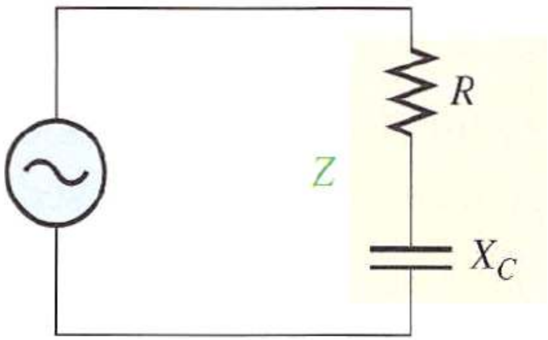
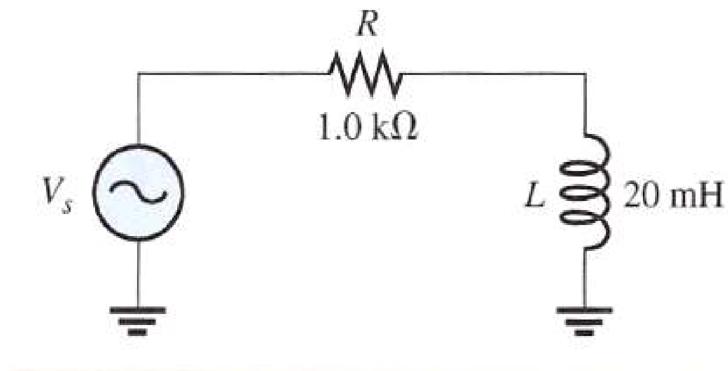
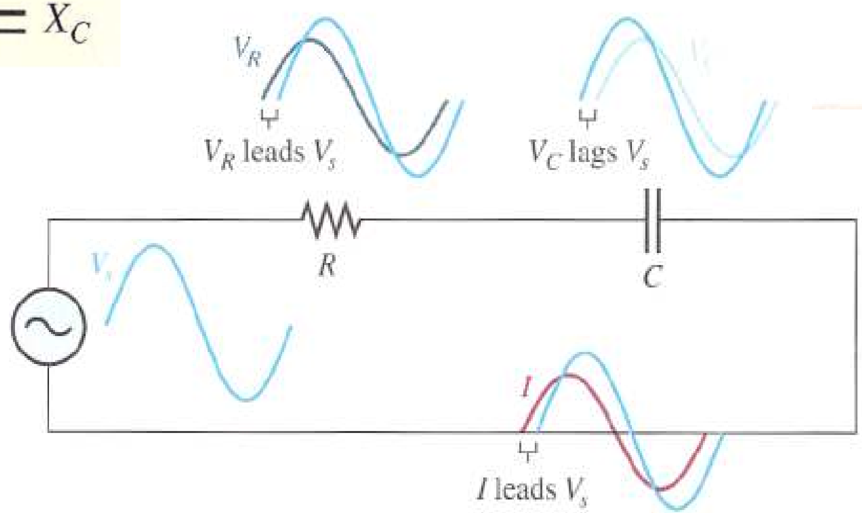
# Electricidade

## Capítulo 7.3. Circuitos em AC





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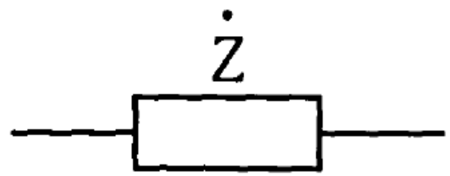
# Fundamentos de circuitos AC

## Impedância

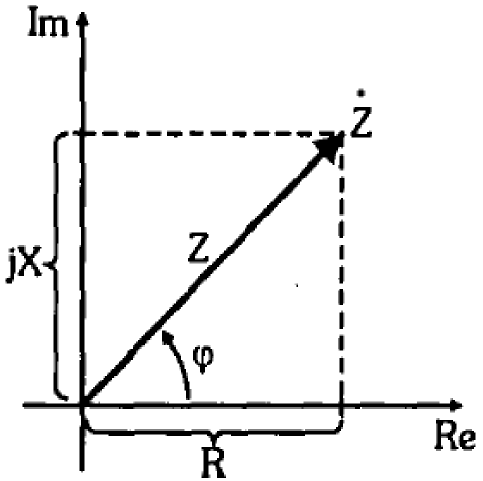
- A impedância Z, em ohm (Ω) é um número complexo que caracteriza um dispositivo ou circuito e reflecte:
  - A **oposição total** que ela impõe à **passagem de corrente alternada**
  - **Desfasamento** total entre a **tensão e a corrente**.




- A impedância Z, é composta por uma componente real denominada ***resistência R*** e uma componente imaginária denominada ***reactância X.***

$$\dot{Z} = R + jX \quad (\text{forma rectangular})$$

$$\dot{Z} = Z\angle\varphi \quad (\text{forma polar})$$

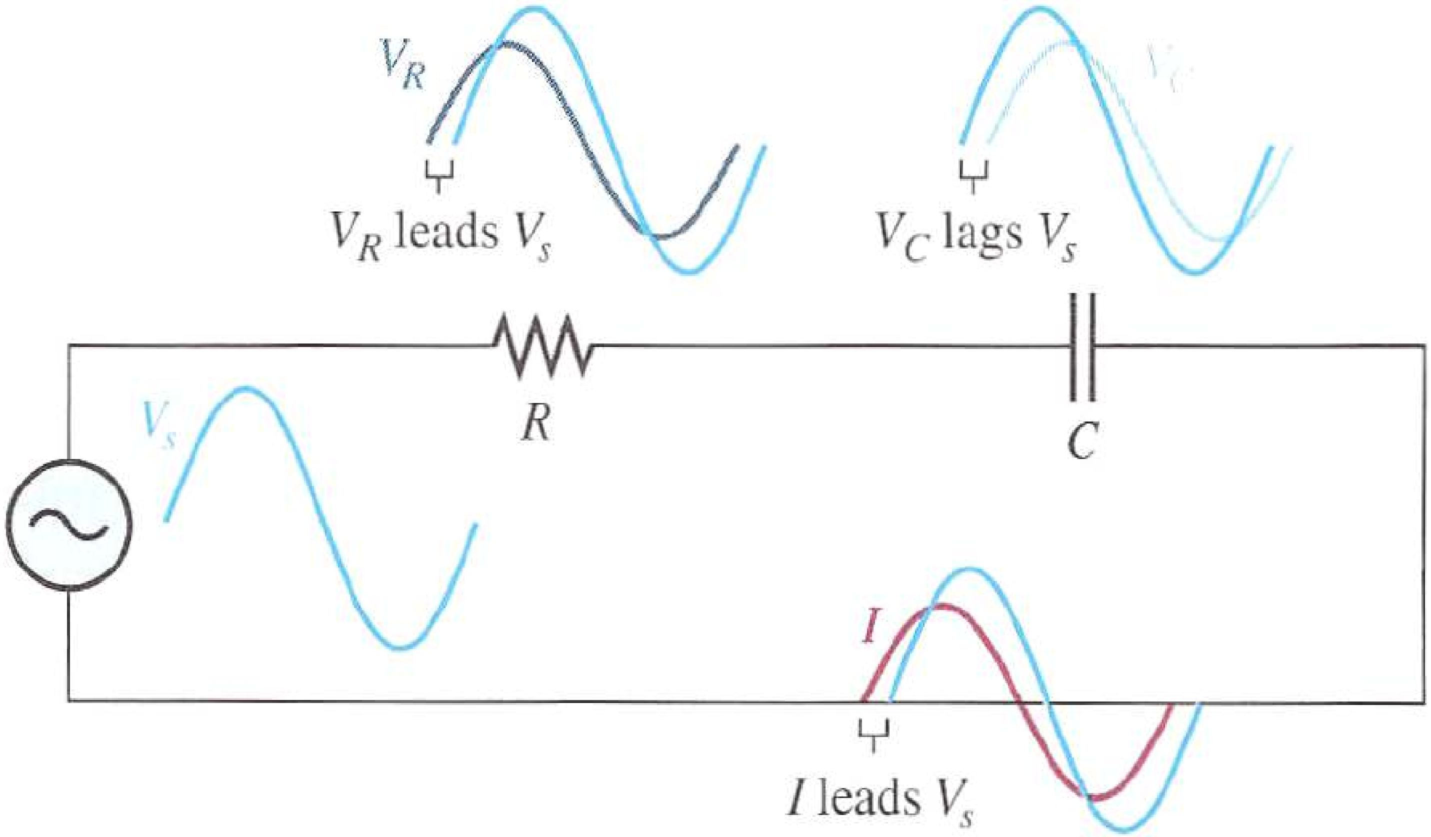



$$\dot{Z} = \sqrt{R^2 + X^2} \Rightarrow \text{Módulo da impedância } \dot{Z}$$

$$\varphi = arctg\,\frac{X}{R} \Rightarrow \text{Fase da impedância } \dot{Z}$$

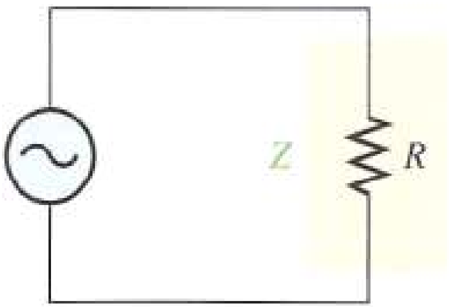
$$R = Z.\cos\varphi \quad \text{e} \quad X = Z.sen\,\varphi$$

# ▪ Circuitos RC

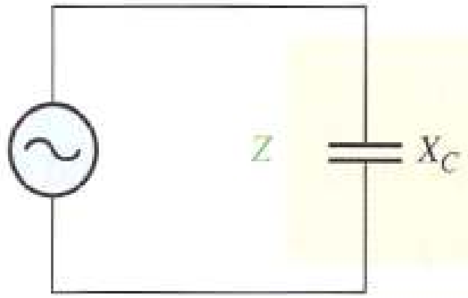


# ▪ Circuitos RC Série

## — Impedância

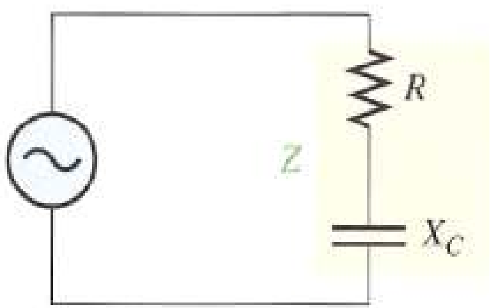


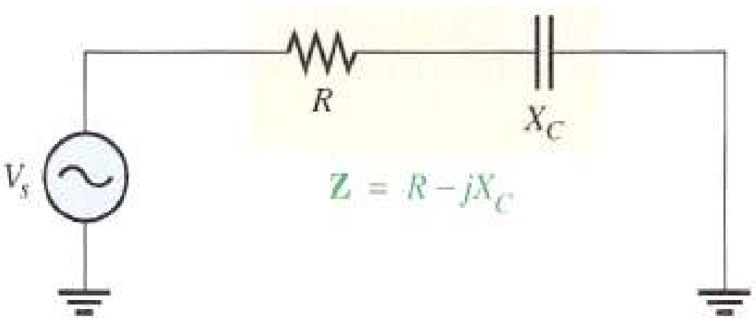

$$\dot{Z} = R$$




$$\dot{Z} = -jX_C$$

## — Impedância de um circuito RC série



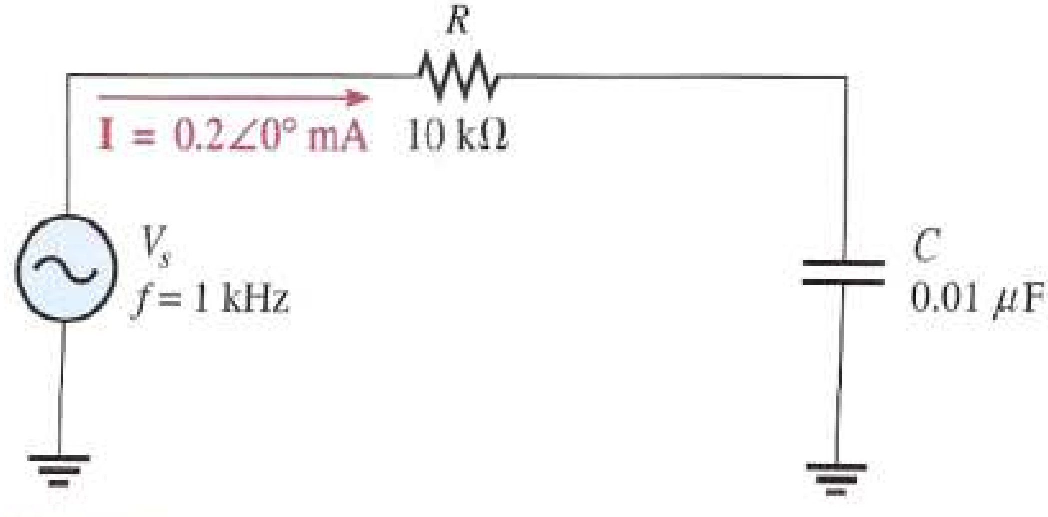

$$\dot{Z} = R - jX_C$$

# ▪ Circuitos RC Série

## — Lei de Ohm

$$\boxed{\dot{V} = \dot{I}.\dot{Z}}$$

$$\boxed{\dot{I} = \frac{\dot{V}}{\dot{Z}}} \qquad \boxed{\dot{Z} = \frac{\dot{V}}{\dot{I}}}$$

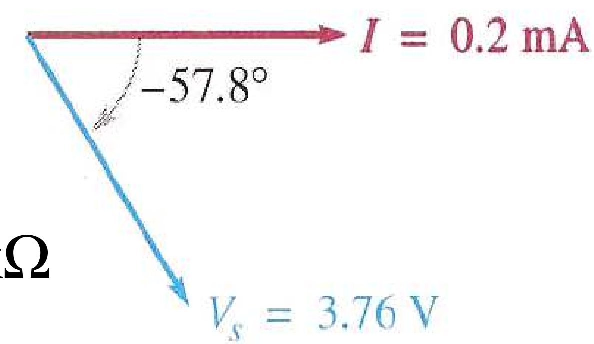



$$X_C = \frac{1}{2\pi.f.C} = \frac{1}{2\pi(1000\,\text{Hz})(0.01\mu F)} = 15{,}9\,\text{k}\Omega$$

$$\dot{Z} = R - jX_C \; = 10\,\text{k}\Omega \text{ - } j15.9\,\text{k}\Omega \Leftrightarrow \dot{Z} = 18.8\angle -57.8^{o}\ \text{k}\Omega$$

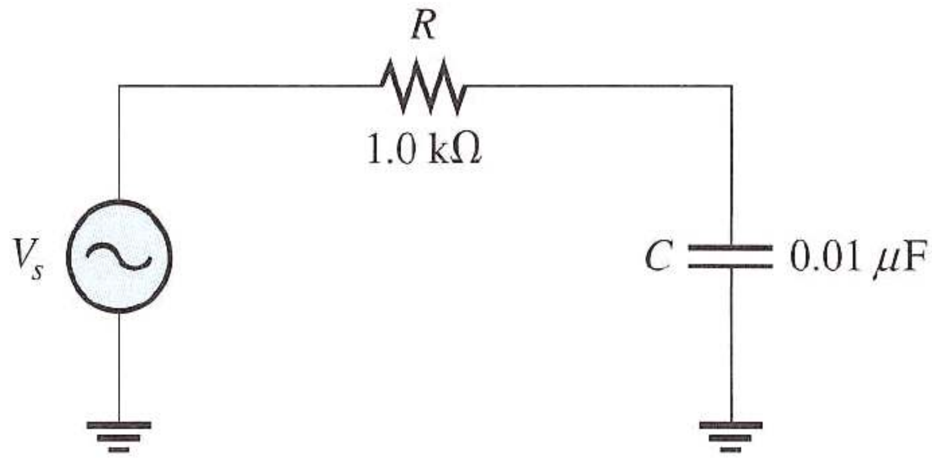
$$\dot{V} = \dot{I}.\dot{Z} = (0.2\angle -0^{o}\,\text{mA})(18.8\angle -57.8^{o}\,k\Omega) = \boxed{3.76\angle -57.8^{o}\ V}$$

**Diagrama de fasores**



# Circuitos RC Série

## Influência da frequência




Para $f = 10$ kHz

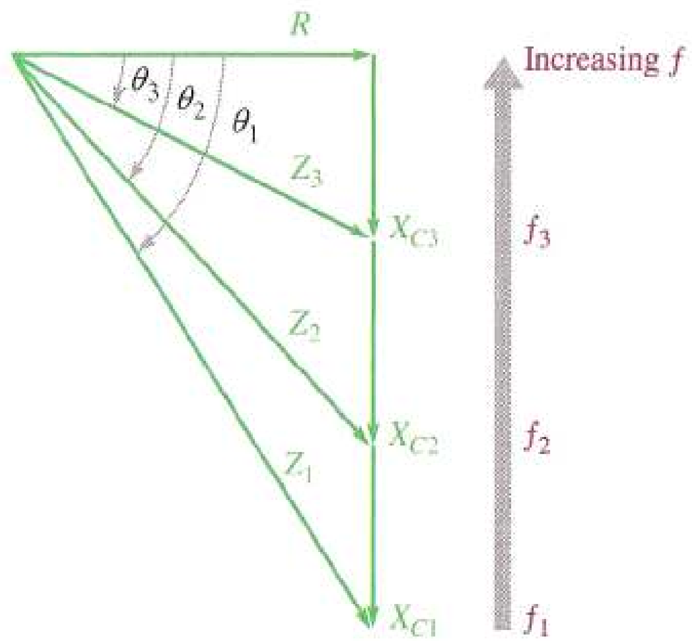
$$X_C = \frac{1}{2\pi . f . C} = \frac{1}{2\pi (10\,\text{kHz})(0.01 \mu F)} = 1{,}59\,\text{k}\Omega$$

$$\boxed{1.88 \angle -57.8 \quad \text{k}\Omega}$$

Para $f = 20$ kHz

$$X_C = \frac{1}{2\pi (20\,\text{kHz})(0.01 \mu F)} = 796\,\Omega$$

$$\boxed{1.28 \angle -38.5^{\circ} \quad \text{k}\Omega}$$



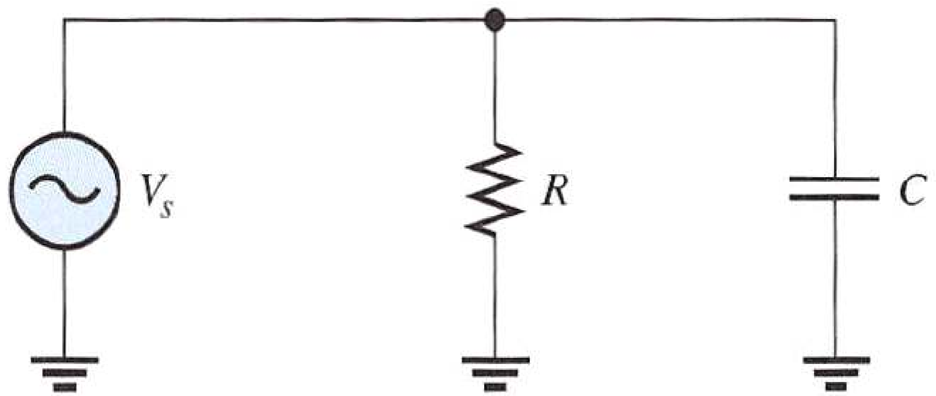

Para $f = 30$ kHz

$$X_C = \frac{1}{2\pi (30\,\text{kHz})(0.01 \mu F)} = 531\,\Omega$$

$$\boxed{1.13 \angle -28.0^{\circ} \quad \text{k}\Omega}$$

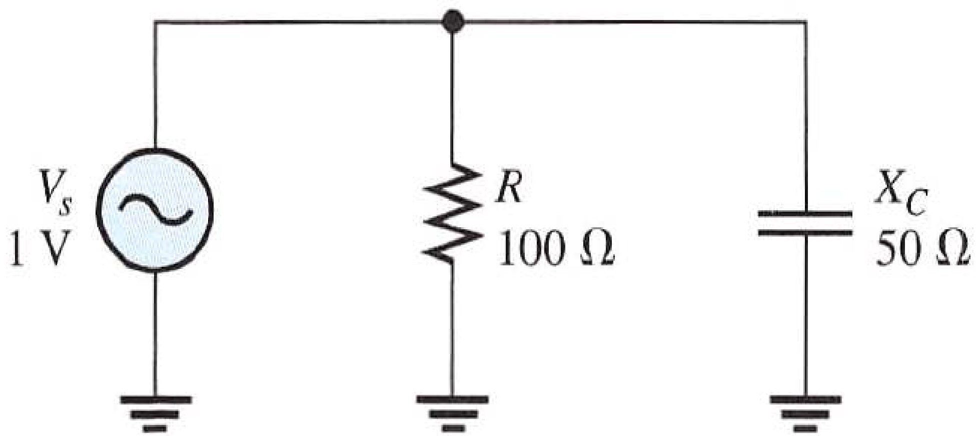
# ▪ Circuitos RC - Paralelo

## Paralelo




$$\dot{Z} = \frac{(R\angle 0^o)(X_C\angle -90^o)}{R - jX_C}$$

$$\dot{Z} = \frac{RX_C}{\sqrt{R^2 + X_C^2}}\angle -\tan^{-1}\left(\frac{R}{X_C}\right)$$

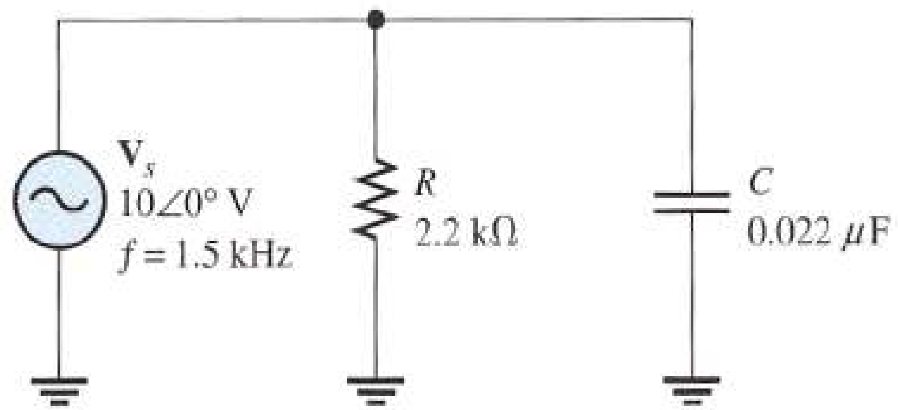



$$\dot{Z} = \frac{(100\,\Omega)(50\,\Omega)}{\sqrt{(100\,\Omega)^2 + (50\,\Omega)^2}}\angle -\tan^{-1}\left(\frac{100\,\Omega}{50\,\Omega}\right)$$

$$\dot{Z} = 44.7\angle -63.4^o \quad \Omega$$

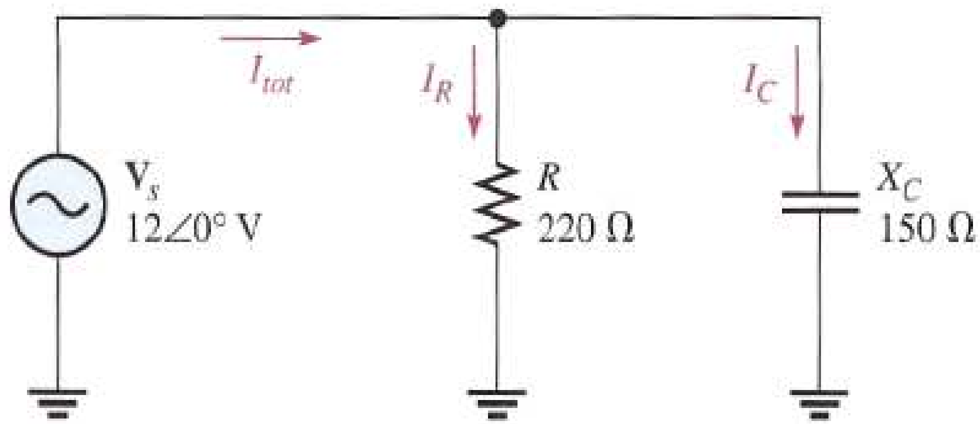
# ▪ Circuitos RC - Paralelo

## LEI de ohm




$$X_C = \frac{1}{2\pi(1.5\,\text{kHz})(0.022\mu F)} = 4.82\,\text{k}\Omega$$

$$\boxed{\dot{I} = 5\angle 24.5^o \quad \text{mA}}$$



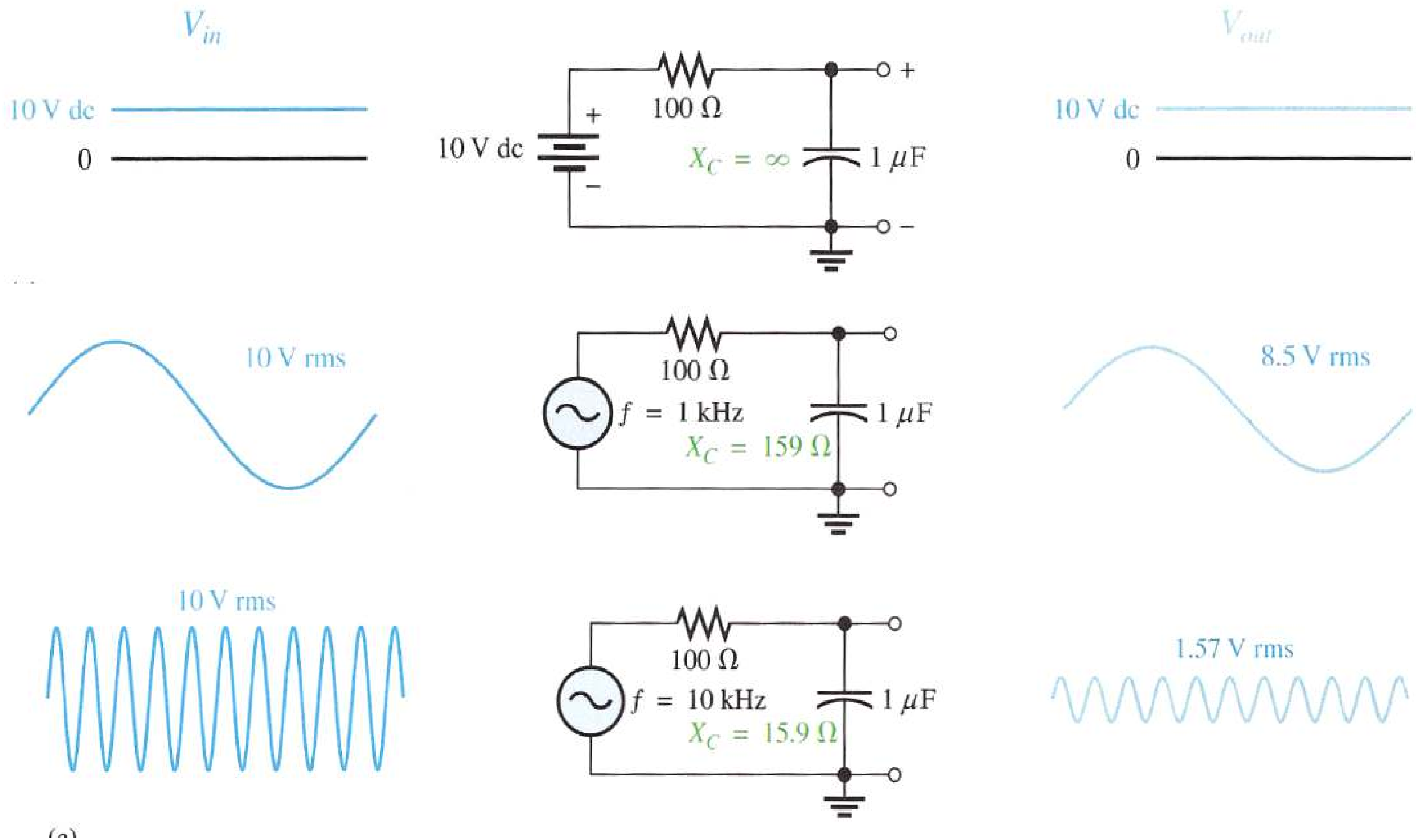

$$\dot{I} = \frac{\dot{V}_S}{\dot{R}} = \frac{12\angle 0^o\ \text{V}}{220\angle 0^o\ \Omega} = 54.5\angle 0^o \quad \text{mA}$$

$$\dot{I} = \frac{\dot{V}_S}{\dot{R}} = \frac{12\angle 0^o\ \text{V}}{150\angle -90^o\ \Omega} = 80\angle 90^o \quad \text{mA}$$

$$\boxed{\dot{I}_{tot} = 54.5\,\text{mA} + j80\text{mA} \Leftrightarrow \dot{I} = 96.8\angle 55.7^o \quad \text{mA}}$$

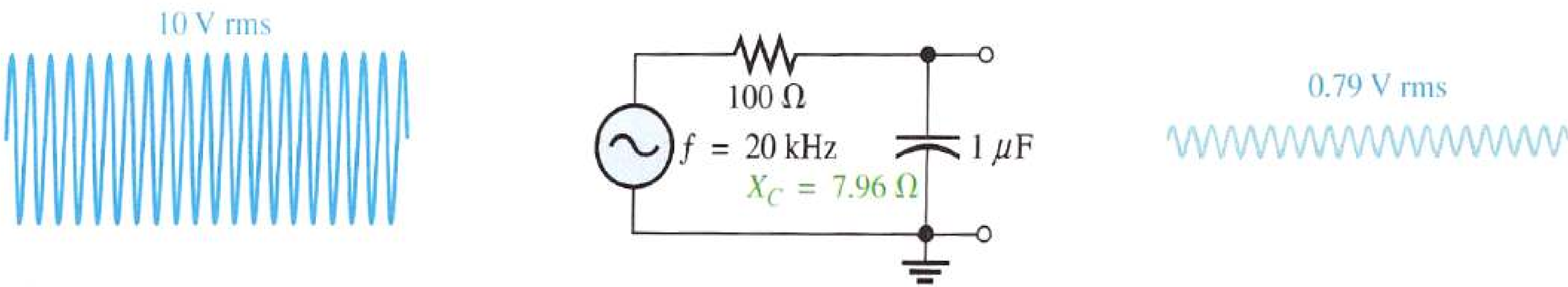
# ▪ Circuitos RC - Filtros

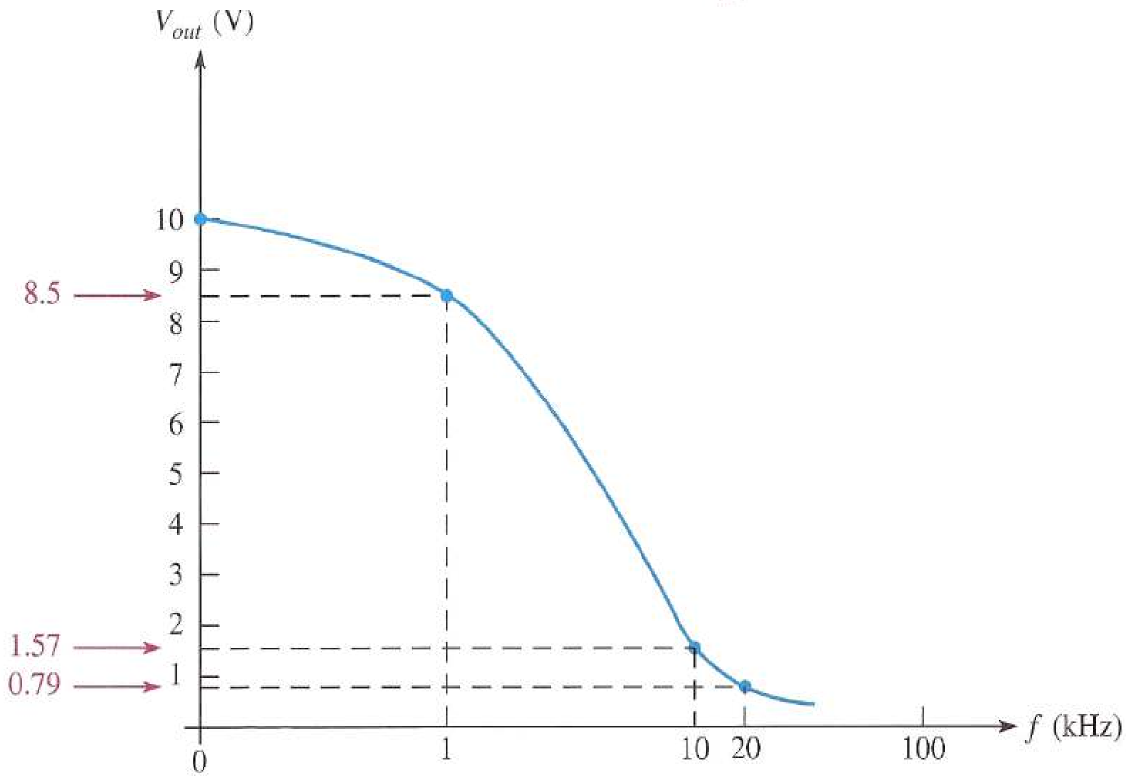
## Filtros passivos- Passa Baixo



# ▪ Circuitos RC - Filtros

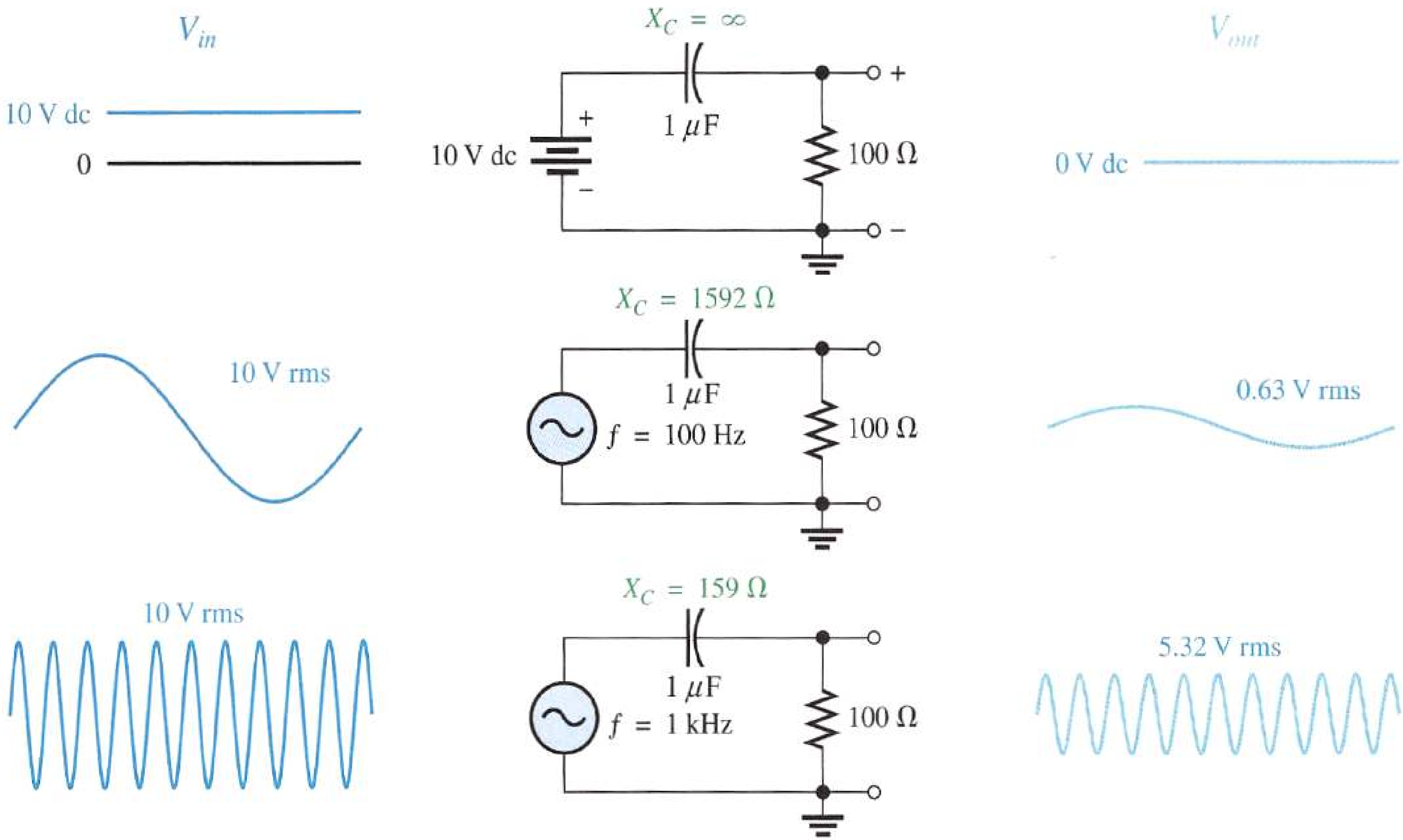
## Filtros passivos- Passa Baixo



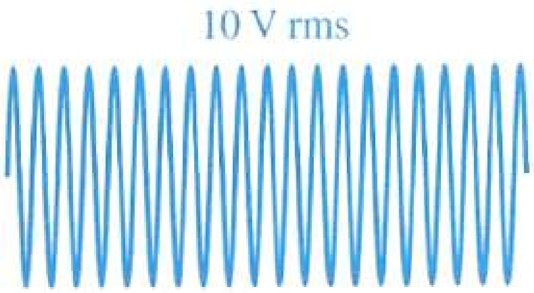
# ▪ Circuitos RC - Filtros

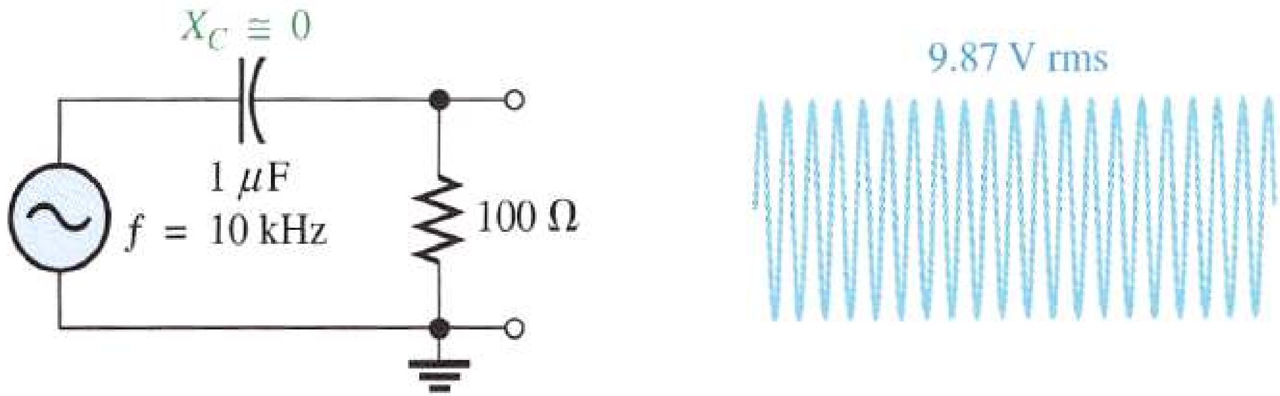
## Filtros passivos- Passa alto



# Circuitos RC - Filtros

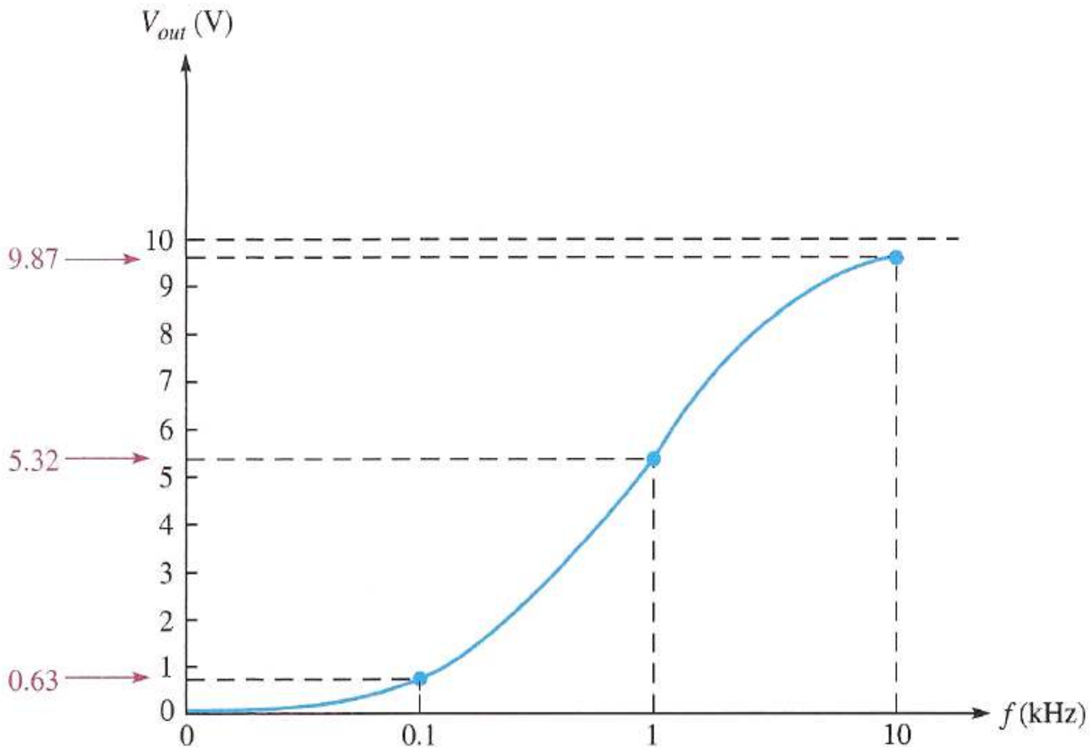
## Filtros passivos- Passa alto



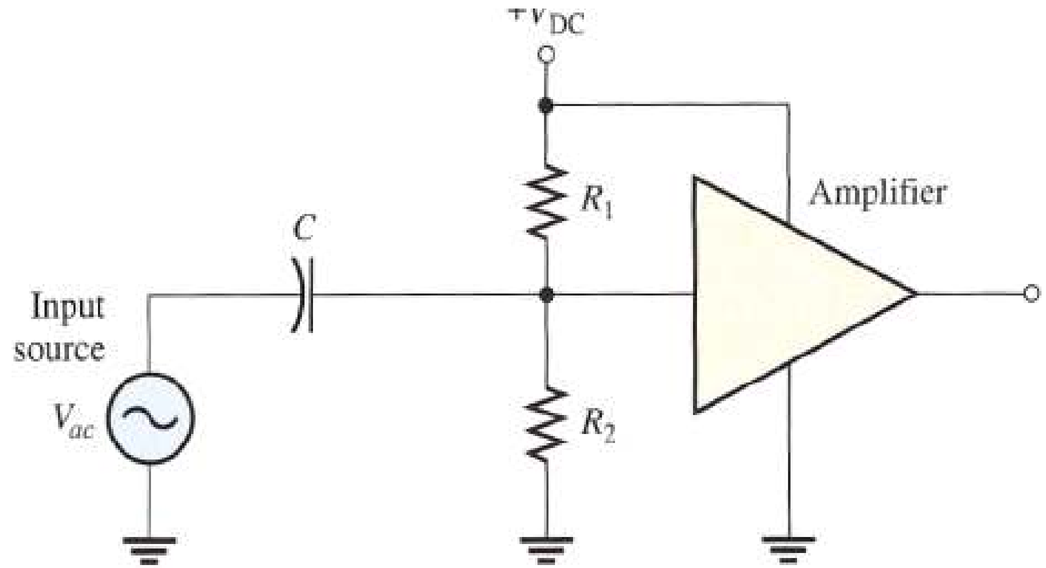

### Frequência de corte

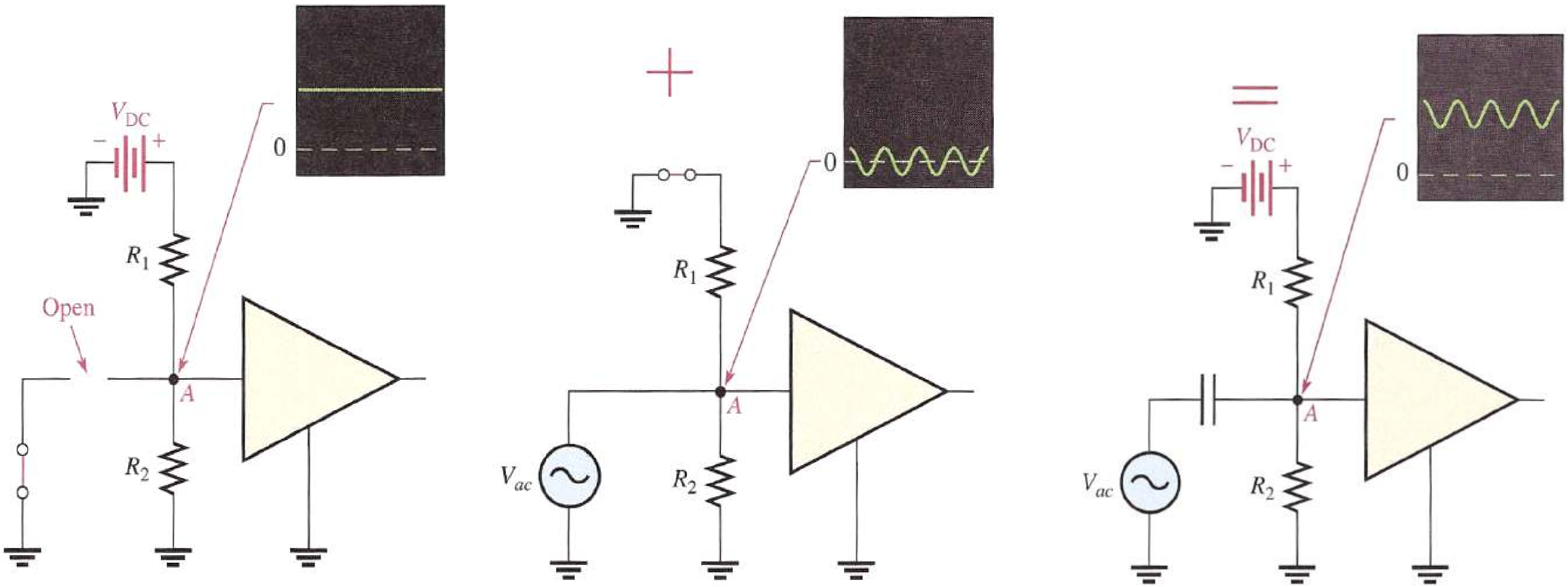
$$f_C = \frac{1}{2\pi.R.C}$$



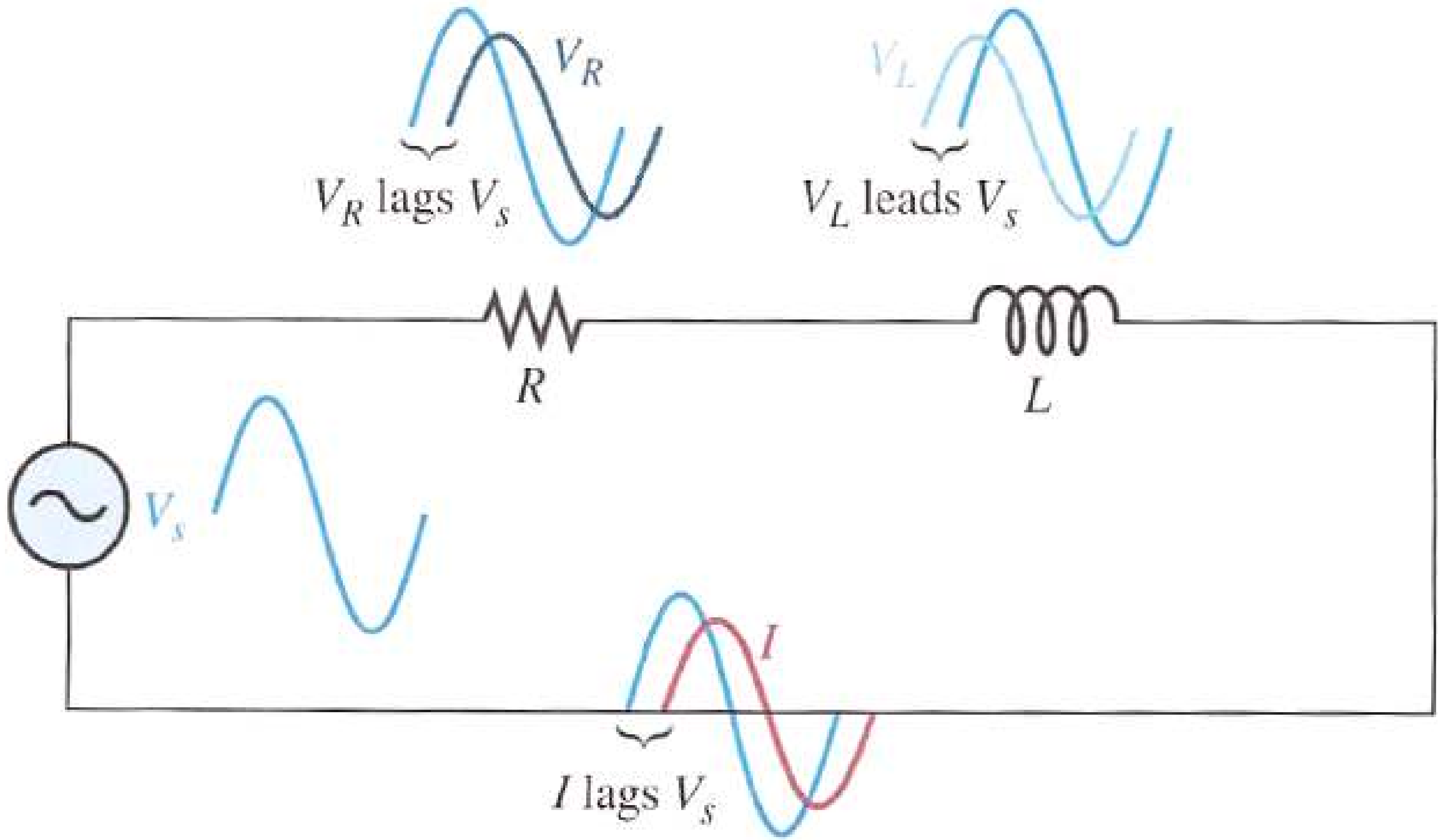
# ▪ Circuitos RC

## Condensadores de acoplamento



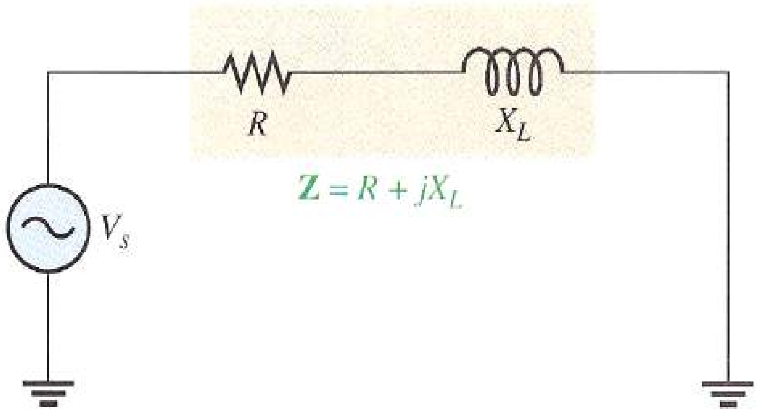



# ▪ Circuitos RL

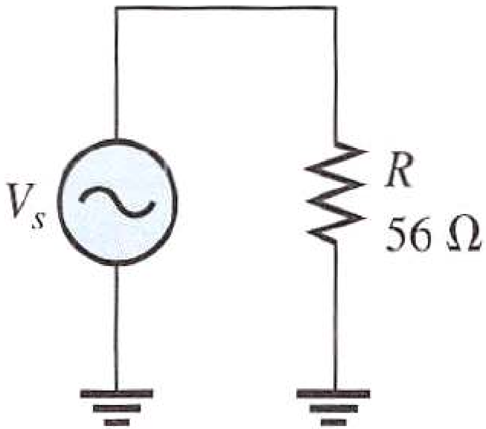


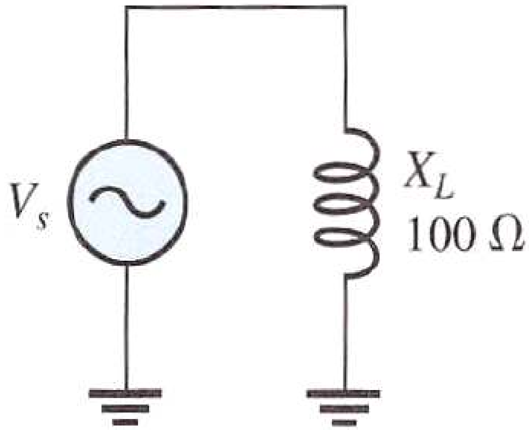
# ▪ Circuitos RL Série

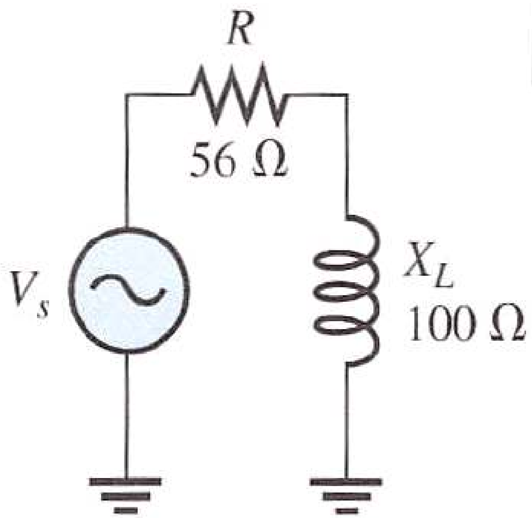
## Impedância de um circuito RL




$$\dot{Z} = R + jX_L$$




$$\dot{Z} = \mathrm{R} + j0 = 56\,\Omega$$

$$\dot{Z} = \mathrm{R}\,\angle 0^{o} = 56\,\angle 0^{o}\quad \Omega$$

$$\dot{Z} = \mathrm{R} + jX_L = j100\,\Omega$$

$$\dot{Z} = X_L\,\angle 90^{o} = 100\,\angle 90^{o}\quad \Omega$$

$$\dot{Z} = \mathrm{R} + jX_L = 56 + j100\,\Omega$$

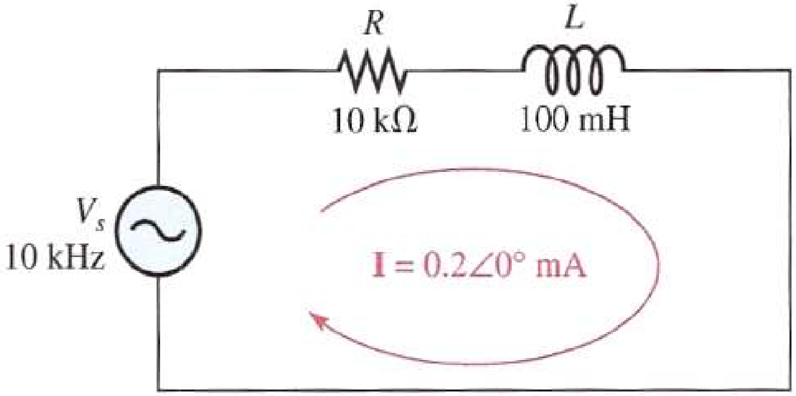
# Circuitos RL Série

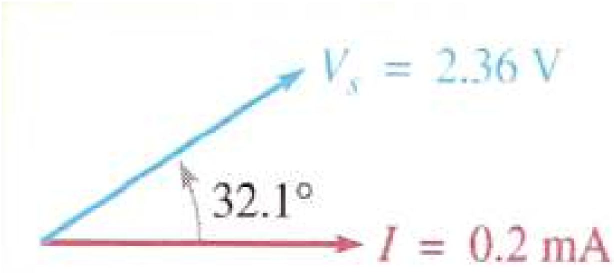
## Lei de ohm

$$\dot{V} = \dot{I} . \dot{Z}$$

$$\dot{I} = \frac{\dot{V}}{\dot{Z}}$$

$$\dot{Z} = \frac{\dot{V}}{\dot{I}}$$




$$X_L = 2\pi . f . L = 2\pi (10 \text{ kHz})(100 \text{ mH}) = 6{,}28 \text{ k}\Omega$$

$$\dot{Z} = \text{R} + jX_L = 10 \text{ k}\Omega + j6.28 \text{ k}\Omega$$

$$\dot{V}_S = \dot{I} . \dot{Z} = (0.2 \angle 0^o \text{ mA})(11.8 \angle 32.1^o \text{ k}\Omega) = 2.36 \angle 32.1^o \text{ V}$$

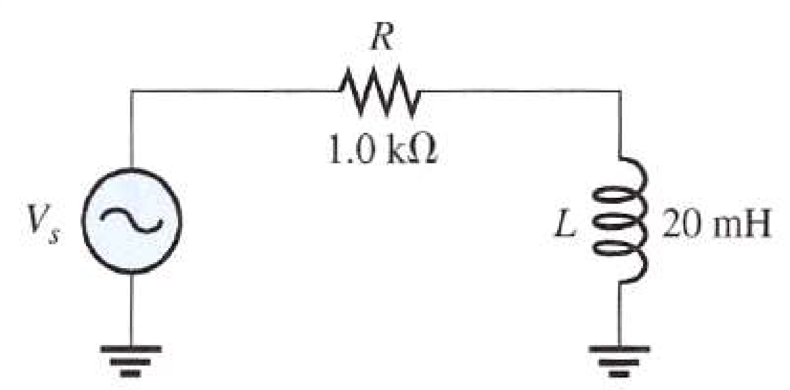
# Circuitos RL Série

## Variação da impedância com a frequência

Para $f = 10\ \text{kHz}$

$$X_L = 2\pi.f.L = 2\pi(10\,\text{kHz})(20\,\text{mH}) = 1{,}26\,\text{k}\Omega$$

$$\boxed{1.66\angle 51.6^{\circ}\ \ \text{k}\Omega}$$




Para $f = 20\ \text{kHz}$

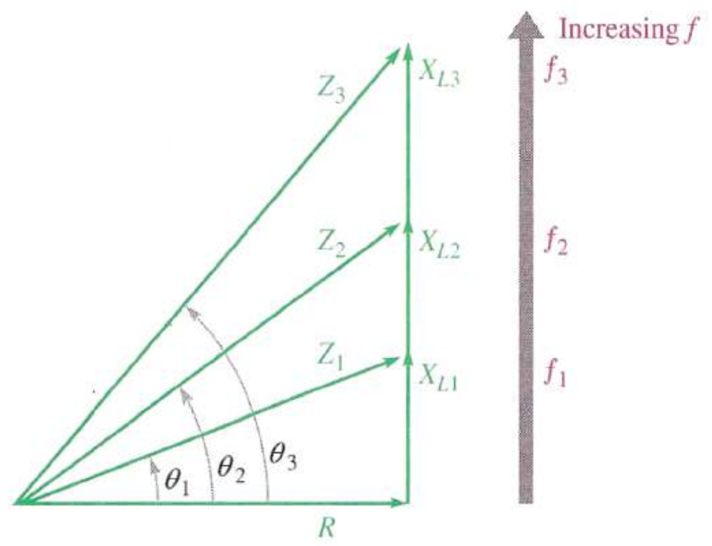
$$X_L = 2\pi.f.L = 2\pi(20\,\text{kHz})(20\,\text{mH}) = 2{,}51\,\text{k}\Omega$$

$$\boxed{2.70\angle 68.3^{\circ}\ \ \text{k}\Omega}$$

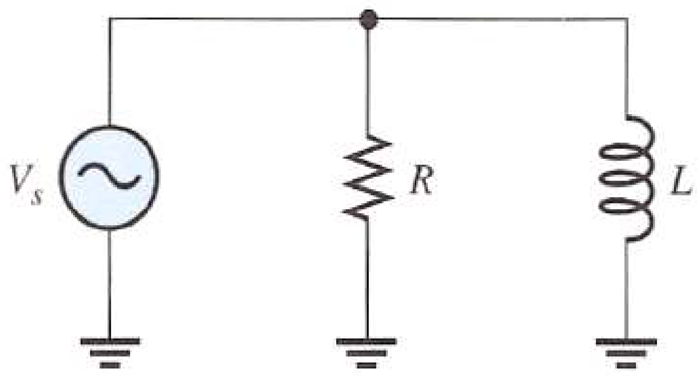
Para $f = 30\ \text{kHz}$

$$X_L = 2\pi.f.L = 2\pi(30\,\text{kHz})(20\,\text{mH}) = 3{,}77\,\text{k}\Omega$$

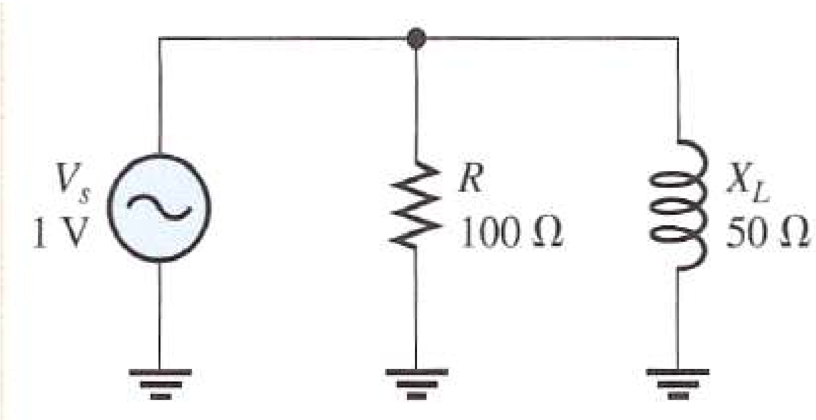
$$\boxed{3.90\angle 75.1^{\circ}\ \ \text{k}\Omega}$$



# ▪ Circuitos RL Paralelo




$$\dot{Z} = \frac{(R\angle 0^{\circ})(X_L\angle 90^{\circ})}{R + jX_L}$$

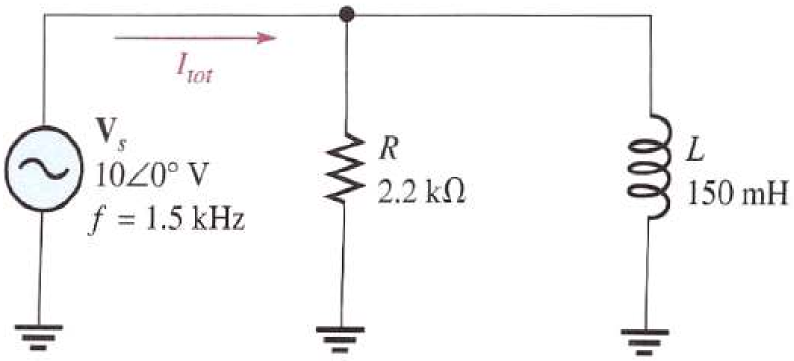



$$\dot{Z} = \frac{RX_L}{\sqrt{R^2 + X_L^2}} \angle \tan^{-1}\left(\frac{R}{X_L}\right)$$

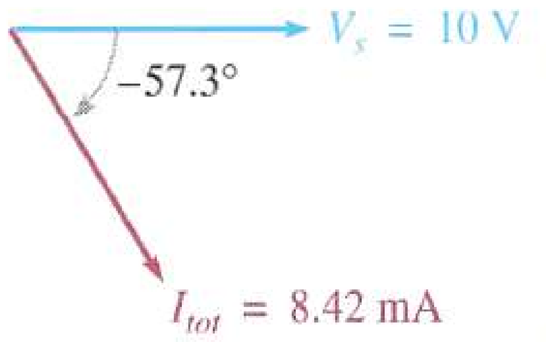
$$\dot{Z} = \frac{(100\,\Omega)(50\,\Omega)}{\sqrt{(100\,\Omega)^2 + (50\,\Omega)^2}} \angle \tan^{-1}\left(\frac{100\,\Omega}{50\,\Omega}\right)$$

$$\dot{Z} = 44.7\angle 63.4^{\circ} \quad \Omega$$

# ▪ Circuitos RL Paralelo

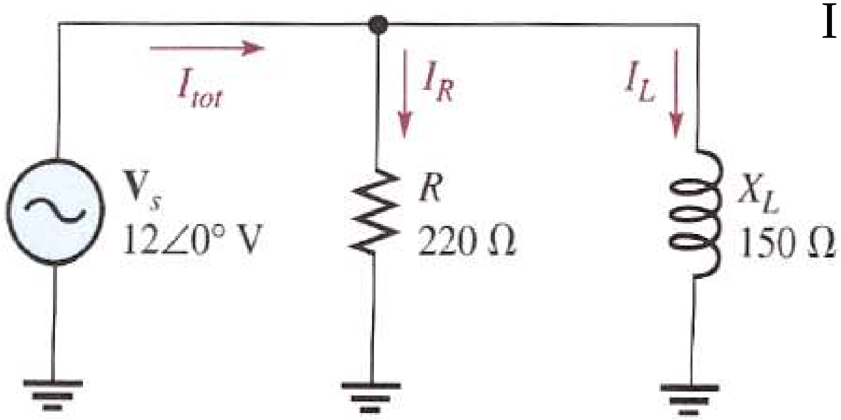



$$X_L = 2\pi . f . L = 2\pi(1.5\,\text{kHz})(150\,\text{mH}) = 1{,}41\,\text{k}\Omega$$




$$\dot{I}_{tot} = \dot{V}_S . \dot{Y}_{tot} = (10\angle 0^o\,\text{V})(842\angle -57.3^o\,\mu\text{S}) = 8.42\angle -57.3^o\,\text{mA}$$

## Exemplo




$$\dot{I}_R = \frac{\dot{V}_S}{\dot{R}} = \frac{12\angle 0^o\,\text{V}}{220\angle 0^o\,\Omega} = 54.5\angle 0^o \quad \text{mA}$$

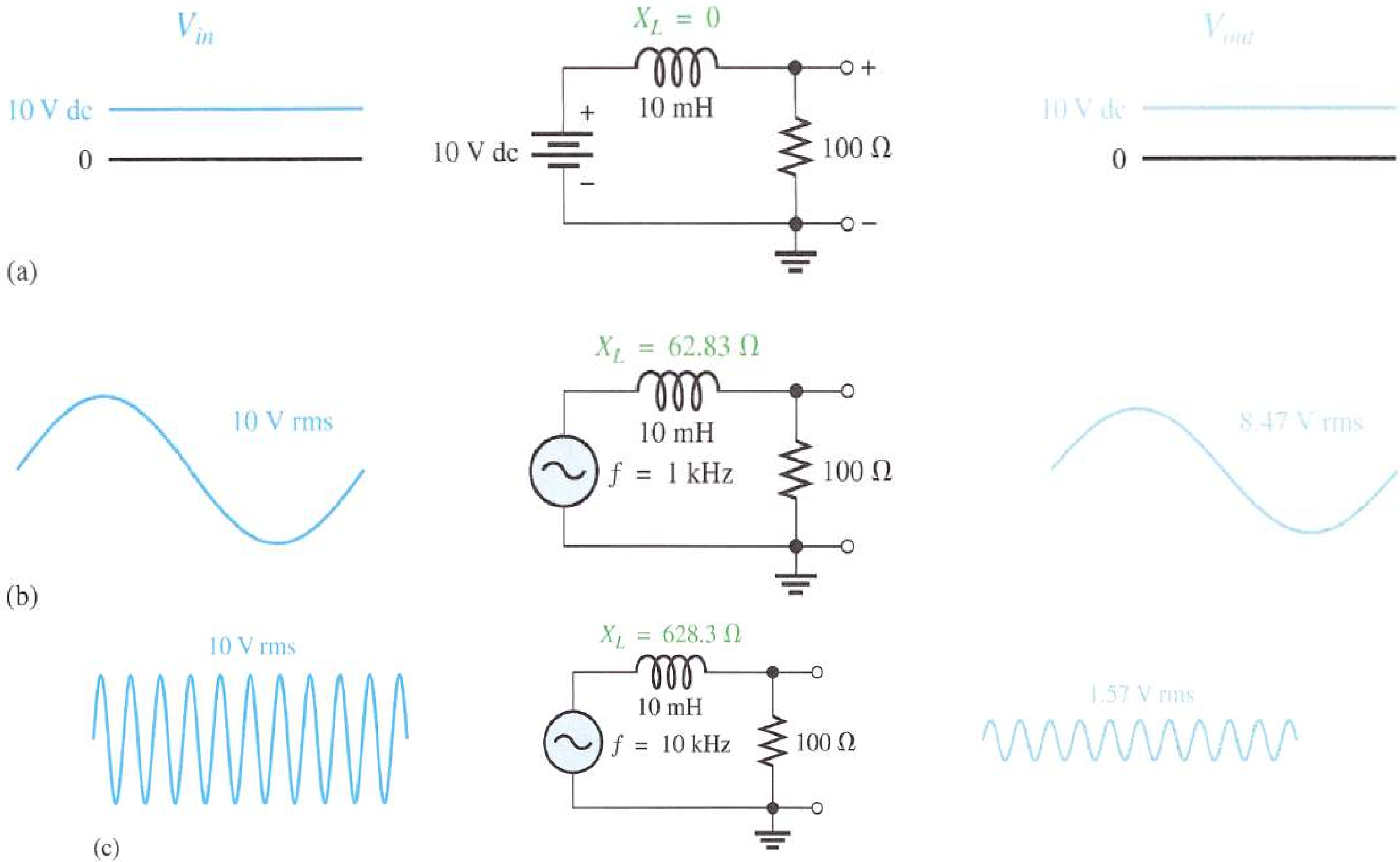
$$\dot{I}_L = \frac{\dot{V}_S}{\dot{X}_L} = \frac{12\angle 0^o\,\text{V}}{150\angle 90^o\,\Omega} = 80\angle -90^o \quad \text{mA}$$

$$\dot{I}_{tot} = 54.5\,\text{mA} - j80\text{mA}$$

$$\dot{Z} = \sqrt{I_R^2 + I_L^2}\angle -\tan^{-1}\left(\frac{I_L}{I_R}\right) = \sqrt{(54.5\,\text{mA}) + (80\,\text{mA})}\angle -\tan^{-1}\left(\frac{80\,\text{mA}}{54.5\,\text{mA}}\right) = 96.8\angle -55.7^o\,\text{mA}$$

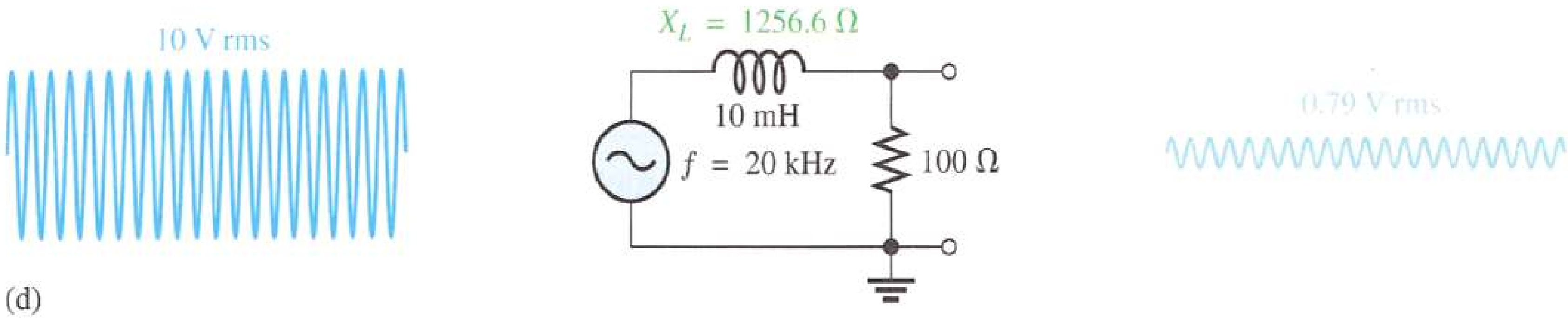
# Circuitos RL – Filtros

## Filtros passa-baixo

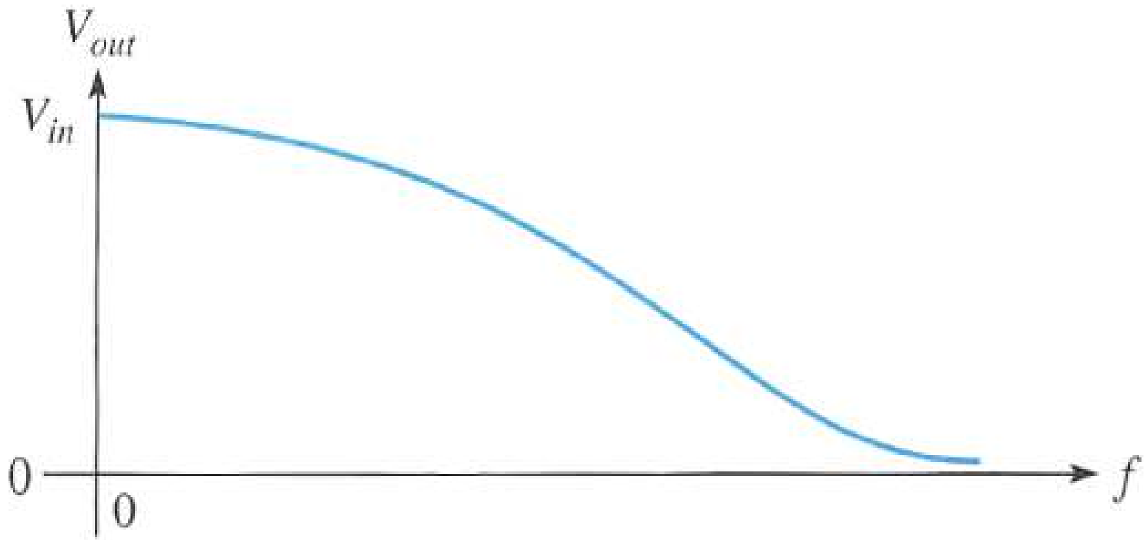


# ▪ Circuitos RL – Filtros

## Filtros passa-baixo

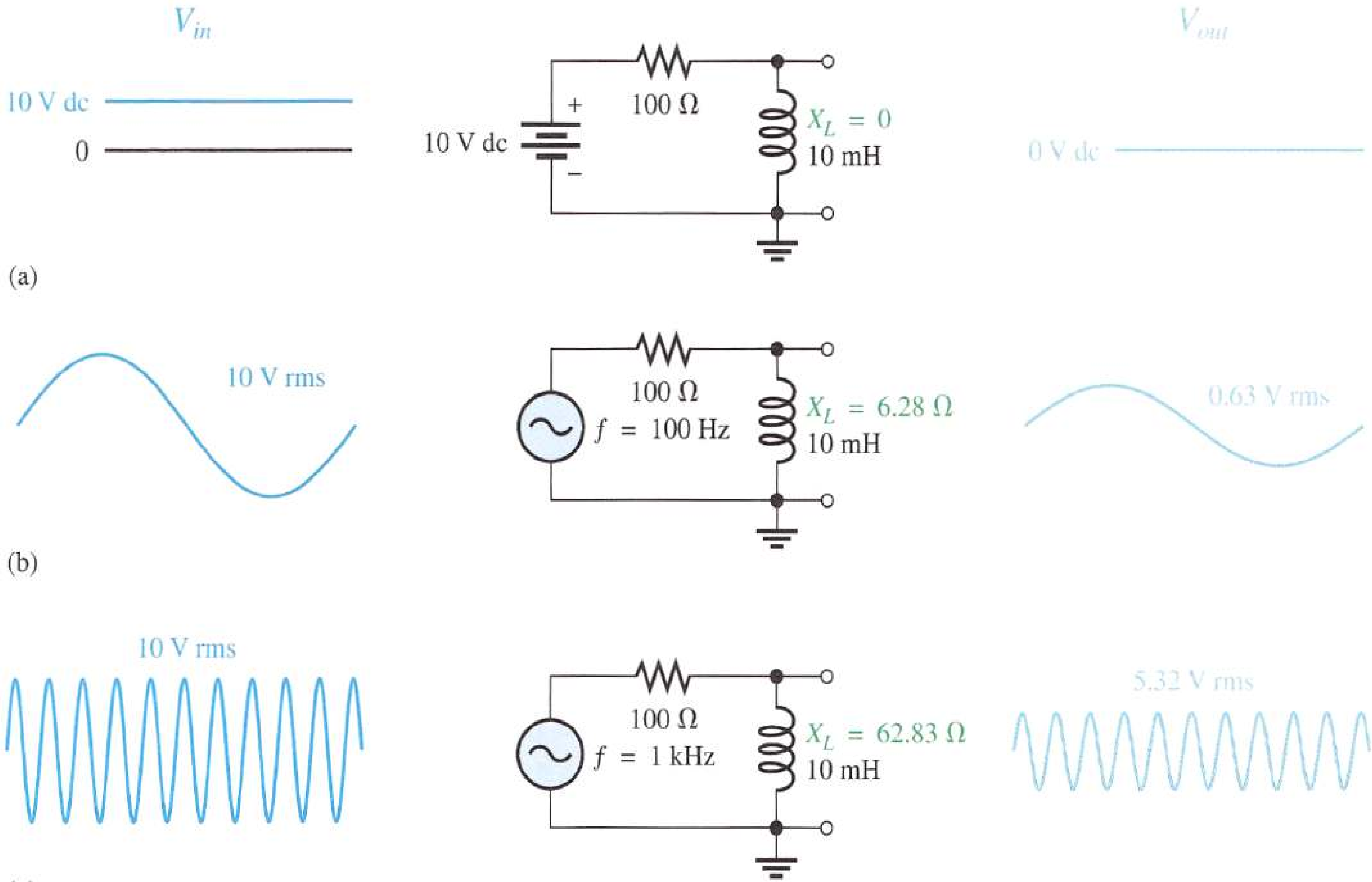



(d)



# ▪ Circuitos RL – Filtros

## – Filtros passa-alto



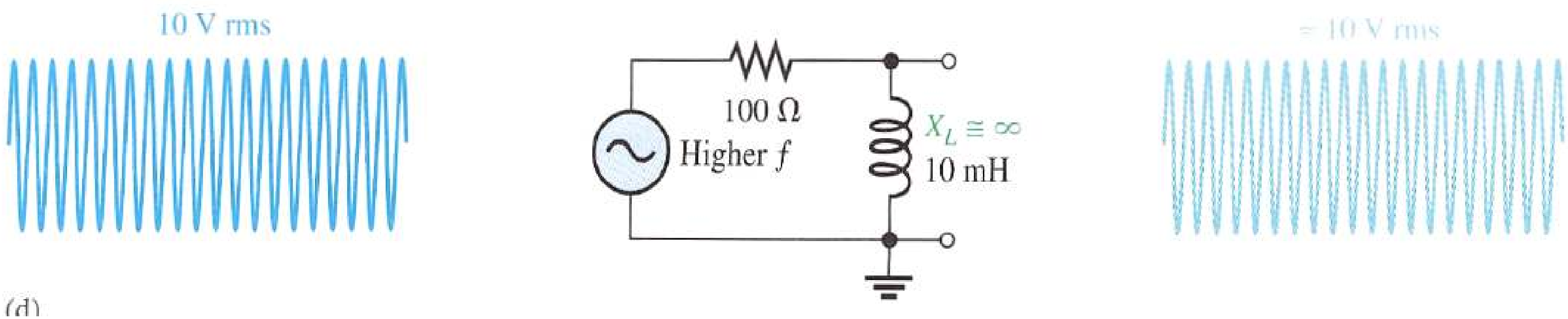

(a)

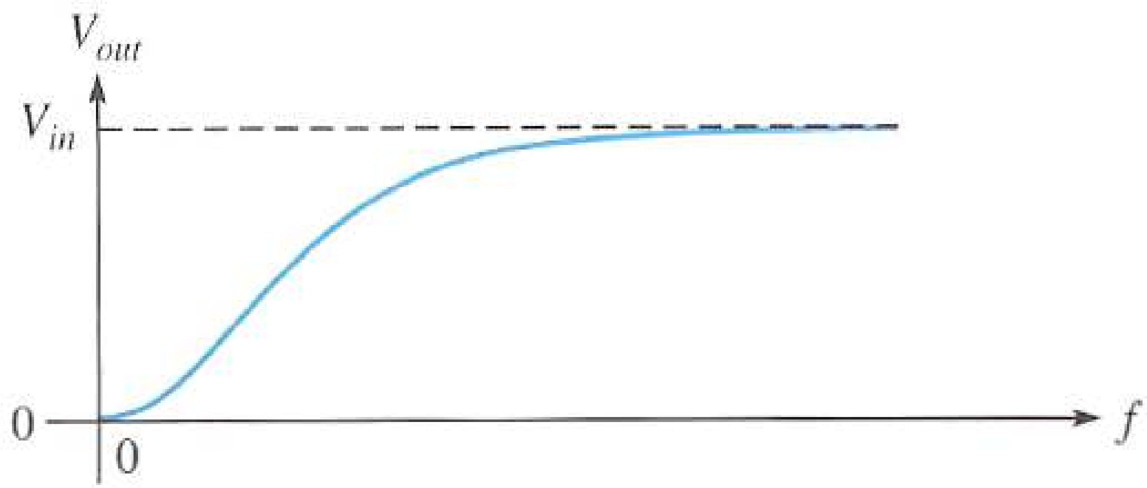
(b)

(c)

# ▪ Circuitos RL – Filtros

## – Filtros passa-alto

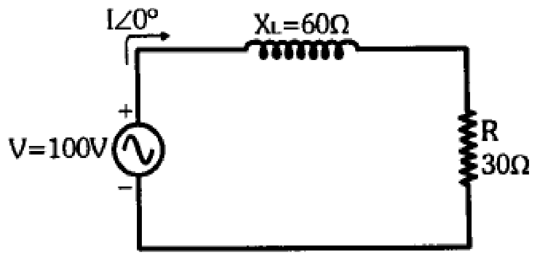


# ▪ Exercícios Propostos

***Circutos RL e RC em Série***

**14.1)** Considere o circuito ao lado.



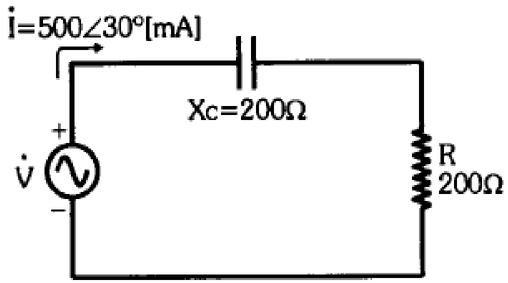

**a)** Determine $\dot{Z}$ , $\dot{V}$ e $\dot{I}$ nas formas polar e retangular;

**b)** Determine $\dot{V}_R$ e $\dot{V}_L$ na forma polar pela Lei de Ohm;

**c)** Esboce o diagrama fasorial com a corrente e as tensões envolvidas no circuito;

**d)** Compare os módulos de $\dot{V}_R$ e $\dot{V}_L$ obtidos no item <u>b</u> com as componentes real e imaginária de $\dot{V}$ obtidas no item <u>a</u> e justifique os resultados.

**14.2)** Considere o circuito ao lado.




**a)** Determine $\dot{Z}$ e $\dot{V}$ nas formas polar e retangular;

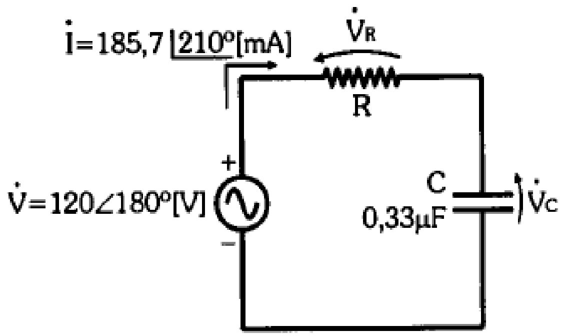
**b)** Determine $\dot{V}_R$ e $\dot{V}_C$ na forma polar pela Lei de Ohm;

**c)** Esboce o diagrama fasorial com a corrente e as tensões envolvidas no circuito;

**d)** Por que os módulos de $\dot{V}_R$ e $\dot{V}_C$ obtidos no item <u>b</u> são diferentes das componentes real e imaginária de $\dot{V}$ obtidas no item <u>a</u>?

# ▪ Exercícios Propostos

**14.3)** Considere o circuito ao lado.




**a)** Determine a defasagem $\varphi$ entre a tensão e a corrente do gerador e esboce o diagrama fasorial com a corrente e as tensões do circuito (sem usar valores);

**b)** Determine $\dot{Z}$, $R$ e $Xc$;

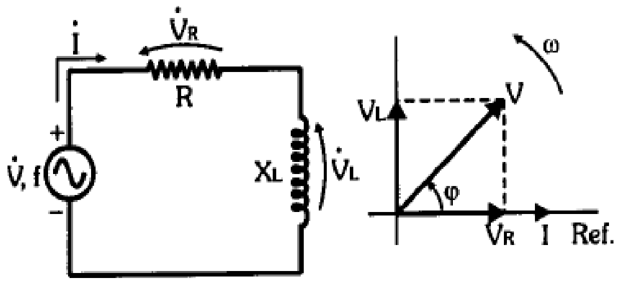
**c)** Determine a freqüência do gerador;

**d)** Determine $\dot{V}_R$ e $\dot{V}_C$;

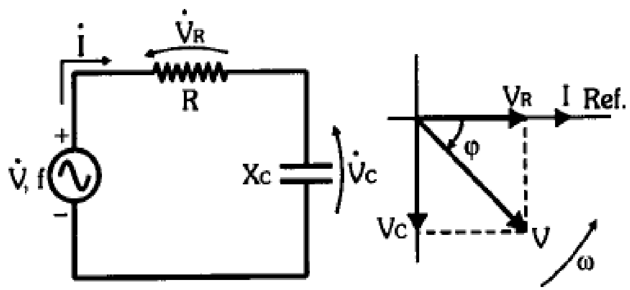
**e)** Compare os resultados obtidos no item d com o diagrama fasorial esboçado no item a.

**14.4)** Considere os circuitos abaixo com os seus respectivos diagramas fasoriais e responda como variarão os fasores se a freqüência do gerador aumentar, mantendo a tensão constante.

Circuito Indutivo:




Circuito Capacitivo:



# Exercícios Propostos

**14.5)** Uma resistência de $600\Omega$ é ligada em série com um indutor de $100mH$. O circuito é alimentado pela tensão $v(t) = 20.\cos 1000\pi t\ [V]$. Determine:

**a)** $\omega$, $f$, $X_L$ e $\dot{Z}$ do circuito;

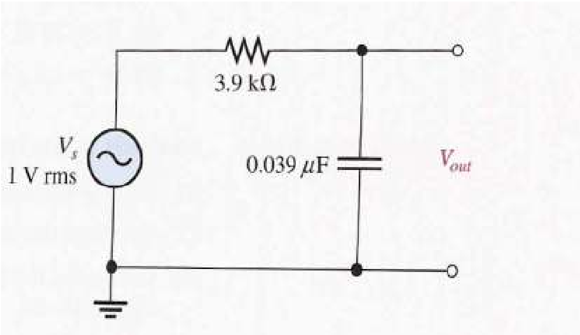
**b)** a corrente $\dot{I}$ e as tensões $\dot{V}_R$ e $\dot{V}_L$;

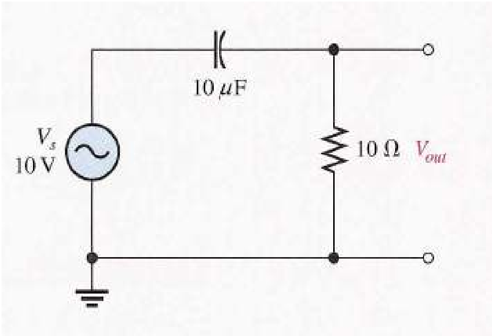
**c)** $\dot{Z}$, $\dot{I}$, $\dot{V}_R$ e $\dot{V}_L$ se a freqüência dobrar e se ela cair pela metade.

**14.6)** Um gerador de $127V / 60Hz$ alimenta uma impedância $\dot{Z} = 60\angle -30^o\ [\Omega]$. Determine, em módulo, a corrente fornecida pelo gerador e a tensão em suas componentes resistiva e reativa.

**35.** The lag circuit in Figure 15–91 also acts as a low-pass filter. Draw a response curve for this circuit by plotting the output voltage versus frequency for 0 Hz to 10 kHz in 1 kHz increments.

**36.** Repeat Problem 34 for the lead circuit in Figure 15–92.

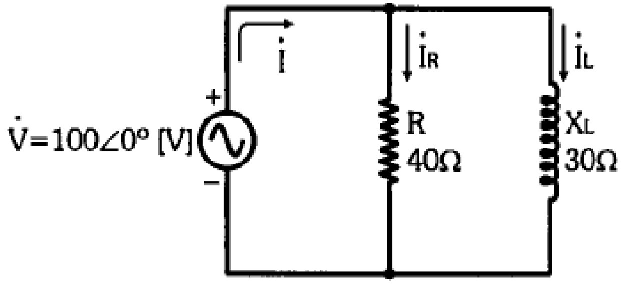


# ▪ Exercícios Propostos

***Circutos RL e RC em Paralelo***

**14.12)** Considere o circuito ao lado.



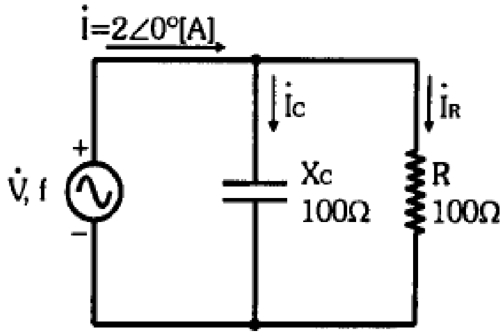

**a)** Determine $\dot{Z}$ e $\dot{I}$ nas formas polar e retangular;

**b)** Determine $\dot{I}_R$ e $\dot{I}_L$ na forma polar pela Lei de Ohm;

**c)** Esboce o diagrama fasorial com a tensão e as correntes envolvidas no circuito;

**d)** Compare os módulos de $\dot{I}_R$ e $\dot{I}_L$ obtidos no item b com as componentes real e imaginária de $\dot{I}$ obtidas no item a e justifique os resultados.

**14.13)** Considere o circuito ao lado.




**a)** Determine $\dot{I}_C$ e $\dot{I}_R$ pela fórmula do divisor de corrente;

**b)** Determine $\dot{V}$ e $\dot{Z}$ ;

**c)** Esboce o diagrama fasorial com a tensão e as correntes envolvidas no circuito.

# ▪ Exercícios Propostos

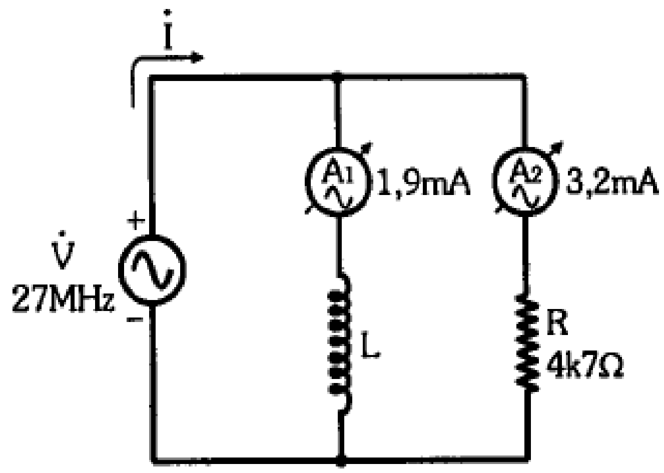
**14.14)** Considere o circuito ao lado com as medidas realizadas pelos amperímetros e determine:

**a)** a corrente I;

**b)** a defasagem $\varphi$;

**c)** a tensão $V$;

**d)** a impedância $Z$, a reatância $X_L$ e a indutância $L$.

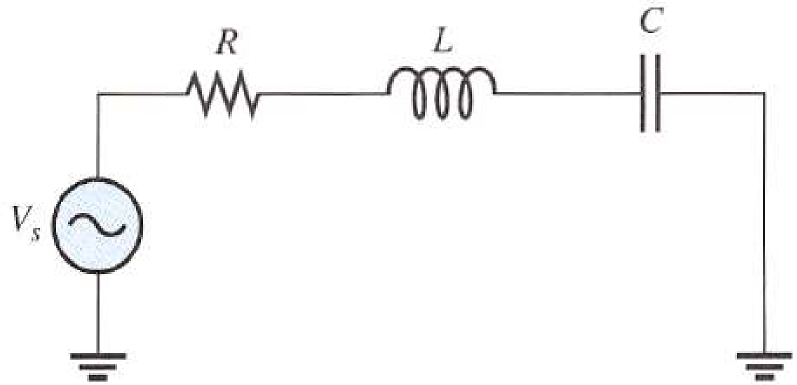



**14.15)** Um gerador $v(t) = 156.\cos(377t - 3\pi/4)$ $[V]$ alimenta uma reatância indutiva de $12\Omega$ em paralelo com um resistor de $22\Omega$. Determine a impedância $\dot{Z}$ do circuito, a defasagem $\varphi$ entre a tensão e a corrente do gerador e a corrente $\dot{I}$ que o gerador fornece ao circuito.

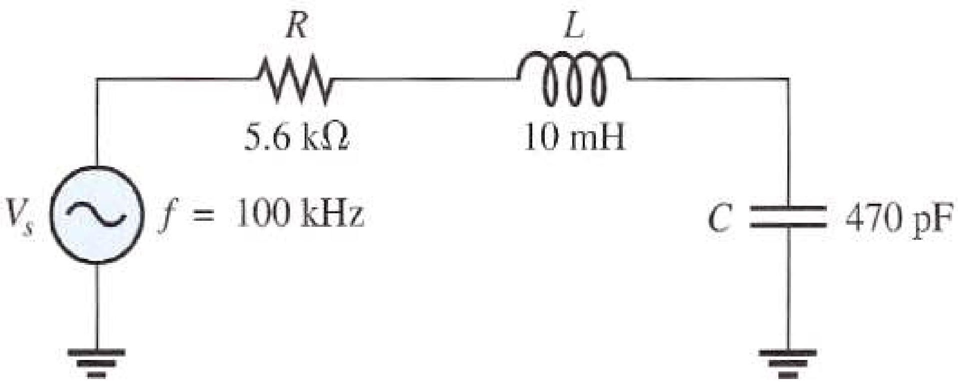
**14.16)** Considere um circuito $RC$ paralelo, em que $C = 120nF$, a freqüência de operação é de $100kHz$ e as correntes valem $I_C = 0{,}5A$ e $I_R = 0{,}8A$. Determine a tensão $V$ do gerador, a resistência $R$, a impedância $Z$ do circuito, a defasagem $\varphi$ entre a tensão e a corrente do gerador e a corrente $I$ que o gerador fornece ao circuito.

# ▪ Circuitos RLC

## Circuito




$$\dot{Z} = R + jX_L - jX_C$$



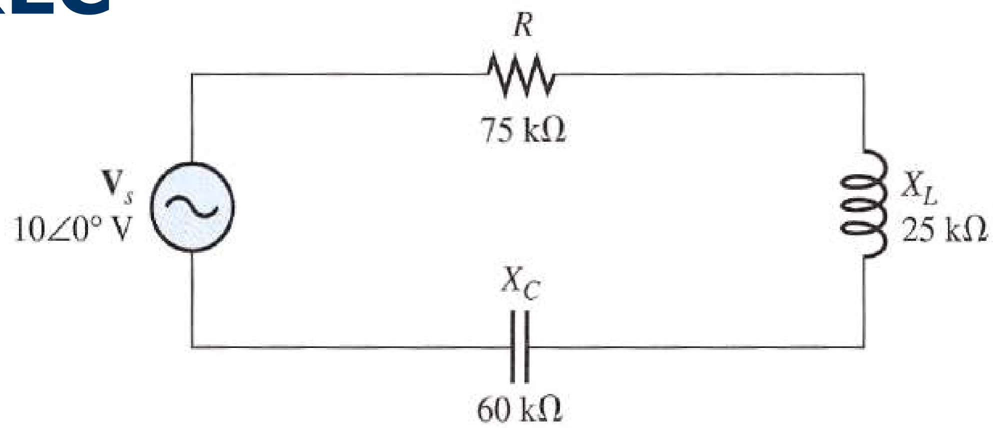

$$X_L = 2\pi . f . L = 2\pi(100\ \text{kHz})(10\ \text{mH}) = 6{,}28\ \text{k}\Omega$$

$$X_C = \frac{1}{2\pi . f . C} = \frac{1}{2\pi(100\ \text{kHz})(470\, pF)} = 3{,}39\ \text{k}\Omega$$

$$\dot{Z} = 5.6\ \text{k}\Omega + j2.89\ \text{k}\Omega$$

# ▪ Circuitos RLC

## — Exercício




$$\dot{Z} = R + jX_L - jX_C = 75\ \text{k}\Omega + j25\ \text{k}\Omega - j60\ \text{k}\Omega = 75\ \text{k}\Omega - j35\ \text{k}\Omega$$

$$\boxed{\dot{I} = \frac{\dot{V}_S}{\dot{Z}} = \frac{10\angle 0^o\ \text{V}}{82.8\angle -25^o\ \text{k}\Omega} = 121\angle 25.0^o\ \ \mu\text{A}}$$

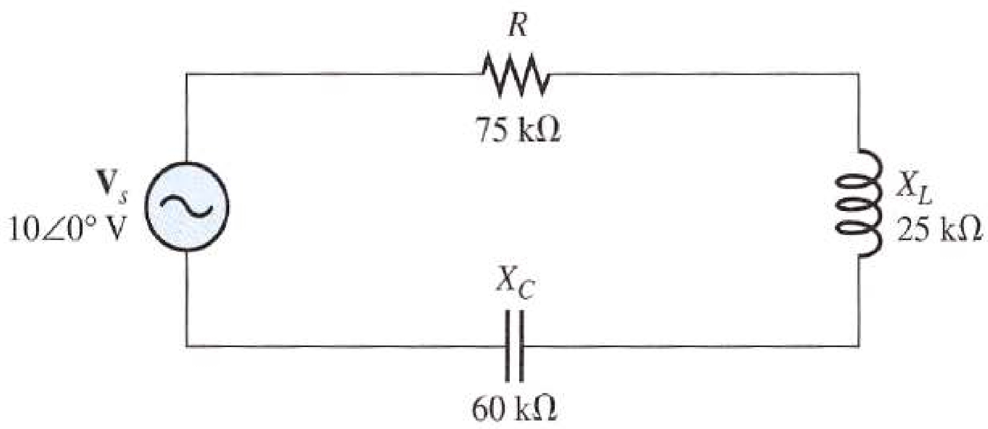
$$\dot{V}_R = \dot{I}.\dot{R} = (121\angle 25^o\ \mu\text{A})(75\angle 0^o\ \text{k}\Omega) = 9.08\angle 25^o\ \text{V}$$

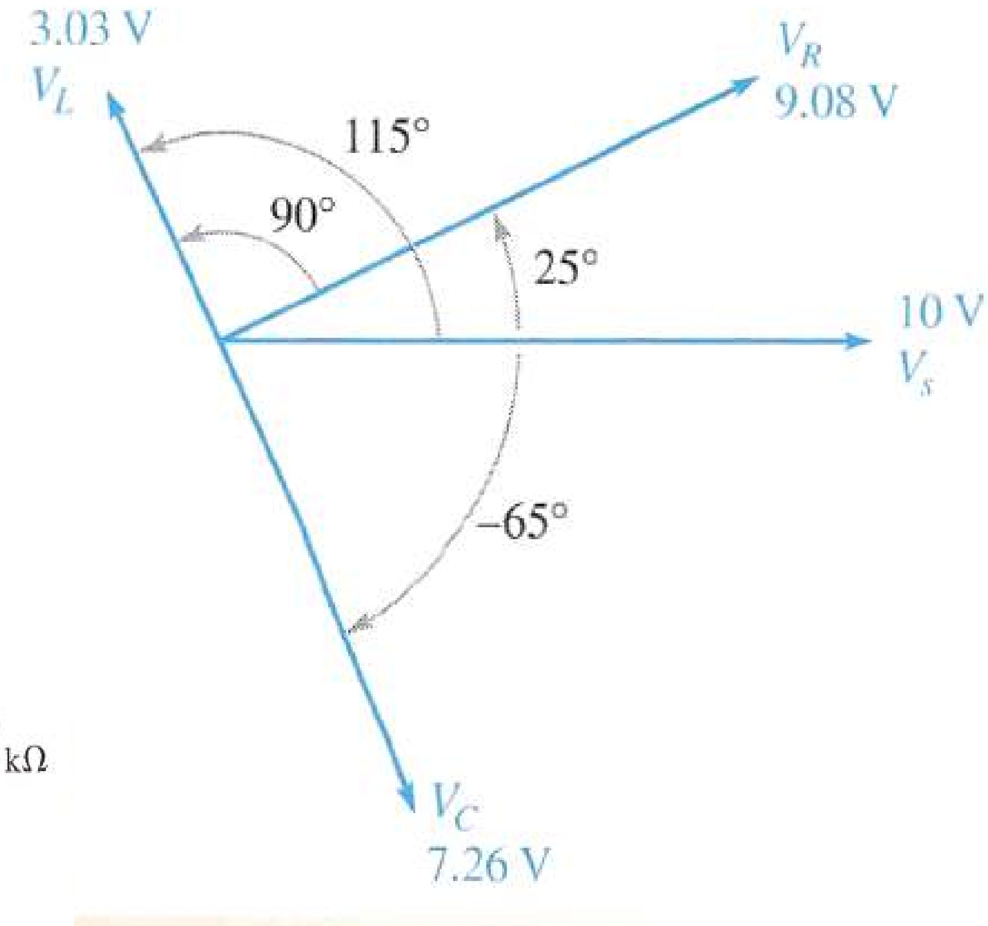
$$\dot{V}_L = \dot{I}.\dot{X}_L = (121\angle 25^o\ \mu\text{A})(25\angle 90^o\ \text{k}\Omega) = 3.03\angle 115^o\ \text{V}$$

$$\dot{V}_C = \dot{I}.\dot{X}_C = (121\angle 25^o\ \mu\text{A})(60\angle -90^o\ \text{k}\Omega) = 7.26\angle -65^o\ \text{V}$$

# ▪ Circuitos RLC

## Exercício



# ▪ Circuitos RLC

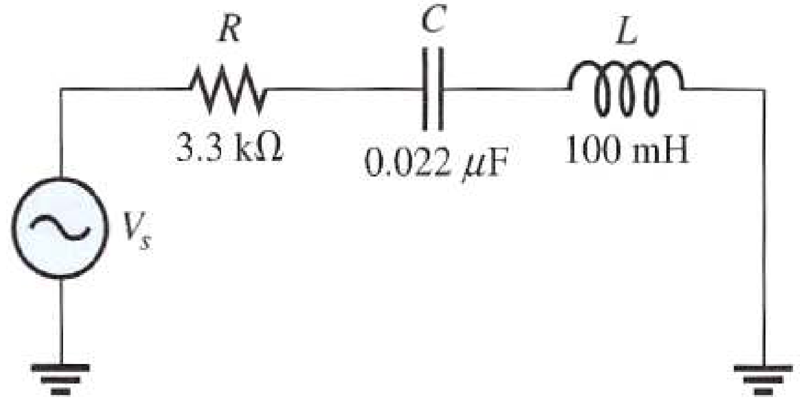
## — Frequência

Para $f = 1$ kHz

$$X_L = 2\pi . f . L = 2\pi(1\,\text{kHz})(100\,\text{mH}) = 628\,\Omega$$

$$X_C = \frac{1}{2\pi . f . C} = \frac{1}{2\pi(1\,\text{kHz})(0.022\mu F)} = 7{,}23\,\text{k}\Omega$$

$$\boxed{7.38\angle -63.4^{\circ}\ \ \text{k}\Omega}$$




Para $f = 2$ kHz

$$X_L = 2\pi . f . L = 2\pi(2\,\text{kHz})(100\,\text{mH}) = 1.26\,\text{k}\Omega$$

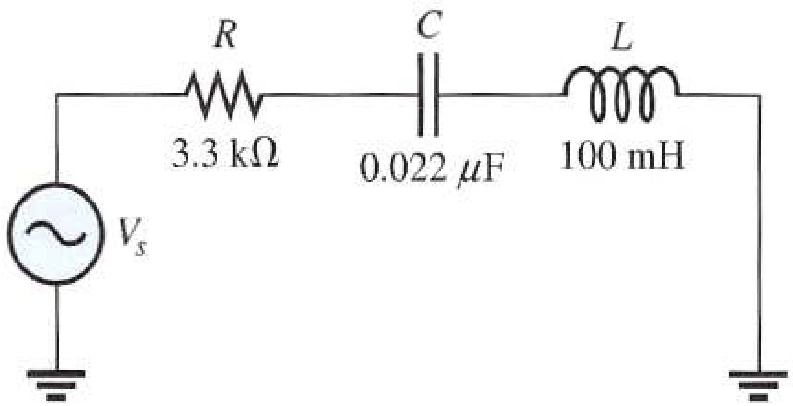
$$X_C = \frac{1}{2\pi . f . C} = \frac{1}{2\pi(2\,\text{kHz})(0.022\mu F)} = 3{,}62\,\text{k}\Omega$$

$$\boxed{4.06\angle -35.6^{\circ}\ \ \text{k}\Omega}$$

The negative sign for the angle is used to indicate that the circuit is capacitive.

# Circuitos RLC

## Frequência




Para $f = 3.5$ kHz

$$X_L = 2\pi.f.L = 2\pi(3.5\,\text{kHz})(100\,\text{mH}) = 2.07\,\text{k}\Omega$$

$$X_C = \frac{1}{2\pi.f.C} = \frac{1}{2\pi(3.5\,\text{kHz})(0.022\mu F)} = 2{,}20\,\text{k}\Omega$$

$$\boxed{3.3\angle 2.26^{\circ}\quad \text{k}\Omega}$$

Para $f = 5$ kHz

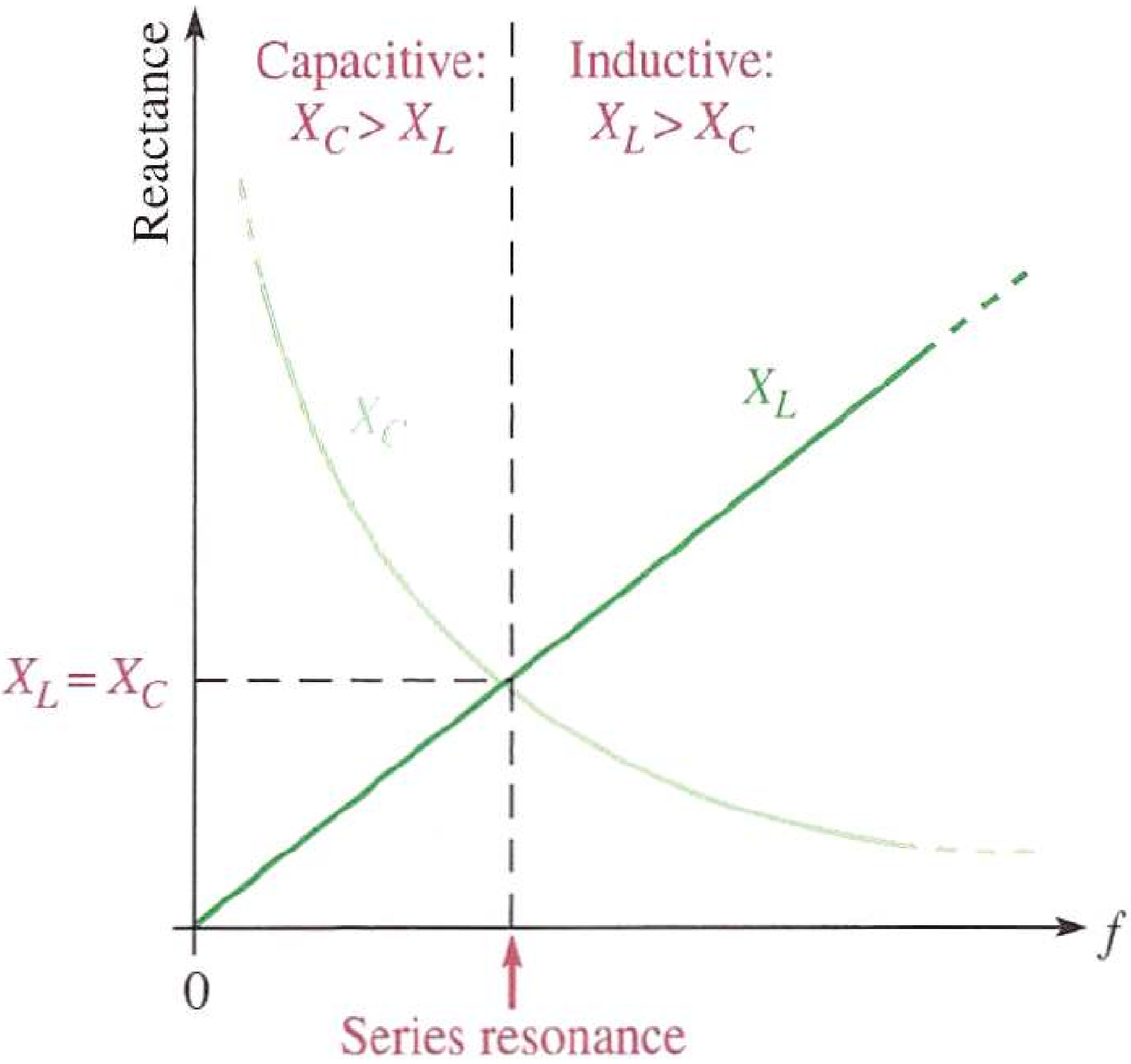
$$X_L = 2\pi.f.L = 2\pi(5\,\text{kHz})(100\,\text{mH}) = 1.45\,\text{k}\Omega$$

$$X_C = \frac{1}{2\pi.f.C} = \frac{1}{2\pi(5\,\text{kHz})(0.022\mu F)} = 3{,}14\,\text{k}\Omega$$

$$\boxed{3.71\angle 27.1^{\circ}\quad \text{k}\Omega}$$

# ▪ Circuitos RLC

## — Frequência



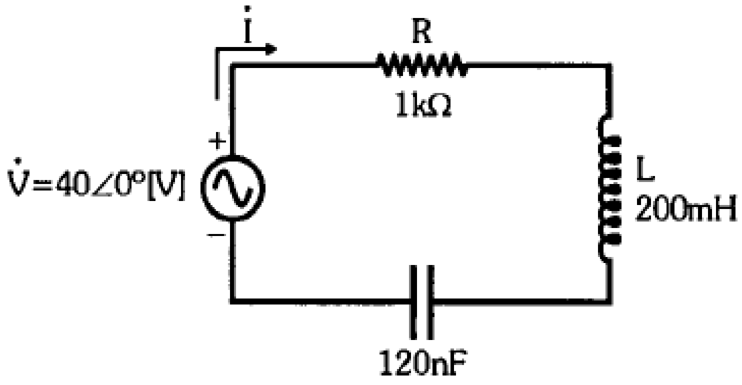

$$X_L = X_C$$

$$\boxed{f_R = \frac{1}{2\pi.\sqrt{LC}}}$$

# Exercícios Propostos

***Circuito RLC Série***

**18.1)** Considere o circuito ao lado:




**a)** Determine $\dot{Z}$ quando o gerador estiver operando na freqüência de *800Hz*;

**b)** Determine $\dot{I}$, $\dot{V}_R$, $\dot{V}_L$ e $\dot{V}_C$;

**c)** Esboce o diagrama fasorial e responda se o circuito é indutivo, capacitivo ou resistivo.

**18.2)** Considere o mesmo circuito do exercício anterior.

**a)** Determine $\dot{Z}$ quando o gerador estiver operando na freqüência de *1200Hz*;

**b)** Determine $\dot{I}$, $\dot{V}_R$, $\dot{V}_L$ e $\dot{V}_C$;

**c)** Esboce o diagrama fasorial e responda se o circuito é indutivo, capacitivo ou resistivo.

**18.3)** Considerando os resultados de $\dot{Z}$ nas freqüências de *800Hz* e *1200Hz* (obtidos nos exercícios 18.1 e 18.2), estime o valor da freqüência de ressonância $f_0$ do circuito.

# ▪ Exercícios Propostos

**18.4)** Um circuito *RLC* série é formado por $R = 220\Omega$, $L = 3{,}3mH$ e $C = 4{,}7nF$ e é alimentado por um gerador de tensão com $\dot{V} = 60\angle 45^{o}\ [V]$. Determine:

**a)** $\dot{Z}$ na freqüência de $30kHz$;

**b)** $\dot{I}$, $\dot{V}_R$, $\dot{V}_L$ e $\dot{V}_C$ na freqüência de $30kHz$;

**c)** $\dot{Z}$ na freqüência de $50kHz$;

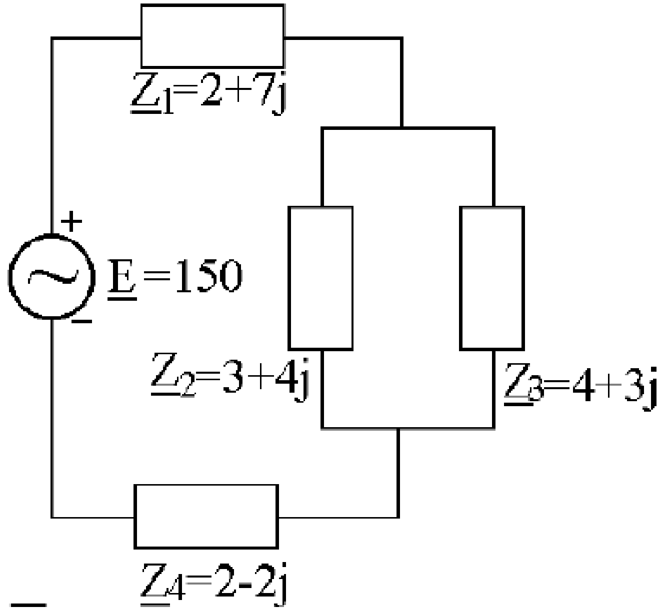
**d)** $\dot{I}$, $\dot{V}_R$, $\dot{V}_L$ e $\dot{V}_C$ na freqüência de $50kHz$;

**e)** $\dot{Z}$ na freqüência de $40{,}4kHz$;

**f)** $\dot{I}$, $\dot{V}_R$, $\dot{V}_L$ e $\dot{V}_C$ na freqüência de $40{,}4kHz$.

# ▪ Método da Impedância Equivalente

Utilizar o Método da Impedância Equivalente para determinar $\underline{I}$ da fonte e $\underline{V}$ em todos os elementos. Verifique a lei de Kirchhoff das malhas.




$$\underline{Z}_{eq} = \underline{Z}_1 + \underline{Z}_4 + (\underline{Z}_2 // \underline{Z}_3) = (2 + j\,7) + (2 - j\,2) + \frac{(3 + j\,4) \times (4 + j\,3)}{(3 + j\,4) + (4 + j\,3)} =$$

$$= 5{,}79 + j\,6{,}79 = 8{,}92 \angle 49{,}5^\circ\ \Omega$$

$$\underline{I} = \frac{\underline{E}}{\underline{Z}_{eq}} = \frac{150}{8{,}92 \angle 49{,}5^\circ} = 16{,}82 \angle -49{,}5^\circ\ \text{A}$$

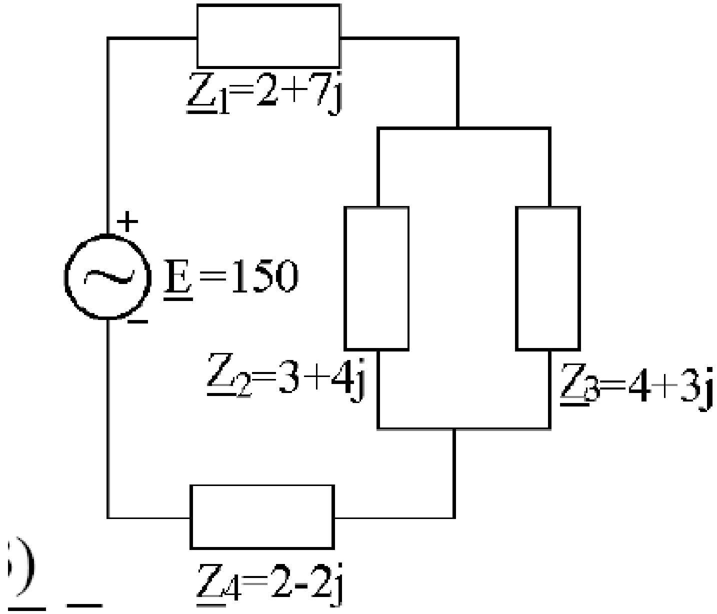
$$\underline{V}_1 = \underline{I}\,\underline{Z}_1 = 16{,}82 \angle -49{,}5^\circ \times (2 + j\,7) = 122{,}45 \angle 24{,}5^\circ\ \text{V}$$

$$\underline{V}_{23} = \underline{I}\,\underline{Z}_{2//3} = 16{,}82 \angle -49{,}5^\circ \times 2{,}53 \angle 45^\circ = 42{,}55 \angle -4{,}5^\circ\ \text{V}$$

$$\underline{V}_4 = \underline{I}\,\underline{Z}_4 = 16{,}82 \angle -49{,}5^\circ \times (2 - j\,2) = 47{,}60 \angle -94{,}5^\circ\ \text{V}$$

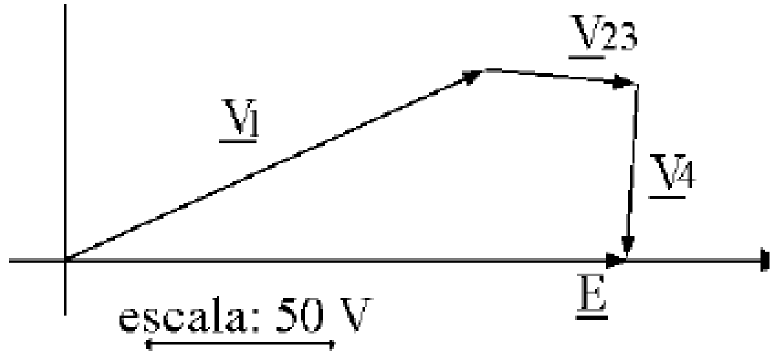
# ▪ Método da Impedância Equivalente

Utilizar o Método da Impedância Equivalente para determinar $\underline{I}$ da fonte e $\underline{V}$ em todos os elementos. Verifique a lei de Kirchhoff das malhas.



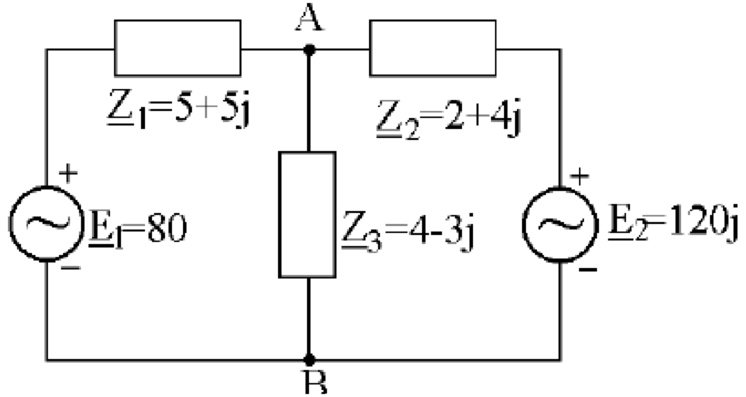

Verificação da lei de Kirchhoff das malhas:

$$\underline{V}_1 + \underline{V}_{23} + \underline{V}_4 = 122{,}45\angle 24{,}5^{\circ} + 42{,}55\angle -4{,}5^{\circ} + 47{,}60\angle -94{,}5^{\circ} =$$
$$=(111{,}39 + j\,50{,}86) + (42{,}42 - j\,3{,}35) + (-3{,}74 - j\,47{,}45) =$$
$$= 150{,}07 + j\,0{,}06 \cong \underline{E}$$ (erros de arredondament

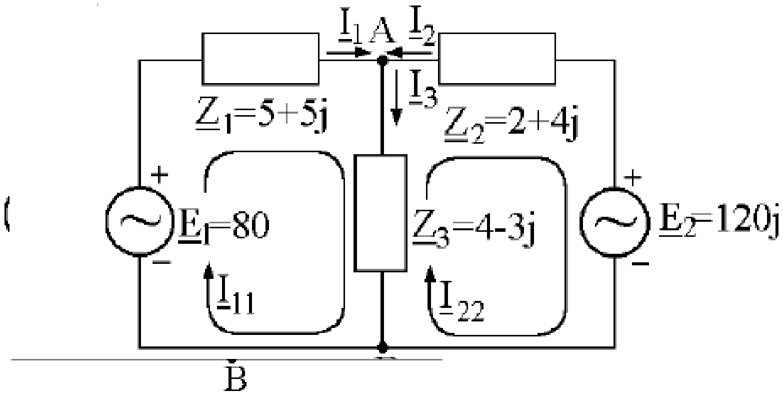


# ▪ Método Malhas Independentes

a) Analisar o circuito.
b) Determinar a $I$ que passaria em $\underline{Z}_4$=(1+$j$ 2) Ω, se fosse ligada entre os pontos **A** e **B**.




a) $\underline{E}_1 = 80\angle 0^o \equiv 80$ e $\underline{E}_2 = 120\angle 90^o \equiv 120\,j$ **R**=3, **N**=2, **M**=2




**Método das Malhas Independentes**

$$\begin{cases} (9+j2)\,\underline{I}_{11} - (4-j3)\,\underline{I}_{22} = 80 \\ -(4-j3)\,\underline{I}_{11} + (6+j)\,\underline{I}_{22} = -j120 \end{cases}$$

$$\Delta = \begin{vmatrix} 9+j2 & -(4-j3) \\ -(4-j3) & 6+j \end{vmatrix} = 54+j\,9+j\,12-2-(16-j\,12-j\,12-9) = 45+j\,45 = 63{,}64\,\angle 45{,}0^o$$

$$\Delta_1 = \begin{vmatrix} 80 & -(4-j3) \\ -j120 & 6+j \end{vmatrix} = 480+j\,80-(j\,480+360) = 120-j\,400 = 417{,}61\,\angle -73{,}3^o$$

$$\Delta_2 = \begin{vmatrix} 9+j2 & 80 \\ -(4-j3) & -j120 \end{vmatrix} = -j\,1080+240-(-320+j\,240) = 560-j\,1320 = 1433{,}88\,\angle -67{,}0^o$$

# ▪ Método Malhas Independentes

$$\underline{I}_{11} = \frac{\Delta_1}{\Delta} = \frac{417{,}61\angle -73{,}3^o}{63{,}64\angle 45{,}0^o} = 6{,}56\ \angle{-118{,}3^o}\ \ \text{A}$$

$$\underline{I}_{22} = \frac{\Delta_2}{\Delta} = \frac{1433{,}88\angle -67{,}0^o}{63{,}64\angle 45{,}0^o} = 22{,}53\ \angle{-112{,}0^o}\ \ \text{A}$$

$$\underline{I}_1 = \underline{I}_{11} = 6{,}56\ \angle{-118{,}3^o}\ \ \text{A}$$

$$\underline{I}_2 = -\underline{I}_{22} = 22{,}53\ \angle(-112{,}0^o + 180^o) = 22{,}53\ \angle 68{,}0^o\ \ \text{A}$$

$$\underline{I}_3 = \underline{I}_{11} - \underline{I}_{22} = 6{,}56\ \angle{-118{,}3^o} - 22{,}53\ \angle 68{,}0^o = 5{,}33 + j\ 15{,}11 = 16{,}02\ \angle 70{,}6^o\ \ \text{A}$$

Verificação a lei de Kirchhoff das malhas:

$$\underline{E}_1 = \underline{Z}_1 \times \underline{I}_1 + \underline{Z}_3 \times \underline{I}_3 =$$
$$= (5 + j\ 5) \times 6{,}56\ \angle{-118{,}3^o} + (4 - j\ 3) \times 16{,}02\ \angle 70{,}6^o =$$
$$= 79{,}97 - j\ 0{,}02 \cong 80\ \ \text{V}$$

$$\underline{E}_2 = \underline{Z}_2 \times \underline{I}_2 + \underline{Z}_3 \times \underline{I}_3 =$$
$$= (2 + j\ 4) \times 22{,}53\ \angle 68{,}0^o + (4 - j\ 3) \times 16{,}02\ \angle 70{,}6^o =$$
$$= 0{,}04 + j\ 119{,}98 \cong j\ \ 120\ \text{V}$$