Math 263 : Discrete Mathematics
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HW-3.1-3.2

1. (24 *points*) Let $Q(x)$ be a statement function "$x^2 + 2x \leq 0$", with domain $D = \mathbb{R}$ the set of all real numbers. Write what each of the following statements mean and circle the correct choice true/false.

$Q(-2.5)$: $(-2.5)^2 + 2(-2.5) \leq 0$ — **True** / **False**

$Q(-2)$: $(-2)^2 + 2(-2) \leq 0$ — **True** / **False**

$Q(-1)$: $(-1)^2 + 2(-1) \leq 0$ — **True** / **False**

$Q(-0.5)$: $(-0.5)^2 + 2(-0.5) \leq 0$ — **True** / **False**

$Q(0)$: $(0)^2 + 2(0) \leq 0$ — **True** / **False**

$Q(0.5)$: $(0.5)^2 + 2(0.5) \leq 0$ — **True** / **False**

2. (12 *points*) Consider the statement:

∀ people x, if x is a mathematician, then x is lazy.

Among the following statements, circle the ones which are equivalent ways of expressing the above statement?

a) Every mathematician is lazy.

b) Among all mathematicians, some are lazy.

c) Some of the lazy people are mathematician.

d) Anyone who is lazy is a mathematician.

e) All people who are mathematicians are lazy.

f) Anyone who is a mathematician is a lazy person.

3. (22 *points*) Let S be the set of all students in your school. Define the following statement functions:

   $M(x) =$"x is a math major.";
   $C(x) =$"x is a computer science student.";
   $E(x) =$"x is an engineering student."

   Write each of the following statements a)-c) using ***only*** the following: variable x, domain S, statement functions $M(x), C(x),$ & $E(x)$, "such that", and symbols $\forall, \exists, \in, \vee, \wedge, \rightarrow, \sim$.

   a) (5 *points*) Every computer science student is an engineering student.

   $\forall x \in S,\ C(x) \rightarrow E(x)$

   b) (5 *points*) There is an engineering student who is a math major.

   $\exists x \in S,\ E(x) \wedge M(x)$

   c) (6 *points*) No computer science students are engineering students.

   $\forall x \in S,\ C(x) \rightarrow \sim E(x)$

   d) (6 *points*) Write the following informally i.e. without using quantifiers, variables, statement functions and **if-then**.

   $$\forall x \in S,\ C(x) \wedge E(x) \rightarrow M(x)$$

   every student that's a computer science student & engineering student is a math major.

4. (72 *points*) Write the formal versions of the following statements, and then write its negation both formally and informally. In the informal version, do not use variables, quantifiers, if-then, for all, there exists and other technical words.

a) (9 *points*) Someone loves John.

Formal Version : $\exists$ a person x such that x loves John

Formal Negation : $\forall$ people x, x does not love John

Informal Negation: no one loves John

b) (9 *points*) Some students completed redemption.

Formal Version : $\exists$ a student x such that x completed redemption

Formal Negation : $\forall$ students x, x did not complete redemption

Informal Negation: no students completed redemption

c) (9 *points*) No valid argument has a false conclusion. (NOT a true statement)

Formal Version : $\forall$ valid arguments x, x has a false conclusion

Formal Negation : $\exists$ a valid argument x such that x does not have a false conclusion

Informal Negation: some arguments do not have a false conclusion

d) (9 *points*) No rock was left unturned.

Formal Version : $\forall$ rocks x, x was not left unturned

Formal Negation : $\exists$ a rock x, such that x was left unturned

Informal Negation: some rocks were left unturned

e) (12 *points*) For every valid argument, if all its premises are true, then its conclusion is true too.

Formal Version : ∀ valid arguments x, if x's premises are true, then x's conclusion is true.

Formal Negation : ∃ a valid argument x, such that x's premises are true & x's conclusion is not true.

Informal Negation: some valid arguments have true premises & a false conclusion.

f) (12 *points*) If the product of any two real numbers is zero, then at least one of them is zero.

Formal Version : ∀ real #'s x & y, if $xy = 0$, then either x or $y = 0$

Formal Negation : ∃ real #'s x & y such that $xy = 0$ & neither x nor $y = 0$

Informal Negation: Some real #'s x & y multiply to 0 & are not 0 themselves.

g) (12 *points*) If the sum of any two integers is at least 10, then both of them must be at least 5. (This is NOT a true statement)

Formal Version : ∀ real #s x & y, if $x+y \geq 10$, then $x \geq 5$ & $y \geq 5$

Formal Negation : ∃ real #s x & y such that $x+y \geq 10$ & either $x < 5$ or $y < 5$

Informal Negation: Some real #'s x & y add to at least 10 & x or y is less than 5.