

Basic Properties and Facts**Arithmetic Operations**

$$ab + ac = a(b + c)$$

$$a\left(\frac{b}{c}\right) = \frac{ab}{c}$$

$$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

$$\frac{a-b}{c-d} = \frac{b-a}{d-c}$$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\frac{ab+ac}{a} = b+c, \quad a \neq 0$$

$$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{ad}{bc}$$

Exponent Properties

$$a^n a^m = a^{n+m}$$

$$(ab)^n = a^n b^n$$

$$(a^n)^m = a^{nm}$$

$$a^0 = 1, \quad a \neq 0$$

$$\frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}}$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = (a^n)^{\frac{1}{m}} \quad \frac{1}{a^{-n}} = a^n$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n} \quad a^{-n} = \frac{1}{a^n}$$

Properties of Radicals

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

$$\sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$$

$$\sqrt[n]{\sqrt[n]{a}} = \sqrt[nm]{a}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\sqrt[n]{a^n} = a \text{ if } n \text{ is odd}$$

$$\sqrt[n]{a^n} = |a| \text{ if } n \text{ is even}$$

Properties of Inequalities

If $a < b$ then $a + c < b + c$ and $a - c < b - c$

If $a < b$ and $c > 0$ then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

If $a < b$ and $c < 0$ then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

Properties of Absolute Value

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

$$|a| \geq 0 \quad |-a| = |a|$$

$$|ab| = |a| |b| \quad \left|\frac{a}{b}\right| = \frac{|a|}{|b|}$$

$$|a + b| \leq |a| + |b| \quad \text{Triangle Inequality}$$

Distance Formula

If $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ are two points the distance between them is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Complex Numbers

$$i = \sqrt{-1} \quad i^2 = -1 \quad \sqrt{-a} = i\sqrt{a}, \quad a \geq 0$$

$$(a + bi) + (c + di) = a + c + (b + d)i$$

$$(a + bi) - (c + di) = a - c + (b - d)i$$

$$(a + bi)(c + di) = ac - bd + (ad + bc)i$$

$$(a + bi)(a - bi) = a^2 + b^2$$

$$|a + bi| = \sqrt{a^2 + b^2} \quad \text{Complex Modulus}$$

$$\overline{(a + bi)} = a - bi \quad \text{Complex Conjugate}$$

$$\overline{(a + bi)}(a + bi) = |a + bi|^2$$

Definition

$y = \log_b(x)$ is equivalent to $x = b^y$

Example

$\log_5(125) = 3$ because $5^3 = 125$

Special Logarithms

$\ln(x) = \log_e(x)$ natural log

$\log(x) = \log_{10}(x)$ common log

where $e = 2.718281828\dots$

Logarithms and Log Properties**Logarithm Properties**

$$\log_b(1) = 0$$

$$\log_b(b) = 1$$

$$\log_b(b^x) = x$$

$$b^{\log_b(x)} = x$$

$$\log_b(x^r) = r \log_b(x)$$

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

The domain of $\log_b(x)$ is $x > 0$

Factoring and Solving**Quadratic Formula**

Solve $ax^2 + bx + c = 0, a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac > 0$ – Two real unequal solns.

If $b^2 - 4ac = 0$ – Repeated real solution.

If $b^2 - 4ac < 0$ – Two complex solutions.

Square Root Property

If $x^2 = p$ then $x = \pm\sqrt{p}$

Absolute Value Equations/Inequalities

If b is a positive number

$$|p| = b \Rightarrow p = -b \text{ or } p = b$$

$$|p| < b \Rightarrow -b < p < b$$

$$|p| > b \Rightarrow p < -b \text{ or } p > b$$

Completing the Square

Solve $2x^2 - 6x - 10 = 0$

(1) Divide by the coefficient of the x^2

$$x^2 - 3x - 5 = 0$$

(4) Factor the left side

$$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$$

(2) Move the constant to the other side.

$$x^2 - 3x = 5$$

(5) Use Square Root Property

$$x - \frac{3}{2} = \pm\sqrt{\frac{29}{4}} = \pm\frac{\sqrt{29}}{2}$$

(3) Take half the coefficient of x , square it and add it to both sides

(6) Solve for x

$$x^2 - 3x + \left(-\frac{3}{2}\right)^2 = 5 + \left(-\frac{3}{2}\right)^2 = 5 + \frac{9}{4} = \frac{29}{4}$$

$$x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$$

Functions and Graphs**Constant Function**

$$y = a \quad \text{or} \quad f(x) = a$$

Graph is a horizontal line passing through the point $(0, a)$.

Line/Linear Function

$$y = mx + b \quad \text{or} \quad f(x) = mx + b$$

Graph is a line with point $(0, b)$ and slope m.

Slope

Slope of the line containing the two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

Slope – intercept form

The equation of the line with slope m and y-intercept $(0, b)$ is

$$y = mx + b$$

Point – Slope form

The equation of the line with slope m and passing through the point (x_1, y_1) is

$$y = y_1 + m(x - x_1)$$

Parabola/Quadratic Function

$$y = a(x - h)^2 + k \quad f(x) = a(x - h)^2 + k$$

The graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at (h, k) .

Parabola/Quadratic Function

$$y = ax^2 + bx + c \quad f(x) = ax^2 + bx + c$$

The graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

Parabola/Quadratic Function

$$x = ay^2 + by + c \quad g(y) = ay^2 + by + c$$

The graph is a parabola that opens right if $a > 0$ or left if $a < 0$ and has a vertex at

$$\left(g\left(-\frac{b}{2a}\right), -\frac{b}{2a}\right).$$

Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

Graph is a circle with radius r and center (h, k) .

Ellipse

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

Graph is an ellipse with center (h, k) with vertices a units right/left from the center and vertices b units up/down from the center.

Hyperbola

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

Graph is a hyperbola that opens left and right, has a center at (h, k) , vertices a units left/right of center and asymptotes that pass through center with slope $\pm\frac{b}{a}$.

Hyperbola

$$\frac{(y - k)^2}{b^2} - \frac{(x - h)^2}{a^2} = 1$$

Graph is a hyperbola that opens up and down, has a center at (h, k) , vertices b units up/down from the center and asymptotes that pass through center with slope $\pm\frac{b}{a}$.

Common Algebraic Errors**Error**

$$\frac{2}{0} \neq 0 \text{ and } \frac{2}{0} \neq 2$$

Division by zero is undefined!

$$-3^2 \neq 9$$

$-3^2 = -9, (-3)^2 = 9$ Watch parenthesis!

$$(x^2)^3 \neq x^5$$

$$(x^2)^3 = x^2 x^2 x^2 = x^6$$

$$\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$$

$$\frac{1}{2} = \frac{1}{1+1} \neq \frac{1}{1} + \frac{1}{1} = 2$$

$$\frac{1}{x^2 + x^3} \neq x^{-2} + x^{-3}$$

A more complex version of the previous error.

$$\frac{d+bx}{d} \neq 1 + bx$$

$$\frac{a+bx}{a} = \frac{a}{a} + \frac{bx}{a} = 1 + \frac{bx}{a}$$

Beware of incorrect canceling!

$$-a(x - 1) \neq -ax - a$$

Make sure you distribute the “-”!

$$(x + a)^2 \neq x^2 + a^2$$

$$(x + a)^2 = (x + a)(x + a) = x^2 + 2ax + a^2$$

$$\sqrt{x^2 + a^2} \neq x + a$$

$$5 = \sqrt{25} = \sqrt{3^2 + 4^2} \neq \sqrt{3^2} + \sqrt{4^2} = 3 + 4 = 7$$

$$\sqrt{x + a} \neq \sqrt{x} + \sqrt{a}$$

See previous error.

$$(x + a)^n \neq x^n + a^n \text{ and } \sqrt[n]{x + a} \neq \sqrt[n]{x} + \sqrt[n]{a}$$

More general versions of previous three errors.

$$2(x + 1)^2 \neq (2x + 2)^2$$

$$(2x + 2)^2 = 4x^2 + 8x + 4$$

Square first then distribute!

$$(2x + 2)^2 \neq 2(x + 1)^2$$

See the previous example. You can not factor out a constant if there is a power on the parenthesis!

$$\sqrt{-x^2 + a^2} \neq -\sqrt{x^2 + a^2}$$

$$\sqrt{-x^2 + a^2} = (-x^2 + a^2)^{\frac{1}{2}}$$

Now see the previous error.

$$\frac{a}{\left(\frac{b}{c}\right)} \neq \frac{ab}{c}$$

$$\frac{a}{\left(\frac{b}{c}\right)} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{b}{c}\right)} = \left(\frac{a}{b}\right) \left(\frac{c}{b}\right) = \frac{ac}{b}$$

$$\frac{\left(\frac{a}{b}\right)}{c} \neq \frac{ac}{b}$$

$$\frac{\left(\frac{a}{b}\right)}{c} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{1}\right)} = \left(\frac{a}{b}\right) \left(\frac{1}{c}\right) = \frac{a}{bc}$$