathbb{Z}$, $f(x) \neq y$

Proof: let $y = 8$. note that $y \in \mathbb{Z}$. let x be any integer.

To prove $f(x) \neq y$, we use method of contradiction.

suppose $f(x) = y$ $(=8)$

$\Rightarrow 7 - 2x = 8$

$\Rightarrow -2x = 1$

$\Rightarrow x = \frac{-1}{2}$

this shows that $x = \frac{-1}{2}$, but $\frac{1}{2} \notin \mathbb{Z}$. This is a contradiction. Thus, our assumption is false & the given statement is true.