南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

Robot Modeling & Control

ME331

16: Trajectory Planning & Control I

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Dept. of MEE， SUSTech

Initiative project

Project Proposal （项目开题）

PPT Presentation Date: May 12，2025

（5 minutes PPT Group Report + 4 minutes discussion）

1. 小组成员（Team member）
2. 研究动机（Motivation）
3. 研究内容（Research contents）
4. 大致思路（General idea）
5. 分工与工作计划（Working plan）

桌面级机械臂建模与控制上位机使用说明

Guest TA：梅杰 18270653359 （支持2个组，最多10个人）

目录


9:56
4月22日, 星期二
acer


摩擦力和重力补偿


定点阻抗控制


基于计算力矩法的
滑模控制
六轴机械臂
AOC

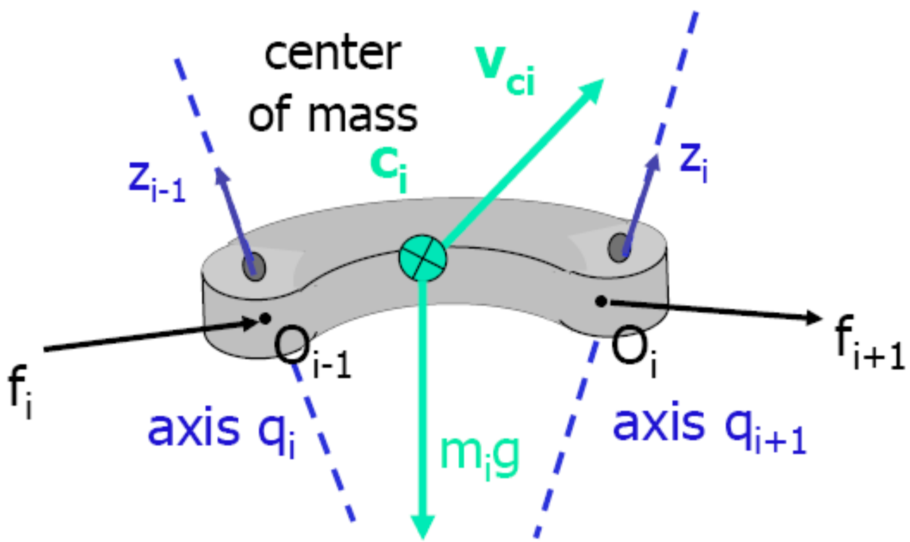
Outline

- Review
  - Newton-Euler method
  - Examples： Double Pendulum
- Trajectory Planning
  - Problem Definition
  - Trajectories for Point-to-Point Motion
  - Trajectories for Paths Specified by Via Points
- Introduction to Control

Newton-Euler Approach

- **Newton equation**

link i



FORCES

f_i force applied from link (i-1) on link i

f_{i+1} force applied from link i on link (i+1)

$m_i g$ gravity force

all vectors expressed in the same RF (better RF_i)

Newton equation

$$\mathbf{f}_i - R^{i}_{i+1}\mathbf{f}_{i+1} + m_i\mathbf{g}_i = m_i\mathbf{a}_{c,i}$$

linear acceleration of c_i

Newton-Euler Approach

- **Euler equation**

link i

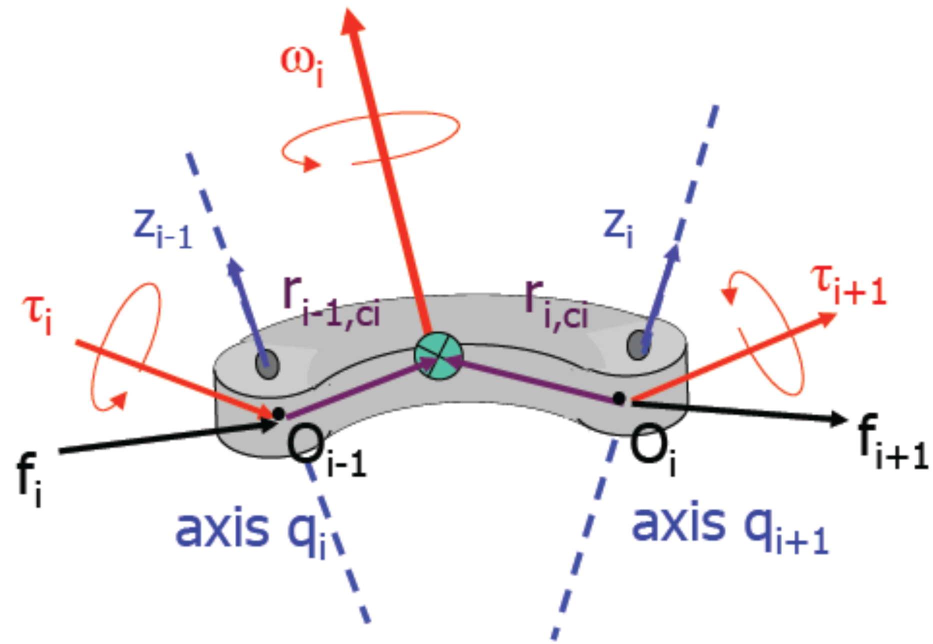
TORQUES

τ_i torque applied from link (i-1) on link i

τ_{i+1} torque applied from link i on link (i+1)

$f_i \times r_{i-1,c}$ torque due to f_i w.r.t. c_i

$- f_{i+1} \times r_{i,c}$ torque due to $- f_{i+1}$ w.r.t. c_i



all vectors expressed in the same RF (RF_i !!)

Euler equation

$$\boldsymbol{\tau}_\mathrm{i} - R^{i}_{i+1}\boldsymbol{\tau}_\mathrm{i+1} + \boldsymbol{f}_\mathrm{i} \times \mathbf{r}_\mathrm{i-1,ci} - R^{i}_{i+1}\boldsymbol{f}_\mathrm{i+1} \times \mathbf{r}_\mathrm{i,ci} = \mathbf{I}_\mathrm{i}\dot{\boldsymbol{\omega}}_\mathrm{i} + \boldsymbol{\omega}_\mathrm{i} \times (\mathbf{I}_\mathrm{i}\boldsymbol{\omega}_\mathrm{i})$$

Dynamics

Lagrangian method (energy-based approach)

- multi-body robot seen as a whole
- Constraint (internal) reaction forces between the links are automatically eliminated: in fact, they do not perform work
- closed-form (symbolic) equations are directly obtained
- best suited for study of dynamic properties and analysis of control schemes

Newton-Euler method (balance of forces/torques)

- dynamic equations written separately for each link/body
- Inverse dynamics in real time
- equations are evaluated in a numeric and recursive way
- best for synthesis (=implementation) of model- based control schemes

Trajectory Planning

1. Problem Definition

– **PATH VS. TRAJECTORY**

- **Path:** A sequence of robot configurations in a particular order without regard to the timing of these configurations.
- **Trajectory:** It concerned about when each part of the path must be attained, thus specifying timing.

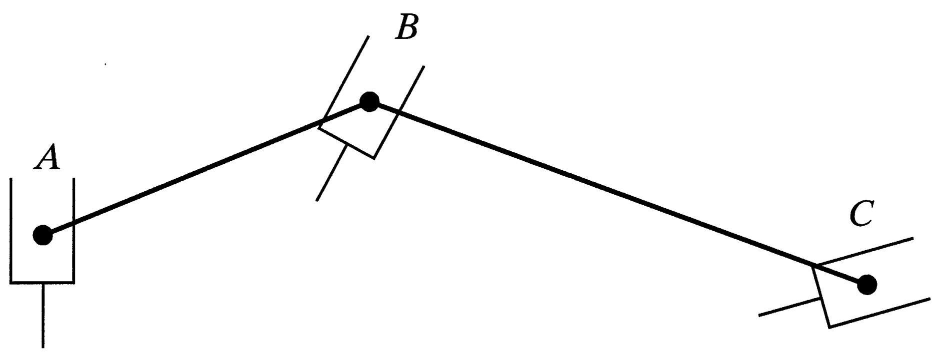
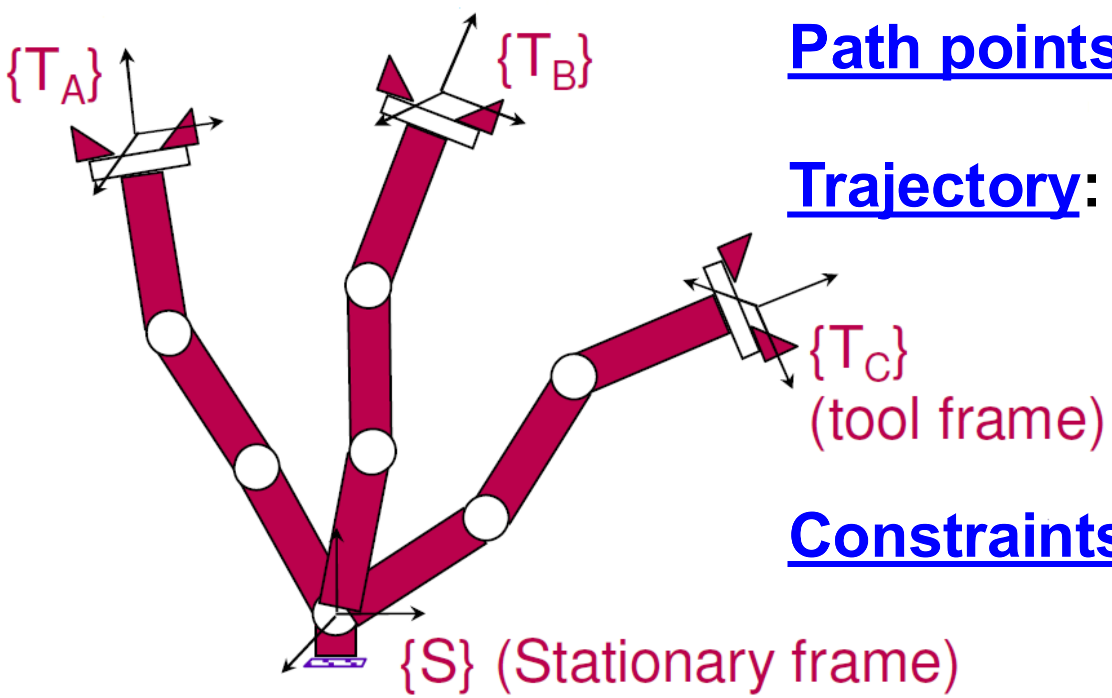


Fig. Sequential robot movements in a path.

Trajectory Planning

1. Problem Definition

– Move the manipulator arm from the initial position {T_A} to the desired final position {T_C}. Maybe going through some via point {T_B}.



Path points: Initial, final and via points

Trajectory: Time history of position, velocity and acceleration for each DOF

Constraints: Spatial, time, stability, smoothness, etc.

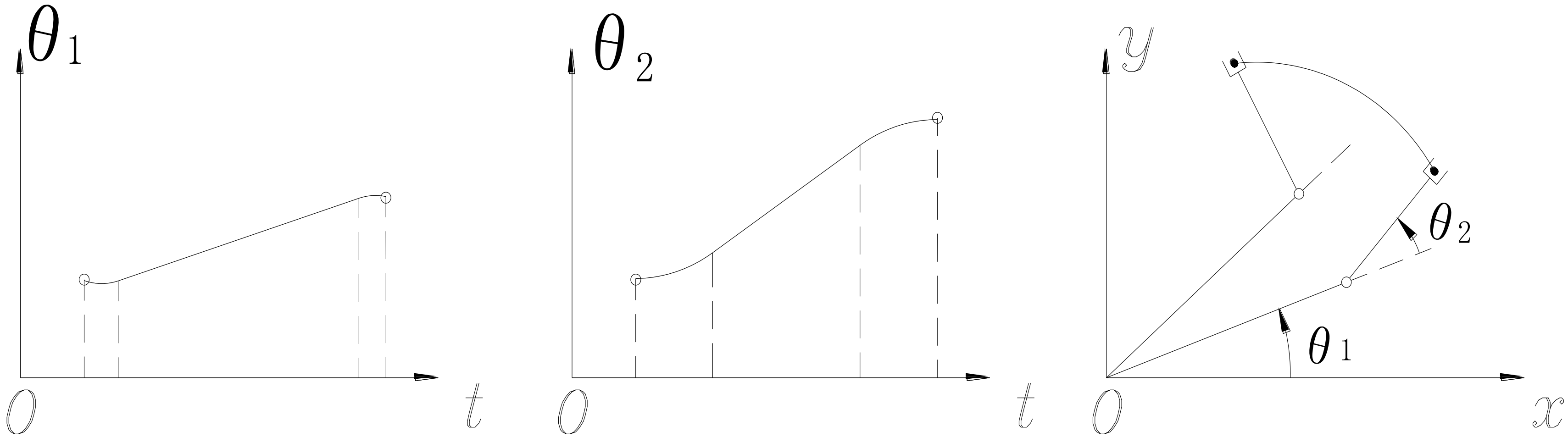
Trajectory Planning

2. Solution Spaces

1）Joint Space

- The description of the motion to be made by joint values.
- The motion of end-effector between the two points is unpredictable.

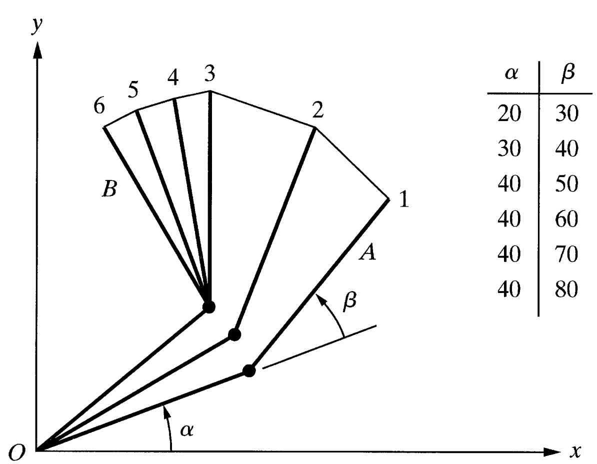
Spec: Cartesian poses is determined indirectly，reduced calculation，no singularity problem



Trajectory Planning

BASICS OF TRAJECTORY PLANNING

- Let's consider a simple 2 degree of freedom robot.
- We desire to move the robot from Point A to Point B.
- Let's assume that both joints of the robot can move at the maximum rate of 10 degree/sec.



α	β
20	30
30	40
40	50
40	60
40	70
40	80

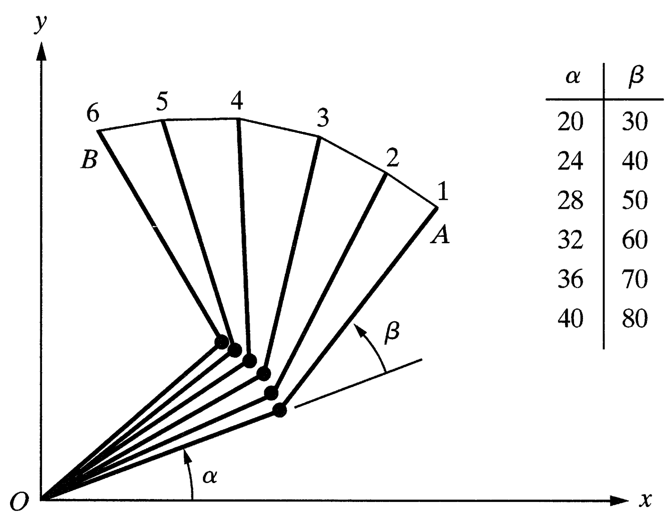
Fig. Joint-space nonnormalized movements of a robot with two degrees of freedom.

- Move the robot from A to B, to run both joints at their maximum angular velocities.
- After 2 [sec], the lower link will have finished its motion, while the upper link continues for another 3 [sec].
- The path is irregular and the distances traveled by the robot's end are not uniform.

Trajectory Planning

BASICS OF TRAJECTORY PLANNING

- **Let's assume that the motions of both joints are normalized by a <u>common factor</u> such that the joint with smaller motion will move proportionally slower and the both joints will start and stop their motion simultaneously.**



α	β
20	30
24	40
28	50
32	60
36	70
40	80

Fig. Joint-space, normalized movements of a robot with two degrees of freedom.

- **Both joints move at different speeds, but move continuously together.**
- **The resulting trajectory will be different.**

Trajectory Planning

2）Cartesian Space

- The motion of end-effector between the two points is known at all times and controllable.
- It is easy to visualize the trajectory, but is is difficult to ensure that singularity.

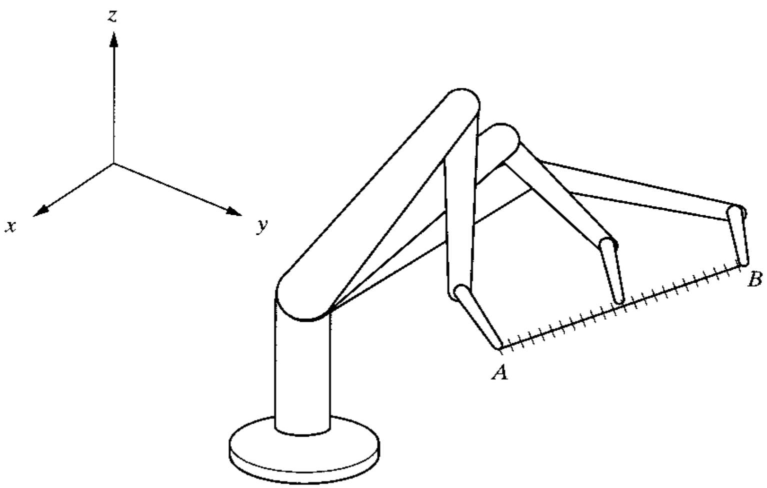


Fig. Sequential motions to follow a straight line.

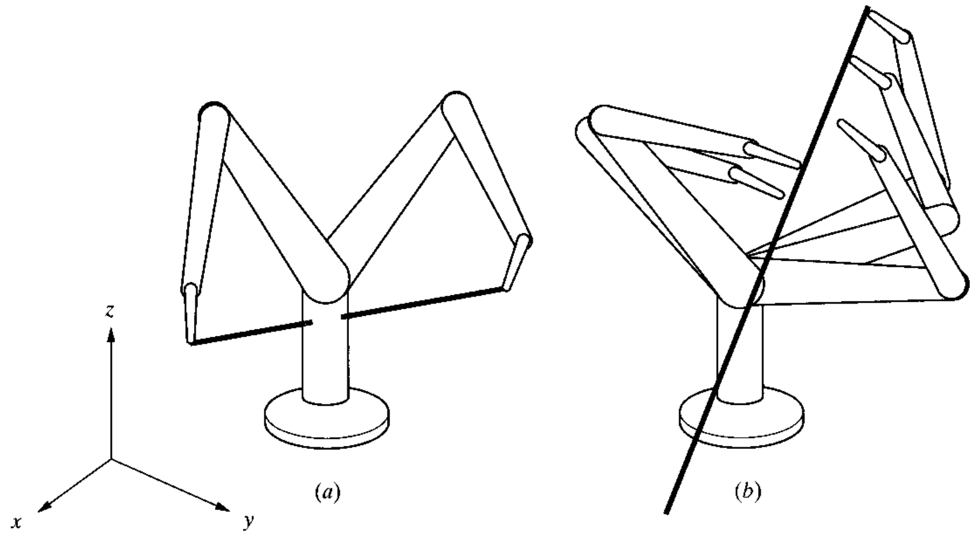


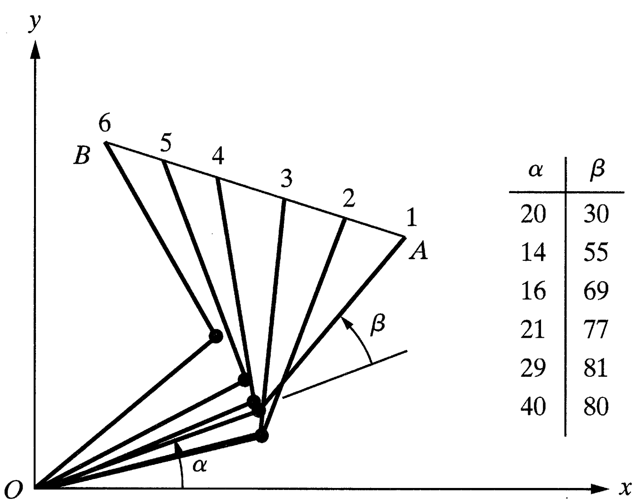
Fig. Cartesian-space trajectory (a) The trajectory may force the robot to run into itself, and (b) the trajectory may requires a sudden change in the joint angles.

Spec： predictable, increased calculation, singularity problem

Trajectory Planning

BASICS OF TRAJECTORY PLANNING

- Let's assume that the robot's hand follow a known path between point A to B with straight line.
- The simplest solution would be to draw a line between points A and B, so called interpolation.



α	β
20	30
14	55
16	69
21	77
29	81
40	80

Fig. Joint-space, normalized movements of a robot with two degrees of freedom.

- Divide the line into five segments and solve for necessary angles α and β at each point.
- The joint angles are not uniformly changing.

Trajectory Planning

BASICS OF TRAJECTORY PLANNING

- Let's assume that the robot's hand follow a known path between point A to B with straight line.
- The simplest solution would be to draw a line between points A and B, so called interpolation.

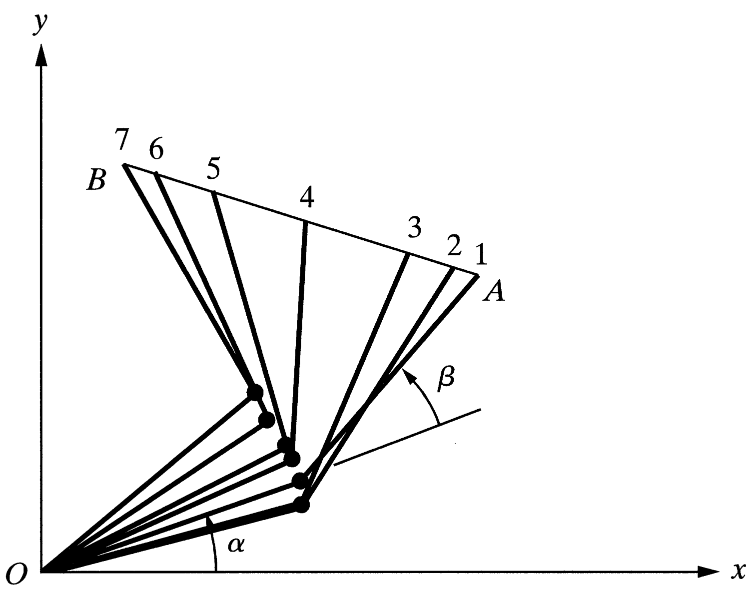


Fig. Joint-space, normalized movements of a robot with two degrees of freedom.

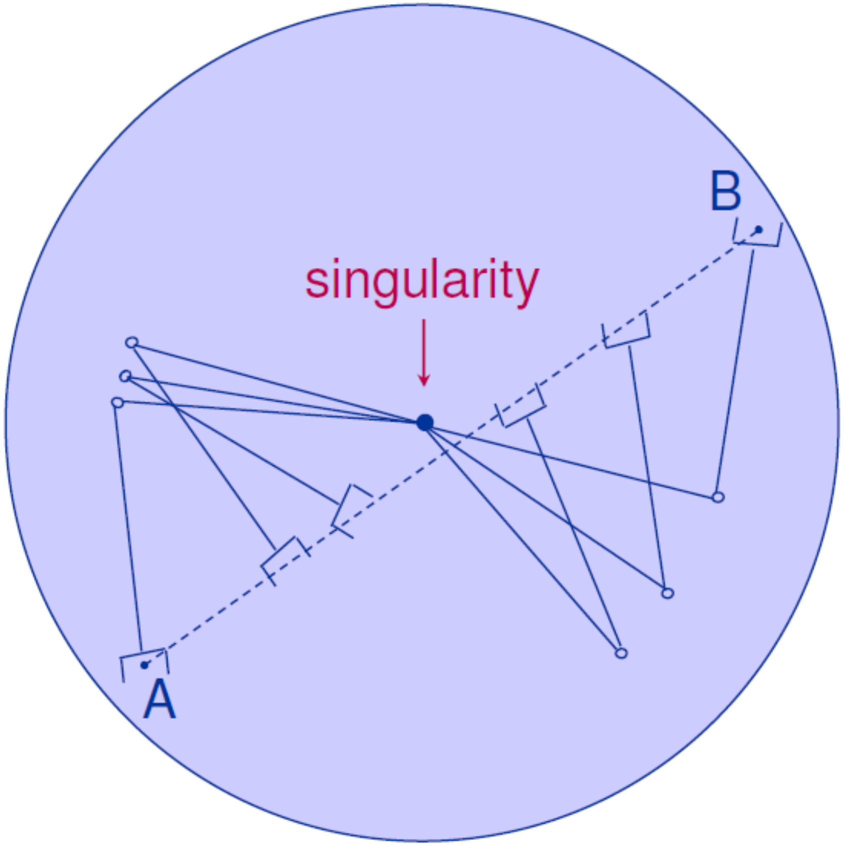
- It is assumed that the robot's actuators are strong enough to provide large forces necessary to accelerate and decelerate the joints as needed.

- Divide the segments differently.
  - The arm move at smaller segments as we speed up at the beginning.
  - Go at a constant cruising rate.
  - Decelerate with smaller segments as approaching point B.

Trajectory Planning

2） Cartesian Space

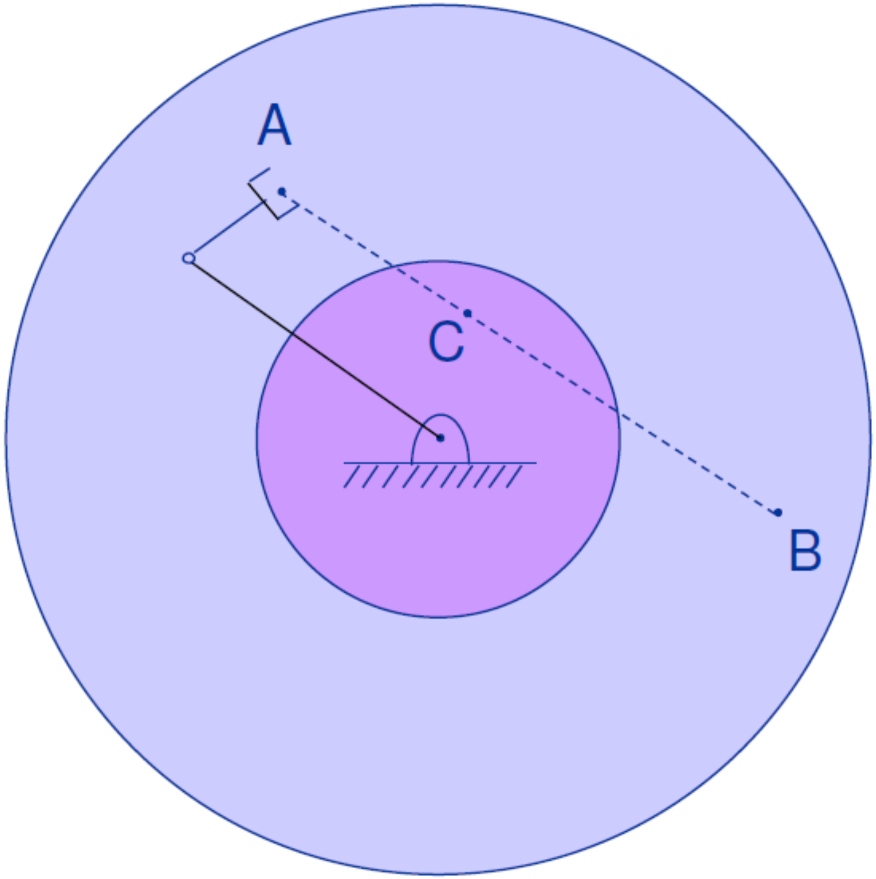
– **Approaching singularities some joint velocities go to ∞ causing deviation from the path.**



Trajectory Planning

2）Cartesian Space

- Initial Point (A) and Goal Point (B) are reachable. But Intermediate Point (C) unreachable.

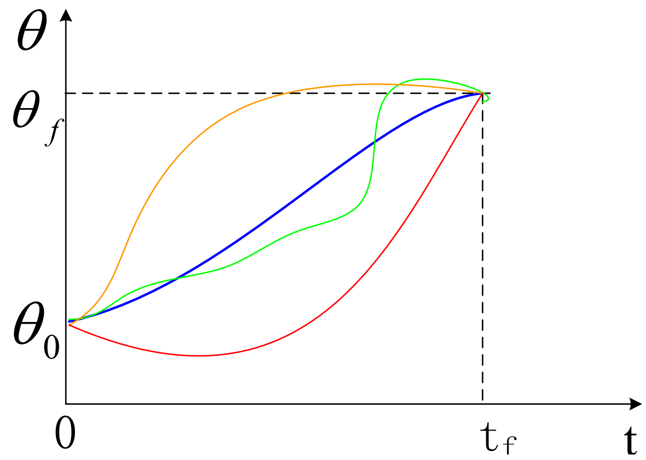


Trajectory Planning

3. Joint-Space Trajectory Planning

-How the motions of a robot can be planned in joint-space with controlled characteristics.

-Choice of interpolation function is not unique.



Several possible path shapes

The initial and final position and orientation of the robot is known, and using the inverse kinematic to find the joint angles for the initial and final position and orientation.

Trajectory Planning

1）Third-Order Polynomial Trajectory Planning

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

To achieve smooth motion, the trajectory needs at least four constraints, namely

Joint angle constraints for starting and ending points——

$$\begin{cases} \theta(0) = \theta_0 \\ \theta(t_f) = \theta_f \end{cases}$$

Joint velocity constraints for starting and ending points——

$$\begin{cases} \dot{\theta}(0) = 0 \\ \dot{\theta}(t_f) = 0 \end{cases}$$

By solving the above four equations, we can obtain：

$$a_0 = \theta_0$$

$$a_1 = 0$$

$$a_2 = \frac{3}{t_f^2}(\theta_f - \theta_0)$$

$$a_3 = -\frac{2}{t_f^3}(\theta_f - \theta_0)$$

Trajectory Planning

Example: For the 1-DoF manipulator, the initial angle $\theta_0 = 15°$, and the initial angular velocity $\dot{\theta}_0 = 0$. In order to reach the desired angle $\theta_f = 75°$, and the desired angular velocity $\dot{\theta}_f = 0$ at 3 second. Find angle, angular velocity and angular acceleration curves.

Substitute the known conditions of the above equation into the following four equations to obtain four coefficients:

$$a_0 = \theta_0 \qquad a_2 = \frac{3}{t_f^2}(\theta_f - \theta_0)$$

$$a_1 = 0 \qquad a_3 = -\frac{2}{t_f^3}(\theta_f - \theta_0)$$

We can obtain：

$$\theta(t) = 15.0 + 20.0t^2 - 4.44t^3$$

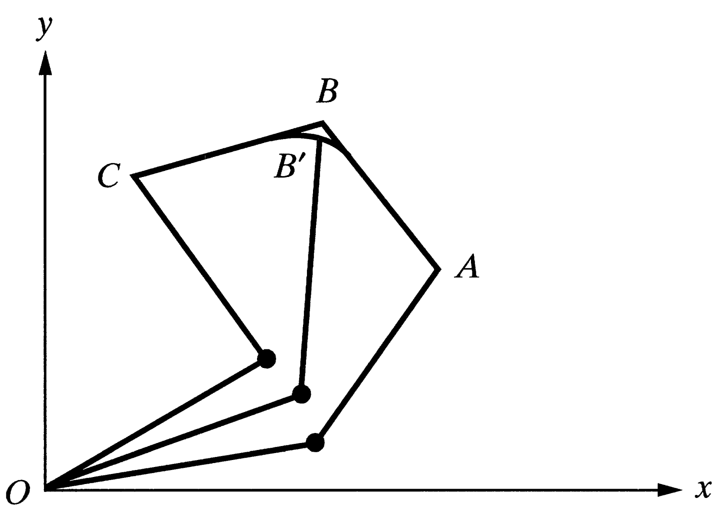
$$\dot{\theta}(t) = 40.0t - 13.32t^2$$

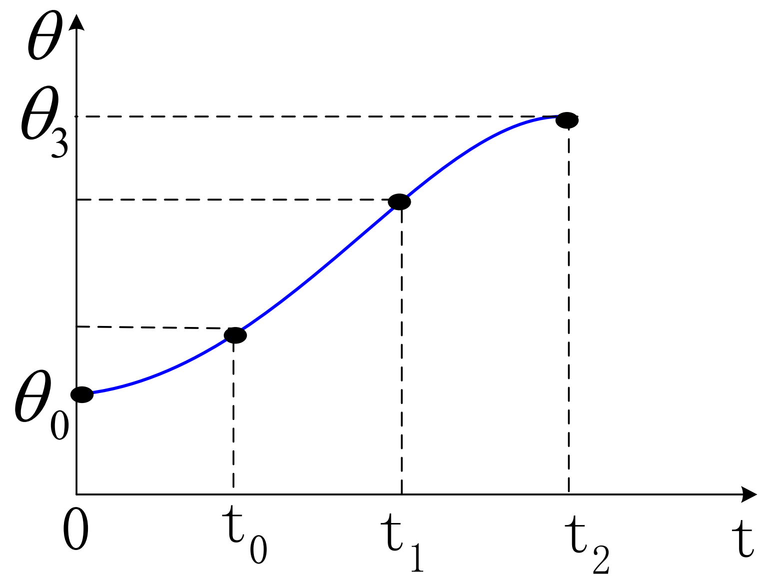
$$\ddot{\theta}(t) = 40.0 - 26.64t$$

Trajectory Planning

2) Via Middle Points

- All middle points can be regarded as "starting points" or "ending points", and therefore the corresponding joint angles and angular velocities can be solved by the inverse kinematics.
- Then the required cubic polynomial interpolation function can be determined to connect the path points smoothly.
- The difference is that these "starting" and "ending" joint velocities are no longer zero.





Trajectory Planning

Similarly, the cubic polynomial coefficient can be obtained:

$$\begin{cases} a_0 = \theta_0 \\ a_1 = \dot{\theta}_0 \\ a_2 = \dfrac{3}{t_f^2}(\theta_f - \theta_0) - \dfrac{2}{t_f}\dot{\theta}_0 - t_f \dot{\theta}_f \\ a_3 = -\dfrac{2}{t_f^3}(\theta_f - \theta_0) + \dfrac{1}{t_f^2}(\dot{\theta}_0 + \dot{\theta}_f) \end{cases}$$

The velocity constraint condition: $\dot{\theta}(0) = \dot{\theta}_0$, $\dot{\theta}(t_f) = \dot{\theta}_f$

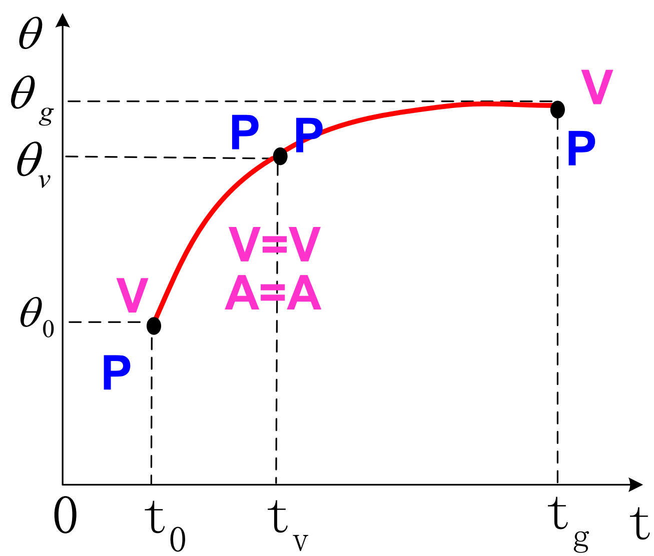
The cubic polynomial determined by the above equation describes the trajectory of the starting and ending points with any given position and speed. The following problem is how to determine the joint velocity at the path point.

Trajectory Planning

（1） Velocity and acceleration continuity constraints

In order to ensure the continuity of velocity and acceleration at the waypoint, two cubic curves can be connected at the waypoint according to certain rules to piece together the desired trajectory.

The constraint condition is: not only the speed of the connection is continuous, but also the acceleration should be continuous.

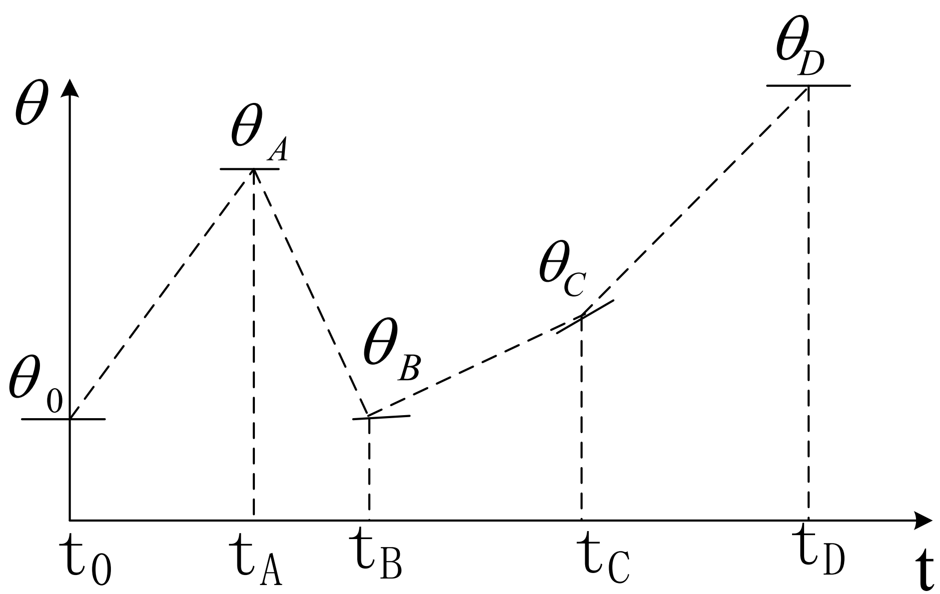


Eight equations
Eight unknowns

Trajectory Planning

（2）Slope designation method

If the slope of the adjacent line segment changes its sign at the via point, the speed is set to zero; If adjacent line segments do not change its sign, the average of the slope of the line segments on either side of the waypoint is chosen as the speed of that point.



The acceleration at the via point may be discontinuous.

Trajectory Planning

- If the requirements for the trajectory are more strict and the constraints are increased. The cubic polynomial cannot meet the needs in this case, and the path segment of the trajectory must be interpolated with higher order polynomials.
- For example, if the joint positions, velocities, and accelerations (with six unknown coefficients) are specified for both the starting and ending points of a path, a quintic polynomial is used to interpolate.

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

Trajectory Planning

4. Cartesian-Space Trajectory Planning

- Cartesian-space trajectories relate to the motions of a robot relative to the Cartesian reference frame.
- In Cartesian-space, the joint values must be repeatedly calculated through the inverse kinematic equations of the robot.
- **Computation Loop Algorithm：**

(1) Increment the time by $t=t+\Delta t$.

(2) Calculate the position and orientation of the hand based on the selected function for the trajectory.

(3) Calculate the joint values for the position and orientation through the inverse kinematic equations of the robot.

(4) Send the joint information to the controller.

(5) Go to the beginning of the loop.

Trajectory Planning

- Trajectory planning can be carried out in both joint space and cartesian space. Many methods can be used in both Spaces.
- Cartesian space trajectory planning is more intuitive, but more difficult to calculate and plan. A specified path (such as a line) must be planned in Cartesian space.
- If the path of the robot is not specified, the trajectory planning of the joint space is easier to calculate.

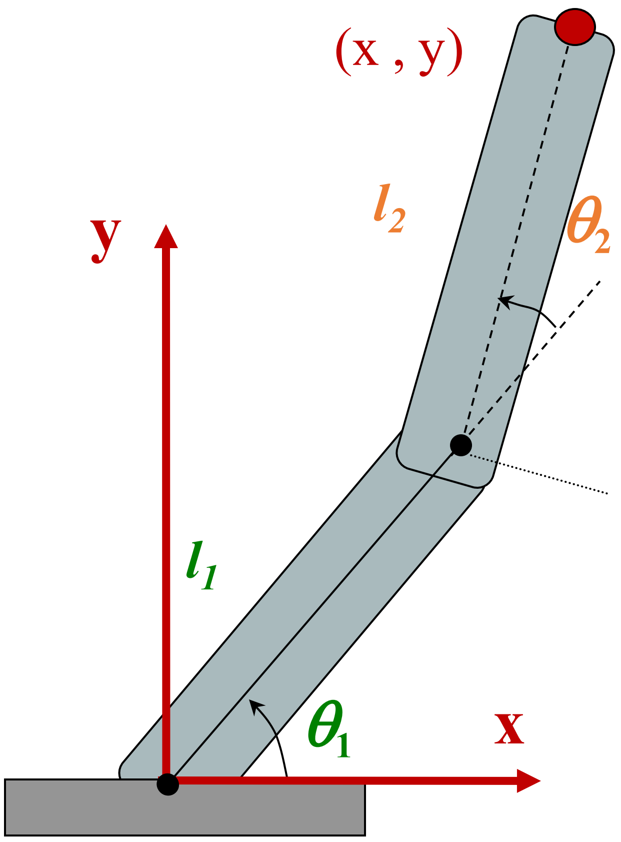
Homework 16

Sakai： http://bb.sustech.edu.cn

Due date: May 7, 2025

For 2-DoF manipulator, the initial angle $\theta_1 = \pi/6$, $\theta_2 = \pi/5$ and the initial angular velocities are both zero. In order to reach the desired angle $\theta_1 = \pi/2$, $\theta_2 = \pi/2$ and the desired angular velocities are also zero at 3 second.

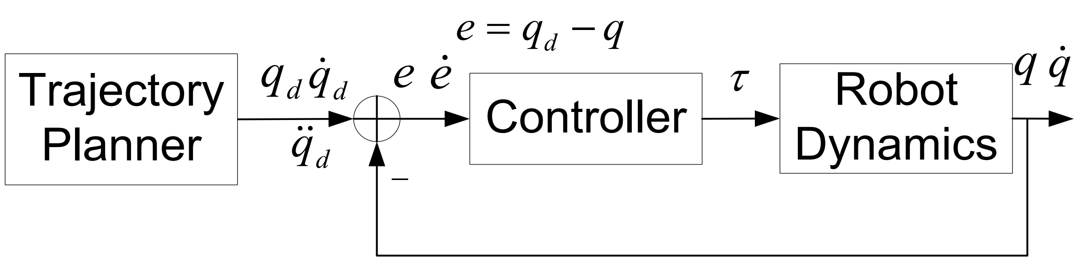
Plot angles, angular velocities and angular accelerations versus time, with the method of third-order polynomial trajectory planning.



Introduction to Control

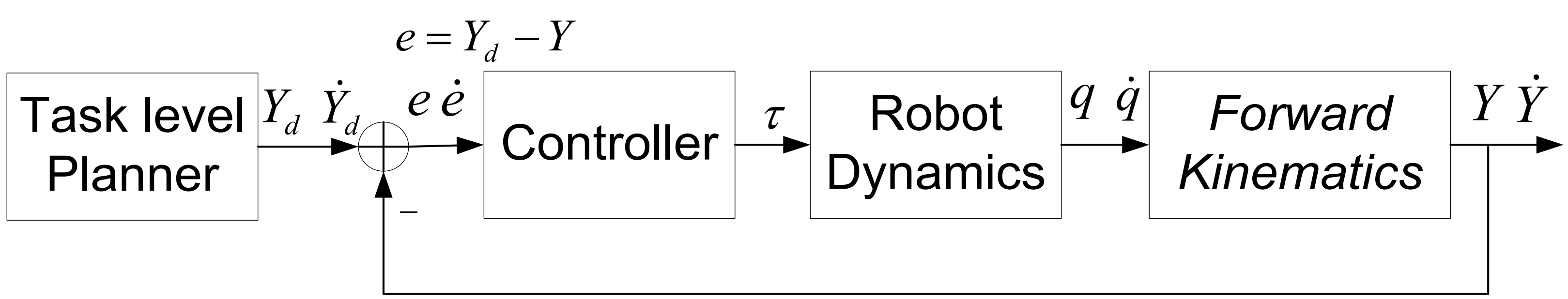
- **Robot model:** $\begin{cases} D(q)\ddot{q} + C(q,\dot{q}) + G(q) = \tau \\ Y = h(q) \end{cases}$

- **Joint space**



Find a control input (τ), $\quad q \to q_d \quad as \quad t \to \infty$

- **Task space**



Find a control input (τ), $\quad Y \to Y_d \quad as \quad t \to \infty \quad e = Y_d - Y \to 0$

Summary

- Trajectory Planning
  - Problem Definition
  - Trajectories for Point-to-Point Motion
  - Trajectories for Paths Specified by Via Points
- Introduction to Control