



南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

Robot Modeling & Control ME331

Section 2: Kinematics I

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Outline

- **Basic Concepts**
 - **What is a robot?**
 - **Why use robots?**
- **Robot Manipulators**
 - **Robot Geometry**
 - **Robot Specification**
 - Number of Axes, DoF (Degree of Freedom)
 - Accuracy (精度), Repeatability (重复定位精度)
 - Workspace
- **Kinematics**
 - **Preliminary**
 - World Frame, Joint Frame, End-effector Frame
 - Rotation Matrix
 - Composite Rotation Matrix

What is a robot?

- **Origin of the word “robot”**
 - Czech word “robota” – labor, “robotnik” – workman
 - 1923 play by Karel Capek – Rossum’s Universal Robots
- **Definition:** (no precise definition yet)
 - Webster’s Dictionary
 - An automatic device that performs functions ordinarily ascribed to human beings. ➔ washing machine = robot?
 - Robotics Institute of American
 - A robot (industrial robot) is a **reprogrammable**, **multifunctional manipulator** designed to move materials, parts, tools, or specialized devices, through variable programmed motions for the performance of a variety of tasks.

What is a robot?

- Hollywood's imagination



STAR WARS 1977



TERMINATOR 1984



I, ROBOT 2004



EX MACHINA 2015

What is a robot?

- **By general agreement, a robot is:**

A programmable machine that imitates the actions or appearance of an intelligent creature—usually a human.

- **To qualify as a robot, a machine must be able to:**
 - 1) Sensing and perception: obtain information from its surroundings
 - 2) Carry out different tasks: locomotion or manipulation, do something physical—such as move or manipulate objects
 - 3) Re-programmable: can do different things
 - 4) Function autonomously and/or interact with human beings

Types of Robots

- Locomotion



Aerial robot



Underwater robot



Legged robot



Humanoid



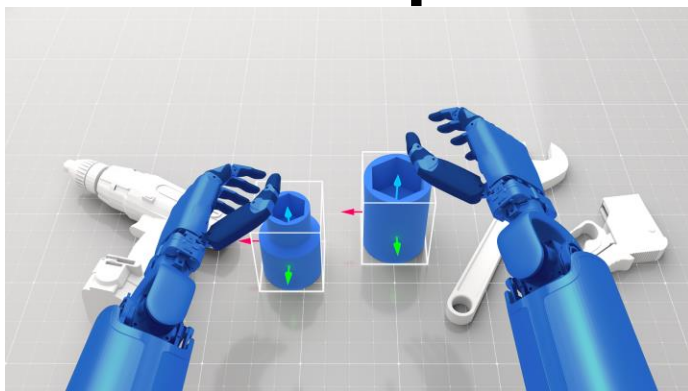
Wheeled mobile robot



Tracked robot

Types of Robots

- **Robot Manipulators**



- **Mobile Manipulators**



Why Use Robots?

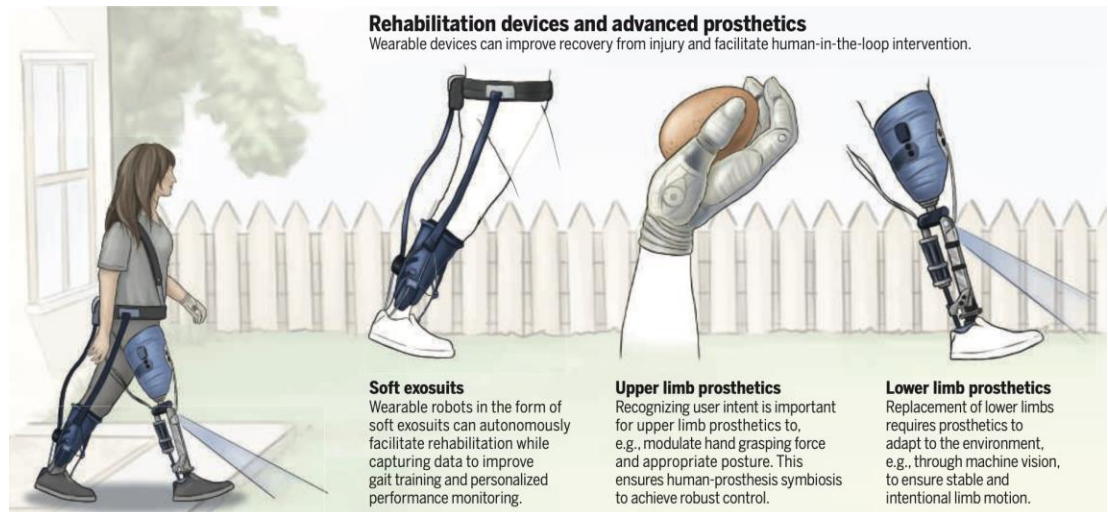
- **Application in 4D environments**

- **Dangerous**
- **Dirty**
- **Dull**
- **Difficult**



- **4A tasks**

- **Automation**
- **Augmentation**
- **Assistance**
- **Autonomous**



Why Use Robots?

- **Increase product quality**

- Superior Accuracies (thousands of an inch, wafer-handling: microinch)
- Repeatable precision → Consistency of products

- **Increase efficiency**

- Work continuously without fatigue
- Need no vacation

- **Increase safety**

- Operate in dangerous environment
- Need no environmental comfort – air conditioning, noise protection, etc

- **Reduce cost**

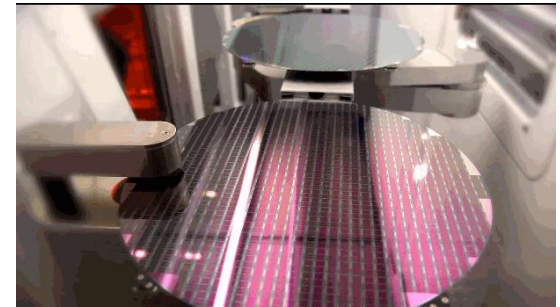
- Reduce scrap rate (废品率)
- Lower in-process inventory (在制品库存)
- Lower labor cost (人工成本)

- **Reduce manufacturing lead time (制造周期)**

- Rapid response to changes in design

- **Increase productivity**

- Value of output per person per hour increases



Why Use Robots?

- **Three Laws of Robotics – by Isaac Asimov**



1. A robot must not harm a human being or, through inaction, allow one to come harm.
2. A robot must always obey human beings unless that is conflict with the first law.
3. A robot protect itself from harm unless that is conflict with the first or second laws.

Why Use Robots?

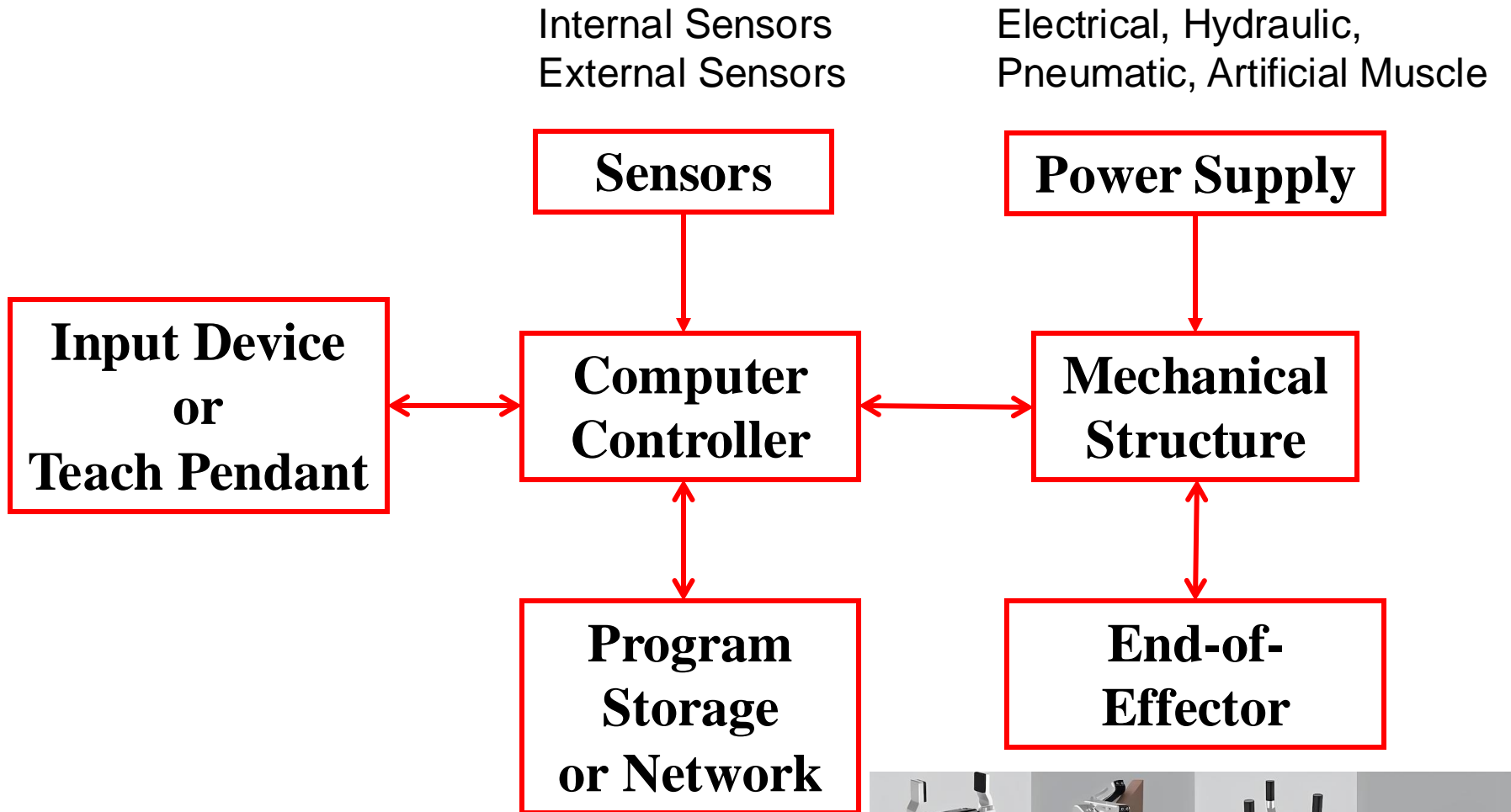
- **Three Laws of Robotics**

The 0th Law of Robotics: A robot may not injure humanity, or, through inaction, allow humanity to come to harm.



In the movie, it is explained that the reason that the robots revolted was because the main computer VIKI explains that to save and protect humanity, it needed to take freedom from humans.

Architecture of Robotic Systems



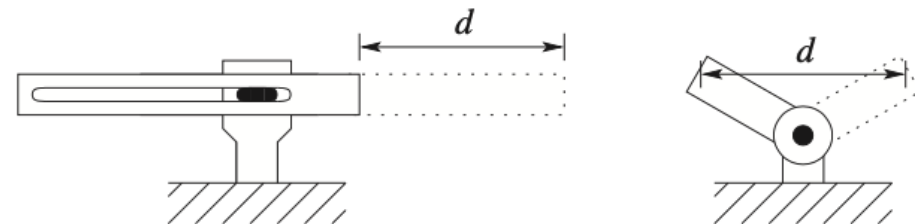
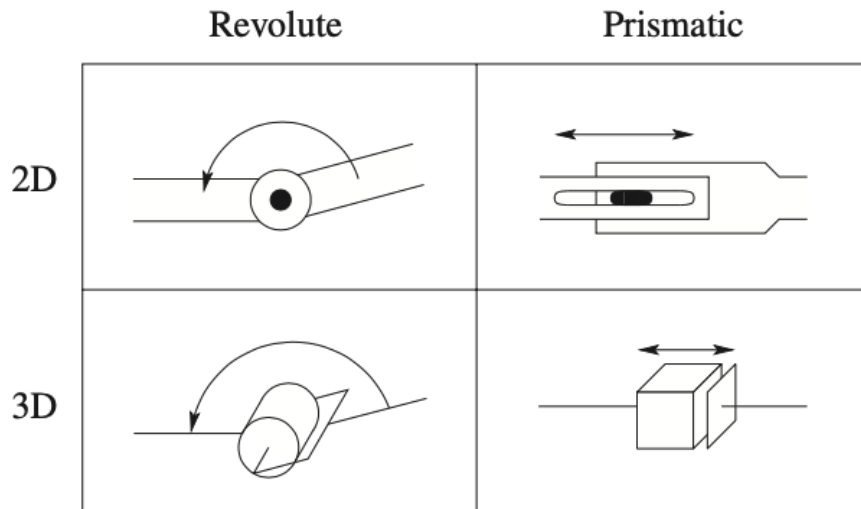
Summary

- **Robotics--interdisciplinary research**
 - Mechanical engineering
 - Computer science and engineering
 - Electrical engineering
 - Materials science and engineering
 - Biomedical engineering, biomechanics
 - Cognitive psychology, perception and neuroscience
- **Research open problems**
 - Manipulation, Locomotion
 - Control, Navigation
 - Human-Robot Interaction
 - Learning & Adaptation (AI)

Manipulators

• Symbolic Representation of Robot Manipulators

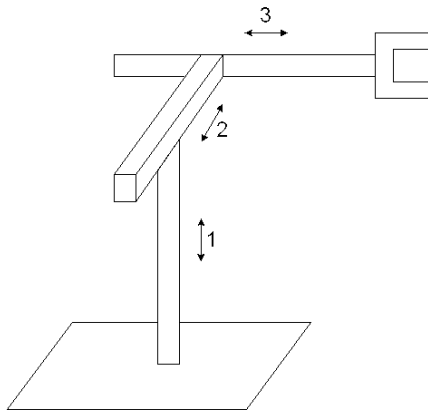
- ✓ Robot manipulators are composed of **links** connected by **joints** to form a **kinematic chain**.
- ✓ Joints are typically rotary (**revolute**) or linear (**prismatic**).
- ✓ A **revolute** is like a hinge and allows relative rotation between two joint links.
- ✓ A **prismatic** joint allows a linear relative motion between two links.
- ✓ We denote revolute joints by **R** and prismatic joints by **P**.



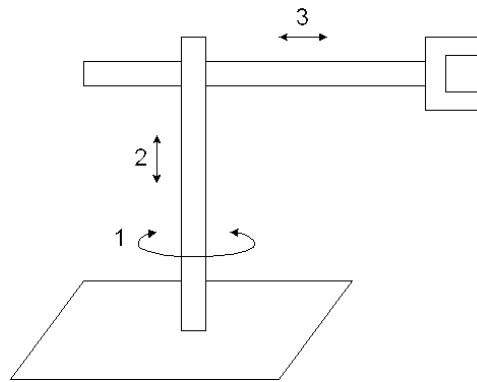
Linear vs. rotational link motion

Manipulators

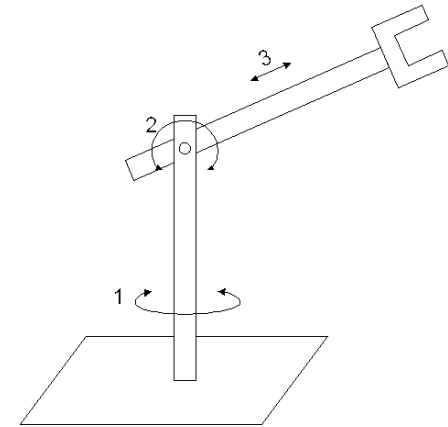
- Geometry:



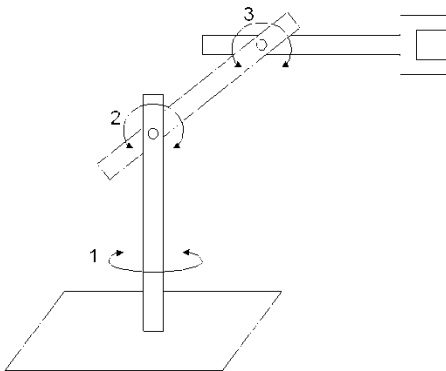
直角坐标式: PPP
(Cartesian)



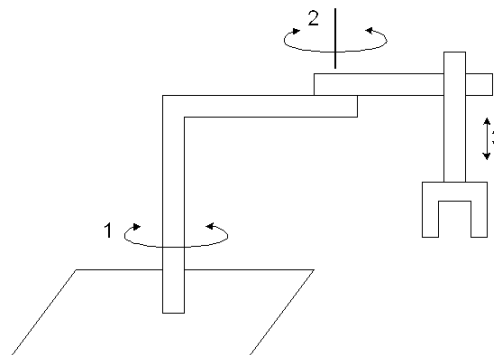
圆柱坐标式: RPP
(Cylindrical)



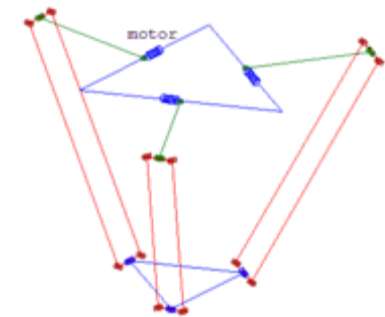
球坐标式: RRP
(Spherical)



多关节式: RRR
(Articulated)

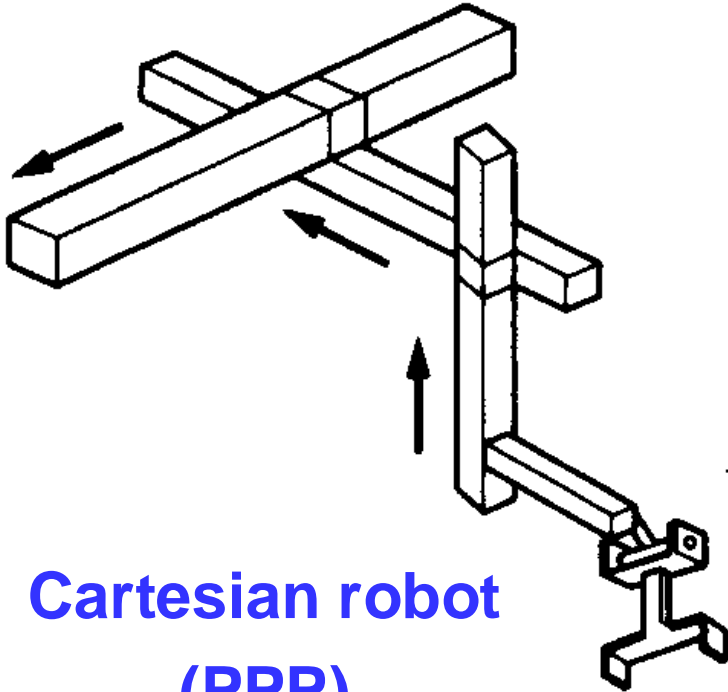


SCARA式: RRP
(**S**elective **C**ompliance
Assembly **R**obot **A**rm)



并联式:
(Parallel Robot)

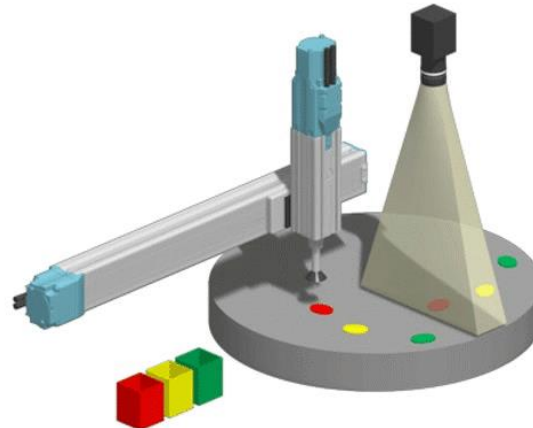
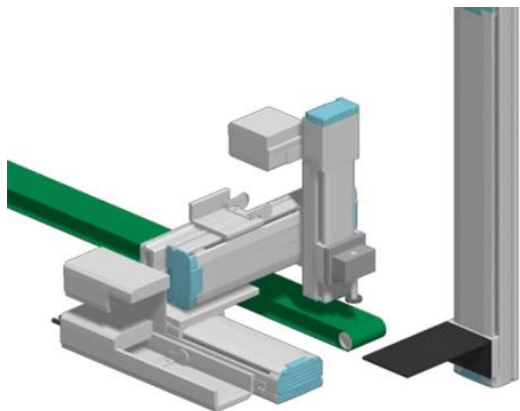
Manipulators



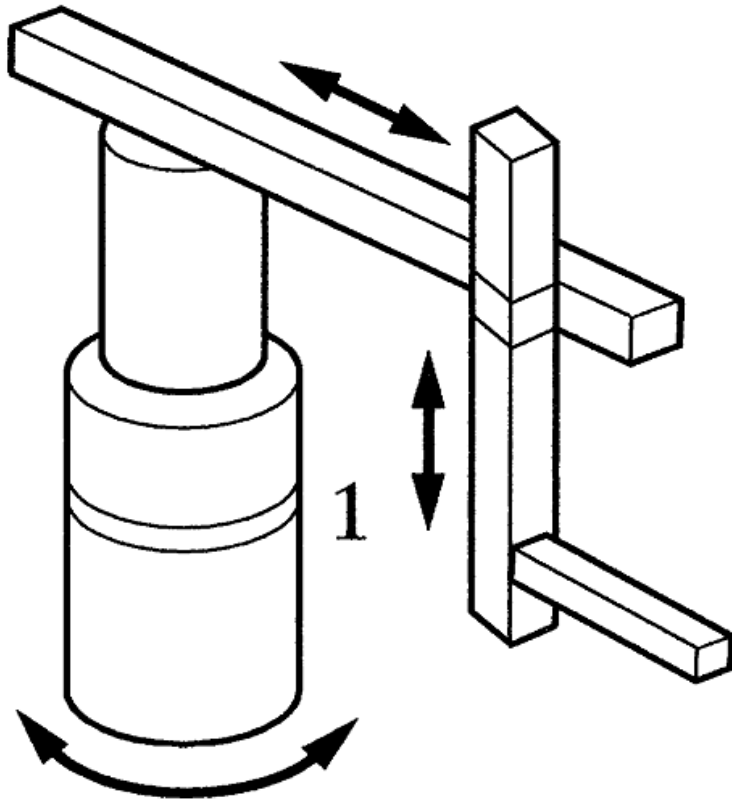
**Cartesian robot
(PPP)**

- ✓ Simple structure
- ✓ Easy to control
- ✓ Dependable and precise
- ✗ Slow (large inertia)
- ✗ Requiring a vast amount of space

Application: pick-and-place



Manipulators



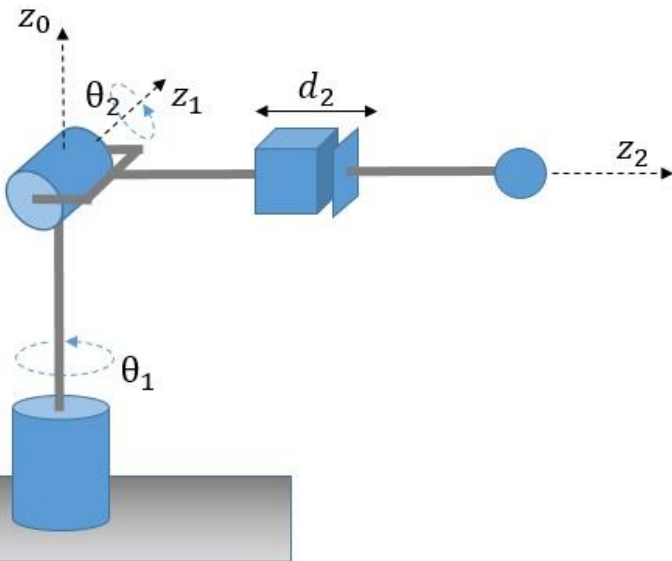
**Cylindrical robot
(RPP)**

- ✓ **Faster than Cartesian robots**
- ✗ **Mechanical rigidity is decreased**
- ✗ **The repeatability and accuracy are also less in the direction of rotation.**
- ✗ **A more sophisticated control scheme is needed**

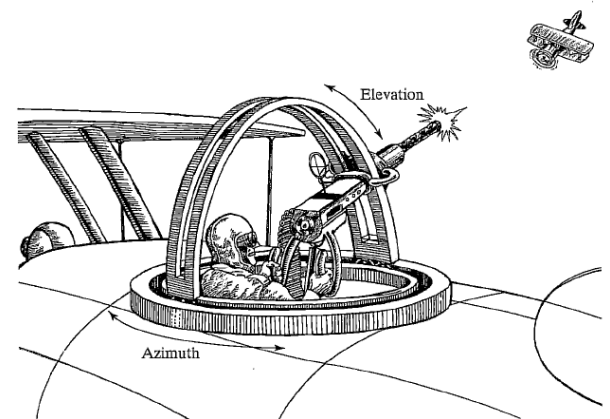
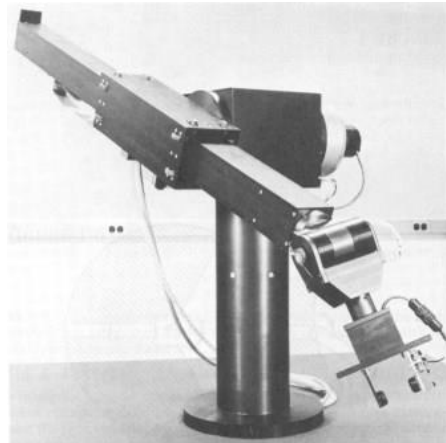
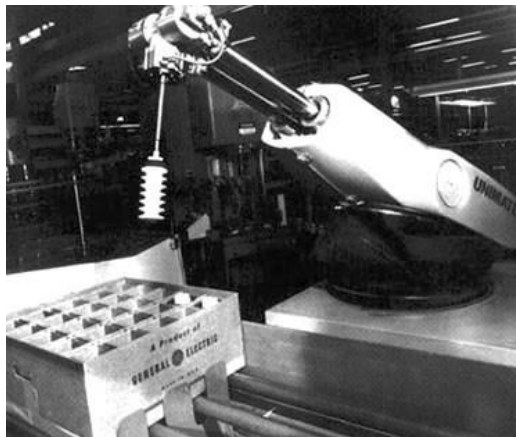
Application: commonly used in small spaces, and they are ideal for objects that require circular symmetry. They're also well-suited to standard pick-and-place work in manufacturing settings.

Manipulators

Spherical robot (RRP)

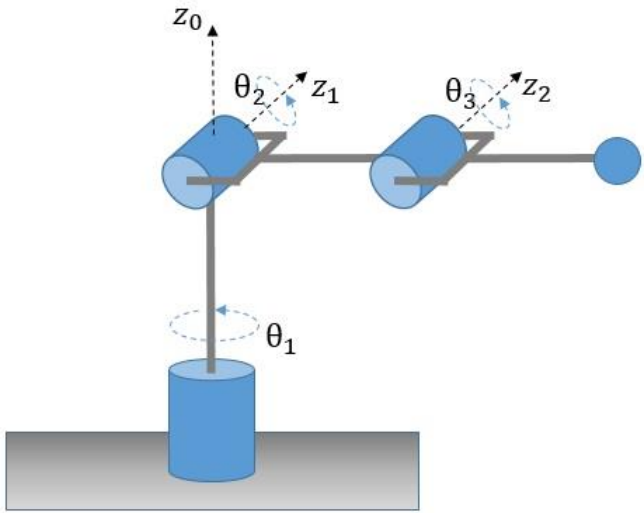


- The spherical manipulator, also called polar manipulator, has three mutually perpendicular axes.
- The joint variables are the spherical coordinates of the end-effector with respect to the base.
- Spherical manipulators have lost practicality in the workplace due to 6 axes articulated manipulators.



Manipulators

Articulated robot (RRR)



- ✓ Faster than Cartesian robots
- ✓ Easy to align to different planes
- ✓ Relatively large movement freedom in a compact space
- ✓ Installation is versatile
- ✓ Reusability
- ✗ Complex kinematics to control
- ✗ Large inertia



ABB



FANUC



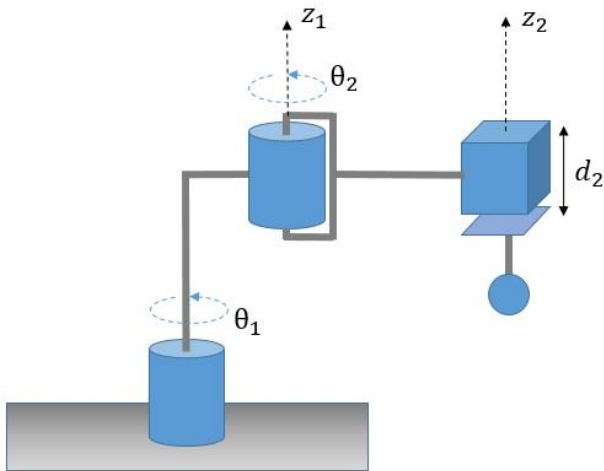
KUKA



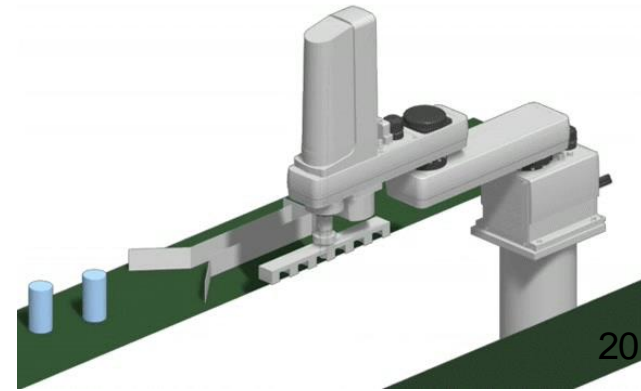
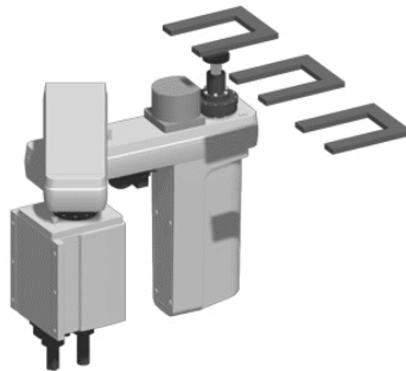
YASKAWA

Manipulators

SCARA robot (RRP)



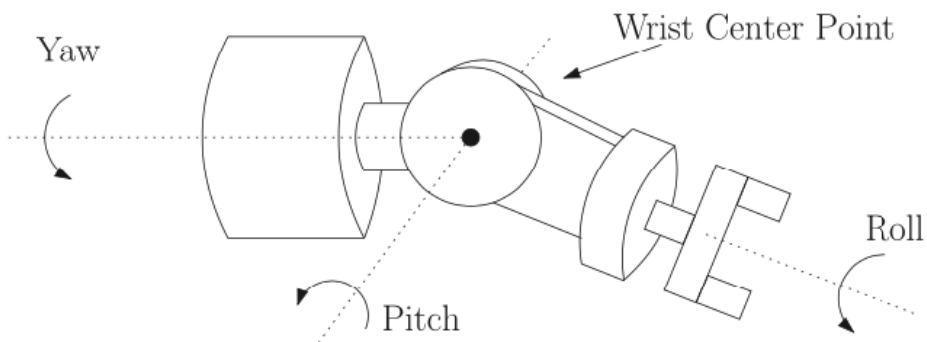
- SCARA is the short for **Selective Compliance Assembly Robot Arm**.
- It is RRP but the axes are parallel (parallel z_0, z_1, z_2).
- Specially designed for assembly operations.
- The SCARA construction allow for precise work and quick assembly or packaging.



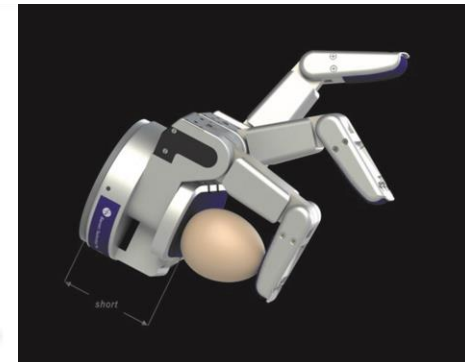
Manipulators

Wrist and End Effectors

- The joints in the kinematic chain between the arm and end effector are referred to as the **wrist**. The wrist joints are nearly always all revolute. It is increasingly common to design manipulators with **spherical wrists**.
- The arm and wrist assemblies of a robot are used primarily for positioning the hand, **end effector**, and any tool it may carry. It is the end effector or tool that actually performs the task.



The spherical wrist



End effectors

Manipulators

- **Motion Control Methods**
 - **Point to point control**
 - a sequence of discrete points
 - spot welding, pick-and-place, loading & unloading
 - **Continuous path control**
 - follow a prescribed path, controlled-path motion
 - spray painting, arc welding, gluing

Manipulators

- **Robot Specifications**

- **Number of Axes**

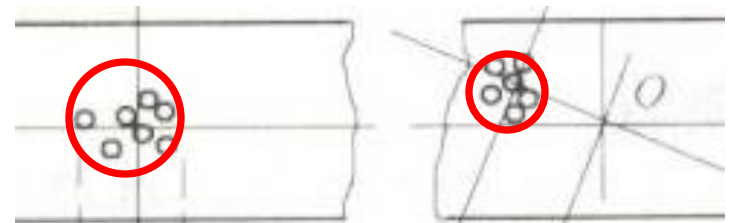
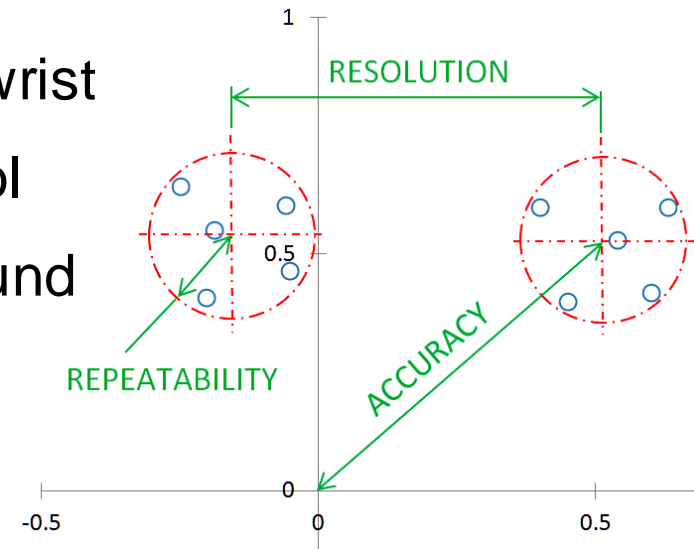
- Major axes, (1-3) => Position the wrist
 - Minor axes, (4-6) => Orient the tool
 - Redundant, (7-n) => reaching around obstacles, avoiding undesirable configuration

- **Degree of Freedom (DoF)**

- **Payload (load capacity)**

- **Accuracy v.s. Repeatability**

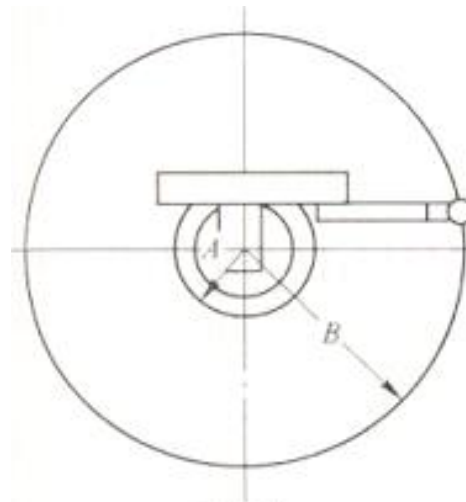
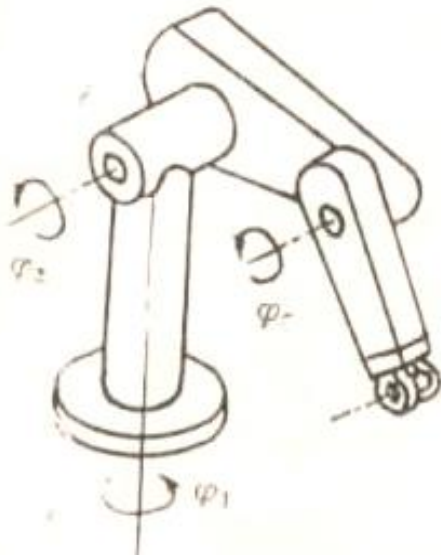
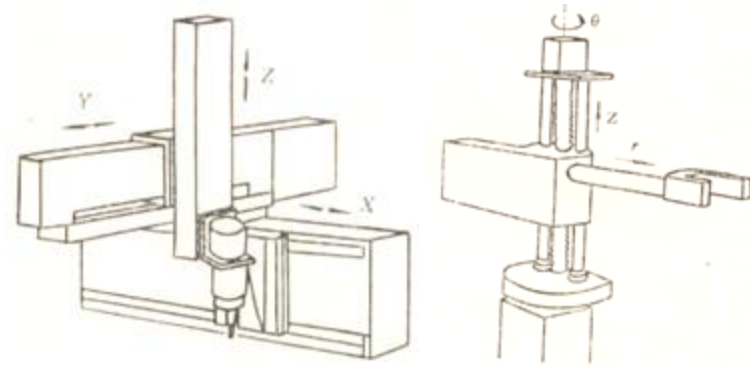
Which one is more important?



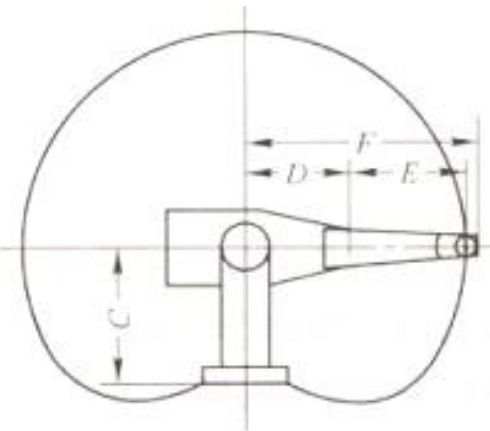
Manipulators

- Robot Specifications

Workspace (工作空间) : the total volume swept out by the end effector as the manipulator executes all possible motions.



顶视图



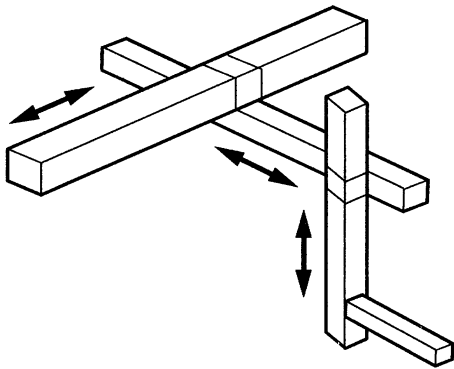
侧视图

PUMA 机器人工作空间

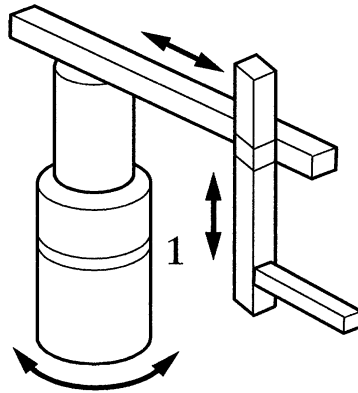
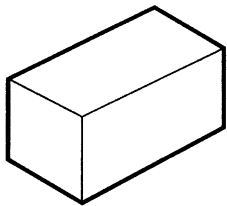
Manipulators

- **Robot Specifications**

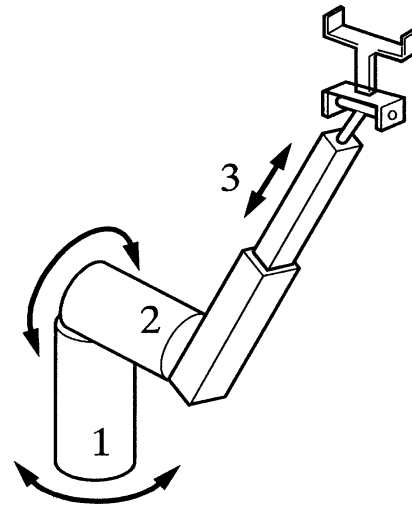
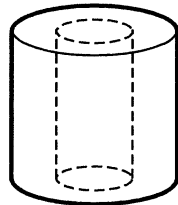
Typical workspace of several geometric types



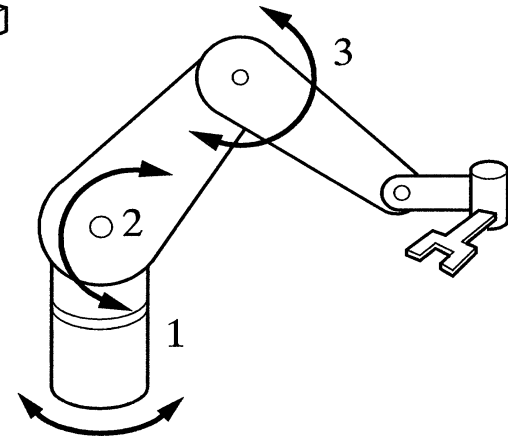
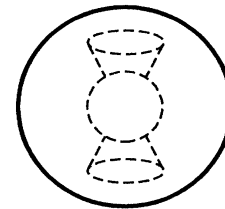
Cartesian



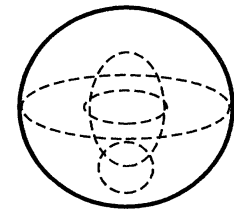
Cylindrical



Spherical



Articulated

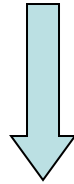


What is Kinematics

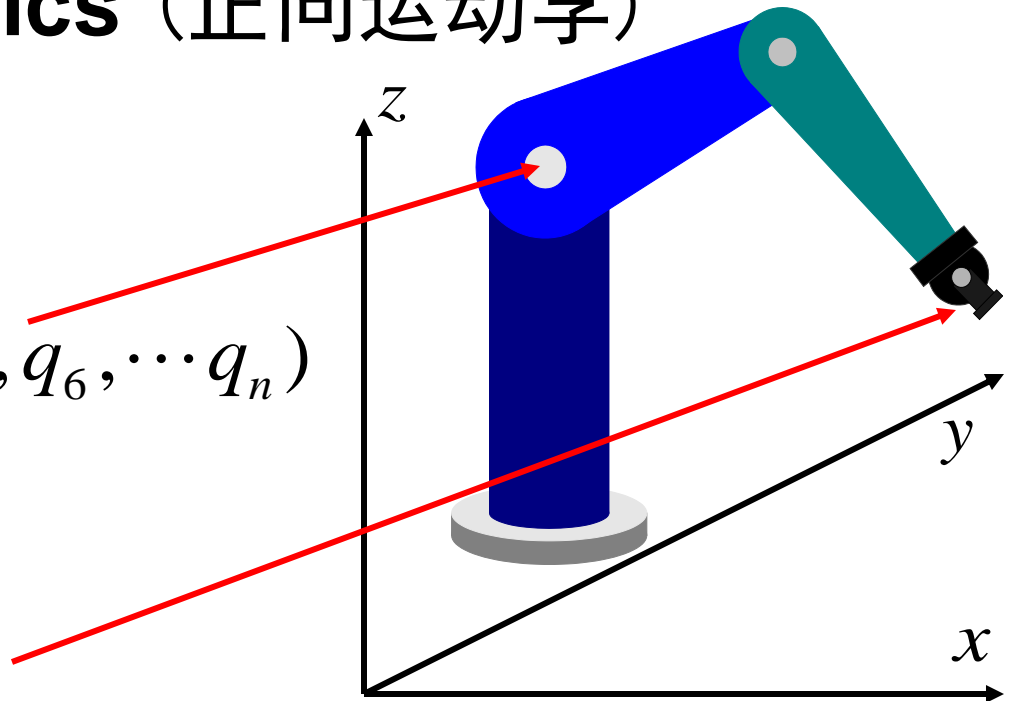
- **Forward kinematics (正向运动学)**

Given joint variables

$$q = (q_1, q_2, q_3, q_4, q_5, q_6, \dots, q_n)$$



$$Y = (x, y, z, O, A, T)$$



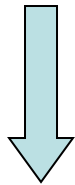
End- effector position and orientation - Formula?

What is Kinematics

- **Inverse kinematics** (逆向运动学)

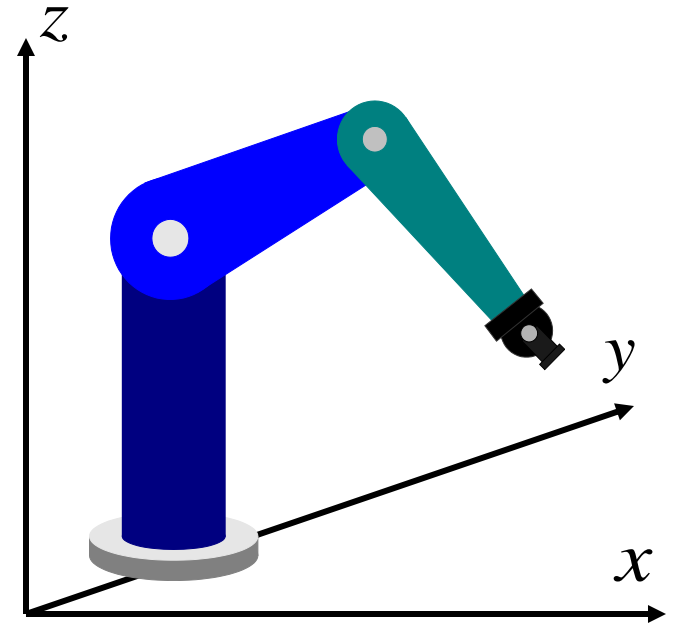
Given end effector position and orientation

$$(x, y, z, O, A, T)$$



$$q = (q_1, q_2, q_3, q_4, q_5, q_6, \dots, q_n)$$

Joint variables - Formula?



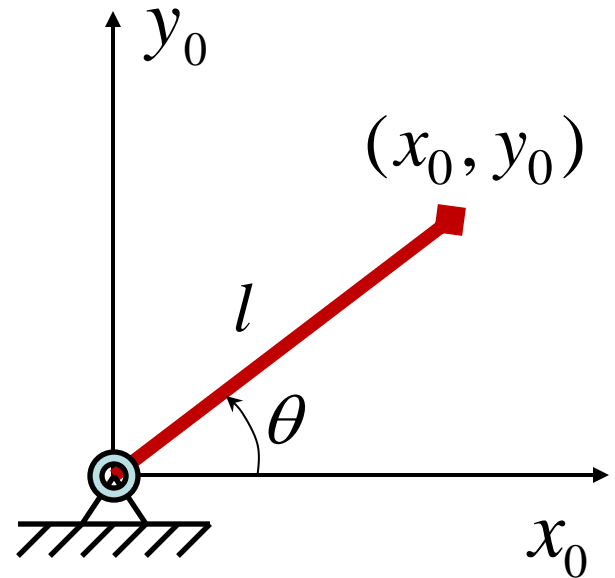
Example 1

- **Forward kinematics**

$$\begin{cases} x_0 = l \cos \theta \\ y_0 = l \sin \theta \end{cases}$$

- **Inverse kinematics**

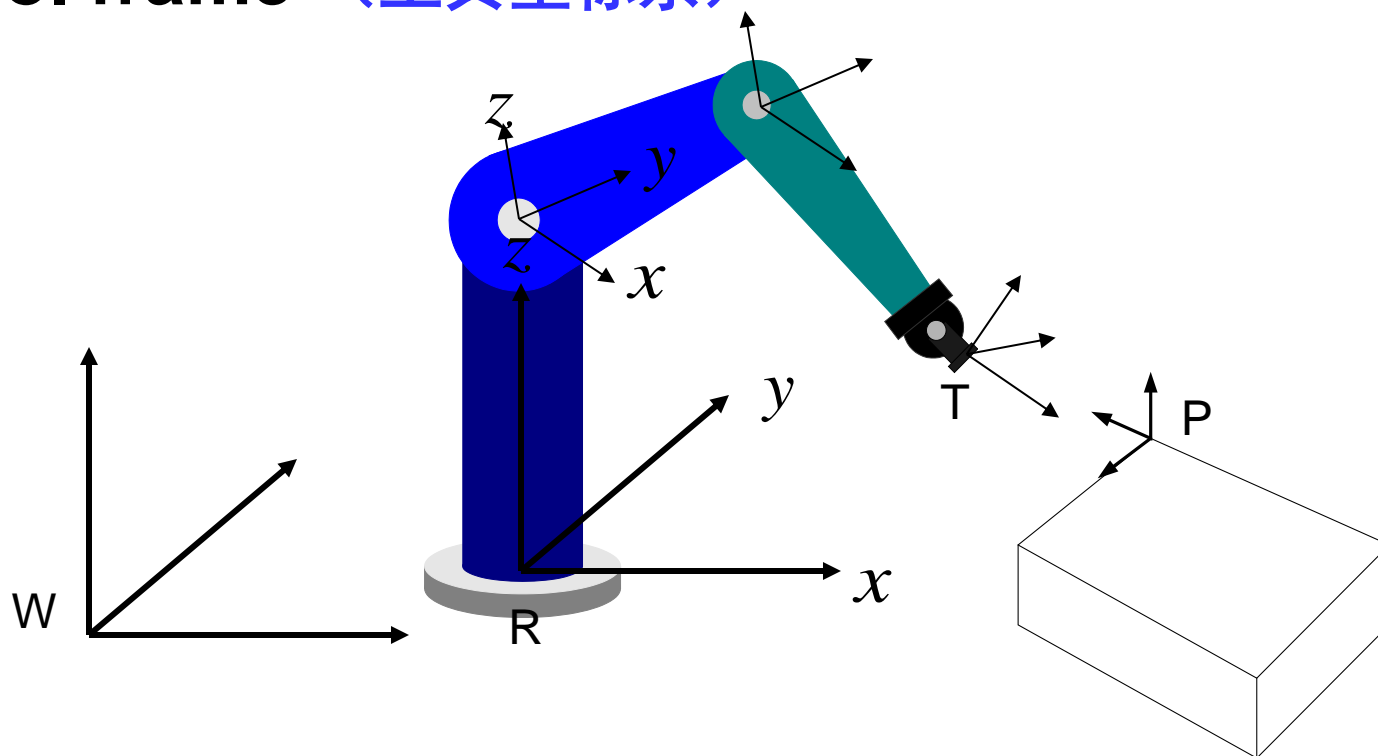
$$\theta = \cos^{-1}(x_0 / l)$$



Preliminary

- **Robot Reference Frames**

- World frame (世界坐标系, 全局坐标系)
- Joint frame (关节坐标系, 结体坐标系)
- Tool frame (工具坐标系)



Preliminary

- **Coordinate Transformation**
 - Reference coordinate frame OXYZ
 - Body-attached frame O'UVW

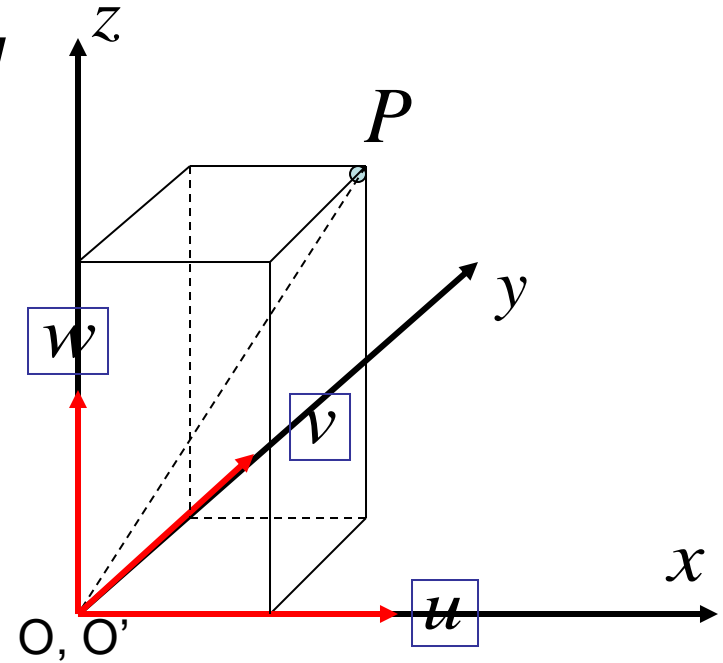
Point *P* represented in OXYZ:

$$P_{xyz} = [p_x, p_y, p_z]^T$$

$$\vec{P}_{xyz} = p_x \mathbf{i}_x + p_y \mathbf{j}_y + p_z \mathbf{k}_z$$

Point *P* represented in O'UVW:

$$\vec{P}_{uvw} = p_u \mathbf{i}_u + p_v \mathbf{j}_v + p_w \mathbf{k}_w$$



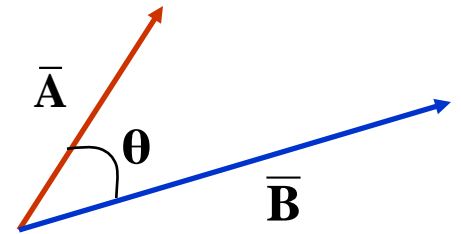
Two frames coincide $\longrightarrow p_u = p_x \quad p_v = p_y \quad p_w = p_z$

Preliminary

Dot Product

Let A and B be arbitrary vectors in R^3 and θ be the angle from A to B , then

$$A \cdot B = \|A\| \cdot \|B\| \cos \theta$$



Properties of orthonormal coordinate frame

- Mutually perpendicular
- Unit vectors

$$\vec{i} \cdot \vec{j} = 0$$

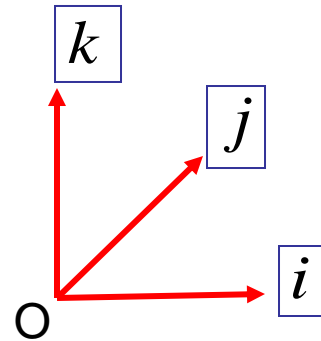
$$\vec{i} \cdot \vec{k} = 0$$

$$\vec{k} \cdot \vec{j} = 0$$

$$|\vec{i}| = 1$$

$$|\vec{j}| = 1$$

$$|\vec{k}| = 1$$



Preliminary

- **Coordinate Transformation**

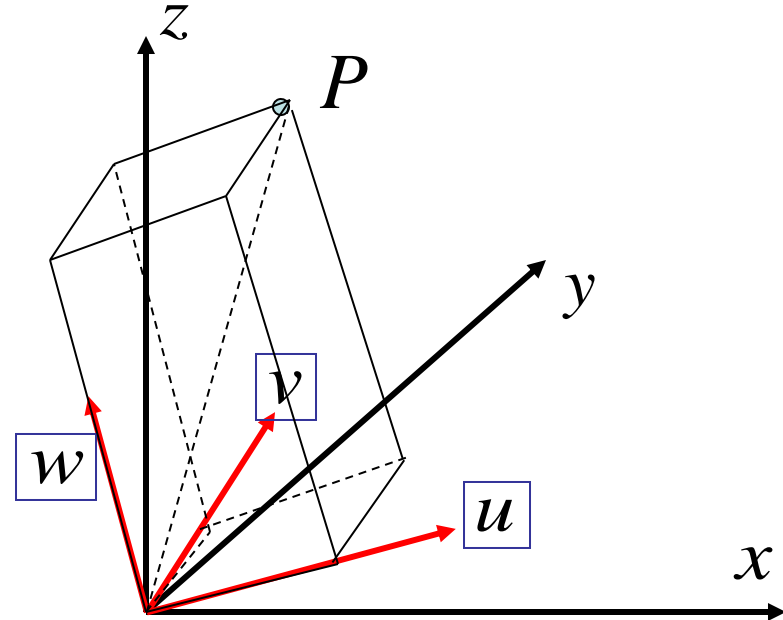
- **Rotation only**

P represented in OXYZ:

$$\vec{P}_{xyz} = p_x \mathbf{i}_x + p_y \mathbf{j}_y + p_z \mathbf{k}_z$$

P represented in OUVW:

$$\vec{P}_{uvw} = p_u \mathbf{i}_u + p_v \mathbf{j}_v + p_w \mathbf{k}_w$$



How to relate the coordinate in these two frames?

$$P_{xyz} = RP_{uvw}$$

 **Basic Rotation Matrix**

Preliminary

- **Basic Rotation**

- p_x , p_y , and p_z represented the projections of P onto **OX**, **OY**, **OZ** axes, respectively.

- Since $P = p_u \mathbf{i}_u + p_v \mathbf{j}_v + p_w \mathbf{k}_w$

- **Then**

$$p_x = \mathbf{i}_x \cdot P = \mathbf{i}_x \cdot \mathbf{i}_u p_u + \mathbf{i}_x \cdot \mathbf{j}_v p_v + \mathbf{i}_x \cdot \mathbf{k}_w p_w$$

$$p_y = \mathbf{j}_y \cdot P = \mathbf{j}_y \cdot \mathbf{i}_u p_u + \mathbf{j}_y \cdot \mathbf{j}_v p_v + \mathbf{j}_y \cdot \mathbf{k}_w p_w$$

$$p_z = \mathbf{k}_z \cdot P = \mathbf{k}_z \cdot \mathbf{i}_u p_u + \mathbf{k}_z \cdot \mathbf{j}_v p_v + \mathbf{k}_z \cdot \mathbf{k}_w p_w$$

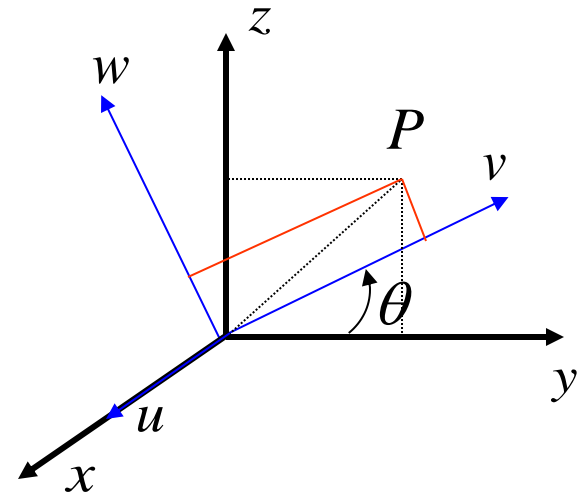
Preliminary

- **Basic Rotation Matrix**

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} \mathbf{i}_x \cdot \mathbf{i}_u & \mathbf{i}_x \cdot \mathbf{j}_v & \mathbf{i}_x \cdot \mathbf{k}_w \\ \mathbf{j}_y \cdot \mathbf{i}_u & \mathbf{j}_y \cdot \mathbf{j}_v & \mathbf{j}_y \cdot \mathbf{k}_w \\ \mathbf{k}_z \cdot \mathbf{i}_u & \mathbf{k}_z \cdot \mathbf{j}_v & \mathbf{k}_z \cdot \mathbf{k}_w \end{bmatrix} \begin{bmatrix} p_u \\ p_v \\ p_w \end{bmatrix}$$

– **Rotation about x-axis with θ**

$$Rot(x, \theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta & -S\theta \\ 0 & S\theta & C\theta \end{bmatrix}$$



Preliminary

- **Is it true?**

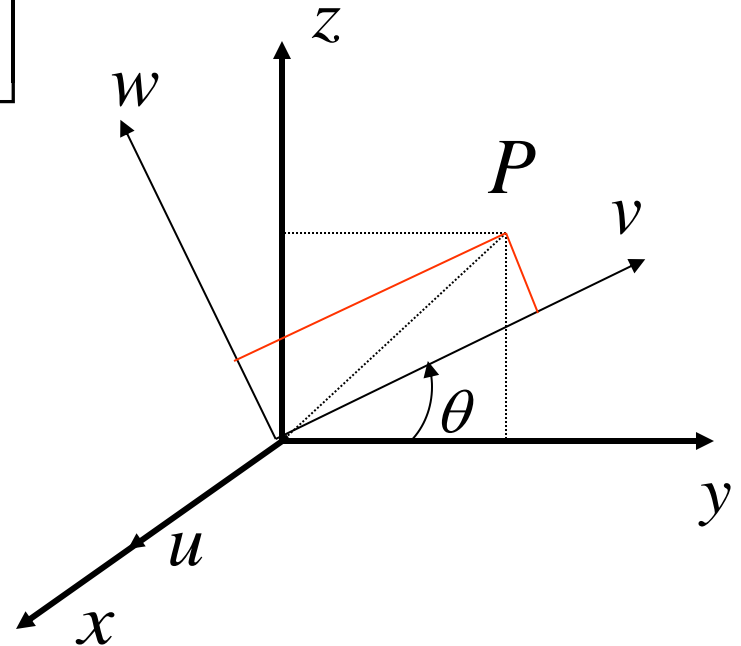
- Rotation about x axis with θ

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} p_u \\ p_v \\ p_w \end{bmatrix}$$


$$p_x = p_u$$


$$p_y = p_v \cos \theta - p_w \sin \theta$$


$$p_z = p_v \sin \theta + p_w \cos \theta$$



Basic Rotation Matrices

– Rotation about x-axis with θ  $Rot(x, \theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta & -S\theta \\ 0 & S\theta & C\theta \end{bmatrix}$

– Rotation about y-axis with θ  $Rot(y, \theta) = \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix}$

– Rotation about z-axis with θ  $Rot(z, \theta) = \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$P_{xyz} = RP_{uvw}$$

Basic Rotation Matrices

- Basic Rotation Matrix

$$R = \begin{bmatrix} \mathbf{i}_x \cdot \mathbf{i}_u & \mathbf{i}_x \cdot \mathbf{j}_v & \mathbf{i}_x \cdot \mathbf{k}_w \\ \mathbf{j}_y \cdot \mathbf{i}_u & \mathbf{j}_y \cdot \mathbf{j}_v & \mathbf{j}_y \cdot \mathbf{k}_w \\ \mathbf{k}_z \cdot \mathbf{i}_u & \mathbf{k}_z \cdot \mathbf{j}_v & \mathbf{k}_z \cdot \mathbf{k}_w \end{bmatrix} \quad P_{xyz} = RP_{uvw}$$

– Obtain the coordinate of P_{uvw} from the coordinate of P_{xyz} :

Dot products are commutative (点积遵守交换律) !

$$\begin{bmatrix} p_u \\ p_v \\ p_w \end{bmatrix} = \begin{bmatrix} \mathbf{i}_u \cdot \mathbf{i}_x & \mathbf{i}_u \cdot \mathbf{j}_y & \mathbf{i}_u \cdot \mathbf{k}_z \\ \mathbf{j}_v \cdot \mathbf{i}_x & \mathbf{j}_v \cdot \mathbf{j}_y & \mathbf{j}_v \cdot \mathbf{k}_z \\ \mathbf{k}_w \cdot \mathbf{i}_x & \mathbf{k}_w \cdot \mathbf{j}_y & \mathbf{k}_w \cdot \mathbf{k}_z \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} \quad P_{uvw} = QP_{xyz}$$

$$Q = R^{-1} = R^T$$

$$QR = R^T R = R^{-1} R = I_3 \quad \Leftarrow \text{3X3 identity matrix}$$

Basic Rotation Matrices

- **special orthogonal group of order n**

$$SO(n) := \{R \in \mathbb{R}^{n \times n} \mid R^T R = R R^T = I, \det R = 1\}$$

Thus, for any $R \in SO(n)$ the following properties hold

- $R^T = R^{-1} \in SO(n)$
- *The columns (the rows) of R are mutually orthogonal*
- *Each column (each row) of R is a unit vector*
- $\det R = 1$

The special case $SO(2)$: $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$

$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}^{-1} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}^T = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix}$$

Example 2

- A point $a_{uvw} = (4,3,2)$ is attached to a rotating frame, and the frame rotates 60 degree about the OZ axis of the reference frame. Find the coordinates of the point relative to the reference frame after the rotation.

$$\begin{aligned} a_{xyz} &= Rot(z, 60) a_{uvw} \\ &= \begin{bmatrix} 0.5 & -0.866 & 0 \\ 0.866 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -0.598 \\ 4.964 \\ 2 \end{bmatrix} \end{aligned}$$

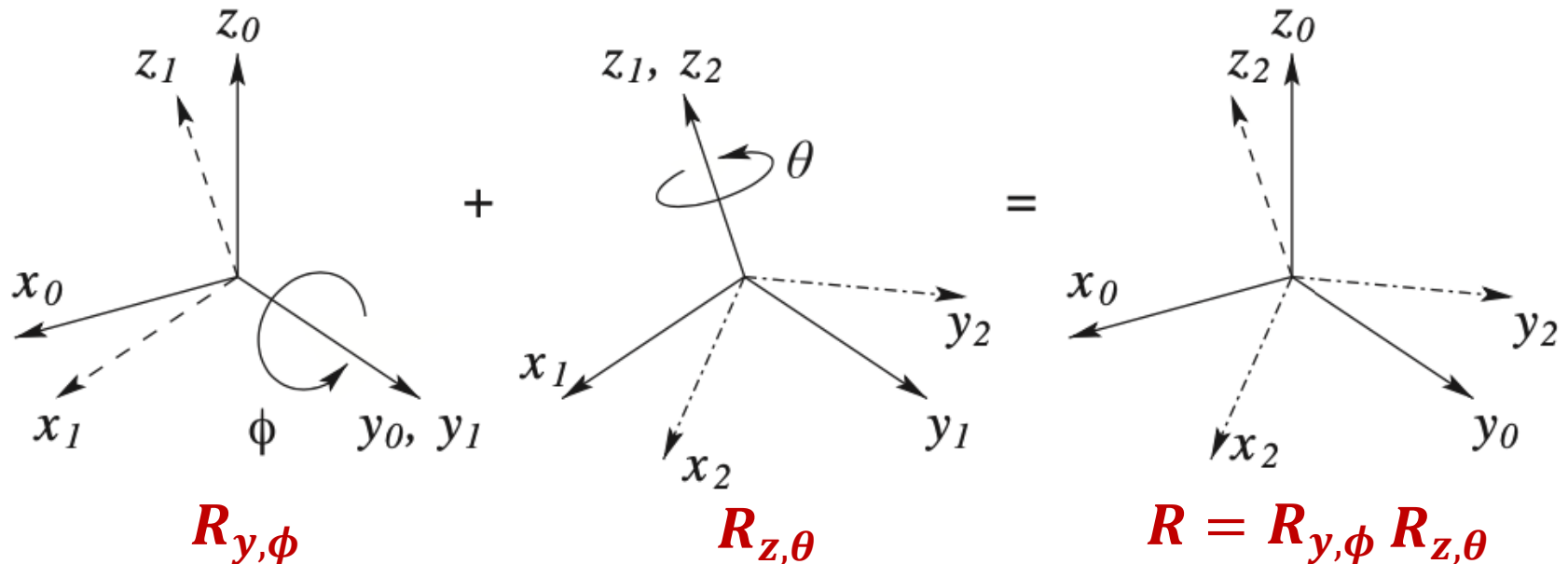
Example 3

- A point $a_{xyz} = (4,3,2)$ is the coordinate w.r.t. the reference coordinate system, find the corresponding point a_{uvw} w.r.t. the rotated OU-V-W coordinate system if it has been rotated 60 degree about OZ axis.

$$\begin{aligned} a_{uvw} &= Rot(z, 60)^T a_{xyz} \\ &= \begin{bmatrix} 0.5 & 0.866 & 0 \\ -0.866 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 4.598 \\ -1.964 \\ 2 \end{bmatrix} \end{aligned}$$

Composition of Rotations

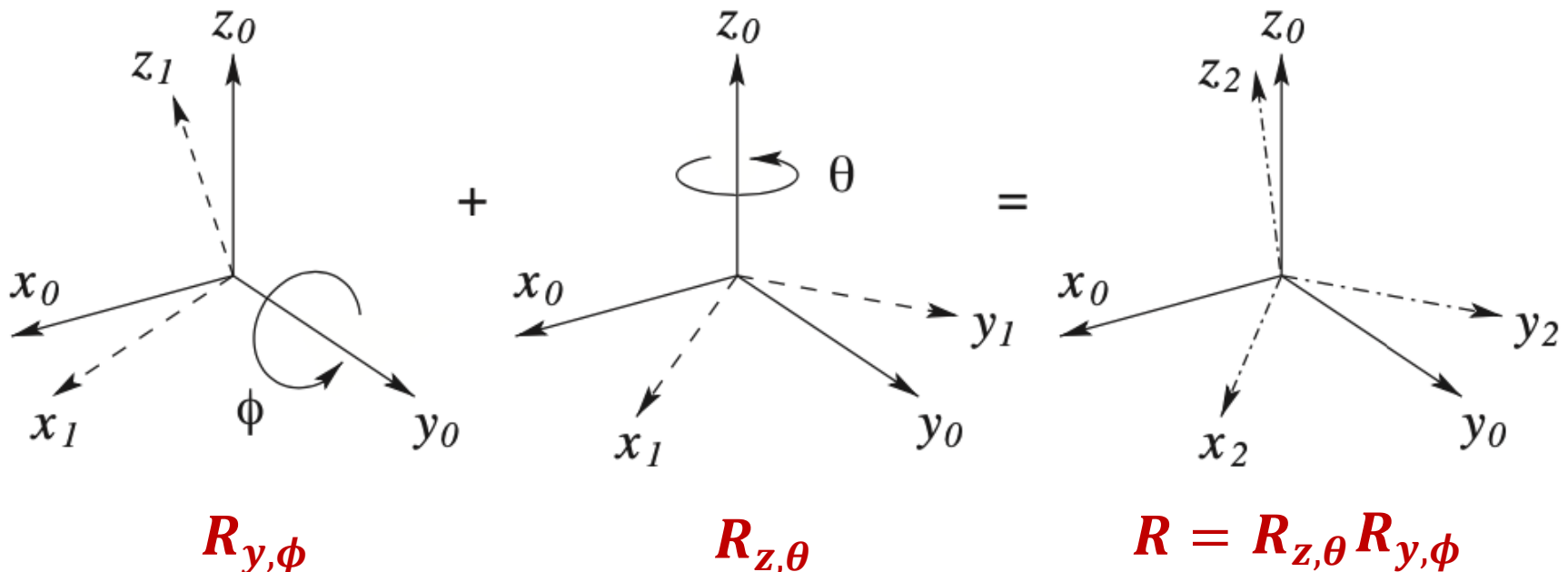
- Matrix multiplications do not commute
 - Rotation with Respect to the Current Frame:
if rotating coordinate OUVW is rotating about its **current principal axes**, then **post-multiply** the previous (resultant) rotation matrix with an appropriate basic rotation matrix



Composition of Rotations

– Rotation with Respect to the Fixed Frame:

- if rotating coordinate O-U-V-W is rotating about **principal axis of OXYZ frame**, then **Pre-multiply** the previous (resultant) rotation matrix with an appropriate basic rotation matrix



Composition of Rotations

- 多次旋转的复合

- 矩阵乘法不符合交换律
- 旋转变换的叠加定律:

- 如果旋转坐标系 $O-U-V-W$ 是绕其自身坐标系的主轴旋转，那么在原有旋转矩阵的右边乘以单轴旋转矩阵。（绕当前坐标系轴）
- 如果旋转坐标系 $O-U-V-W$ 绕参考坐标系 $OXYZ$ 的主轴旋转，那么在原有旋转矩阵的左边乘以单轴旋转矩阵。（绕原来坐标系轴）

（推导过程见参考书1：P28-29）

Example 4

- Find the rotation matrix for the following operations:

Rotation ϕ about OY axis

Rotation θ about OW axis

Rotation α about OU axis

Answer...

$$\begin{aligned}
 R &= \text{Rot}(y, \phi) I_3 \text{Rot}(w, \theta) \text{Rot}(u, \alpha) \\
 &= \begin{bmatrix} C\phi & 0 & S\phi \\ 0 & 1 & 0 \\ -S\phi & 0 & C\phi \end{bmatrix} \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\alpha & -S\alpha \\ 0 & S\alpha & C\alpha \end{bmatrix} \\
 &= \begin{bmatrix} C\phi C\theta & S\phi S\alpha - C\phi S\theta C\alpha & C\phi S\theta S\alpha + S\phi C\alpha \\ S\theta & C\theta C\alpha & -C\theta S\alpha \\ -S\phi C\theta & S\phi S\theta C\alpha + C\phi S\alpha & C\phi C\alpha - S\phi S\theta S\alpha \end{bmatrix}
 \end{aligned}$$

Pre-multiply if rotate about the OXYZ axes

Post-multiply if rotate about the OUVW axes

Summary

Manipulator

- Robot Geometry
- Robot Specifications

Kinematics - Preliminary

- Basic Rotation Matrix (X,Y,Z)
- Composite Rotations

Homework 2

Homework 2 is posted at <http://bb.sustech.edu.cn>

Due date: February 26, 2025

Next class: Homogeneous Transformation、 Euler Angles
(February 24, Next Monday)