

# The Pigeonhole Principle

Ben Haines

August 29, 2023

The Pigeonhole Principle is a key topic of Discrete Mathematics, but is also used in Computer Science as a broad field, and is often used to compare the performance of algorithms. The Principle itself states that if you have  $y$  containers and  $x$  items, and  $x$  is greater than  $y$ , then in at least one container there will be two or more items. This is often explained as being  $x$  pigeons and  $y$  holes, thus it is called the Pigeonhole Principle.

Pigeonhole Principle:

$$\left\lceil \frac{|X|}{|Y|} \right\rceil$$

In order to learn how the Pigeonhole Principle functions, it is best to learn the basics of how it works first. The formula for the Pigeonhole Principle is the cardinality of the set  $x$ , which represents the total number of objects, over the cardinality of the set  $y$ , which represents the total number of containers, rounded up. The result from this equation gives you the minimum amount of objects that will go in a container.

For an example of this works, assume that set  $X$  has items one through six in it, and set  $Y$  has containers numbered one through five.

$$X = 1, 2, 3, 4, 5, 6$$

$$Y = 1, 2, 3, 4, 5$$

The cardinality of a set is just the number of elements within the set.

$$|X| = 6$$

$$|Y| = 5$$

So then the final step that needs to be done is to divide the total elements in set  $X$  by the total elements in set  $Y$ , then round up.

$$\left\lceil \frac{6}{5} \right\rceil$$

$$\lceil 1.2 \rceil$$

$$2$$

So, by the Pigeonhole Principle, there will be at least two items that go in the same container. This applies in every situation with these numbers, with six items in one box, or with all items equally distributed.

Extended Pigeonhole Principle:

$$f : X \rightarrow Y, |X| > |Y| \Rightarrow \exists x_1, x_2 \in X, x_1 \neq x_2 \wedge f(x_1) = f(x_2)$$

The Extended Pigeonhole Principle goes further in-depth than its shorter counterpart. When function  $f$  maps elements from set  $X$  into  $Y$ , and there are more elements in  $X$  than  $Y$ , then there are two elements,  $x_1$  and  $x_2$ , in set  $X$  that would share the same  $Y$  value. In other words, If there are more elements in the set of  $X$  than the set of  $Y$ , then at least two objects in set  $X$  will share the same  $Y$  value. So in essence, the Pigeonhole Principle gets across the same message as the equation from before, but it is more descriptive, which also makes the extended Pigeonhole Principle a good option for learning what some of the symbols mean, since you already have the roadmap for how the Pigeonhole Principle works.