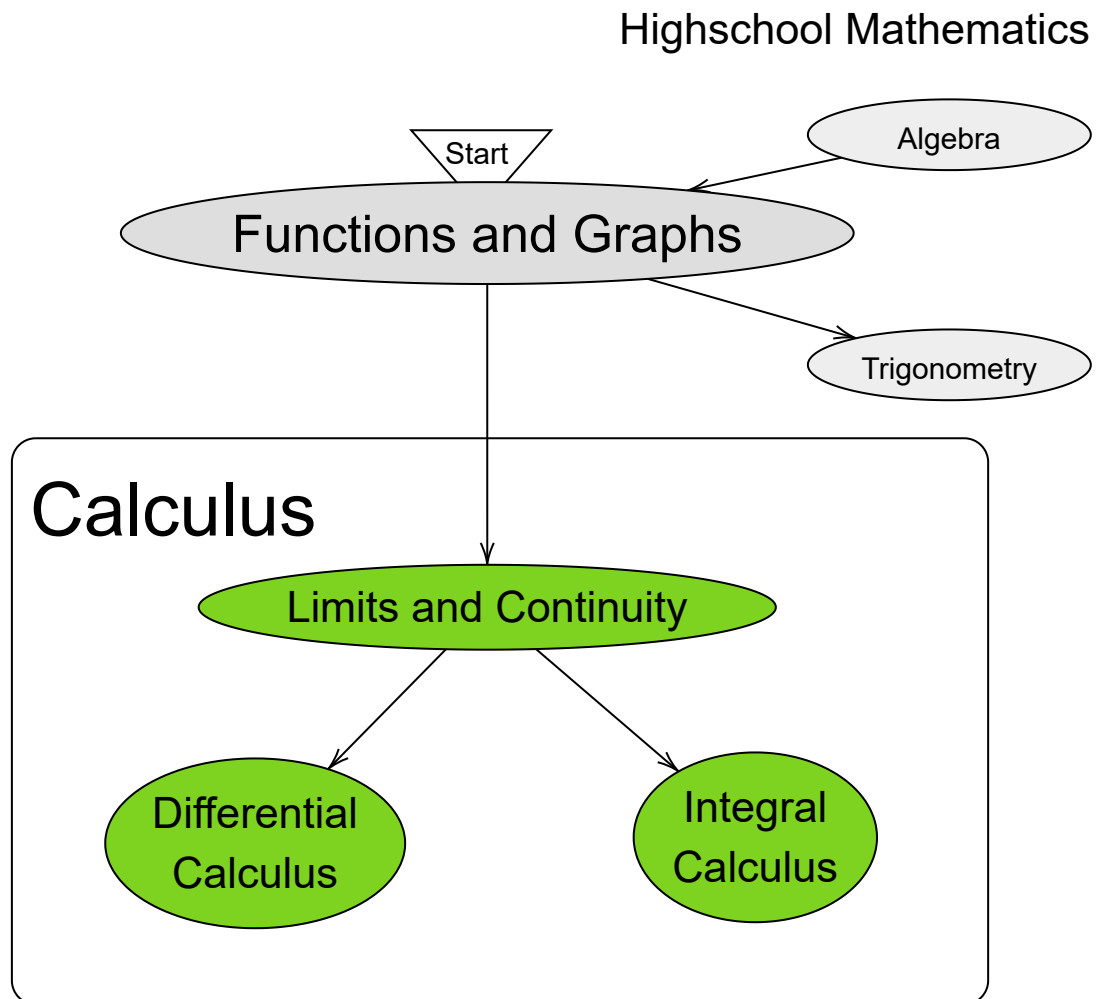


# 1. Map of Calculus

## 1.1. Flowchart

—> means 'prerequisite to'



## 1.2. Lesson Plan

### Lesson 1: Functions and Graphs

- 1) Definition of Function
- 2) Evaluating a Function
- 3) Graphing a Function
- 4) Main Types of Functions
- 5) Finding Domain and Range of a Function

- 6)  $x$ -intercept and  $y$ -intercept
- 7) Operations on Functions
- 8) Symmetry of Functions
- 9) Inverse of a Function
- 10) Types of Functions
- 11) Graphical Translation of a Function
- 12) Analysis of Polynomial Functions
- 13) Analysis of Radical Functions
- 14) Analysis of Rational Functions

## Lesson 2: Limits and Continuity

- 1) Informal Definition of a Limit
- 2) Graphical Approach to Limits of Functions
- 3) Properties and Formulas of Limits
- 4) One-sided Limits
- 5) Evaluation Techniques of Limits
- 6) Infinite Limits
- 7) Limits at Infinity
- 8) Continuity of a Function at a Number
- 9) Continuity of a Composite Function
- 10) Continuity on an Interval
- 11) Formal Definition of a Limit

## Lesson 3: Differential Calculus

- 1) Definition of Derivative
- 2) Notation of Derivative
- 3) Differentiability vs Continuity
- 4) Derivatives of Algebraic Functions
- 5) Derivatives of Transcendental Functions
- 6) Chain Rule
- 7) Higher-order Derivatives
- 8) Implicit Differentiation

## Lesson 4: Integral Calculus

- 1) Definition of Integrals
- 2) Notation of Integrals
- 3) Fundamental Theorems of Calculus
- 4) Indefinite Integrals
- 5) Definite Integrals
- 6) Integrals of Algebraic Functions
- 7) Integrals of Transcendental Functions
- 8) Techniques of Integration
  - a)  $u$ -substitution
  - b) Products and Some Quotients of Trigonometric Functions

- b) Products and some Quotients of Trigonometric Functions
- c) Integration by Parts
- d) Trigonometric Substitutions
- e) Partial Fraction
- 9) Improper Integral
- 10) Approximating Definite Integrals

## Lesson 5: Real-life Application

### Differential Calculus

- 1) Slope of a Tangent Line
- 2) Rate of Change
- 3) Increasing and Decreasing Functions
- 4) Concavity and Points of Inflection
- 5) Marginal Analysis
- 6) Mean Value Theorem
- 7) Newton's Method
- 8) Related Rates
- 9) Optimization

### Integral Calculus

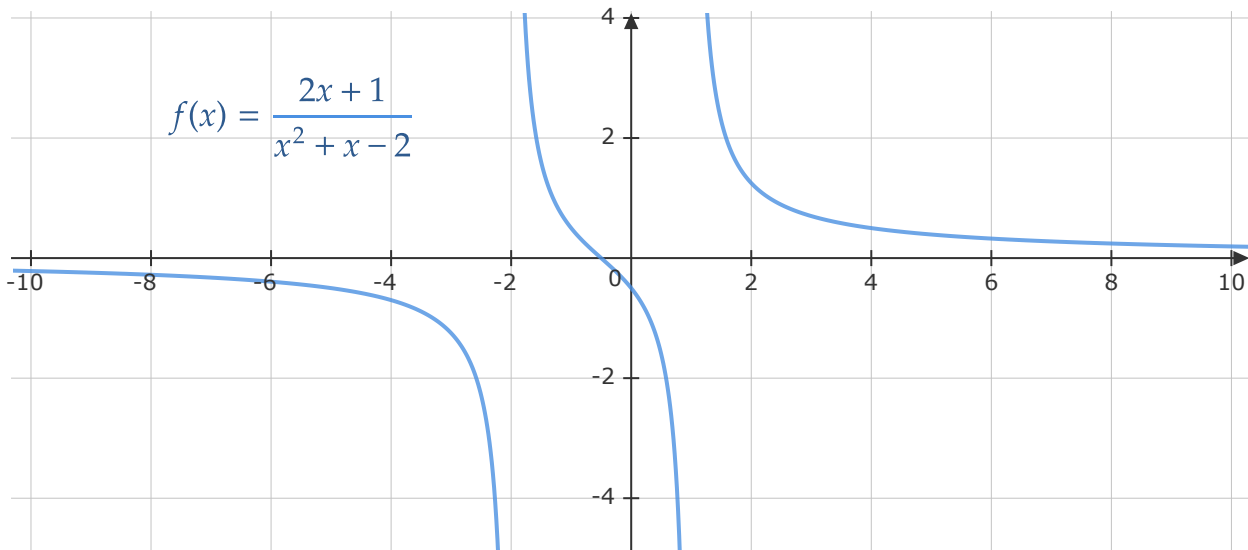
- 1) Net Area
- 2) Area Between Curves
- 3) Volumes of Revolution
- 4) Work
- 5) Average Function Value
- 6) Arc Length and Surface Area

## 2. Functions and Graphs

Function is the core prerequisite to calculus. It is the mathematical representation of an input  $x$  plugging it into a function  $f$  to produce an output  $f(x)$ .

A function value  $f(x)$  tells the exact output value when plugging  $x$  as an input

### 2.1. Diagnostic Exercises



Exercise 1. Referencing to a graph above, evaluate the following:

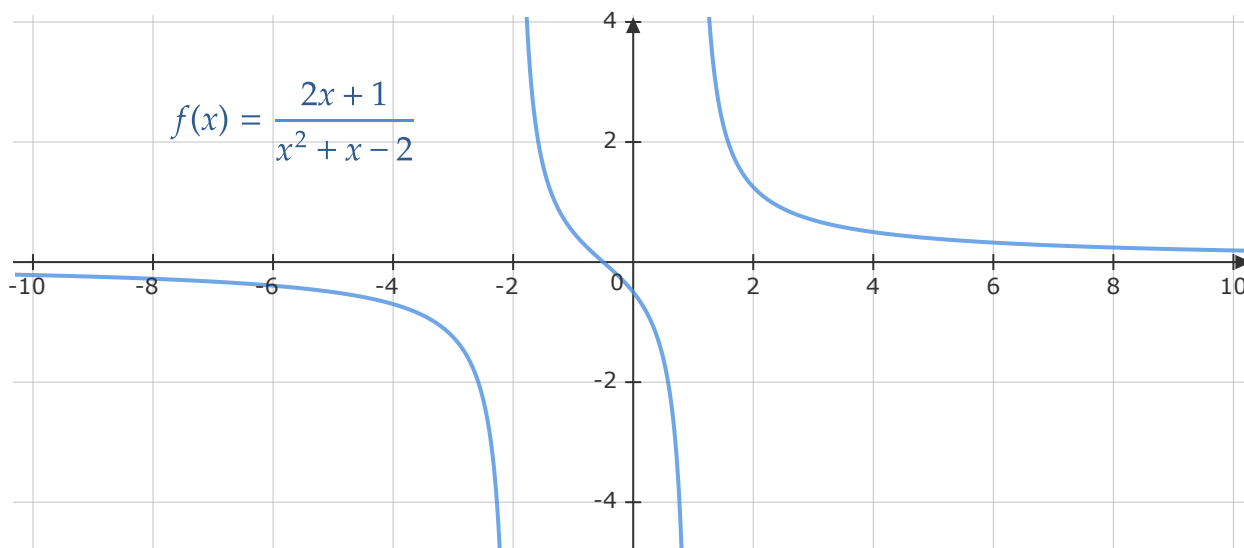
- 1)  $f(4)$
- 2)  $f(1)$
- 3)  $y$  - intercept
- 4)  $x$  - intercept
- 5) Domain
- 6) Range
- 7) Horizontal asymptote/s
- 8) Vertical asymptote/s

Exercise 2. Using  $f(x) = x^2 + 3$  and  $g(x) = x - 1$  Evaluate the following:

- 1)  $(f + g)(x)$
- 2)  $(f \circ g)(2)$

### 3. Limits and Continuity

The limit of a function  $\lim_{x \rightarrow a} f(x)$  tells what is the approaching output value to  $f(a)$ .



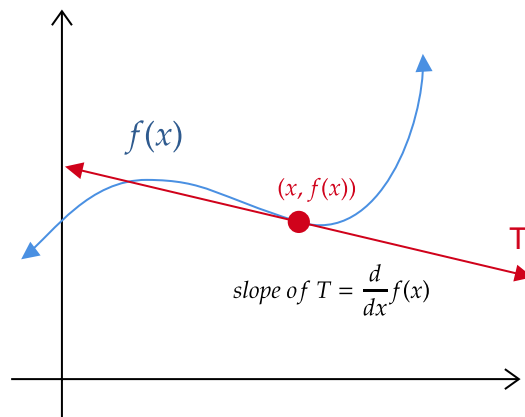
### 3.1. Diagnostic Exercises

Exercise 1. Referencing to a graph above, evaluate the following:

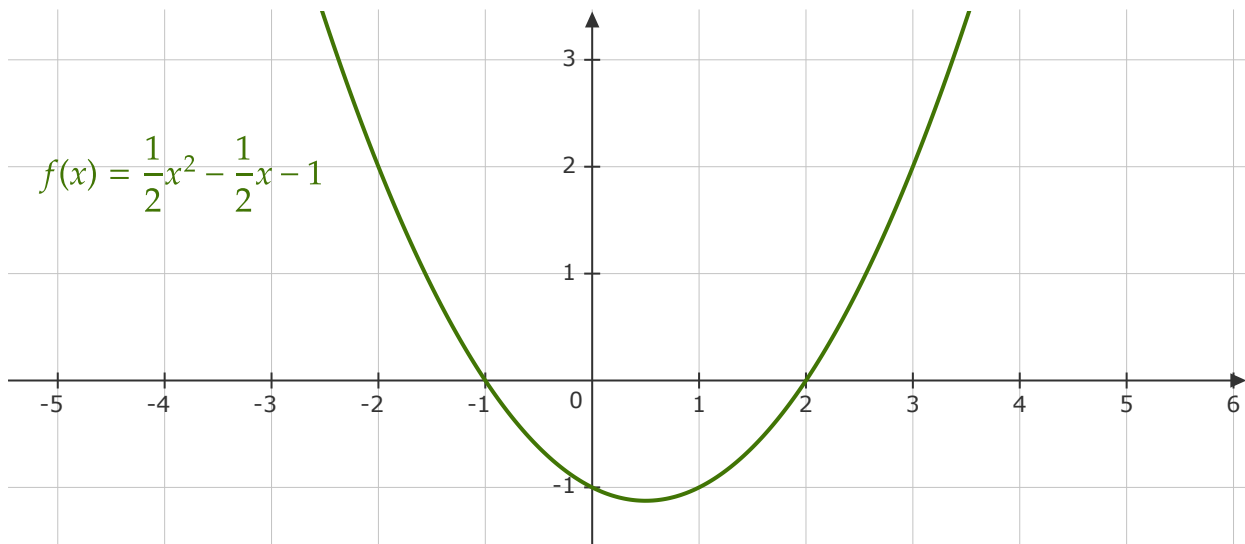
- 1)  $\lim_{x \rightarrow 4} f(x)$
- 2)  $\lim_{x \rightarrow 1^+} f(x)$
- 3)  $\lim_{x \rightarrow 1^-} f(x)$
- 4)  $\lim_{x \rightarrow 1} f(x)$
- 5)  $\lim_{x \rightarrow 2^+} f(x)$
- 6)  $\lim_{x \rightarrow 2^-} f(x)$
- 7)  $\lim_{x \rightarrow 2} f(x)$
- 8)  $\lim_{x \rightarrow \infty} f(x)$
- 9)  $\lim_{x \rightarrow -\infty} f(x)$
- 10) Is the function  $f(x)$  continuous at  $x = 0$ ?

## 4. Differential Calculus

The derivative of a function  $f(x)$  with respect to  $x$ , written as  $\frac{d}{dx}f(x)$ , produces the instantaneous rate of change of  $f(x)$  as  $x$  changes. In a graphical approach,  $\frac{d}{dx}f(x)$  is the slope of a function's tangent line  $T$  at any point  $(x, f(x))$ .



## 4.1. Diagnostic Exercises



Exercise 1. Referencing to a graph above, evaluate the following:

- 1)  $\frac{d}{dx}f(x)$
- 2)  $\frac{d}{dx}f(3)$

$$3) \frac{d^2}{dx^2} f(3)$$

4) Minimum value of  $f(x)$

Exercise 2. Evaluate the following:

$$1) \frac{d}{dx} [2 \sin(x) + 3 \tan(x)]$$

$$2) \frac{d}{dx} [3(x^3 - 2x + 1)^2 + x]$$

$$3) \frac{d^2}{dx^2} (4x^6 + 7x - 2)$$

$$4) \frac{d}{dx} [(x^2 - 2)(x + 3)]$$

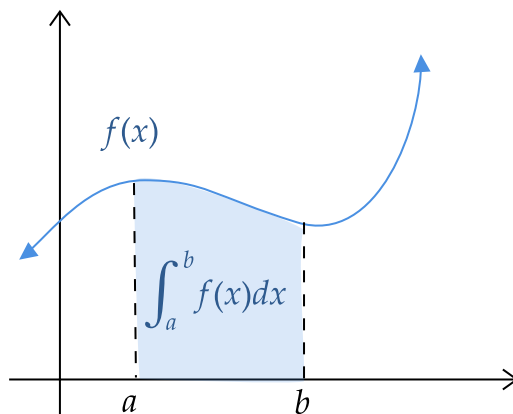
$$5) \frac{d}{dx} \left( \frac{x+1}{x+2} \right)$$

$$6) \frac{d}{da} [3a^5 + 2y^2 + 4y]$$

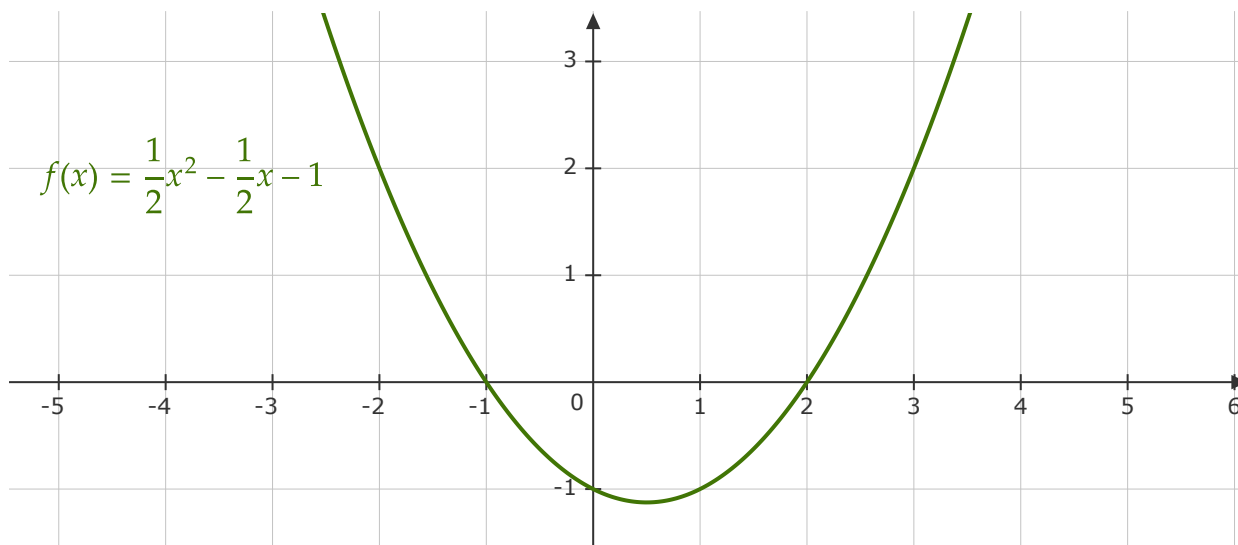
## 5. Integral Calculus

Integrating  $f(x)$  with respect to its differential  $dx$  from left bound  $a$  to right bound  $b$  ( $a < b$ ), written as  $\int_a^b f(x)dx$ , produces  $F(b) - F(a)$  where  $F(x)$  is the antiderivative of  $f(x)$ . In a graphical approach,

$\int_a^b f(x)dx$  is the accumulated area bounded by  $f(x)$ ,  $x = a$ ,  $x = b$ , and the  $x$ -axis.



## 5.1. Diagnostic Exercises



Exercise 1. Referencing to a graph above, evaluate the following:

- 1)  $\int f(x)dx$
- 2)  $\int_2^4 f(x)dx$
- 3)  $\int_0^3 f(x)dx$
- 4)  $\int_4^2 f(x)dx$

Exercise 2. Without using the graph, evaluate the following:

- 1)  $\int [3 \sin(u) + u^2] du$
- 2)  $\int_{-7}^{-7} (x^2 + 3x + 1) dx$
- 3) If  $f(x) = 2x^2 - 3$  and  $g(x) = 5x - 2$ , what is  $\int [(f + g)(x)] dx$ ?
- 4)  $\int (3x + 2)^2 dx$
- 5)  $\int \frac{1}{(x^2 + 5x)^7} dx$
- 6)  $\int x^2 (x^3 - 1)^4 dx$