

### Math 330 Practice questions for Test 1

1. A factory has two assembly lines, each of which is shut down(S), at partial capacity(P), or at full capacity(F). For example (S, P) denotes that the first assembly line is shut down and the second one is operating at partial capacity.

(a) Find the probability that both assembly lines are shut down?

(b) Find the probability that the first assembly line is shut down?

c. Write the sample space.

|              |              |              |
|--------------|--------------|--------------|
| $P(S,S)=.02$ | $P(S,P)=.06$ | $P(S,F)=.05$ |
| $P(P,S)=.07$ | $P(P,P)=.14$ | $P(P,F)=.20$ |
| $P(F,S)=.06$ | $P(F,P)=.21$ | $P(F,F)=.19$ |

2. Let A be the event that a person is female, let B be the event that a person has black hair, and let C be the event that a person has brown eyes. Describe the kinds of people in the following events: (a).  $A \cap B$  (b)  $A \cup C'$  (c)  $A' \cap B \cap C$  (d)  $A \cap (B \cup C)$

3. A bag contains 200 balls that are either red or blue and either dull or shiny. There are 55 shiny red balls, 91 shiny balls, and 79 red balls. If a ball is chosen at random, what is the probability that it is either a shiny ball or a red ball? What is the probability that it is a dull blue ball?

4. A ball is chosen at random from a bag containing 150 balls that are either red or blue and either dull or shiny. There are 36 red shiny balls and 54 blue balls. What is the probability of the chosen ball being shiny conditional on it being red? What is the probability of the chosen ball being dull conditional on it being red?
5. A gene can be either type A or type B, and it can be either dominant or recessive. If the gene is type B, then there is a probability of 0.31 that it is dominant. There is also a probability of 0.22 that a gene is type B and it is dominant. What is the probability that a gene is of type A?
6. Suppose that two cards are drawn from a pack of cards without replacement. What is the probability that exactly one card from the heart suit is obtained?
7. Suppose that in the taste test, each participant samples eight products and is asked to select the three best products, but not in any particular order. What is the number of possible answers?
8. Suppose that four cards are taken at random without replacement from a pack of cards. What is the probability that two kings and two queens are chosen?

9. A discrete random variable takes values 2,6,7, and 12 with the following CDF values:

$$F(2)=.2, F(6)=.45, F(7)=.83, F(12)=1$$

a. Find  $P(X=x)$  for all  $x$  values

b.  $E[X]$

c.  $\text{Var}[X]$

d.  $E[2X+3]$

e.  $\text{Var}\{2X+3\}$

10. Suppose that the battery failure time, measured in hours, has a probability density function given by  $f(x)=2(x+1)^{-3}$ ,  $x \geq 0$

a. Show that  $f(x)$  is in fact a pdf.

b. Find the general formula for CDF  $F(x)$ , and use the CDF to find  $P(2 < X \leq 10)$

11. Suppose that the probability density function of the amount of milk deposited in a milk container is  $f(x) = 40.976 - 16x - 30e^{-x}$  for  $1.95 \leq x \leq 2.20$

a. Show that  $f(x)$  in fact is a pdf

b. Find  $E[X]$

c. Find  $P(2.10 < X < 2.15)$

12. The bending capabilities of plastic sheets are investigated by bending sheets at increasingly large angles until deformity appears in the sheet. The angle  $\theta$  at which the deformity first appears is then recorded. Suppose that this angle takes values between  $0^\circ$  and  $10^\circ$  with a probability density function  $f(\theta) = A(\text{Exp}(10 - \theta) - 1)$  for  $0 \leq \theta \leq 10$  and  $f(\theta) = 0$  elsewhere.

a. Find the Value of  $A$ .

b. What is the probability that a plastic sheet can be bent up to an angle of  $8^\circ$  without deforming?

c. Find the 95<sup>th</sup> percentile of  $\theta$ .

d. Suppose  $\mu = E[X] = .997729$ ,  $\sigma^2 = \text{Var}(X) = .984854$ . Use Chebyshev's rule to find the smallest value for  $P(\mu - 3\sigma < X < \mu + 3\sigma)$ . Is the answer consistent with the actual value for the probability  $P(\mu - 3\sigma < X < \mu + 3\sigma)$ ?

13. There is a probability of 0.93 that a visitor to a website will bounce (leave the website without clicking on any links).

a. What is the probability that at least 10 of the next 12 visitors to the website will bounce? Use the Binomial probability formula.

b. Is it possible to use the Binomial Table in the textbook to answer Part a?

c. Use a calculator such as TI-84 to answer Part a?

d. Let  $X$  be the number of visitors out of 50 who will bounce. Find  $E[X]$  and interpret its value.