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1. Definition of Function

1.1. Definition

A function f is a rule that assigns to each element x in a set A exactly one element, called $f(x)$, in a set B.

- $f(x)$ is read as "f of x", or "f at x"
- Sets A and B are sets of real numbers.
- Set A is called the *domain* of the function
- Set B is called the *codomain* of the function
- Range of f is the set of all possible values of $f(x)$ as x varies throughout the domain, that is,

$$range\ of\ f = \{f(x) |\ x \in A\}$$

- Range and codomain are different terms because $range \subseteq codomain$
- The symbol that represents an arbitrary number in the domain of a function f is called an *independent variable*.
- The symbol that represents a number in the range of *f* is called a *dependent variable*.

1.2. Representations of a function

- Although in convention, $f(x)$ is used as notation to denote functions for many of mathematical problems, we are not limited in using that symbol; It can be $g(x)$, $p(k)$, $up(x)$, $fire(w)$, etc.

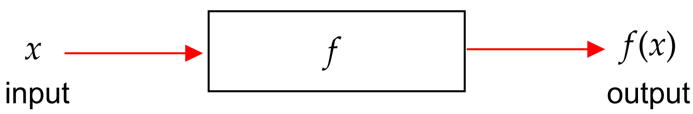


Figure 1.2.1. General Representation of a function

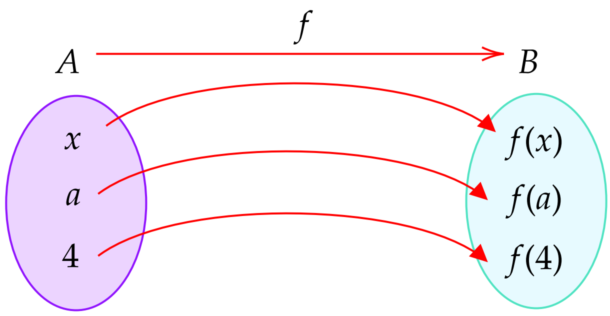


Figure 1.2.2 Mapping Diagram of a function

1.2.1. Differences between $f(x)$ and y

- Although can be used interchangeably of representing same values in a graph, they represent two different ways of thinking about a relationship.

y (the variable): focuses on the *output* value itself. It represents a quantity or coordinate that depends on another. Aka, the name of the result.

$f(x)$ (the function): focuses on the *process* or rule that creates the output. It tells you are applying a specific operation f to an input x.

	Using y (Equation Notation)	Using $f(x)$ (Function Notation)
Naming	Hard to distinguish multiple lines (e.g. $y = 2x$ and $y = 3x$)	Easy to name multiple function (e.g. $f(x)$, $g(x)$, $h(x)$, etc.)
Evaluating	Requires a sentence (e.g. "What is the value of y when $x = 4$?"	Concise notation: $f(4)$
Context	Better for general equations that are not functions (like a circle: $x^2 + y^2 = 25$)	Strictly used for functions (where one input only maps to one output)
Transformations	$y + 2 = x$ (less intuitive for shifts)	$f(x) + 2$ (clearly shows a vertical shifts for whole process)

Table 1.2.1.

As one moves into higher math, one may realize that using $f(x)$ is more powerful for some reasons:

1) Substitution: One can easily show complex inputs. If $f(x) = x^2$, then $f(a + b)$ clearly means

$(a+b)^2$. Using y for this is clunky.

2) Calculus: Notation like $f'(x)$ tells one exactly which function they are differentiating.
3) Dimensionality: In multivariate math, one might encounter $z = f(x, y)$. Here, y is an input, not an output. Function notation prevents confusion by listing the inputs inside the parentheses.

1.3. Determining if it is a function or not

1.3.1. From set of ordered pairs

Applying from Section 1.1., the set of n ordered pairs $\{(x_1, f(x_1)), (x_2, f(x_2)), \ldots, (x_n, f(x_n))\}$ is a function if and only if each distinct input element $x_1, x_2, \ldots, x_n$ results to exactly one corresponding output $f(x_1), f(x_2), \ldots, f(x_n)$.

Exercise 1. Determine if it is a function or not.

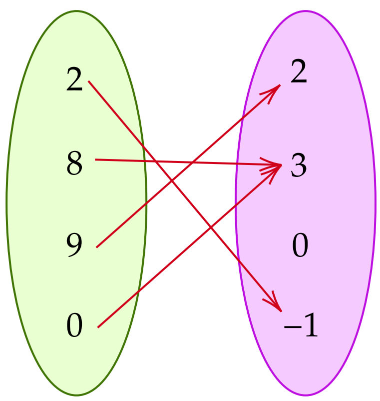
1) $g(x) = \{(a, 1), (b, 2), (c, 4), (c, 3)\}$
2) $h(x) = \{(-4, 2), (0, 2), (1, 2), (3, 2)\}$
3) $j(x) = \{(1, 1), (3, 3), (5, 5), (5.5, 2)\}$
4) $a(x) = \{(b, l), (b, j)\}$
5) $t(x) = \{(0, h), (0, 3), (0, t), (0, y)\}$

1.3.2. From mapping diagram

Applying from Section 1.1., a mapping diagram is a function if and only if each element from input maps to exactly one output.

Exercise 1. Determine if it is a function or not.

Item 1



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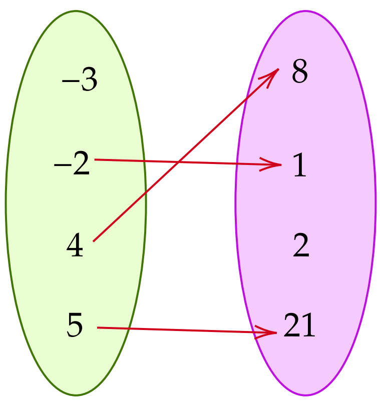

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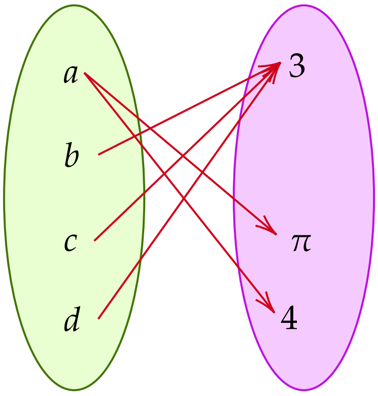

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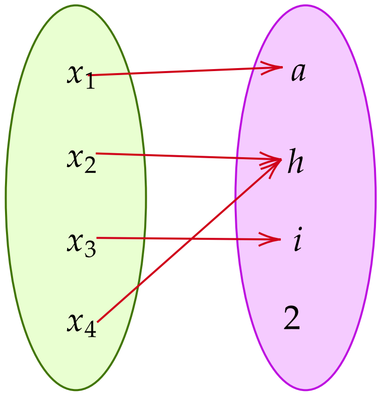

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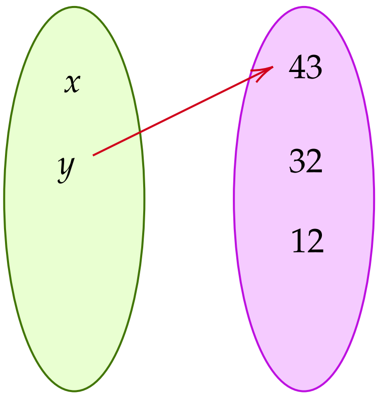


1.3.3. Vertical line test

Vertical line test determines graphically whether a function is one-to-one or not.

Passing vertical line test

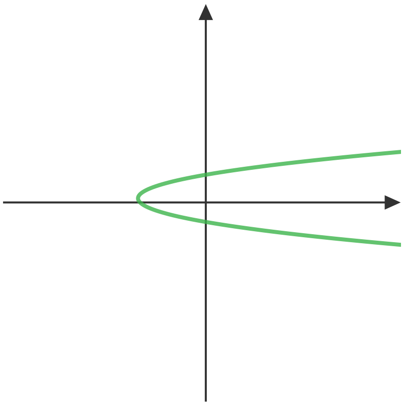
- If a vertical line only makes at most one intersection in any part of a graph, then it passes vertical line test.
- Therefore, that graph is a function.

Failing vertical line test

- If a vertical line can make two or more intersections in <u>any</u> part of a graph defined by a function, then it fails vertical line test.
- Therefore, that graph is <u>not</u> a function.

Exercise 1. Determine visually whether each graph is a function or not.

Item 1



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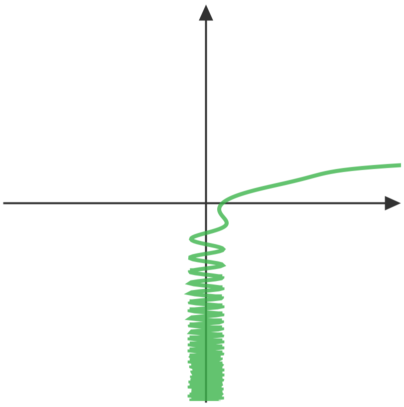

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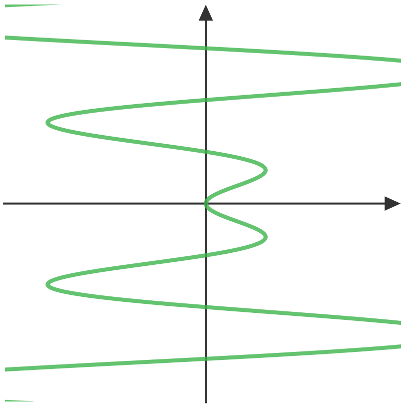

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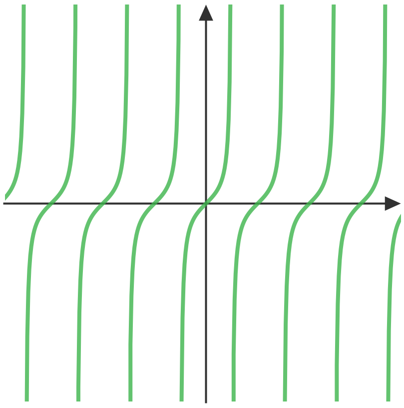

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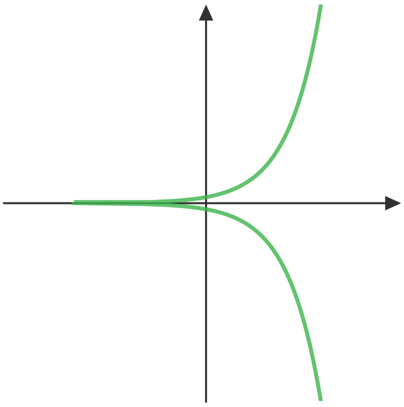

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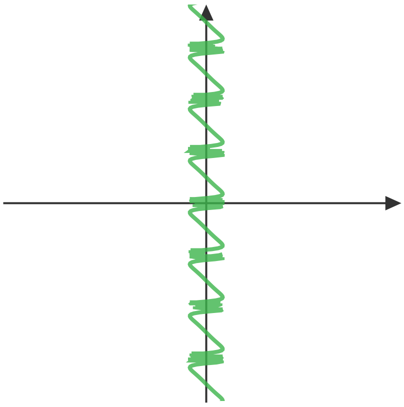

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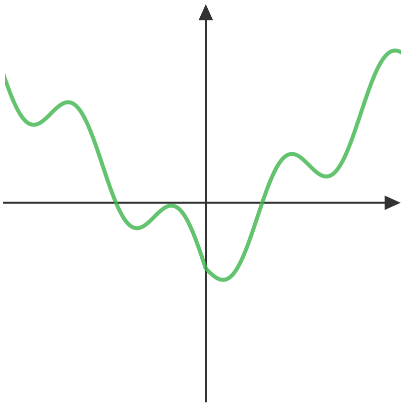

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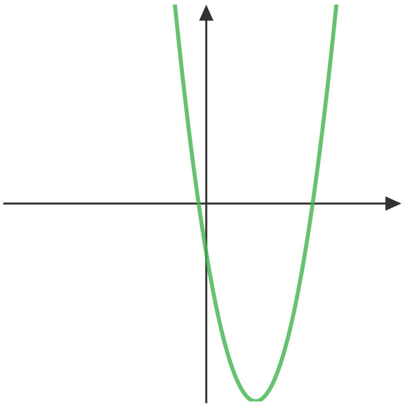

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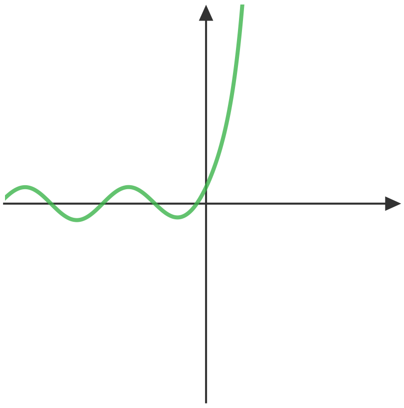

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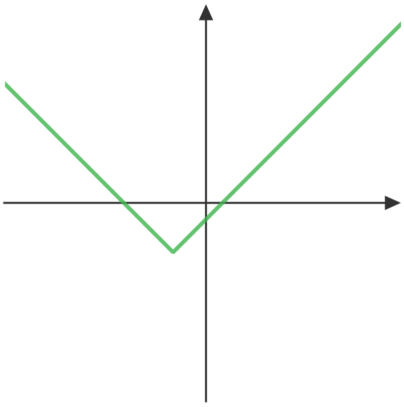

1.3.4. Proving algebraically

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To prove if it is a function algebraically, isolate y in terms of x, and determine if resulting expression is unique. The following are common red flags to say that it is not a function:

1) Even-powers of y (e.g. y^2, y^4, ...)
2) $|y|$
3) $\sin(y)$, $\cos(y)$, $\tan(y)$, $\csc(y)$, $\sec(y)$, and $\cot(y)$

Exercise 1. Determine algebraically if it is a function or not.

1) $x = 3 + y^2$
2) $x = 2x + y - 1$
3) $3 = \dfrac{x + y - 2}{4}$
4) $x = 3sin(y)$
5) $0 = 5x + y^3$
6) $3 = (y + 2)(x + y)$
7) $y = 3|x| + x$
8) $x = y - 2x^4 - 4x^2$
9) $x = \dfrac{3x + 4}{y + 2}$
10) $\sqrt[3]{x} + 3 = y^5$

2. Evaluating a Function

Exercise 2.1. Let $f(x) = 5x^3 - x^2 - 3x + 1$.
Evaluate each function value.

a) $f(2)$
b) $f(-1)$
c) $f(3)$
d) $f(5)$
e)

Exercise 2.2. Consider the piecewise function.

$$h(x) = \begin{cases} x - 3 & if\ -2 \le x < 3 \\ x^3 + x + 1 & if\ x \ge 3 \end{cases}$$

Evaluate each function value.

a) $h(-2)$
b) $h(3)$
c) $h(4)$
d) $h(-5)$

3. Graphing a Function

If f is a function with domain A, then the graph of f is the set of ordered pairs

$$\{(x, f(x) \mid x \in A\}$$

plotted in a coordinate plane. In other words, the graph of f is the set of all points (x, y) such that $y = f(x)$; that is, the graph of f is the graph of the equation $y = f(x)$.

4. Main Types of Functions

4.1. Main types of functions

- There are two main types: one-to-one and many-to-one
- One-to-many functions do not exist.

4.1.1. One-to-one function

Each distinct first element x in set A corresponds to one distinct second element $f(x)$ in set B.

4.1.2. Many-to-one function

Ordered pairs with different first elements correspond to a same second element $f(x)$ in set B.

4.2. Determining if it is a one-to-one function or not

4.2.1. From set of ordered pairs

Applying from Section 4.1.1., the set of n ordered pairs $\{(x_1, f(x_1)), (x_2, f(x_2)),..., (x_n, f(x_n))\}$ is a one-to-one function if and only if each distinct input element $x_1, x_2,..., x_n$ results to exactly one unique corresponding output $f(x_1), f(x_2),..., f(x_n)$.

Exercise 1. Determine if it is a one-to-one function or not.

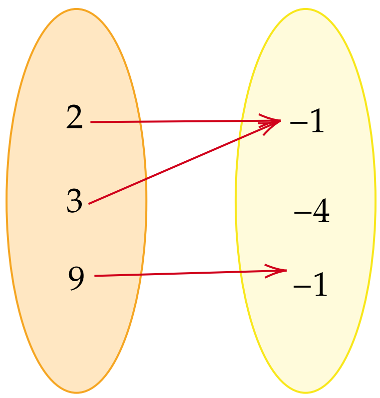
1) $p(x) = \{(a, b), (n, a), (g, h), (j, s), (r, g)\}$
2) $s(x) = \{(4, 5)\}$
3) $t(x) = \{(2, 4), (4, 6), (5, 2), (8, 4)\}$
4) $k(x) = \{(-2, 5), (-1, 3), (5, 3), (6, 3)\}$
5) $f(x) = \{(3, 4), (4, 5), (5, 6), (6, 7), (7, 8)\}$

4.2.2. From mapping diagram

Applying from Section 4.1.1., a mapping diagram is a function if and only if each element from input maps to exactly one output.

Exercise 1. Determine if it is a function or not.

Item 1



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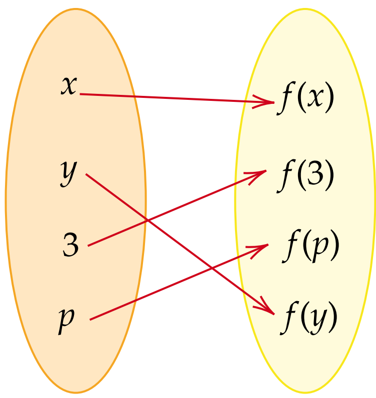

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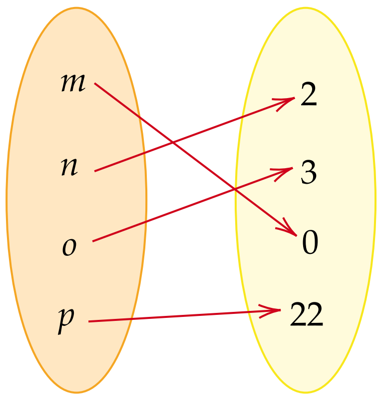

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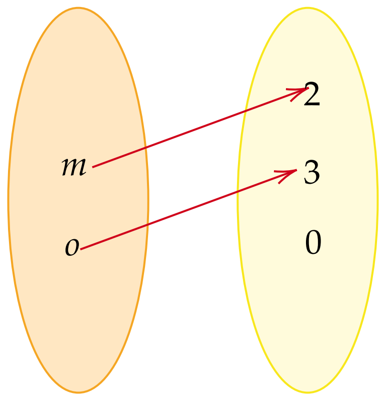

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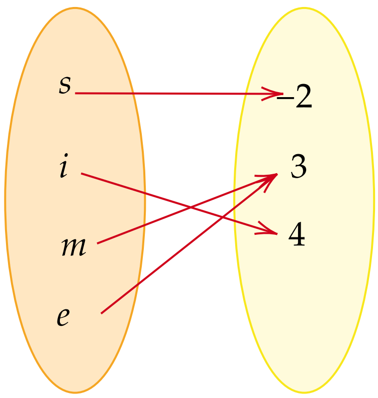


4.2.3. Horizontal line test

Horizontal line test determines graphically whether a function is one-to-one or not.

Passing Horizontal line test

- If a horizontal line only makes at most one intersection in any part of a graph defined by a function, then it passes horizontal line test.
- Therefore, that function is one-to-one.

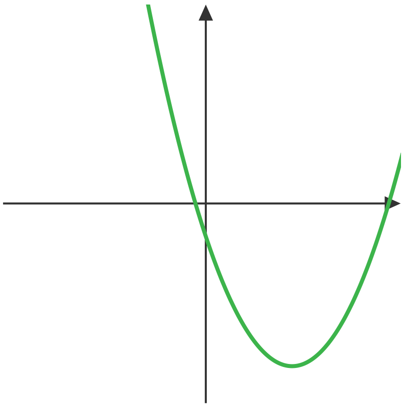
Failing horizontal line test

- If a horizontal line can make two or more intersections in <u>any</u> part of a graph defined by a

function, then it fails horizontal line test.
- Therefore, that function is not one-to-one

Exercise 1. Determine visually whether each graph is a one-to-one function or not.

Item 1



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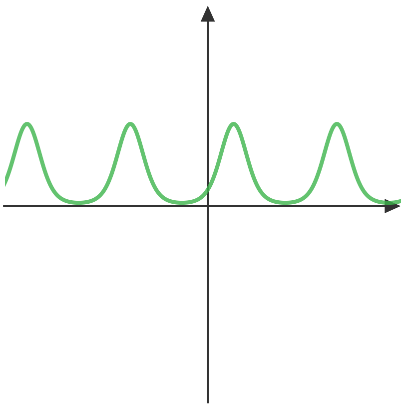

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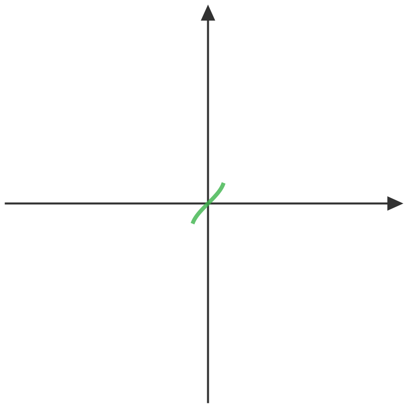

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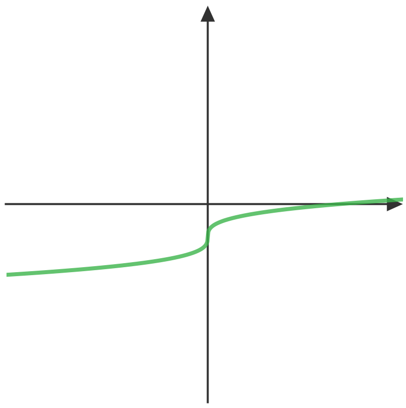

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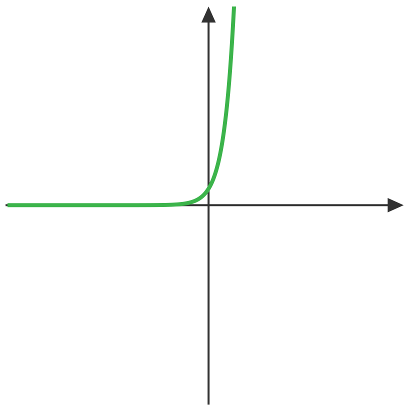

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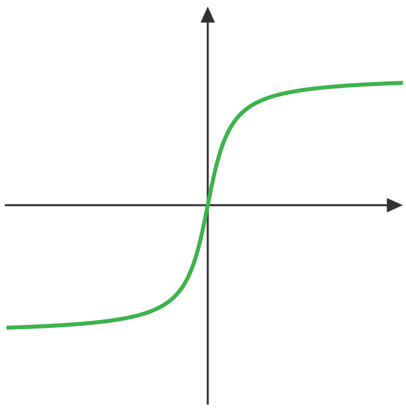

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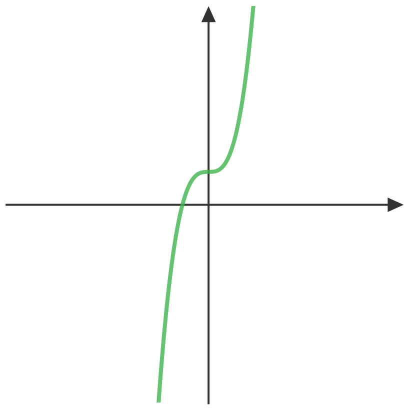

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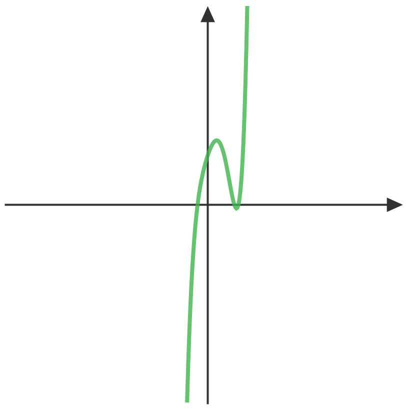

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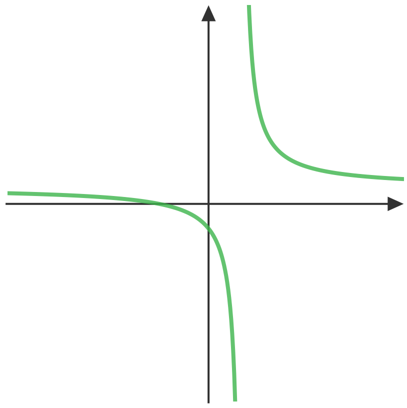

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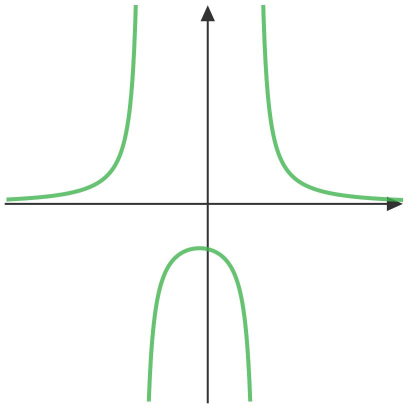

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4.2.4. Proving algebraically

To prove the function f as one-to-one function, the following must uphold:

$$f(x_1) = f(x_2) \text{ whenever } x_1 = x_2$$

Exercise 1.5.2. Determine if the following functions are one-to-one:

1) $f(x) = 4x - 5$
2) $g(x) = 5x^2 - 5$
3) $h(x) = 9x^3$
4) $i(x) = \sqrt[3]{x - 6}$
5) $j(x) = \dfrac{x}{x - 3}$
6) $k(x) = \dfrac{6}{x^2 - 6x + 9}$
7) $l(x) = |x| - 3$
8) $m(x) = 3$
9) $n(x) = 3x^5$
10) $o(x) = \sqrt{x + 3}$

5. Finding Domain and Range of a Function

The *domain* D of a function $y = f(x)$ is the set of all allowed numbers that we can substitute to x.

The *range* R of a function is the set of all possible results (y) when all allowed values of x are substituted.

- Range requires first its domain before determining set of allowable values in range.

5.1. Domain exclusion criteria

In general, the domain of a function excludes <u>all</u> values of x that result in either:

1) division by zero
2) even root of a negative number

5.2. Notations for Domain and Range

- Let D denote domain and R denote range.
- For graph, any values under blue line or shaded dot are considered as part of domain.

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To properly notate the set for domain, use "$D:$",or "$domain:$" succeeded by expression based on the table below

- examples: $D:[a,b]$, $D:\{x \in \Re | a < x \leq b\}$, $D: x < a \; or \; x \geq b$, $domain:[a,b]$

To properly notate the set for range, use "$R:$" or "$range:$" succeeded by expression based on the table below but this time, use $f(x)$ or y instead of x and the graph must be a vertical line to denote y-coordinate.

- examples: $R:(a,b)$, $R:\{f(x) \in \Re | a < f(x) < b\}$. $y:\Re$, $range:(a,b]$

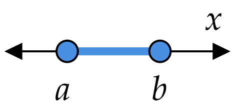
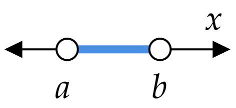
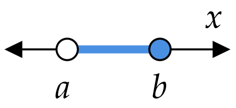
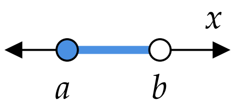
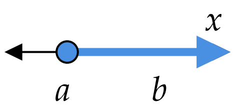
Graph	Inequality	Set Builder	Interval
	$a \leq x \leq b$	$\{x \in \Re \| a \leq x \leq b\}$	$[a,b]$
	$a < x < b$	$\{x \in \Re \| a < x < b\}$	(a,b)
	$a < x \leq b$	$\{x \in \Re \| a < x \leq b\}$	$(a,b]$
	$a \leq x < b$	$\{x \in \Re \| a \leq x < b\}$	$[a,b)$

Table 3.2.1. Closed intervals

Graph	Inequality	Set Builder	Interval
	$x \geq a$	$\{x \in \Re \| x \geq a\}$	$[a,\infty)$

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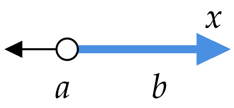
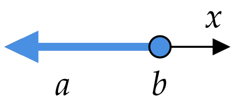
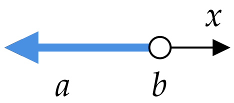
	$x > a$	$\{x \in \Re \vert x > a\}$	(a, ∞)
	$x \leq b$	$\{x \in \Re \vert x \leq b\}$	$(-\infty, b]$
	$x < b$	$\{x \in \Re \vert x < b\}$	$(-\infty, b)$

Table 3.2.2. One-sided infinite interval

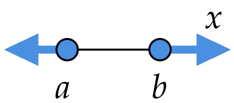
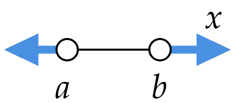
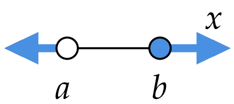
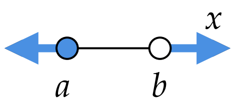
Graph	Inequality	Set Builder	Interval
	$x \leq a \; or \; x \geq b$	$\{x \in \Re \vert x \leq a \; or \; x \geq b\}$	$(-\infty, a] \cup [b. \infty)$
	$x < a \; or \; x > b$	$\{x \in \Re \vert x < a \; or \; x > b\}$	$(-\infty, a) \cup (b. \infty)$
	$x < a \; or \; x \geq b$	$\{x \in \Re \vert x < a \; or \; x \geq b\}$	$(-\infty, a) \cup [b. \infty)$
	$x \leq a \; or \; x > b$	$\{x \in \Re \vert x \leq a \; or \; x > b\}$	$(-\infty, a] \cup (b. \infty)$

Table 3.2.3. Two-sided infinite interval

Graph	Inequality	Set Builder	Interval

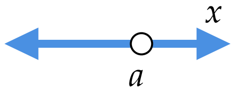
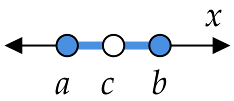
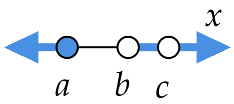
	$x \neq a$	$\{x \in \Re \mid x \neq \{a\}\}$ or $\{x \in \Re \setminus \{a\}\}$	$(-\infty, a) \cup (a. \infty)$
	$(x \geq a \; and \; x < c)$ $or \; (x > c \; and \leq b)$	$\{x \in \Re \mid (x \geq a \; and$ $x < c) \; or$ $(x > c \; and \; x \leq b)\}$	$[a, c) \cup (c. b]$
	$(x \leq a) \; or \; (x > b \; and$ $x < c) \; or \; (x > c)$	$\{x \in \Re \mid (x \leq a) \; or$ $(x > b \; and \; x < c)$ $or \; (x > c)\}$	$(-\infty, a] \cup (b, c)$ $\cup (c. \infty)$
	$\Re$	$\{x \in \Re\}$	$(-\infty, \infty)$

Table 3.2.4. Other cases

Exercise 3.2. Find the domain and range of each function. Express both domain and range in inequality, set builder, and interval notation

1) $h(t) = \dfrac{t+2}{t-1}$
2. $d(a) = \sqrt{a-2} + 4$
3. $k(p) = p^2 + 4p - 2$
4. $f(c) = \dfrac{c}{c^2 - 9}$

6. X-intercept and Y-intercept

x-intercepts are points where the graph of $f(x)$ touches the horizontal x-axis in the Cartesian plane.

- It is obtained by setting all values of y to 0 then solve for x.

y-intercepts are points where the graph of $f(x)$ touches the vertical y-axis in the Cartesian plane.

- It is obtained by setting all values of x to 0 then solve for y.

Exercise 4.1. Find the x- and y- intercepts of $f(x) = x^2 + 7x^2 + 12$.

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Exercise 4.2. If the x-intercepts of $i(x)$ are $(4, 0)$, $(3, 0)$, $(-2, 0)$ and $(1, 0)$, what is the original equation of a function in expanded form?

7. Operations on Functions

Let f and g be functions with their domains A and B respectively. The following are operations that functions can uphold:

$$(f+g)(x) = f(x) + g(x), D: A \cap B \tag{1}$$

$$(f-g)(x) = f(x) - g(x), D: A \cap B \tag{2}$$

$$(f \cdot g)(x) = f(x) \cdot g(x), D: A \cap B \tag{3}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)};\ where\ g(x) \neq 0, D: \{x \in A \cap B | g(x) \neq 0\} \tag{4}$$

Exercise 5.1. Let $f(x) = x^2 - 9$ and $g(x) = x + 3$. Find the following:

a) $(f+g)(x)$
b) $(f-g)(x)$
c) $(f \cdot g)(x)$
d) $\left(\frac{f}{g}\right)(x)$

7.1. Composition of functions

- Let f and g be functions with their domains A and B respectively.
- The *composite function* $f \circ g$, read as "f composed with g", is defined as:

$$(f \circ g)(x) = f(g(x)),\ D: \{x \in B |\ g(x) \in (x \in A)\}$$

- In this composite function, every instance of x in $f(x)$ will be replaced with the expression $g(x)$.
- $(f \circ g)(x)$ is defined whenever both $f(x)$ and $g(x)$ are defined.

Exercise 3.1. Let $f(x) = 2x - 8$ and $g(x) = x^3 - 1$. Find the following

a) $(f \circ g)(x)$

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b) $(g \circ f)(x)$
c) $(f \circ f)(x)$
d) $(g \circ g)(x)$

Add Exercise Variation

8. Symmetry of Functions

8.1. Even function

A function is symmetric with respect to the y-axis if $f(-x) = f(x)$ for all x in the domain of f. A function of this type is called an *even function.*

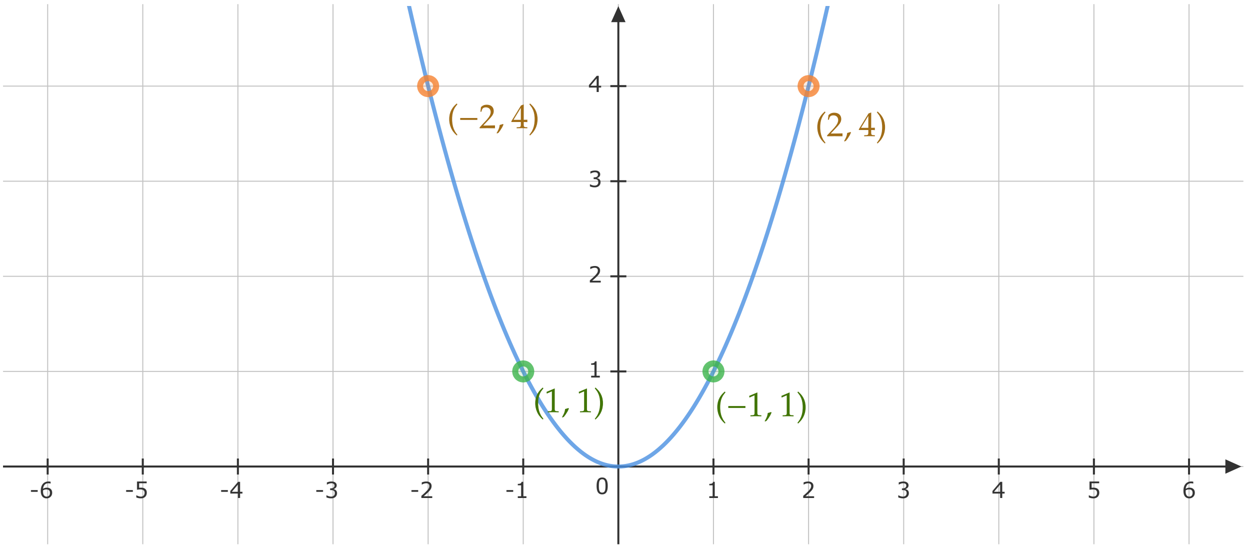

Figure 6.1.1

Exercise 6.1.1. Determine if the following functions are even.

a) $g(x) = -2x^2 + 3$
b) $j(r) = \dfrac{1}{2r}$
c) $y = -2\sqrt{1-x}$
d) $y = x^4 + 1$

8.2. Odd function

A function is symmetric with respect to the origin if $f(-x) = -f(x)$ for all x in the domain of f. A function of this type is called an

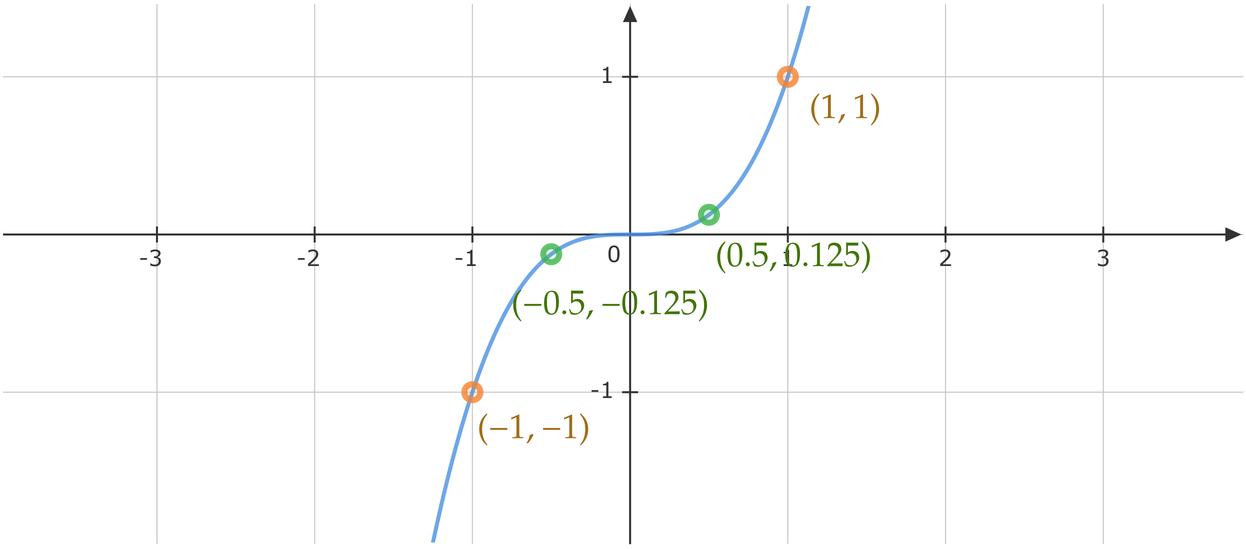

Figure 6.2.1

Exercise 6.2.1. Determine if the following functions are odd.

a) $g(x) = -2x^2 + 3$

b) $j(r) = \dfrac{1}{2r}$

c) $y = -2\sqrt{1-x}$

d) $y = x^4 + 1$

9. Inverse of a Function

9.1. Definition

Let f be a one-to-one function with domain A and range B. Then its inverse function f^{-1} has domain B and range A defined by:

$$f^{-1}(y) = x \Leftrightarrow f(x) = y, \ for\ any\ y\ in\ B$$

- Not all functions have their inverses.
- Only all one-to-one functions can have their inverses.
- Since x and y are swapped, domain and range are also swapped.

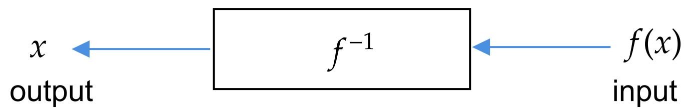

Figure 7.2.1. General Representation of an inverse function

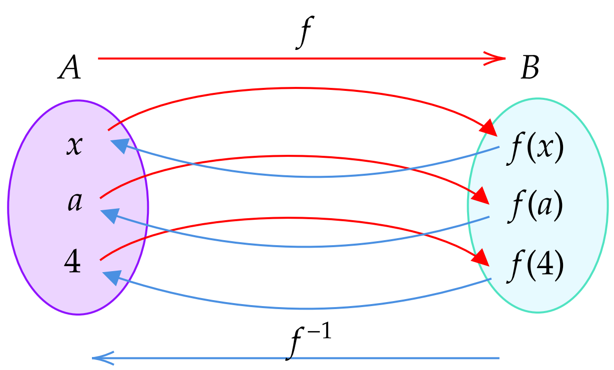

Figure 7.2.2. Comparison Between function and inverse function

9.2. Solving for inverse of a function

1) Replace $f(x)$ with y.
2) Swap all x with y and vice versa.
3) Solve by isolating y on one side of equation.
4) Replace y with $f^{-1}(x)$.

Exercise 7.2.1 Find the inverse of the following functions:

a) $o(x) = \dfrac{x-2}{x+3}$

b) $f(x) = 3x^3 - 2$

9.3. Inverse function property

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Let f be a one-to-one function with domain A and range B. The inverse function f^{-1} satisfies the cancellation properties:

$$f^{-1}(f(x)) = x, \; for\ every\ x\ in\ A$$
$$f\left(f^{-1}(x)\right) = x, \; for\ every\ x\ in\ B$$

Conversely, any function f^{-1} satisfying these equations is the inverse of f.

Exercise 7.3.1. If $f^{-1}(x) = 3x + 15$, what is the value of $f(2)$?

Exercise 7.3.2. If $f^{-1}(x) = x^3 + 2x - 1$, what is f(x)?

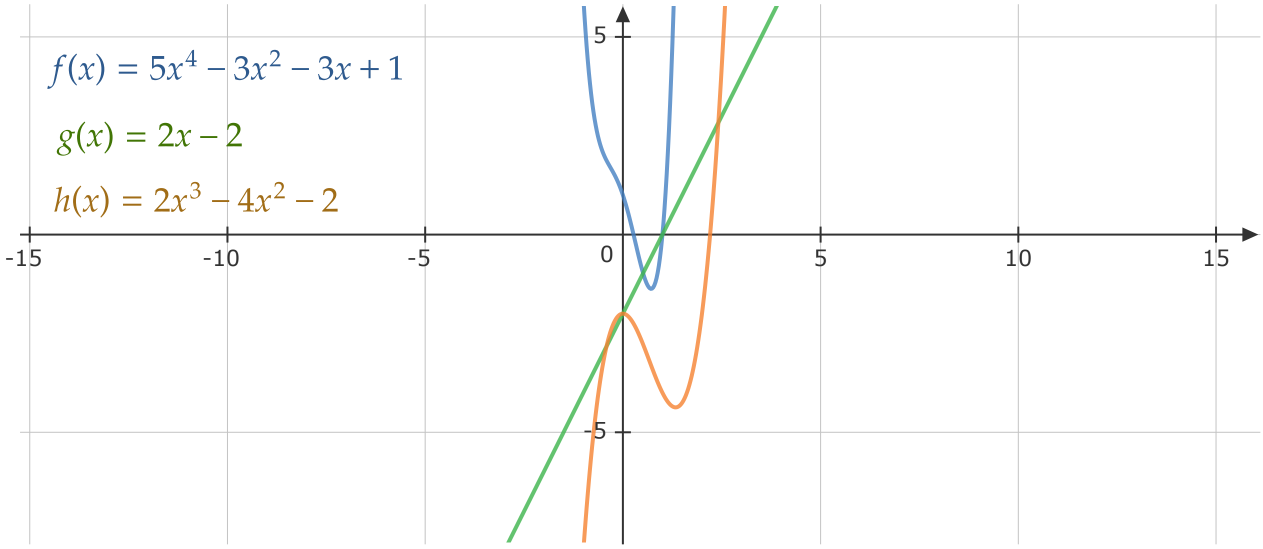
Add Exercise Variation

10. Types of Functions

10.1. Polynomial function

Functions defined by polynomial expressions are called *polynomial functions*. A polynomial function P of a degree n is defined by:

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0, \; a_n \neq 0$$



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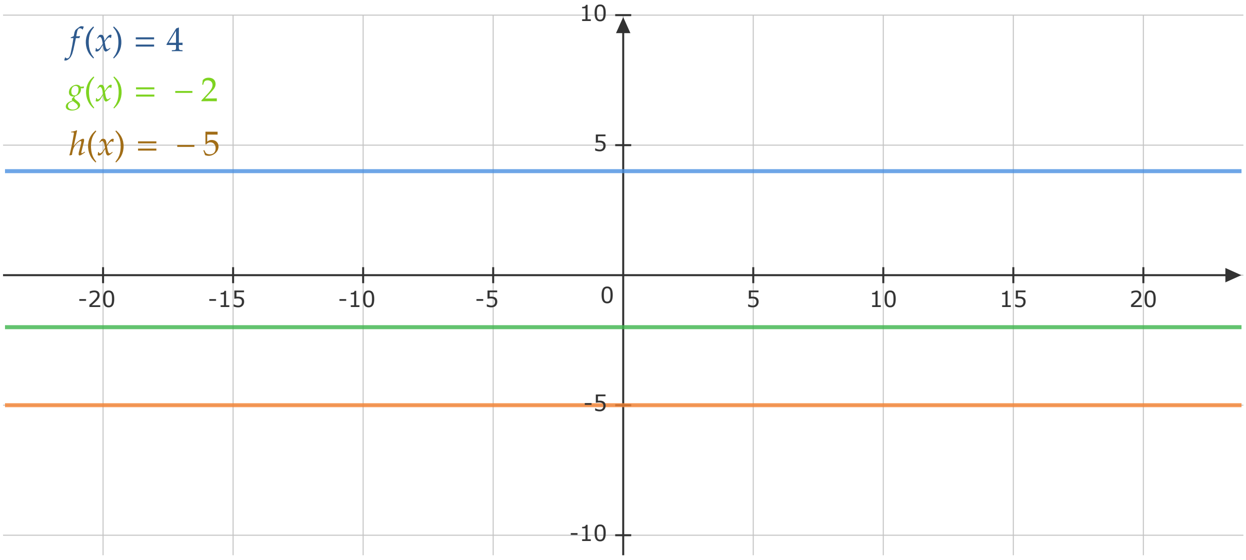
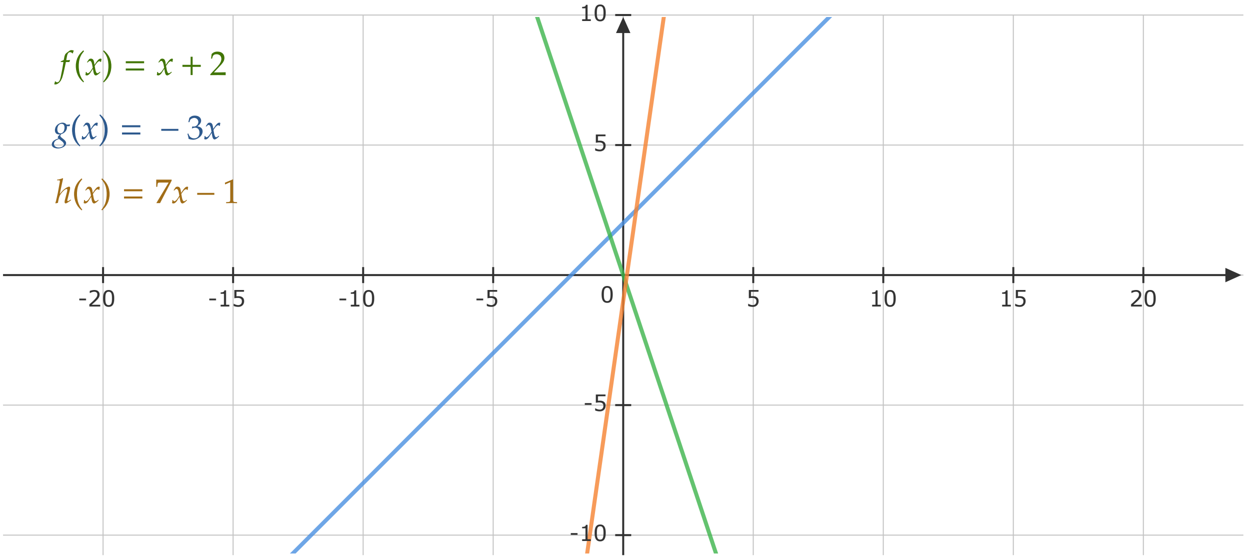
Domain	**Range**
Set of all real numbers $D: \{x \mid x \in \Re\}$	If n is odd: set of all real numbers. $R: \{f(x) \mid f(x) \in \Re\}$ If n is even: set of numbers above the minima/below the maxima.

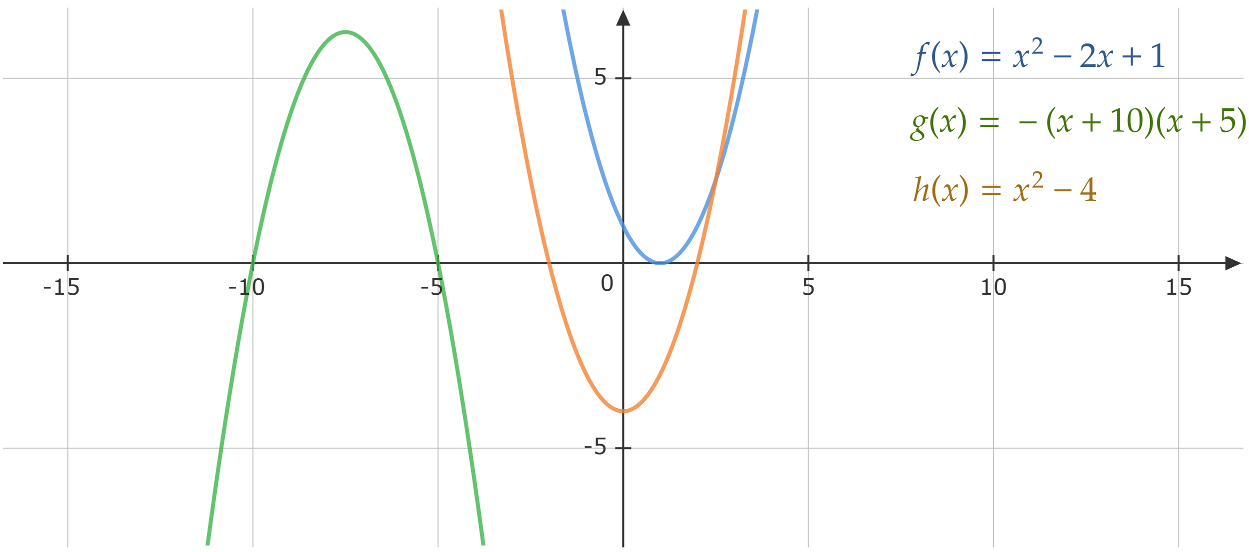
10.1.1. Constant function



10.1.2. Linear function



10.1.3. Quadratic function

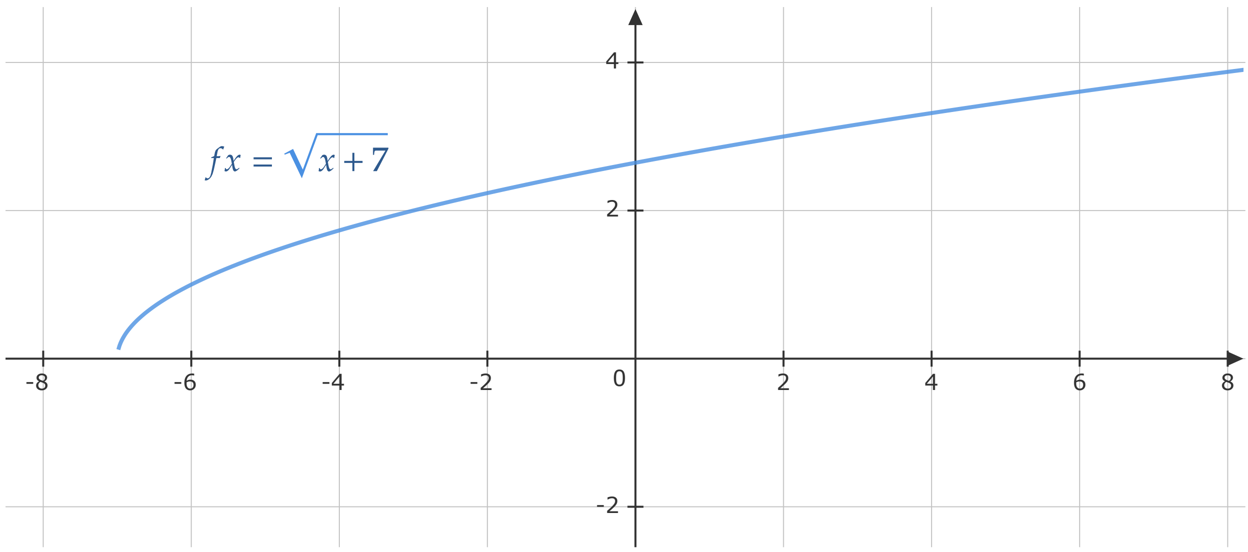


10.2. Radical function

Radical functions are of the form

$$f(x) = \sqrt[n]{P(x)}$$

where $P(x)$ is a polynomial function with $degree \geq 1$ and n as nth-root.



Domain	Range
If n is odd: set of all real numbers $D:\{x \mid x \in \Re\}$ If n is even: set of all values of x that makes $P(x) \geq 0$ $R:\{x \mid P(x) \geq 0\}$	If n is odd: set of all real numbers. $R:\{f(x) \mid f(x) \in \Re\}$ If n is even: all real numbers greater than or equal to 0 $R:\{f(x) \mid f(x) \geq 0\}$

10.3. Rational function

Rational functions are of the form

$$f(x) = \frac{P(x)}{Q(x)}$$

where P and Q are both polynomial functions and $Q(x) \neq 0$.

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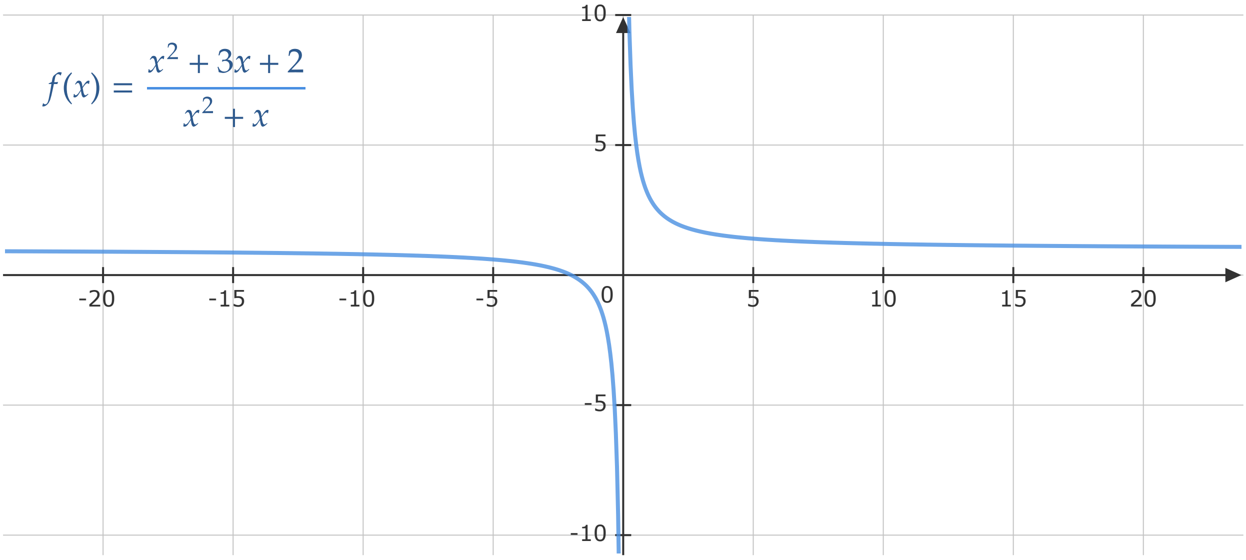

Domain	Range
Set of all real numbers except values of x that makes denominator equal to 0. $D: \{x \mid x \in \Re \; and \; Q(x) \neq 0\}$	Set of all number except those resulting to horizontal asymptote/s. $R: \{f(x) \in \Re \mid f(x) \neq value/s \; of \; horizontal \; asymptote/s\}$

10.4. Absolute value function

Absolute value functions are of the form

$$f(x) = |x|$$

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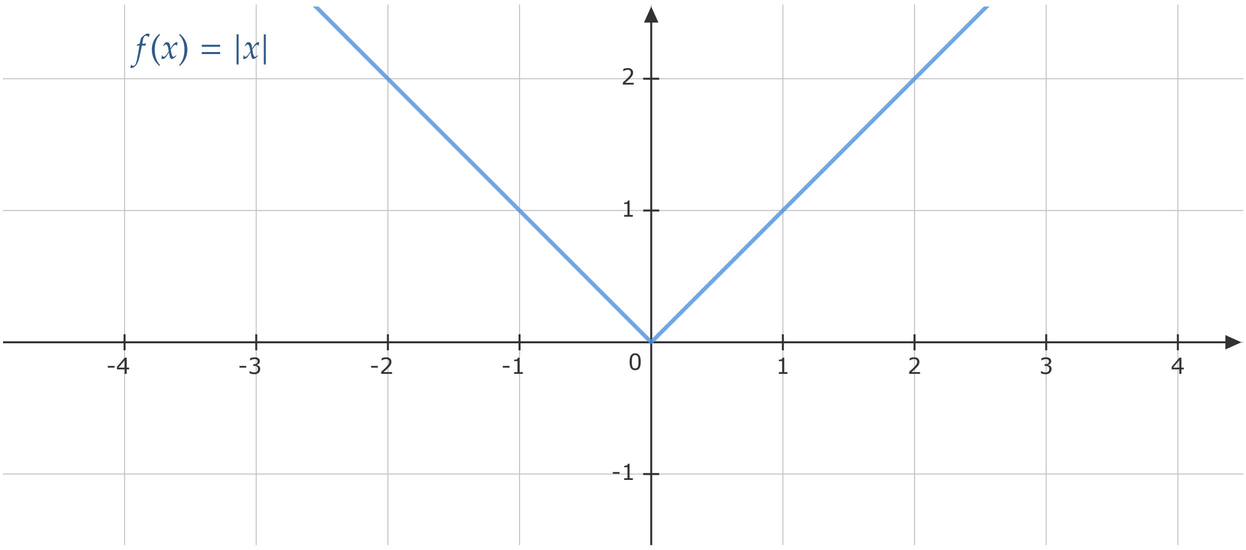

Domain	**Range**
Set of all real numbers $D:\{x \mid x \in \mathfrak{R}\}$	Set of all real numbers greater than or equal to 0. $D:\{f(x) \in \mathfrak{R} \mid f(x) \geq 0\}$

10.5. Floor function

Floor functions are of the form

$$f(x) = \lfloor x \rfloor$$

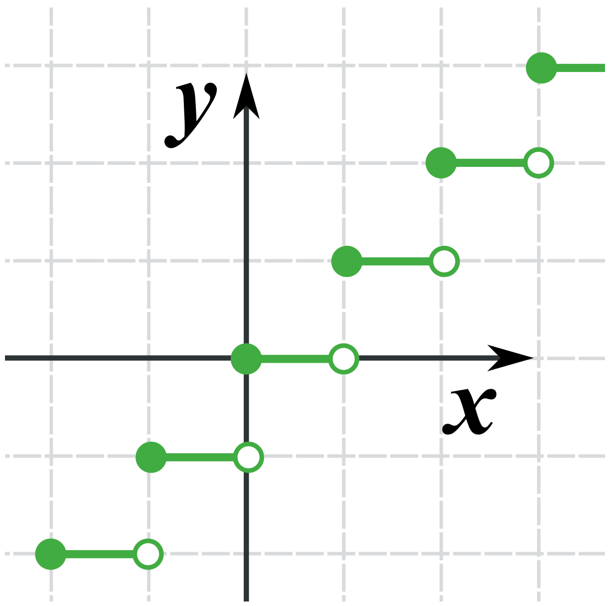

Figure 1: https://www.mathsisfun.com/sets/function-floor-ceiling.html

Domain	Range
Set of all real numbers $D: \{x \mid x \in \Re\}$	Set of all integers $D: \{f(x) \in \mathbb{Z}\}$

10.6. Ceiling function

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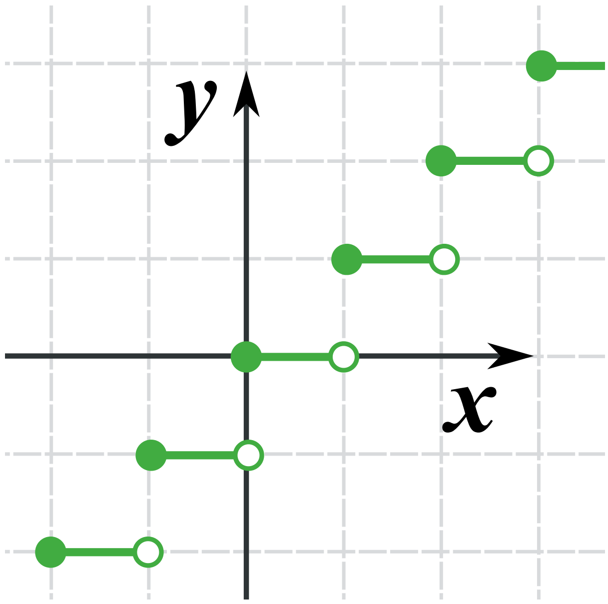

Figure 2: https://www.mathsisfun.com/sets/function-floor-ceiling.html

Domain	Range
Set of all real numbers $D: \{x \vert x \in \Re\}$	Set of all integers $D: \{f(x) \in \mathbb{Z}\}$

10.7. Piecewise function

Piecewise functions are of the form

$$f(x) = \begin{cases} f_1(x) & , x < a_1 \\ f_2(x) & , a_1 \leq x < a_2 \\ \vdots & \vdots \\ f_n(x) & , x \geq a_k \end{cases}$$

where n is number of pieces, k is number of boundaries.

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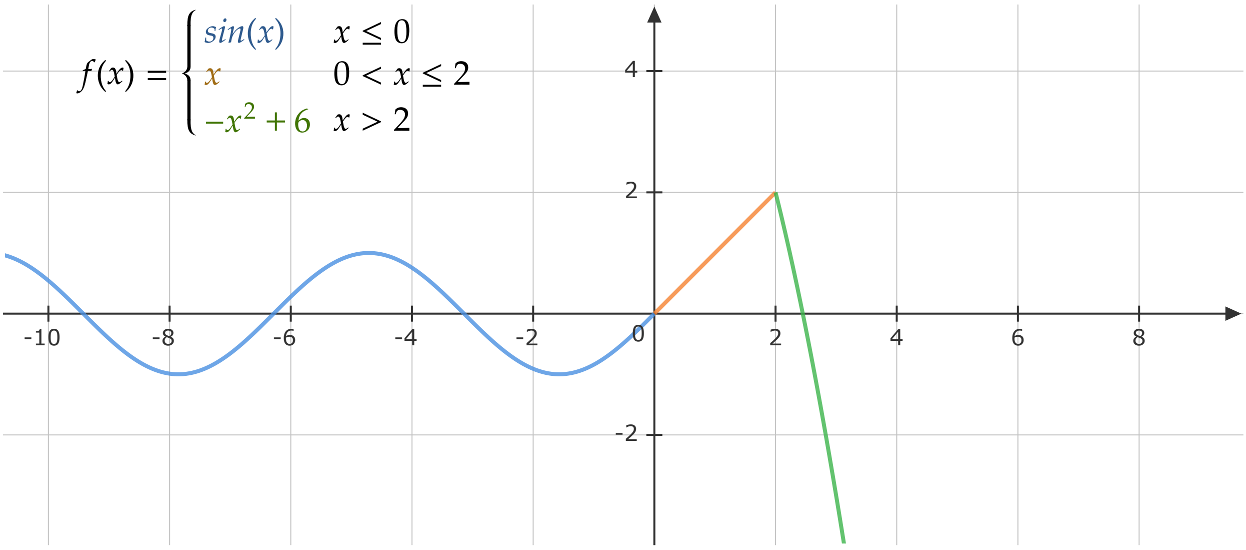

Domain	Range
varies	varies

10.8. Exponential function

The exponential function f with base a is denoted by

$$f(x) = a^x$$

where $a > 0$, $a \neq 1$, and x is any real number.

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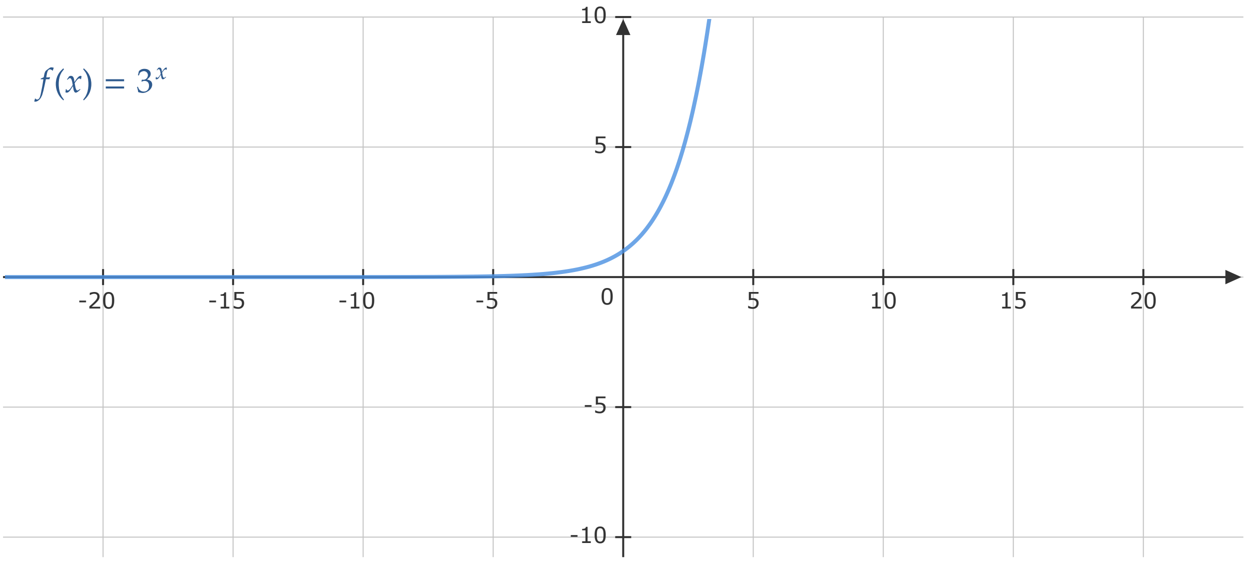

Domain	Range
Set of all real numbers $D: \{x \mid x \in \Re\}$	Set of all positive real numbers $R: \{f(x) \mid f(x) > 0\}$

Other properties of exponential functions:

- The y-intercept is at point $(0, 1)$, that is, $f(0) = 1$.
- $f(1) = a$
- If $0 < a < 1$, as the value of x increases, the value of y decreases.
- If $a > 1$, as the value of x increases, the value of y increases.
- The function f is one-to-one, that is, $a^x = a^y$ if and only if $x = y$.
- It has horizontal asymptote at $y = 0$ or the x-axis.

10.9. Logarithmic function

For $x > 0$, $a > 0$, and $a \neq 1$, $y = \log_a x$ if and only if $x = a^y$. The function

$$f(x) = log_a(x)$$

is called a logarithmic function with base a.

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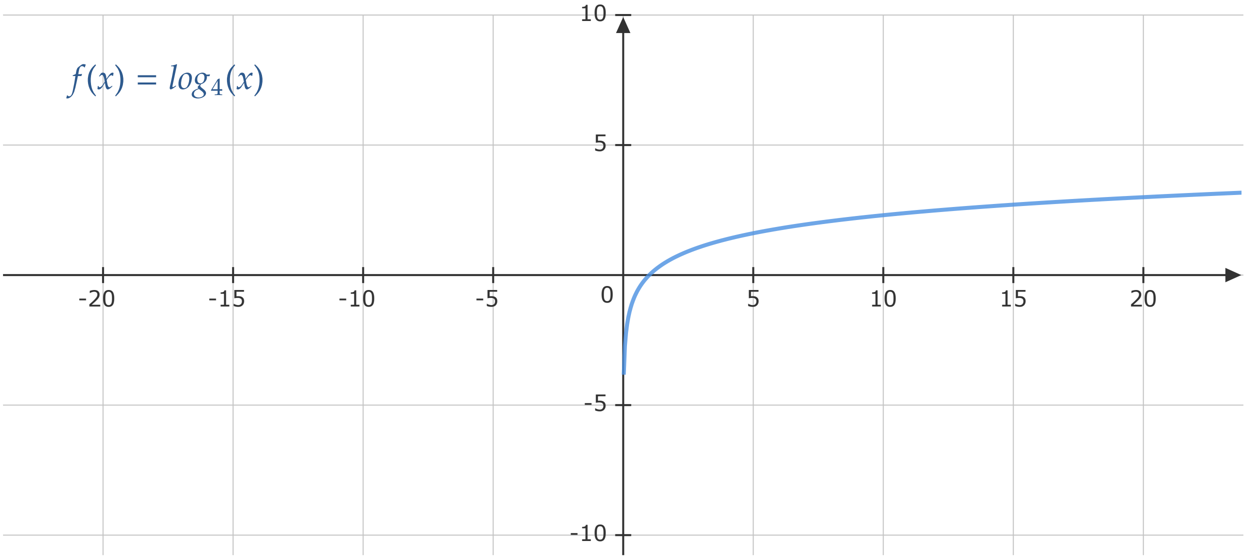

Domain	**Range**
Set of all positive real numbers $D: \{x \in \Re \mid x > 0\}$	Set of all real numbers $R: \{f(x) \mid f(x) \in \Re\}$

Other properties of logarithmic function

- The x-intercept is at point $(1, 0)$, that is, $f(1) = 0$.
- $f(a) = 1$
- If $0 < a < 1$, as the value of x increases, the value of y decreases.
- If $a > 1$, as the value of x increases, the value of y increases.
- The function f is one-to-one, that is, $log_a x = log_a y$ if and only if $x = y$.
- Has vertical asymptote $y = 0$ or the y-axis.

Special kinds of logarithmic function

- The logarithmic function with base 10 is called the *common logarithm.* The log without its base written is understood to be 10. As such, we commonly write it as:
$$f(x) = \log(x)$$
- The *natural logarithmic function* is as $\ln$ without writing its base. It is understood that its base is e. We write it as
$$f(x) = \ln(x)$$

10.10. Trigonometric function

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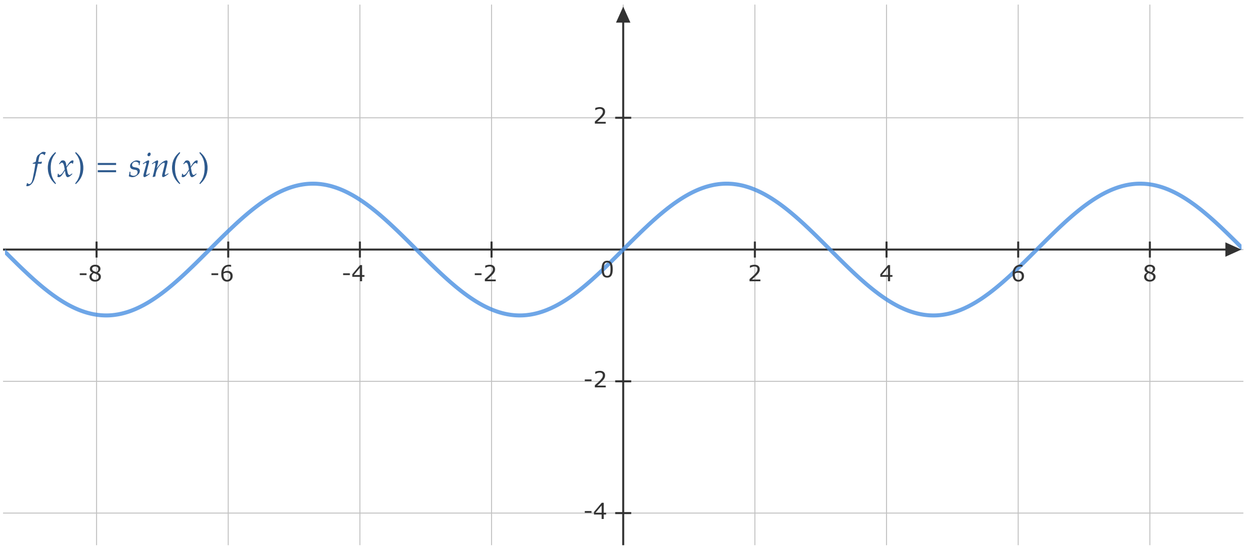
Trigonometric function f is any of the following:

$$f(x) = \sin(x) \qquad f(x) = \csc(x)$$

$$f(x) = \cos(x) \qquad f(x) = \sec(x)$$

$$f(x) = \tan(x) \qquad f(x) = \cot(x)$$



Domain	Range
varies	varies

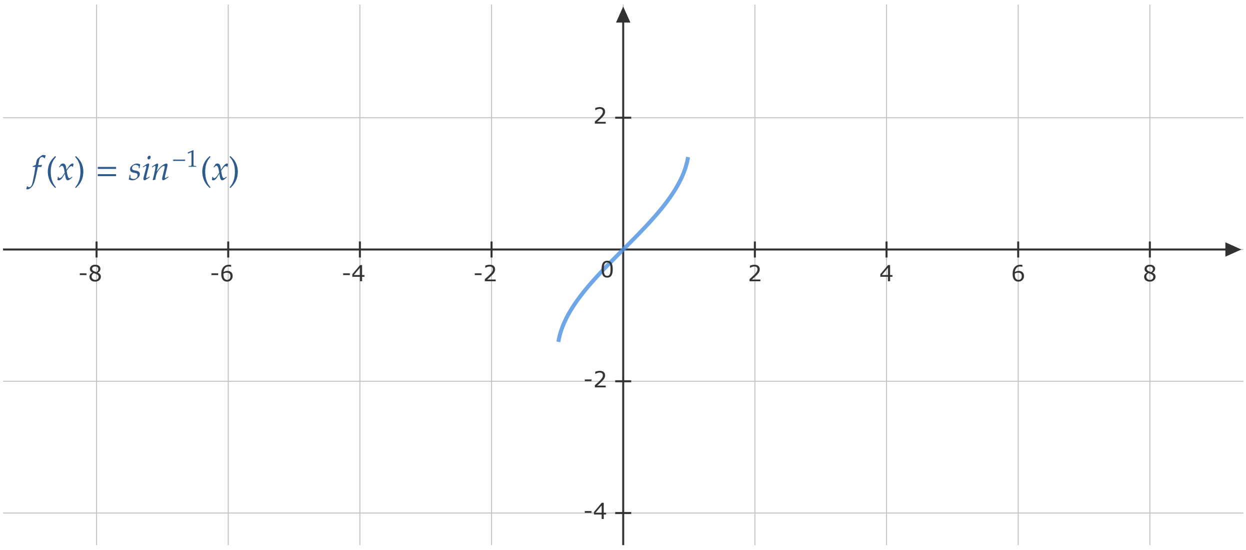
10.11. Inverse trigonometric function

Inverse trigonometric function f is any of the following:

$$f(x) = \sin^{-1}(x) \qquad f(x) = \csc^{-1}(x)$$

$$f(x) = \cos^{-1}(x) \qquad f(x) = \sec^{-1}(x)$$

$$f(x) = \tan^{-1}(x) \qquad f(x) = \cot^{-1}(x)$$



Domain	**Range**
varies	varies

11. Graphical Translation of a Function

The graph of $f(x)$ may be moved or translated according to the following conditions where $a > 0$:

Translation	Function	Direction of movement
Horizontal	$f(x+\alpha)$	a units to the left
Horizontal	$f(x-\alpha)$	a units to the right
Vertical	$f(x)+\alpha$	a units upward
Vertical	$f(x)-\alpha$	a units downward

- The table above are examples of *rigid transformations,* meaning it transforms without changing the shape of the graph.

Exercise 9.1. Suppose $u(p) = 8p^2 + 3p - 1$ is shifted 4 units to the right and 1 unit upward. What will be the equation of newly shifted $u(p)$?

Exercise 9.2. Given the function $f(x) = sin(x) - 6$. Express the equation where the function shifts 2 units to the left.

12. Analysis of Polynomial Functions

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13. Analysis of Radical Functions

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14. Analysis of Rational Functions

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Sources

For outlining content of my work:

1. Dr. Yvette Lim's MTH101a Notes
2. Dr. Morales' first CSMATH1 meeting

For extracting text

- Same as #1
- UPCAT review Book White
- ACAD1 Review Book
- Google Gemini
- Radical Function: https://math.libretexts.org/Courses/Fresno_City_College/Math_3A%3A_College_Algebra_-_Fresno_City_College/05%3A_Other_Functions/5.02%3A_Radical_Functions
- Rational Function: https://www.cuemath.com/calculus/rational-function/

Images extracted online

- Ceiling function: https://www.mathsisfun.com/sets/function-floor-ceiling.html
- Floor function: https://www.mathsisfun.com/sets/function-floor-ceiling.html