Math 263 : Discrete Mathematics Towson University

Name: NATHAN KETTERLINUS **Roll #:** 16 **Section:** 8

HW-6.1-6.3

1. (22 *points*) Prove the following using set identities in the Reference Sheet under Review Material. For each step, use only one set identity and justify the step by citing the name of the set identity.

$$((A^c - B) \cup A^c)^c = A$$

Direct Proof let A, B be any set

Let $x \in ((A^c - B) \cup A^c)^c$

$\Rightarrow x \in ((A^c \cap B^c) \cup A^c)^c$ by Set Difference Law

$\Rightarrow x \in (A^c \cup (A^c \cap B^c))^c$ by Commutative Law

$\Rightarrow x \in (A^c)^c$ by Absorption Law

$\Rightarrow x \in A$ by Double Complement Law $\to$ This proves that $((A^c - B) \cup A^c)^c \subseteq A$ by definition of $\subseteq$

Let $x \in A$

$\Rightarrow x \in (A^c)^c$ by Double Complement Law

$\Rightarrow x \in (A^c \cup (A^c \cap B))^c$ by Absorption Law

$\Rightarrow x \in ((A^c \cap B) \cup A^c)^c$ by Commutative Law

$\Rightarrow x \in ((A^c - B) \cup A^c)^c$ by Set Difference Law $\to$ This proves that $A \subseteq ((A^c - B) \cup A^c)^c$ by definition of $\subseteq$

This proves that $((A^c - B) \cup A^c)^c = A$ by definition of $=$

2. (23 *points*) Complete the following proof of De Morgan's Law: For all sets A and B,

$$(A \cap B)^c = A^c \cup B^c$$

Proof: Let A and B be any sets.

Proof of $(A \cap B)^c \subseteq A^c \cup B^c$ ---- (1) (given in Lecture-Slides-6.1-6.3 Slide # 25)

Proof of $A^c \cup B^c \subseteq (A \cap B)^c$ ---- (2)

Let $x \in A^c \cup B^c$

$\Rightarrow x \in A^c \lor x \in B^c$ by definition of $\cup$

$\Rightarrow x \notin A \lor x \notin B$ by definition of (set)^c

$\Rightarrow \sim(x \in A) \lor \sim(x \in B)$ by definition of $\notin$

$\Rightarrow \sim(x \in A \land x \in B)$ by De Morgan's law of statements

$\Rightarrow \sim(x \in A \cap B)$ by definition of $\cap$

$\Rightarrow x \notin A \cap B$ by definition of $\notin$

$\Rightarrow x \in (A \cap B)^c$ by definition of (set)^c

Thus, $A^c \cup B^c \subseteq (A \cap B)^c$ by definition of $\subseteq$

Conclusion of (1) & (2) is $(A \cap B)^c = A^c \cup B^c$.

3. (30 *points*) Prove the following statement using only the definitions of "∩", "∪", "set difference", "complement of a set", "⊆", "∉" and the **Standard Logical Equivalences** (NO set identities):

For all sets $A, B, \& C$, if $C \subseteq (B^c - A)$, then $C \cap (A \cup B) = \phi$

Proof by Method of Contradiction

suppose the negation, ie)

$\exists$ sets $A, B, \& C$ such that $C \subseteq (B^c - A)$ and $C \cap (A \cup B) \neq \phi$ is true

Thus, there exists an element $x \in C \cap (A \cup B)$

$\Rightarrow x \in C \wedge (x \in A \cup B)$ — by definition of ∩

$\Rightarrow x \in C \wedge (x \in A \vee x \in B)$ — by definition of ∪

$\Rightarrow (x \in C \wedge x \in A) \vee (x \in A \wedge x \in B)$ — by distributive Law

Case 1: $x \in C \wedge x \in A$

$\Rightarrow x \in B^c - A \wedge x \in A$ — since $C \subseteq B^c - A$

$\Rightarrow x \in B^c \wedge x \notin A \wedge x \in A$ — by definition of −

$\Rightarrow x \notin A \wedge x \in A$ — by specialization b

$\Rightarrow \sim(x \in A) \wedge x \in A$ — by definition of ∉

$\Rightarrow x \in A \wedge \sim(x \in A)$ — by Commutative Law

$\Rightarrow c$, a contradiction — by negation law

Case 2: $x \in A \wedge x \in B$

$\Rightarrow x \in C \vee (x \in A \wedge x \in B)$ — by generalization a

$\Rightarrow (x \in C \vee x \in A) \wedge (x \in C \vee x \in A)$ — by distributive law

$\Rightarrow x \in C \vee x \in A$ — by specialization a

$\Rightarrow x \in B^c - A \wedge x \in A$ — since $C \subseteq B^c - A$

$\Rightarrow x \in B^c \wedge x \notin A \wedge x \in A$ — by definition of −

$\Rightarrow x \notin A \wedge x \in A$ — by specialization b

$\Rightarrow \sim(x \in A) \wedge (x \in A)$ — by definition of ∉

$\Rightarrow x \in A \wedge \sim(x \in A)$ — by commutative Law

$\Rightarrow c$, a contradiction — by negation law

Case 1 & 2 are the only possibilities, & both lead to a contradiction.
Thus, our assumption is false & the original statement is true.