Math 263 : Discrete Mathematics Towson University

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HW 2.1

1. (24 *points*) Consider the following two statement forms:
   - $A: p \to (\sim q \wedge r)$
   - $B: (\sim p \vee \sim q) \wedge r$

   a) (15 *points*) Construct a **single** truth table that displays the truth values of the above two statement forms. Use as many rows & columns as you need in the following table.

p	q	r	$\sim p$	$\sim q$	$\sim q \wedge r$	$p \to (\sim q \wedge r)$	$\sim p \vee \sim q$	$(\sim p \vee \sim q) \wedge r$	
T	T	T	F	F	F	F	F	F	
T	T	F	F	F	F	F	F	F	
T	F	T	F	T	T	T	T	T	
T	F	F	F	T	F	F	T	F	
F	T	T	T	F	F	T	T	T	
F	T	F	T	F	F	T	T	F	
F	F	T	T	T	T	T	T	T	
F	F	F	T	T	F	T	T	F	
						A		B	

   b) (9 *points*) Define what it means for two statement forms *A* & *B* to be logically equivalent. Are the above *A* & *B* logically equivalent? Prove or disprove the following **using only the definition (i.e. without using the truth table)**.

$$p \to (\sim q \wedge r) \equiv (\sim p \vee \sim q) \wedge r$$

2 statement forms are logically equivalent if and only if they have the same truth values for ALL truth values of their statement variables.

$p \to (\sim q \wedge r) \not\equiv (\sim p \vee \sim q) \wedge r$ because they do NOT have the same truth values for all truth values of $p, q, \& r$, such as when $(p, q, r) = (F, T, F)$

$p \to (\sim q \wedge r) = T$

$(\sim p \vee \sim q) \wedge r = F$

2. (22 *points*) Consider the following statement:

"If Trevor didn't pass quiz 1, then he needs to redo quiz 1 and study harder for quiz 2"

a) (5 *points*) Define 3 statement variables $p, q, \& r$, and write the **statement form** of the above conditional statement involving only "$\rightarrow$, $\vee$ or $\wedge$". Do not use negation "$\sim$".

$p =$ Trevor didn't pass quiz 1

$q =$ Trevor needs to redo quiz 1

$r =$ Trevor needs to study harder for quiz 2

Statement form: $p \rightarrow (q \wedge r)$

b) (5 *points*) Using the same $p, q, \& r$ as above, write the statement in words that corresponds to:

$$\sim p \vee (q \wedge r)$$

Either Trevor did pass quiz 1 or Trevor needs to redo quiz 1 and Trevor needs to study harder for quiz 2.

c) (6 *points*) Apply the negation "$\sim$" to the statement form given in part b), and ***simplify it*** using logical equivalences laws. Use only one law in each step and include a name for each law.

$$\begin{aligned} \sim(\sim p \vee (q \wedge r)) &\equiv \sim(\sim p) \wedge \sim(q \wedge r) && \text{by DeMorgans Law} \\ &\equiv \sim(\sim p) \wedge (\sim q \vee \sim r) && \text{by DeMorgans Law} \\ &\equiv p \wedge (\sim q \vee \sim r) && \text{by Double Negative Law} \end{aligned}$$

d) (6 *points*) Write in words the negation of the statement that you found in part b). (**Hint**: use part c))

Trevor didn't pass quiz 1 & Either Trevor does not need to redo quiz 1 or Trevor does not need to study harder for quiz 2.

3. (24 *points*) Prove the following logical equivalence using **standard logical equivalences**. Justify each step by stating the name of the standard logical equivalence law & use only one law in each step.

$$(p \wedge q) \wedge \sim r \quad \equiv \quad \sim(p \wedge r) \wedge (q \wedge p)$$

RHS $\equiv \sim(p \wedge r) \wedge (q \wedge p)$

$\equiv (\sim p \vee \sim r) \wedge (q \wedge p)$ by De Morgans Law

$\equiv (q \wedge p) \wedge (\sim p \vee \sim r)$ by Commutative Law

$\equiv q \wedge (p \wedge (\sim p \vee \sim r))$ by Associative Law

$\equiv q \wedge ((p \wedge \sim p) \vee (p \wedge \sim r))$ by Distributive Law

$\equiv q \wedge (c \vee (p \wedge \sim r))$ by Negation Law

$\equiv q \wedge ((p \wedge \sim r) \vee c)$ by Commutative Law

$\equiv q \wedge (p \wedge \sim r)$ by Identity Law

$\equiv (q \wedge p) \wedge \sim r$ by Associative Law

$\equiv (p \wedge q) \wedge \sim r$ by commutative Law

$\equiv$ LHS