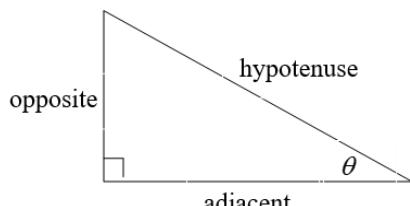


Right triangle definition

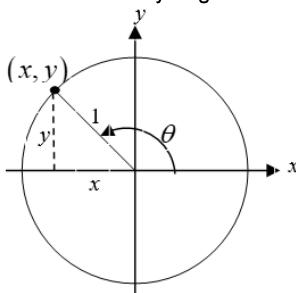
For this definition we assume that
 $0 < \theta < \frac{\pi}{2}$ or $0^\circ < \theta < 90^\circ$.



$$\begin{aligned}\sin(\theta) &= \frac{\text{opposite}}{\text{hypotenuse}} & \csc(\theta) &= \frac{\text{hypotenuse}}{\text{opposite}} \\ \cos(\theta) &= \frac{\text{adjacent}}{\text{hypotenuse}} & \sec(\theta) &= \frac{\text{hypotenuse}}{\text{adjacent}} \\ \tan(\theta) &= \frac{\text{opposite}}{\text{adjacent}} & \cot(\theta) &= \frac{\text{adjacent}}{\text{opposite}}\end{aligned}$$

Unit Circle Definition

For this definition θ is any angle.



$$\begin{aligned}\sin(\theta) &= \frac{y}{1} = y & \csc(\theta) &= \frac{1}{y} \\ \cos(\theta) &= \frac{x}{1} = x & \sec(\theta) &= \frac{1}{x} \\ \tan(\theta) &= \frac{y}{x} & \cot(\theta) &= \frac{x}{y}\end{aligned}$$

Facts and Properties**Domain**

The domain is all the values of θ that can be plugged into the function.

$\sin(\theta)$, θ can be any angle

$\cos(\theta)$, θ can be any angle

$\tan(\theta)$, $\theta \neq \left(n + \frac{1}{2}\right)\pi$, $n = 0, \pm 1, \pm 2, \dots$

$\csc(\theta)$, $\theta \neq n\pi$, $n = 0, \pm 1, \pm 2, \dots$

$\sec(\theta)$, $\theta \neq \left(n + \frac{1}{2}\right)\pi$, $n = 0, \pm 1, \pm 2, \dots$

$\cot(\theta)$, $\theta \neq n\pi$, $n = 0, \pm 1, \pm 2, \dots$

Range

The range is all possible values to get out of the function.

$$-1 \leq \sin(\theta) \leq 1$$

$$-\infty < \tan(\theta) < \infty$$

$$\sec(\theta) \geq 1 \text{ and } \sec(\theta) \leq -1$$

$$-1 \leq \cos(\theta) \leq 1$$

$$-\infty < \cot(\theta) < \infty$$

$$\csc(\theta) \geq 1 \text{ and } \csc(\theta) \leq -1$$

Formulas and Identities**Tangent and Cotangent Identities**

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \quad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

Reciprocal Identities

$$\begin{aligned}\csc(\theta) &= \frac{1}{\sin(\theta)} & \sin(\theta) &= \frac{1}{\csc(\theta)} \\ \sec(\theta) &= \frac{1}{\cos(\theta)} & \cos(\theta) &= \frac{1}{\sec(\theta)} \\ \cot(\theta) &= \frac{1}{\tan(\theta)} & \tan(\theta) &= \frac{1}{\cot(\theta)}\end{aligned}$$

Pythagorean Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\tan^2(\theta) + 1 = \sec^2(\theta)$$

$$1 + \cot^2(\theta) = \csc^2(\theta)$$

Even/Odd Formulas

$$\begin{aligned}\sin(-\theta) &= -\sin(\theta) & \csc(-\theta) &= -\csc(\theta) \\ \cos(-\theta) &= \cos(\theta) & \sec(-\theta) &= \sec(\theta) \\ \tan(-\theta) &= -\tan(\theta) & \cot(-\theta) &= -\cot(\theta)\end{aligned}$$

Periodic Formulas

$$\begin{aligned}\text{If } n \text{ is an integer then,} \\ \sin(\theta + 2\pi n) &= \sin(\theta) & \csc(\theta + 2\pi n) &= \csc(\theta) \\ \cos(\theta + 2\pi n) &= \cos(\theta) & \sec(\theta + 2\pi n) &= \sec(\theta) \\ \tan(\theta + \pi n) &= \tan(\theta) & \cot(\theta + \pi n) &= \cot(\theta)\end{aligned}$$

Degrees to Radians Formulas

$$\begin{aligned}\text{If } x \text{ is an angle in degrees and } t \text{ is an angle in radians then} \\ \frac{\pi}{180} = \frac{t}{x} \Rightarrow t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}\end{aligned}$$

Double Angle Formulas

$$\begin{aligned}\sin(2\theta) &= 2 \sin(\theta) \cos(\theta) \\ \cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta) \\ &= 2 \cos^2(\theta) - 1 \\ &= 1 - 2 \sin^2(\theta) \\ \tan(2\theta) &= \frac{2 \tan(\theta)}{1 - \tan^2(\theta)}\end{aligned}$$

Half Angle Formulas

$$\sin\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos(\theta)}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 + \cos(\theta)}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos(\theta)}{1 + \cos(\theta)}}$$

Half Angle Formulas (alternate form)

$$\begin{aligned}\sin^2(\theta) &= \frac{1}{2}(1 - \cos(2\theta)) & \tan^2(\theta) &= \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)} \\ \cos^2(\theta) &= \frac{1}{2}(1 + \cos(2\theta))\end{aligned}$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta)$$

$$\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha) \tan(\beta)}$$

Product to Sum Formulas

$$\sin(\alpha) \sin(\beta) = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos(\alpha) \cos(\beta) = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin(\alpha) \cos(\beta) = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos(\alpha) \sin(\beta) = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

Sum to Product Formulas

$$\sin(\alpha) + \sin(\beta) = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\sin(\alpha) - \sin(\beta) = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos(\alpha) + \cos(\beta) = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

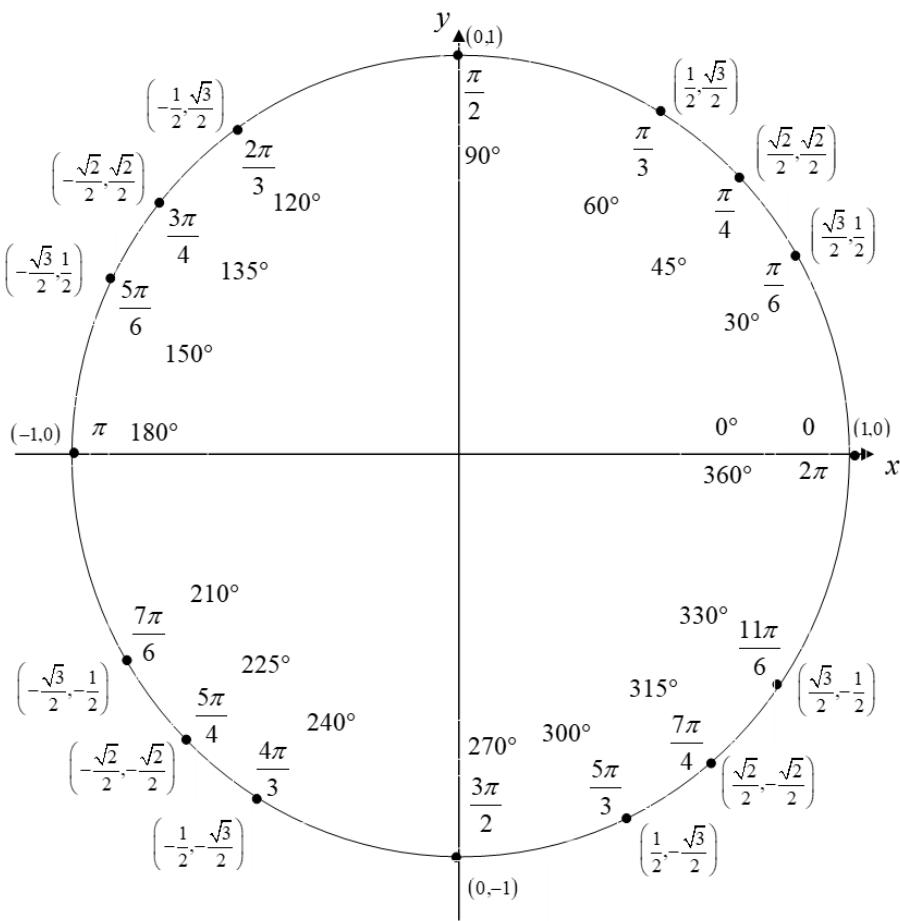
$$\cos(\alpha) - \cos(\beta) = -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

Cofunction Formulas

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta) \quad \cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec(\theta) \quad \sec\left(\frac{\pi}{2} - \theta\right) = \csc(\theta)$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot(\theta) \quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan(\theta)$$



For any ordered pair on the unit circle (x, y) : $\cos(\theta) = x$ and $\sin(\theta) = y$

Example

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$$

$$\sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

Inverse Trig Functions

Definition

$y = \sin^{-1}(x)$ is equivalent to $x = \sin(y)$

$y = \cos^{-1}(x)$ is equivalent to $x = \cos(y)$

$y = \tan^{-1}(x)$ is equivalent to $x = \tan(y)$

Inverse Properties

$\cos(\cos^{-1}(x)) = x$ $\cos^{-1}(\cos(\theta)) = \theta$

$\sin(\sin^{-1}(x)) = x$ $\sin^{-1}(\sin(\theta)) = \theta$

$\tan(\tan^{-1}(x)) = x$ $\tan^{-1}(\tan(\theta)) = \theta$

Domain and Range

Function	Domain	Range
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$y = \sin^{-1}(x)$ $-1 \leq x \leq 1$ $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$y = \cos^{-1}(x)$ $-1 \leq x \leq 1$ $0 \leq y \leq \pi$

$y = \tan^{-1}(x)$ $-\infty < x < \infty$ $-\frac{\pi}{2} < y < \frac{\pi}{2}$

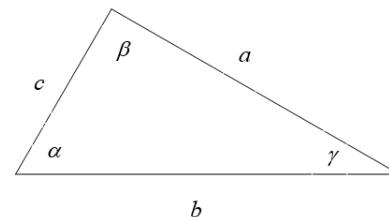
Alternate Notation

$\sin^{-1}(x) = \arcsin(x)$

$\cos^{-1}(x) = \arccos(x)$

$\tan^{-1}(x) = \arctan(x)$

Law of Sines, Cosines and Tangents



Law of Sines

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos(\alpha)$$

$$b^2 = a^2 + c^2 - 2ac \cos(\beta)$$

$$c^2 = a^2 + b^2 - 2ab \cos(\gamma)$$

Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{1}{2}(\alpha-\beta)\right)}{\tan\left(\frac{1}{2}(\alpha+\beta)\right)}$$

$$\frac{b-c}{b+c} = \frac{\tan\left(\frac{1}{2}(\beta-\gamma)\right)}{\tan\left(\frac{1}{2}(\beta+\gamma)\right)}$$

$$\frac{a-c}{a+c} = \frac{\tan\left(\frac{1}{2}(\alpha-\gamma)\right)}{\tan\left(\frac{1}{2}(\alpha+\gamma)\right)}$$

Mollweide's Formula

$$\frac{a+b}{c} = \frac{\cos\left(\frac{1}{2}(\alpha-\beta)\right)}{\sin\left(\frac{1}{2}\gamma\right)}$$