3. (40 *points*) Consider the following statements:
   1. *"If John either came late or did not send recommendation, then he did not get the job"*
   2. *"If John did not get the job, then either he came late or did not send recommendation"*
   3. *"Either John came late or he did not send recommendation, only if he did not get the job"*
   4. *"John did get the job if neither did he come late nor did he not send recommendation"*
   5. *"Not coming late and sending recommendation is necessary for John to get the job"*
   6. *"Coming late and not sending recommendation is sufficient for John to not get the job"*

a) (15 *points*) Define 3 statement variables $p, q, \& r$ (based on stat. 1.), and write the **statement** ***form*** of the above 7 statements involving "$\sim, \rightarrow, \vee$ or $\wedge$". In statement *form* for 1, DO NOT USE "~".

$p =$ John came late.

$q =$ John did not send reccommendation.

$r =$ John did not get the job.

Statement *form* for 1

$(p \vee q) \rightarrow r$

Statement *form* for 2

$r \rightarrow (p \vee q)$

Statement *form* for 3

$(p \vee q) \rightarrow r$

Statement *form* for 4

$(\sim p \wedge \sim q) \rightarrow \sim r$ (can't use ←)

Statement *form* for 5

$\sim r \rightarrow (\sim p \wedge \sim q)$ (can't use ←)

Statement *form* for 6

$(p \wedge q) \rightarrow r$

Scratch work for b)

SF4

$(\sim p \wedge \sim q) \rightarrow \sim r$

$\equiv \sim(p \vee q) \rightarrow \sim r$ by DML

SF5

$\sim r \rightarrow (\sim p \wedge \sim q)$

$\equiv \sim r \rightarrow \sim(p \vee q)$ by DML

b) (18 *points*) For each sentence below, fill in

- the 1st blank with either "$\equiv$" or "$\not\equiv$",
- the 2nd blank with either "**converse of**", "**inverse of**", "**contrapositive of**", or "**negation of**".
- and the 3rd blank with the name of supporting logical equivalence law(s) if needed, otherwise write none.

Note: "S" and "SF" represent "Statement" and "Statement Form" respectively.

S2 $\not\equiv$ S1 because SF2 is the (converse of SF1) by none

S4 $\not\equiv$ S1 because SF4 is the (Inverse of SF1) by DeMorgan's Law

S5 $\equiv$ S1 because SF5 is the (contrapositive of SF1) by De Morgan's Law

c) (7 *points*) Prove/disprove the following **without** using truth tables: Statement 1 $\equiv$ Statement 6.

when $(p\,q\,r) = (T\ F\ F)$, $SF1 = F$, but $SF6 = T$, thus $S1 \not\equiv S6$

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HW 2.2

1. (*8 points*) Suppose n is a fixed integer. Write the negation of the following statement:

"*If $\underbrace{n \text{ is prime}}_{p}$, then $\underbrace{n \text{ is odd}}_{q}$*"

$p \wedge \sim q$ $\boxed{\text{n is prime and n is not odd.}}$

2. (*22 points*) Consider the following statement:

"*If it rained today then not only did Sean's car break down, but also he did not go to school*"

a) (*5 points*) Define 3 statement variables $p, q, \& r$, and write the **statement form** of the above conditional statement involving "$\rightarrow$, $\vee$ or $\wedge$" WITHOUT using "$\sim$".

$p =$ it rained today.

$q =$ Sean's car did break down.

$r =$ Sean did not go to school.

Statement form: $p \rightarrow (q \wedge r)$

b) (*7 points*) Apply the negation "$\sim$" to the statement form found in part a), and ***simplify it*** using logical equivalences laws. Use only one law in each step & include a name for each law.

$\sim(p \rightarrow (q \wedge r)) \equiv p \wedge \sim(q \wedge r)$ by negation of "$\rightarrow$" law

$\equiv p \wedge (\sim q \vee \sim r)$ by DeMorgan's law

c) (*10 points*) Write in words the negation of the conditional statement given at the beginning.

it rained today & either Sean's car did not break down or Sean did go to School.