

HW-2.3

1. (34 points) Prove that the following argument form is valid using only the standard valid argument forms and logical equivalences. Make sure you arrange your arguments in the given order 1., 2., 3., 4., 5. & then 6.:

- A. $p \rightarrow (t \wedge r)$
 B. $\sim q$
 C. $u \vee \sim p$
 D. $(u \wedge r) \rightarrow \sim s$
 E. $\sim p \rightarrow q$
 $\therefore \sim s$

Practice Problem:

- A. $t \rightarrow s$
 B. $(\sim t \vee p) \rightarrow q$
 C. $\sim s$
 D. $r \vee \sim p$
 E. $(\sim t \wedge q) \rightarrow \sim r$
 $\therefore \sim p$

1.

$\sim p \rightarrow q$ by E
 $\sim q$ by B
 $\therefore p$ by Modus Tollens

2.

$u \vee \sim p$ by C
 p by 1
 $\therefore u \wedge r$ by elimination a

3.

$p \rightarrow (t \wedge r)$ by A
 p by 2
 $\therefore t \wedge r$ by Modus Ponens

4.

$t \wedge r$ by 3
 $\therefore r$ by specialization b

5.

u by 2
 r by 4
 $\therefore u \wedge r$ by Conjunction

6.

$(u \wedge r) \rightarrow \sim s$ by D
 $(u \wedge r)$ by 5
 $\therefore \sim s$ by Modus Ponens

2. (24 points) Do as directed:

a) (2 points) Define what it means to say an argument (not argument form) is valid.

an argument is valid if and only if its argument form is valid.

b) (4 points) Consider the following argument form where A. & B. are the premises and C. is the conclusion.

A. $p \rightarrow (\sim q \vee r)$

B. $p \wedge \sim q$

C. $\therefore \sim r$

Define what it means for the above argument form to be valid.

The above argument form is valid if and only if, when all premises are true, the conclusion is also true for all truth values of statement variables.

c) (18 points) Construct a single complete truth table with A, B, & C (in that order) as defined in part b) above in the last 3 columns. Then determine if the argument form given in b) is valid or invalid and justify your answer using only the definition of a valid argument form. Use as many rows & columns as you need in the following table.

premise					premise	conclusion	
p	q	r	$\sim q$	$\sim q \vee r$	$p \rightarrow (\sim q \vee r)$	$p \wedge \sim q$	$\sim r$
T	T	T	F	T	T	F	F
T	T	F	F	F	F	F	T
T	F	T	T	T	T	T	F
T	F	F	T	T	F	T	T
F	T	T	F	T	T	F	F
F	T	F	F	F	T	F	T
F	F	T	T	T	T	F	F
F	F	F	T	T	T	F	T

The argument form is invalid because when $(p, q, r) = (T, F, T)$, all premises are true but the conclusion is false.

3. (62 points) For each of the following arguments, define the statement variables p, q & r etc. and write its argument form in the box provided. State if the argument is valid or invalid and prove your answer without using the truth tables. **USE COMPLETE SENTENCES.**

a) Premise 1: If my son finished his food, then he got the dessert.

Premise 2: My son didn't finish his food.

Conclusion: \therefore My son didn't get the dessert.

$p =$ my son finished his food

$q =$ my son got the dessert

Argument form

Premise 1: $p \rightarrow q$

Premise 2: $\neg p$

Conclusion: \therefore $\neg q$

Answer: The argument is invalid because when $(p, q) = (F, T)$, all premises are true but the conclusion is false.

(inverse error)

b) Let x & y be fixed real numbers:

Premise 1: If $xy = 0$, then either x or y is zero.

Premise 2: Either x or y is zero.

Conclusion: $\therefore xy = 0$.

$p =$ $xy = 0$

$q =$ either x or y is 0

Argument form

Premise 1: $p \rightarrow q$

Premise 2: q

Conclusion: \therefore p

Answer: The argument is invalid because when $(p, q) = (F, T)$, all premises are true but the conclusion is false.

(converse error)

c) Premise 1: If I studied hard, then either I got A or took a vacation.

Premise 2: I neither got A nor took a vacation.

Conclusion: \therefore I didn't study hard.

$p =$ I studied hard

$q =$ I got A

$r =$ I took a vacation

Argument form

Premise 1: $p \rightarrow (q \vee r)$

Premise 2: $\neg q \wedge \neg r$

Conclusion: \therefore $\neg p$

Answer: The argument is valid because after applying DeMorgan's law to $(q \vee r)$ treating it as B , the argument is in the standard argument form of Modus Tollens,

d) Premise 1: Melanie either took the quiz or provided documentation for absence.

Premise 2: Melanie provided documentation for absence.

Conclusion: \therefore Melanie did not take the quiz.

$p =$ Melanie took the quiz

$q =$ Melanie provided documentation for absence

Argument form

Premise 1: $p \vee q$

Premise 2: q

Conclusion: \therefore $\sim p$

Answer: The argument is invalid because when $(p, q) = (T, T)$, all premises are true, but the conclusion is false.

(this would be valid using \oplus instead of \vee)

e) Premise 1: If 4 GB is better than no memory at all, then we will either buy more memory or buy a new computer.

Premise 2: If we will buy a new computer, then we will not buy more memory.

Conclusion: \therefore If 4 GB is better than no memory at all, then we will buy a new computer.

$p =$ 4 GB is better than no memory at all

$q =$ we will buy more memory

$r =$ we will buy a new computer

Argument form

Premise 1: $p \rightarrow (q \vee r)$

Premise 2: $r \rightarrow \sim q$

Conclusion: \therefore $p \rightarrow r$

Answer: The argument is invalid because when $(p, q, r) = (T, T, F)$, all premises are true, but the conclusion is false.

p	q	r
T	T	F

f) Premise 1: If I studied hard, then I got A.

Premise 2: If I got A, then I took a vacation.

Conclusion: \therefore I took a vacation.

$p =$ I studied hard

$q =$ I got A

$r =$ I took a vacation

Argument form

Premise 1: $p \rightarrow q$

Premise 2: $q \rightarrow r$

Conclusion: \therefore r

Answer: The argument is invalid because when $(p, q, r) = (F, F, F)$, all premises are true, but the conclusion is false.