南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

Robot Modeling & Control ME331

Section 17: Control I

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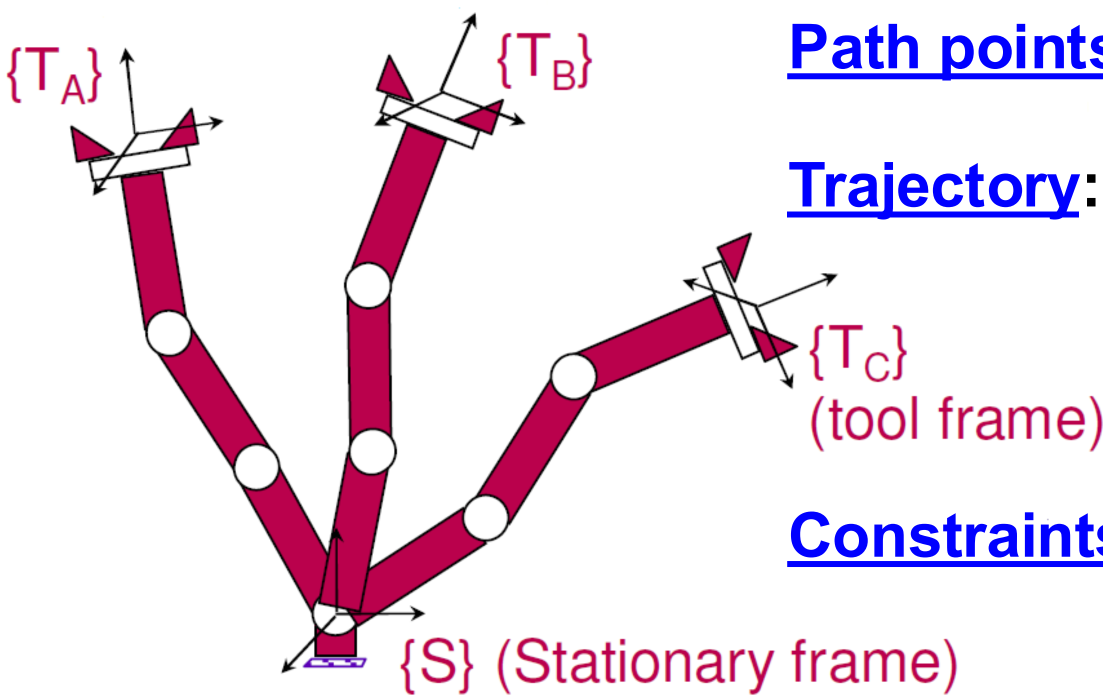
Outline

- **Review**
  - **Trajectory Planning**
- **Control of Manipulators**
  - **Introduction to Control**
  - **Actuator Dynamics**
  - **Load Dynamics**
  - **Driven-Train Dynamics (joint elasticity)**
  - **Independent Joint Control (PID+Forward)**

Trajectory Planning

Problem Definition

– Move the manipulator arm from the initial position $\{T_A\}$ to the desired final position $\{T_C\}$. Maybe going through some via point $\{T_B\}$.



Path points: Initial, final and via points

Trajectory: Time history of position, velocity and acceleration for each DOF

Constraints: Spatial, time, stability, smoothness, etc.

Trajectory Planning

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

The cubic polynomial coefficient can be obtained:

$$\begin{cases} a_0 = \theta_0 \\ a_1 = \dot{\theta}_0 \\ a_2 = \dfrac{3}{t_f^2}(\theta_f - \theta_0) - \dfrac{2}{t_f}\dot{\theta}_0 - t_f \dot{\theta}_f \\ a_3 = -\dfrac{2}{t_f^3}(\theta_f - \theta_0) + \dfrac{1}{t_f^2}(\dot{\theta}_0 + \dot{\theta}_f) \end{cases}$$

The velocity constraint condition:

$$\dot{\theta}(0) = \dot{\theta}_0$$
$$\dot{\theta}(t_f) = \dot{\theta}_f$$

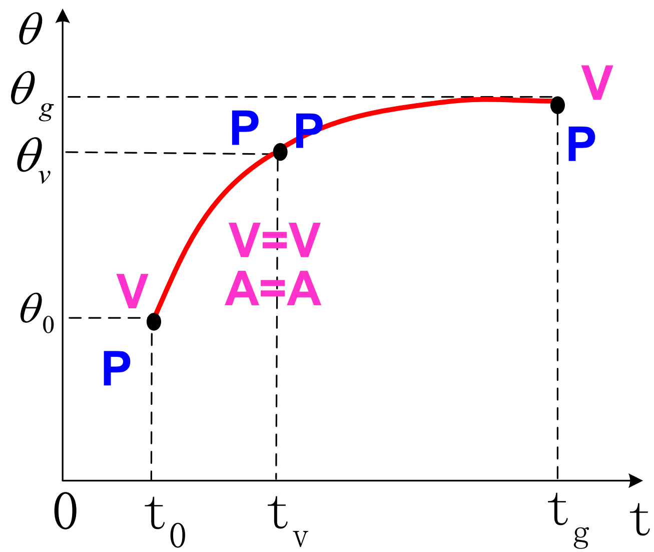
The cubic polynomial determined by the above equation describes the trajectory of the starting and ending points with any given position and speed. The following problem is how to determine the joint velocity at the path point.

Trajectory Planning

Acceleration continuity constraints

In order to ensure the continuity of acceleration at the waypoint, two cubic curves can be connected at the waypoint according to certain rules to piece together the desired trajectory.

The constraint condition is: not only the speed of the connection is continuous, but also the acceleration should be continuous.

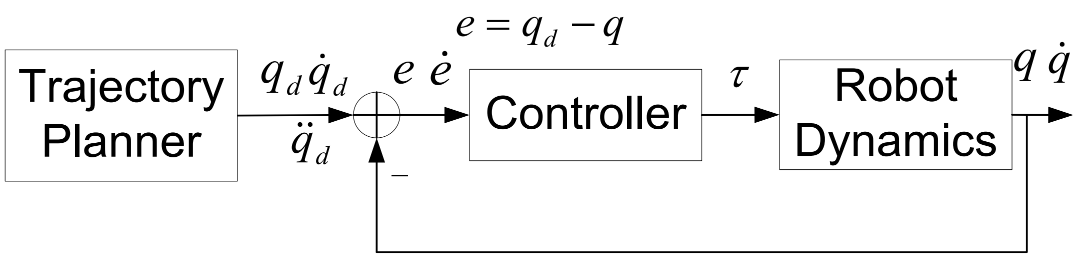


Eight equations
Eight unknowns

Introduction to Control

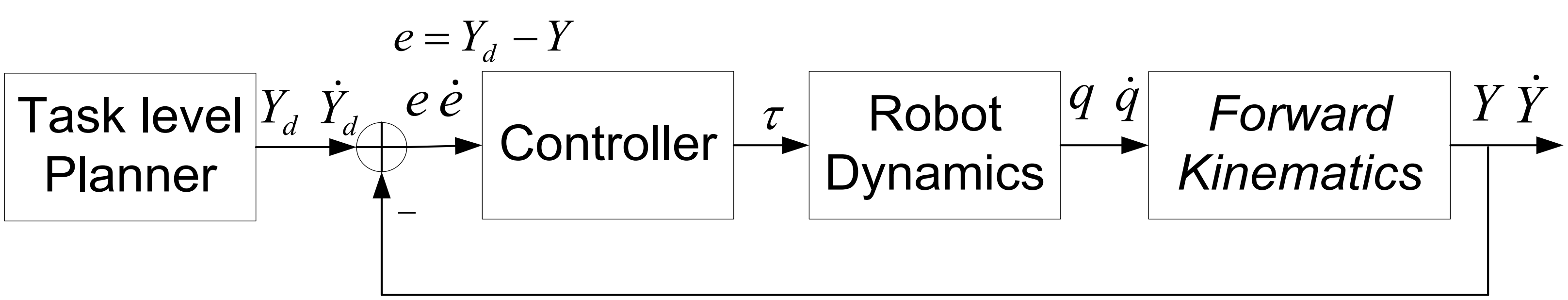
- **Robot model:** $\begin{cases} D(q)\ddot{q} + C(q,\dot{q}) + G(q) = \tau \\ Y = h(q) \end{cases}$

- **Joint space**



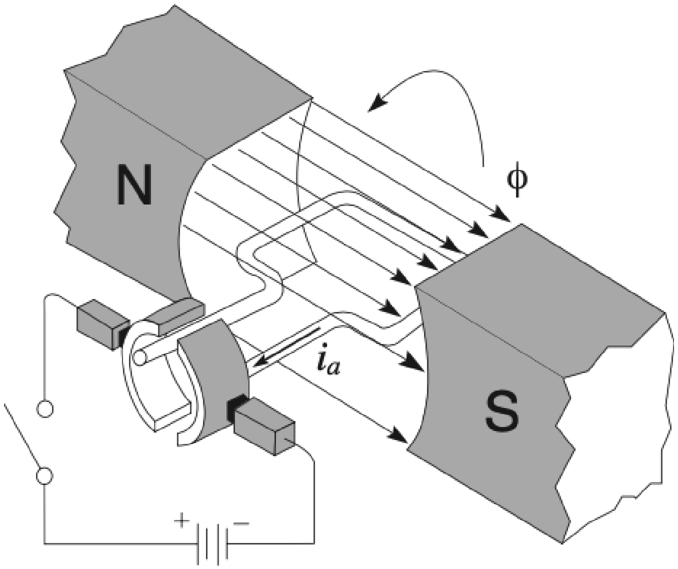
Find a control input (τ), $\quad q \to q_d \qquad as \quad t \to \infty$

- **Task space**



Find a control input (τ), $\qquad Y \to Y_d \qquad as \quad t \to \infty \qquad e = Y_d - Y \to 0$

Actuator Dynamics



Principle of operation of a permanent magnet DC motor.

Working principle of DC motor: a current-carrying conductor in a magnetic field experiences a force $F = i \times \phi$, where ϕ is the magnetic flux, i is the current in the conductor. The motor itself consists of a fixed **stator** and a movable **rotor** that rotates inside the stator.

If the current in the rotor (also called the **armature** 电枢) is i_a, then there will be a torque on the rotor causing it to rotate. The magnitude of this torque is

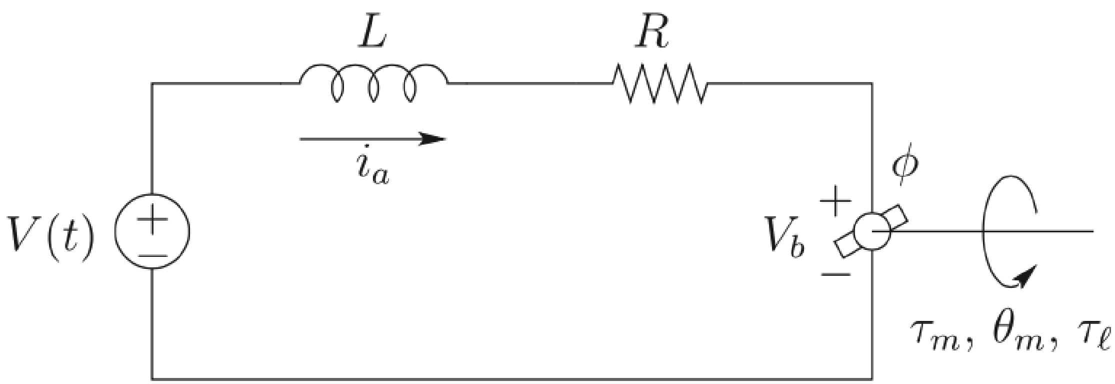
$$\tau_m = K_1 \phi i_a = K_m i_a$$

Whenever a conductor moves in a magnetic field, a voltage V_b is generated across its terminals that is proportional to the velocity of the conductor in the field. This voltage, called the **back emf** 反电动势, will tend to oppose the current flow in the conductor. we have the back emf relation

$$V_b = K_2 \phi \omega_m = K_b \omega_m$$

where ω_m is the angular velocity of the rotor, and K_b is the back emf constant.

Actuator Dynamics



Circuit diagram for an armature controlled DC motor. The rotor windings have an effective inductance *L* and effective resistance *R*. The applied voltage *V* is the control input.

V = armature voltage
L = armature inductance
R = armature resistance
V_b = back emf
i_a = armature current
θ_m = rotor position
τ_m = generated torque
τ_l = load torque
ϕ = magnetic flux

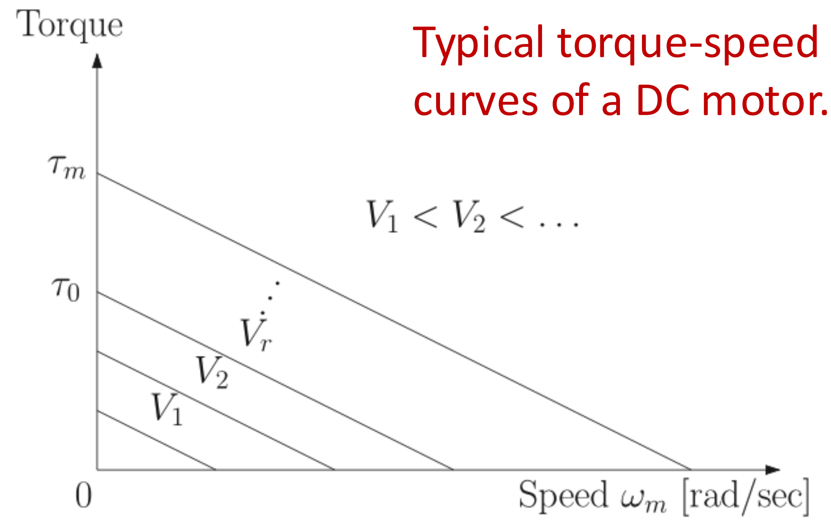
The differential equation for the armature current is then

$$\left.\begin{aligned} L\frac{di_a}{dt} + Ri_a &= V - V_b \\ \tau_m &= K_1\phi i_a = K_m i_a \\ V_b &= K_2\phi\omega_m = K_b\omega_m \end{aligned}\right\} \tau_m = \frac{K_m}{R}\left(V - K_b\omega_m - \cancel{L\frac{di_a}{dt}}\right)$$

Actuator Dynamics

$$\tau_m \approx \frac{K_m}{R}(V - K_b \omega_m)$$

Typical torque-speed curves of a DC motor.



When motor is stalled, the blocked-rotor torque at the rated voltage V_r is denoted by τ_0.

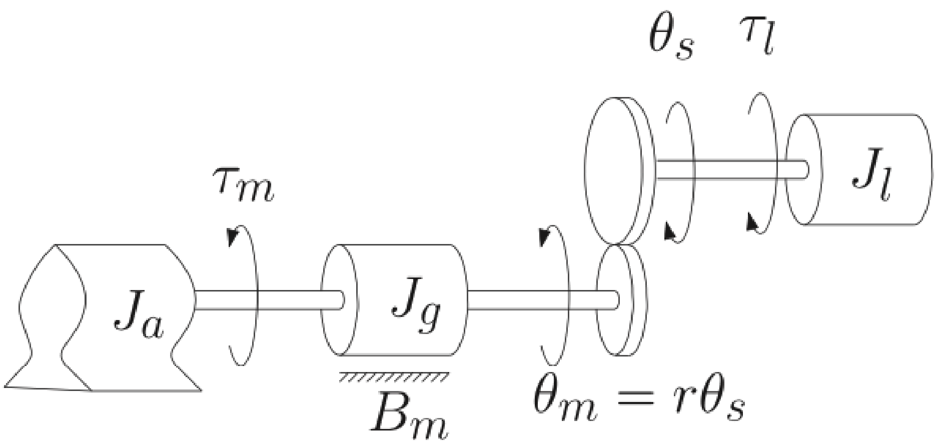
Using $L\frac{di_a}{dt} + Ri_a = V - V_b$ and $\tau_m = K_m i_a$ with $V_b = 0$ and $\frac{di_a}{dt} = 0$, we have

$$V_r = Ri_a = \frac{R\tau_0}{K_m}$$

Therefore the torque constant is

$$K_m = \frac{R\tau_0}{V_r}$$

Load Dynamics



J_a = actuator inertia
J_g = gear inertia
J_l = load inertia
B_m = the coefficient of motor friction
r = the gear ratio ($r \gg 1$)

We set $J_m = J_a + J_g$ the sum of the actuator and gear inertias.

In terms of the motor angle θ_m, the equation of motion of this system is then

$$J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} = \tau_m - \tau_l/r$$
$$= K_m i_a - \tau_l/r$$

$$\Rightarrow (J_m s^2 + B_m s)\Theta_m(s) = K_m I_a(s) - \tau_l(s)/r$$

$$L\frac{di_a}{dt} + Ri_a = V - V_b$$

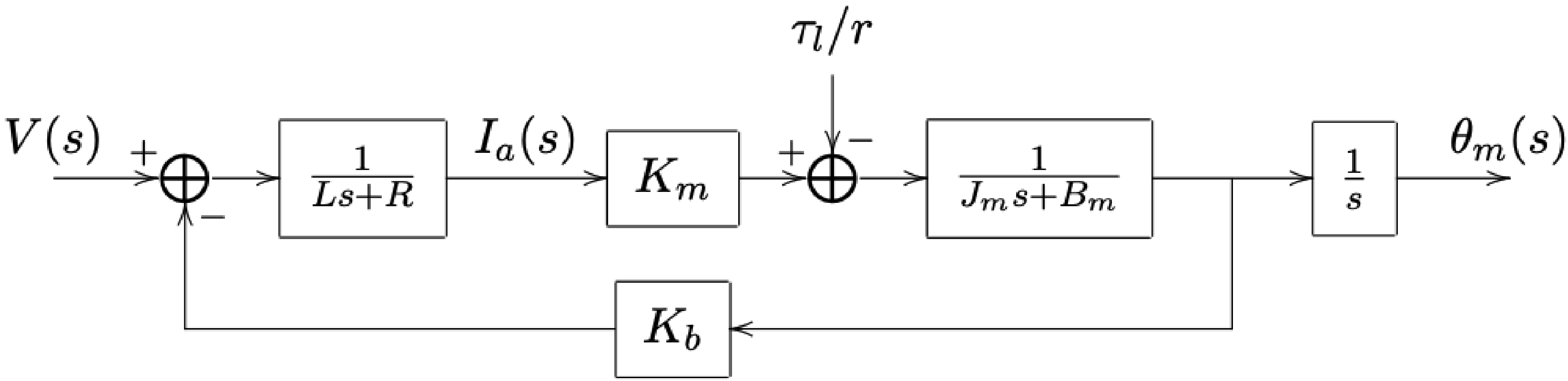
$$V_b = K_b\omega_m = K_b \frac{d\theta_m}{dt}$$

$$\Rightarrow (Ls + R)I_a(s) = V(s) - K_b s\Theta_m(s)$$

Load Dynamics

$$(J_m s^2 + B_m s)\Theta_m(s) = K_m I_a(s) - \tau_l(s)/r$$

$$(Ls + R)I_a(s) = V(s) - K_b s\Theta_m(s)$$



The transfer function from $V(s)$ to $\Theta_m(s)$ is given, with $\tau_l = 0$, by

$$\frac{\Theta_m(s)}{V(s)} = \frac{K_m}{s[(Ls+R)(J_m s + B_m) + K_b K_m]}$$

Homework (1)

The transfer function from the load torque $\tau_l(s)$ to $\Theta_m(s)$ is given, with $V = 0$, by

$$\frac{\Theta_m(s)}{\tau_l(s)} = \frac{-(Ls+R)/r}{s[(Ls+R)(J_m s + B_m) + K_b K_m]}$$

Homework (1)

Load Dynamics

$$\frac{\Theta_m(s)}{V(s)} = \frac{K_m}{s[(Ls+R)(J_m s + B_m) + K_b K_m]}$$

$$\frac{\Theta_m(s)}{\tau_l(s)} = \frac{-(Ls+R)/r}{s[(Ls+R)(J_m s + B_m) + K_b K_m]}$$

Frequently it is assumed that the "electrical time constant" L/R is much smaller than the "mechanical time constant" J_m/B_m. This is a reasonable assumption for many electromechanical systems and leads to a reduced order model of the actuator dynamics.

If we divide numerator and denominator of the above two equations by R and neglect the electrical time constant by setting L/R equal to zero, the transfer functions become, respectively,

$$\frac{\Theta_m(s)}{V(s)} \approx \frac{K_m/R}{s(J_m s + B_m + K_b K_m/R)}$$

$$\frac{\Theta_m(s)}{\tau_l(s)} \approx \frac{-1/r}{s(J_m s + B_m + K_b K_m/R)}$$

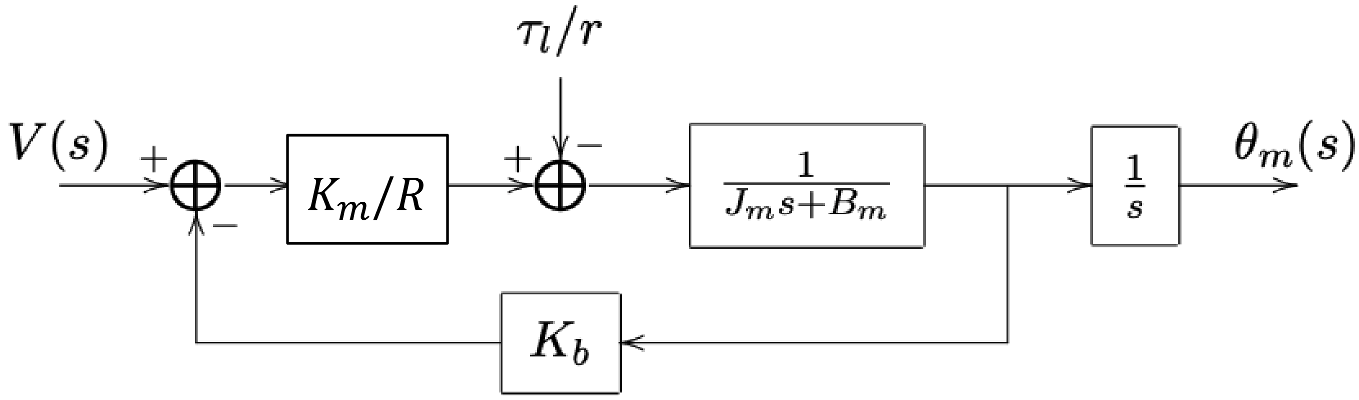
Homework (2)

Load Dynamics

$$\frac{\Theta_m(s)}{V(s)} \approx \frac{K_m/R}{s(J_m s + B_m + K_b K_m/R)}$$

$$\frac{\Theta_m(s)}{\tau_l(s)} \approx \frac{-1/r}{s(J_m s + B_m + K_b K_m/R)}$$

The block diagram corresponding to the reduced-order system is shown in the following figure.



In the time domain, the above two equations represent, by superposition, the second-order differential equation

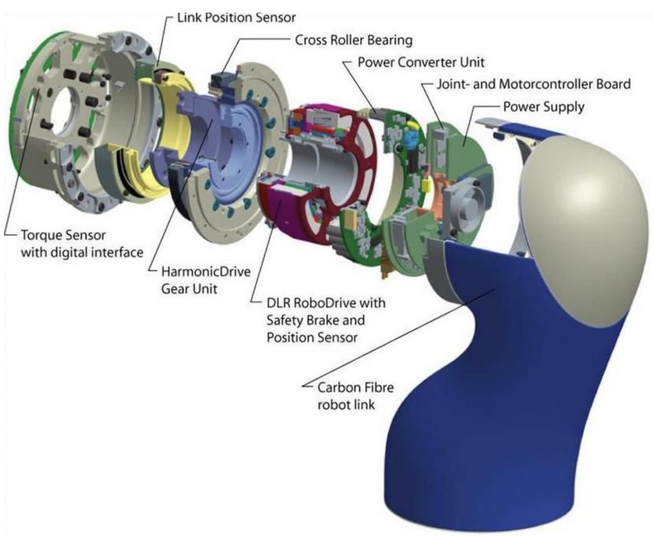
$$J_m\ddot{\theta}_m(t) + \left(B_m + \frac{K_b K_m}{R}\right)\dot{\theta}_m(t) = \left(\frac{K_m}{R}\right)V(t) - \frac{\tau_l(t)}{r}$$

Drive-Train Dynamics

- **in industrial robots, use of motion transmissions based on**
  - belts
  - harmonic drives
  - long shafts

  introduces flexibility between actuating motors (input) and driven links (output)
- **in research robots for human cooperation, compliance in the transmissions is introduced on purpose for safety**
  - actuator relocation by means of (compliant) cables and pulleys
  - harmonic drives and lightweight (but rigid) link design
  - redundant (macro-mini or parallel) actuation, with elastic couplings
- **in both cases, flexibility is modeled as concentrated at the joints**
- **most of the times, assuming small joint deformation (elastic domain)**

Robots with Joint Elasticity



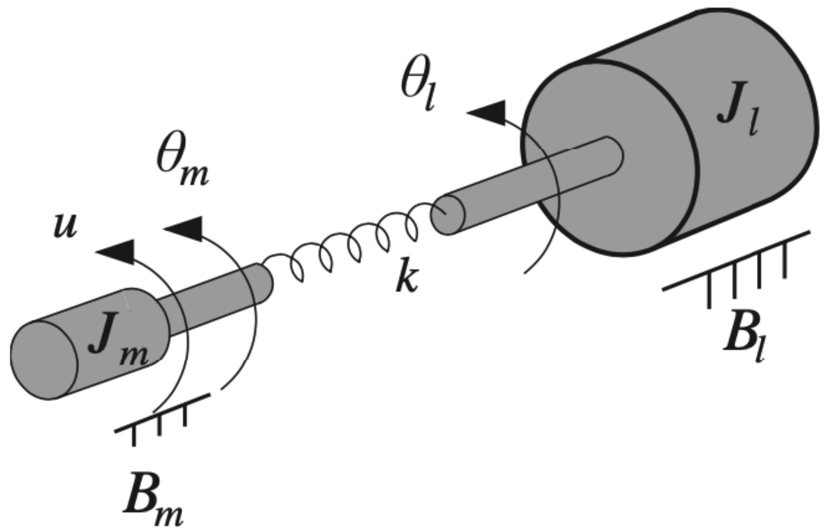
DLR LWR-III with harmonic drives

Dexter with cable transmissions

Robot with series elastic actuator

Drive-Train Dynamics

Idealized model to represent joint flexibility



- Consider the idealized situation of the above figure, consisting of an actuator connected to a load through a torsional spring representing the joint flexibility.
- The stiffness constant k represents the effective torsional stiffness of the harmonic gear.
- Need two generalized coordinates to describe one joint with flexibility.

Drive-Train Dynamics

- introduce 2N generalized coordinates
  - q = N link positions
  - θ = N motor positions (after reduction, $\theta_i = \theta_{mi}/n_{ri}$)
- add motor kinetic energy T_m to that of the links $T_q = \frac{1}{2}\dot{q}^T B(q)\dot{q}$

$$T_{mi} = \frac{1}{2} I_{mi}\dot{\theta}_{mi}^2 = \frac{1}{2} I_{mi} n_{ri}^2 \dot{\theta}_i^2 = \frac{1}{2} B_{mi}\dot{\theta}_i^2 \qquad T_m = \sum_{i=1}^{N} T_{mi} = \frac{1}{2}\dot{\theta}^T \underset{\text{diagonal, >0}}{B_m} \dot{\theta}$$

- add elastic potential energy U_e to that due to gravity $U_g(q)$
  - K = matrix of joint stiffness (diagonal, >0)

$$U_{ei} = \frac{1}{2} K_i \left(q_i - (\theta_{mi}/n_{ri})\right)^2 = \frac{1}{2} K_i (q_i - \theta_i)^2 \quad U_e = \sum_{i=1}^{N} U_{ei} = \frac{1}{2}(q-\theta)^T K\,(q-\theta)$$

- apply Euler-Lagrange equations w.r.t. (q, θ)

2N 2nd-order differential equations

$$\begin{cases} B(q)\ddot{q} + c(q,\dot{q}) + g(q) + K(q-\theta) = 0 \\ B_m\ddot{\theta} + K(\theta - q) = \tau \end{cases}$$

← no external torques performing work on q

Independent Joint Control

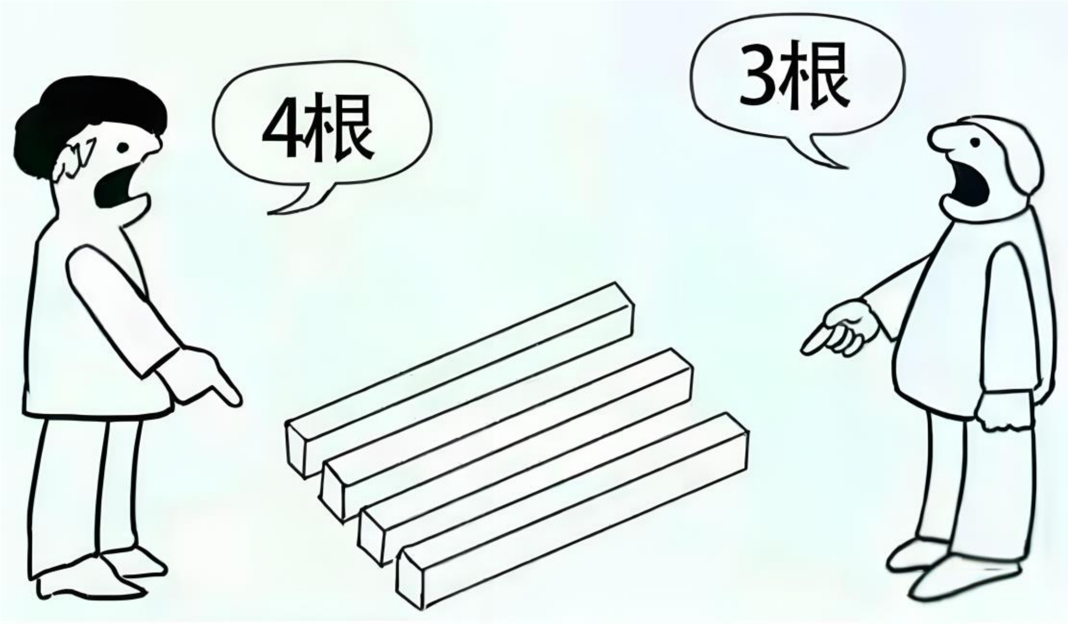
The equations of motion of the manipulator can be written component-wise, for $k = 1, \cdots, n$ as

$$J_{m_k}\ddot{\theta}_{m_k} + \left(B_{m_k} + K_{b_k}K_{m_k}/R_k\right)\dot{\theta}_{m_k} = \left(K_{m_k}/R_k\right)V_k - \frac{\tau_k}{r_k} \quad (1)$$

$$\sum_{j=1}^{n} d_{jk}(q)\ddot{q}_j + \sum_{i,j=1}^{n} c_{ijk}(q)\dot{q}_i\dot{q}_j + g_k(q) = \tau_k \quad (2)$$

Nonlinear and Multivariable Control

Independent Joint Control？



Independent Joint Model

$$J_{m_k}\ddot{\theta}_{m_k} + \left(B_{m_k} + K_{b_k}K_{m_k}/R_k\right)\dot{\theta}_{m_k} = \left(K_{m_k}/R_k\right)V_k - \frac{\tau_k}{r_k} \quad (1)$$

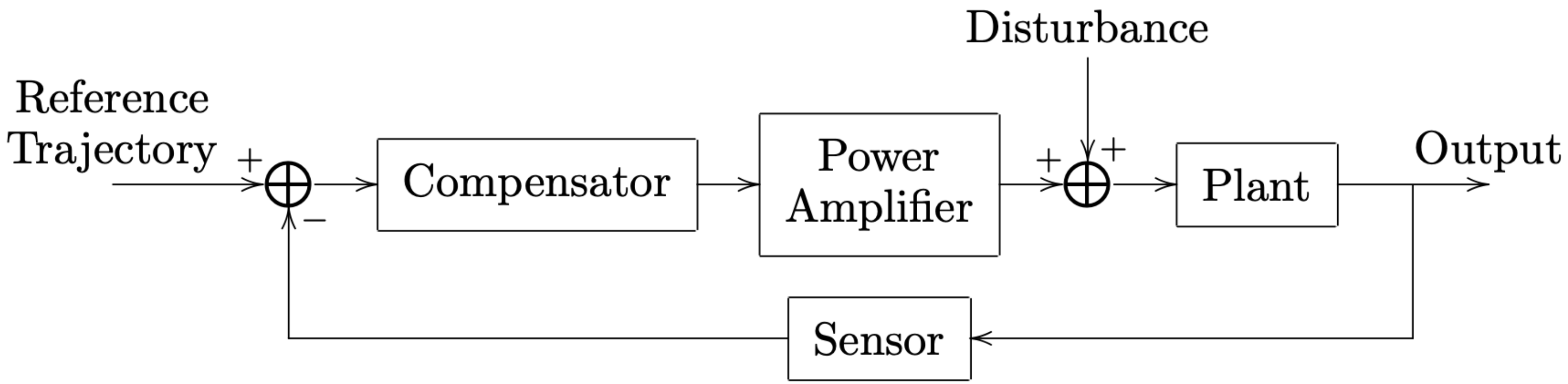
$$\sum_{j=1}^{n} d_{jk}(q)\ddot{q}_j + \sum_{i,j=1}^{n} c_{ijk}(q)\dot{q}_i\dot{q}_j + g_k(q) = \tau_k \quad (2)$$

The simplest approach to the control of the above system is to consider the nonlinear term τ_k entering (1) and defined by (2) as an input disturbance to the motor and design an independent controller for each joint according to the model (1).

- The advantage of this approach is its **simplicity** since the motor dynamics represented by (1) are linear.
- Note that the term τ_k in (1) is divided by the gear ratio r_k. The effect of the gear reduction is to reduce magnitude of the coupling nonlinearities.
- For high speed motion or for direct-drive manipulators, the coupling nonlinearities have a much larger effect on the control performance.

Independent Joint Control

We consider the simplest type of control strategy, namely, **independent joint control**. In this type of control each axis of the manipulator is controlled as a single-input/single-output (SISO) system. Any coupling effects due to the motion of the other links are treated as disturbances.



Basic structure of a feedback control system. The compensator measures the error between a reference and a measured output and produces signals to the plant that are designed to drive the error to zero despite the presence of disturbances.

Independent Joint Model

$$J_{m_k}\ddot{\theta}_{m_k} + \left(B_{m_k} + K_{b_k}K_{m_k}/R_k\right)\dot{\theta}_{m_k} = \left(K_{m_k}/R_k\right)V_k - \frac{\tau_k}{r_k} \quad (1)$$

Setting

$$J_{eff_k} := J_{m_k}$$
$$B_{eff_k} := B_{m_k} + K_{b_k}K_{m_k}/R_k$$
$$u_k := \left(K_{m_k}/R_k\right)V_k$$

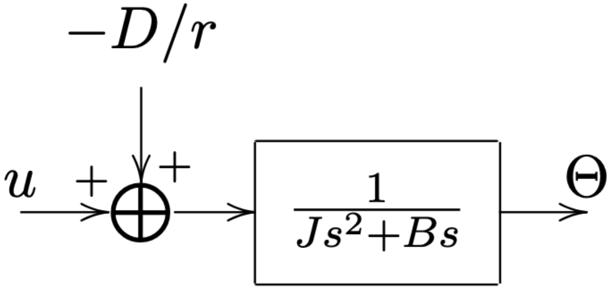
We write Equation (1) as

$$J_{eff_k}\ddot{\theta}_{m_k} + B_{eff_k}\dot{\theta}_{m_k} = u_k - \frac{d_k}{r_k}$$

where d_k is treated as a disturbance and defined by

$$d_k = \sum_{j=1}^{n} d_{jk}(q)\ddot{q}_j + \sum_{i,j=1}^{n} c_{ijk}(q)\dot{q}_i\dot{q}_j + g_k(q)$$

Block diagram of the simplified:
(open-loop system.)



PID Control

The **PID** (**P**roportional, **I**ntegral, **D**erivative) compensator is the most common type of controller used in most manipulators. The general form of a PID controller $u(t)$ is

$$u(t) = K_P e(t) + K_I \int_0^t e(\sigma)\, d\sigma + K_D \frac{de}{dt}$$

where $e(t) = \theta(t) - \theta^d(t)$ is the tracking error. The controller parameters are the proportional gain K_P, integral gain K_I, and derivative gain K_D. In the Laplace domain we write the equation as

$$U(s) = \left(K_P + K_D s + \frac{K_I}{s}\right) E(s) = C(s)E(s)$$

and we call

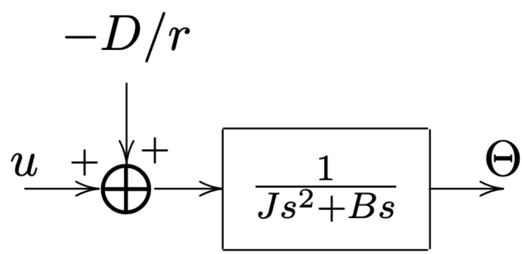
$$C(s) = K_P + K_D s + \frac{K_I}{s} = \frac{K_D s^2 + K_P s + K_I}{s}$$

the PID **compensator**.

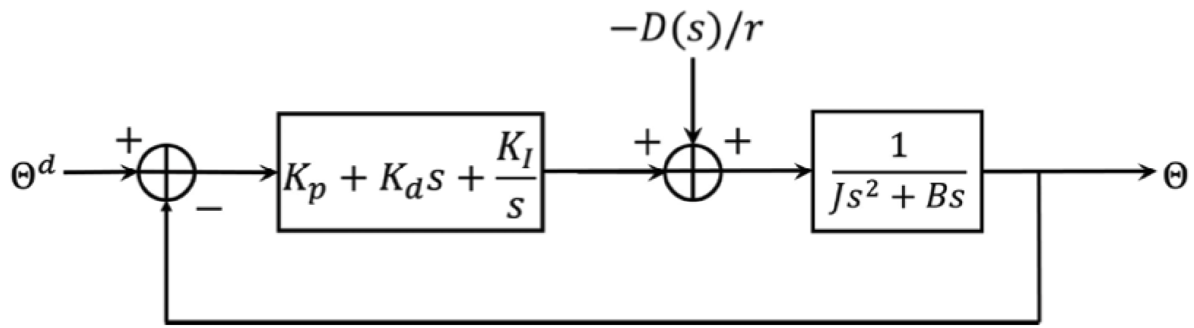
The design problem, known as **tuning**, is then to choose the PID gains K_P, K_D, and K_I to achieve the desired performance. There are numerous results available for tuning PID compensators.

PID Control (1-DOF & 2-DOF)

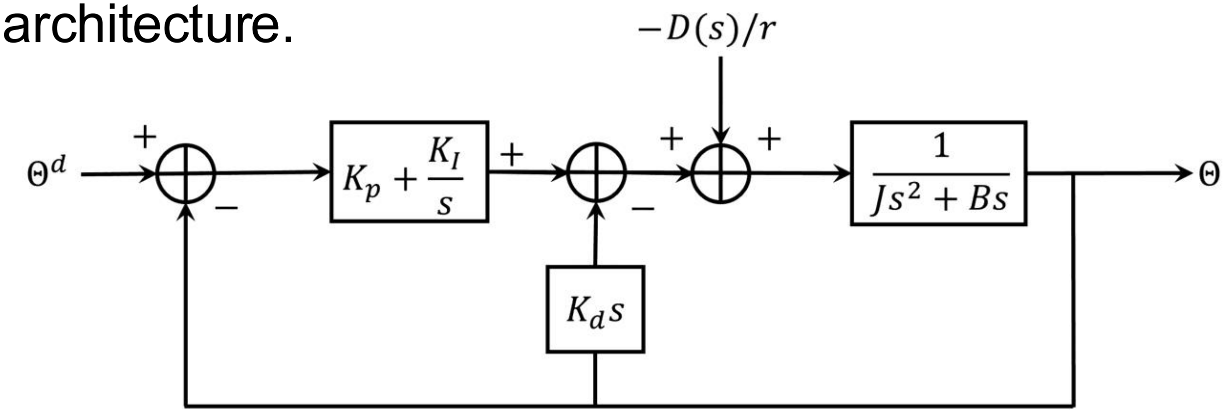
Open-loop system:



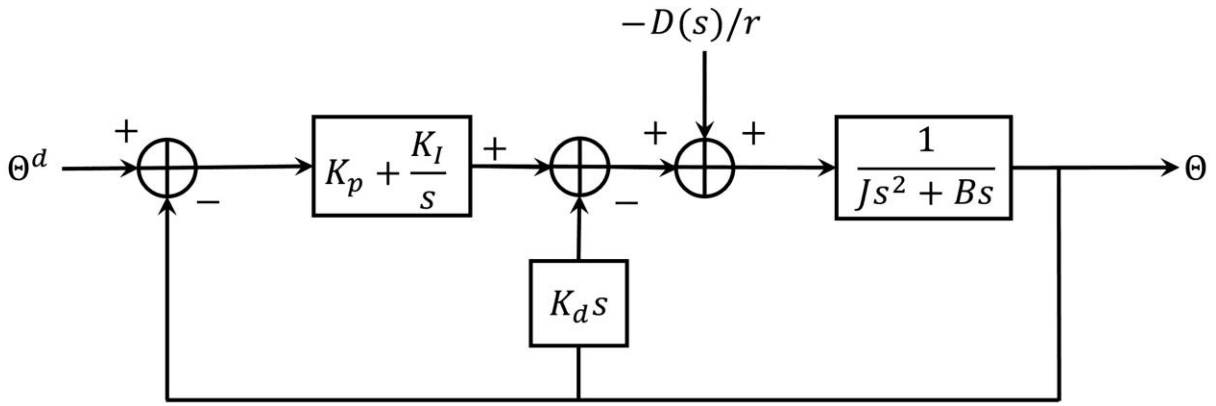
Closed-loop system:
(a PID compensator)



If θ^d is a step reference, then the derivative term will introduce an impulse at $t = 0$. To avoid this we use a **two-degree-of-freedom controller** architecture.



PID Control (Closed-Loop Systems)



The transfer function from the reference to the output:

$$G_1(s) = \frac{\Theta(s)}{\Theta^d(s)}$$

and the transfer function from the disturbance to the output:

$$G_2(s) = \frac{\Theta(s)}{D(s)}$$

By the **Principle of Superposition** then, the overall system response, in each case, is given by

$$\Theta(s) = G_1(s)\Theta^d(s) + G_2(s)D(s)$$

PID Control (Closed-Loop Systems)

Homework 3

	With PID Compensator	With PD Compensator
$G_1(s)$ 1-DoF	a) $\dfrac{K_D s^2 + K_P s + K_I}{Js^3 + (B + K_D)s^2 + K_P s + K_I}$	d) $\dfrac{K_D s + K_P}{Js^2 + (B + K_D)s + K_P}$
$G_1(s)$ 2-DoF	b) $\dfrac{K_P s + K_I}{Js^3 + (B + K_D)s^2 + K_P s + K_I}$	e) $\dfrac{K_P}{Js^2 + (B + K_D)s + K_P}$
$G_2(s)$	c) $\dfrac{-s/r}{Js^3 + (B + K_D)s^2 + K_P s + K_I}$	f) $\dfrac{-1/r}{Js^2 + (B + K_D)s + K_P}$

Closed-loop transfer functions using both the one- and two-degree-of-freedom architectures. a) and d) represent $G_1(s)$ in the one-degree-of-freedom architecture. b) and e) represent $G_1(s)$ in the two-degree-of-freedom architecture. c) and f) represent $G_2(s)$.

PID Control (Closed-Loop Systems)

Remark (Derivative Filter). *Note that the derivative term $K_D s$ in the PID compensator involves differentiation of the signal $\theta(t)$. In practice, a differentiator will amplify high-frequency signals and thus will perform poorly in the presence of noise.*

1) *One may avoid using a pure differentiator by using a velocity sensor to measure $\dot{\theta}$ directly.*

2) *More commonly, use a filter of the form*

$$K_D f(s) = K_D \frac{Ns}{s+N} = K_D \frac{s}{\epsilon s + 1}$$

where $\epsilon = 1/N$. Note that $f(s) \to s$, an ideal differentiator, as $\epsilon \to 0$ and thus represents an approximate differentiator for small values of ϵ and low frequencies in s. The effect of such a derivative filter is both to introduce phase lag into the estimate of $\theta(t)$ but also to mitigate the effect of noise amplification caused by ideal differentiation.

PID Control (Set-Point Tracking)

We consider the problem of set-point tracking, which is the problem of tracking a constant or step reference command θ^d and arises in point-to-point motion. Since the reference input is a step signal, we will use the two-degree-of-freedom compensator.

We first consider a PD compensator by setting the integral gain K_I equal to zero. In this case, the closed-loop system is given by

$$\Theta(s) = \frac{K_P}{\Omega(s)}\Theta^d(s) - \frac{1/r}{\Omega(s)}D(s)$$

where $\Omega(s)$ is the closed-loop characteristic polynomial

$$\Omega(s) = Js^2 + (B + K_D)s + K_P$$

The closed-loop system will be stable for all positive values of K_P and K_D and bounded disturbances, and the tracking error $E(s)$ is given by

$$\begin{aligned} E(s) &= \Theta^d(s) - \Theta(s) \\ &= \frac{Js^2+Bs}{\Omega(s)}\Theta^d(s) + \frac{1/r}{\Omega(s)}D(s) \end{aligned}$$

PID Control (Set-Point Tracking)

For a step reference input

$$\Theta^d(s) = \frac{\Theta^d}{s}$$

and a constant disturbance

$$D(s) = \frac{D}{s}$$

it follows directly from the final value theorem that the steady-state error e_{ss} satisfies

$$e_{ss} = \lim_{s \to 0} sE(s) = -\frac{D/r}{K_P}$$

Since the magnitude of the disturbance is proportional to the gear ratio $1/r$, the steady state error is smaller for larger gear ratio and can be made arbitrarily small by making the position gain K_P large.

Using a PD compensator, the closed-loop system is second-order and hence the step response is determined by the closed-loop natural frequency ω and damping ratio ζ.

PID Control (Set-Point Tracking)

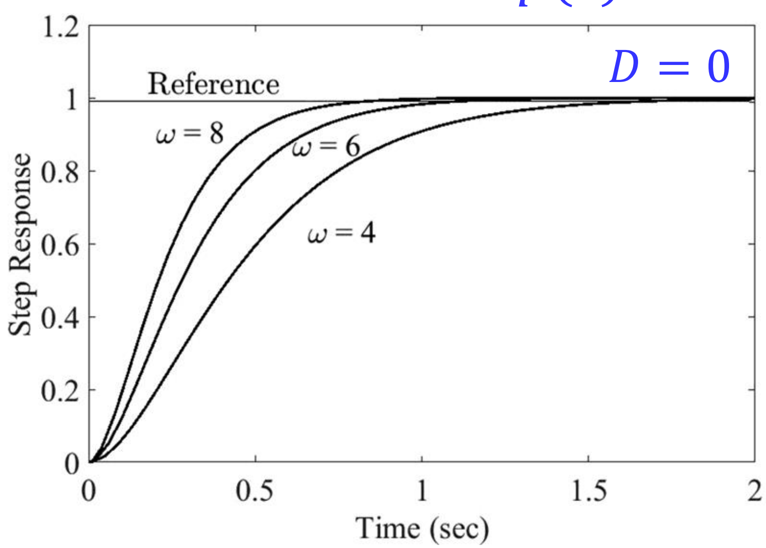
Given a desired value for these quantities, the gains K_D and K_P can be found from the expression $s^2 + \frac{(B+K_D)}{J}s + \frac{K_P}{J} = s^2 + 2\zeta\omega s + \omega^2$ as

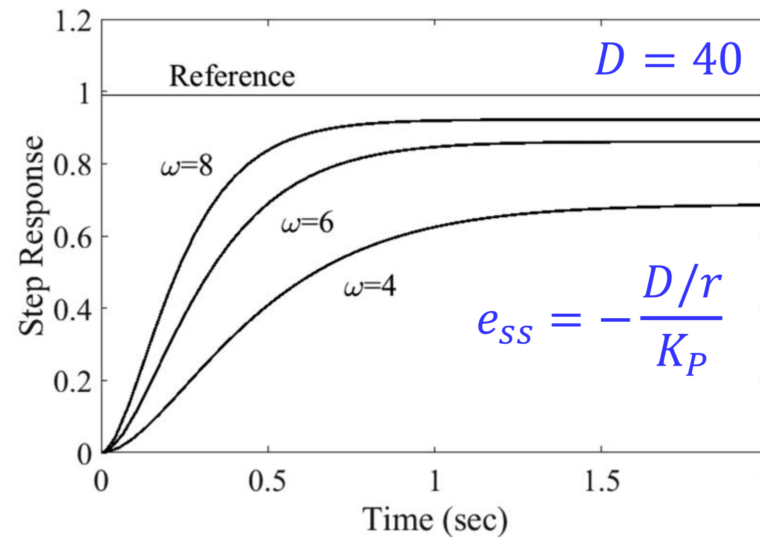
$$K_P = \omega^2 J,\ \ K_D = 2\zeta\omega J - B.$$

It is customary in robotics applications to take the damping ratio $\zeta = 1$ so that the response is critically damped. In this context ω determines the speed of response.

Example 8.2. Taking $J = B = 1$, for illustrative purposes, the closed-loop characteristic polynomial is

$$p(s) = s^2 + (1 + K_D)s + K_P$$



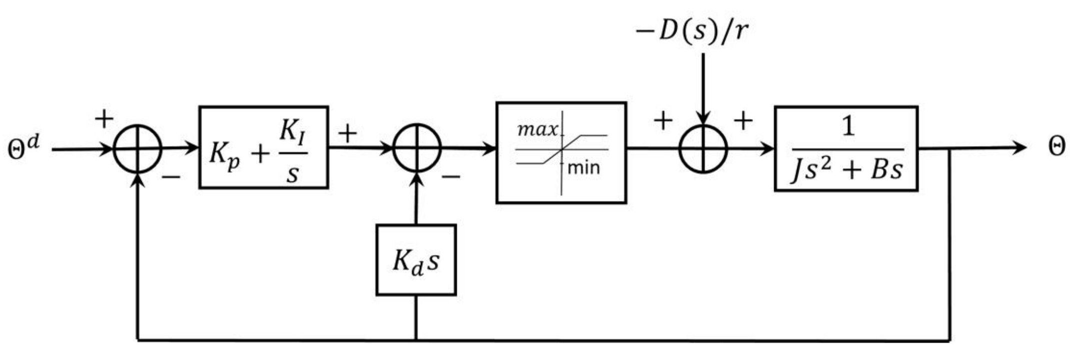


PID Control (The Effect of Saturation)

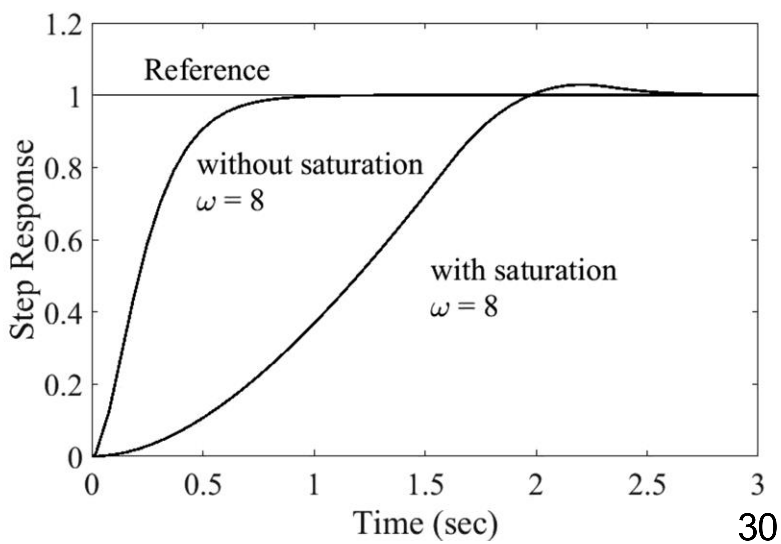
In theory, an arbitrarily fast response and arbitrarily small steady-state error to a constant disturbance could be achieved by simply increasing the gains in the PD compensator.

In practice, however, there is a maximum speed of response achievable from the system due to limits on the maximum torque (or current) **saturation**, due to limits on the maximum torque output of the motors.

Many manipulators, in fact, incorporate current limiters in the servo-system to prevent damage to the motors that might result from overdrawing current.



Second-order system with input saturation limiting the magnitude of the input signal.



PID Control

In order to remove the steady-state error due to the disturbance entirely we can employ **integral control**. Adding an integral term K_I/s to the above results in the closed-loop transfer functions. The PID control achieves exact steady tracking of step inputs while rejecting step disturbances, provided of course that the closed-loop system is stable.

With the PID compensator

$$U(s) = \left(K_P + \frac{K_I}{s}\right)(\Theta^d(s) - \Theta(s)) - K_D s\Theta(s)$$

the closed-loop system is now the third order system

$$\Theta(s) = \frac{K_P s + K_I}{\Omega_2(s)}\Theta^d(s) - \frac{s}{\Omega_2(s)}D(s)$$

with characteristic polynomial

$$\Omega_2(s) = Js^3 + (B + K_D)s^2 + K_P s + K_I$$

Applying the Routh–Hurwitz criterion to this polynomial, it follows that the closed-loop system is stable if the gains are positive, and in addition,

$$K_I < \frac{(B + K_D)K_P}{J}$$

PID Control

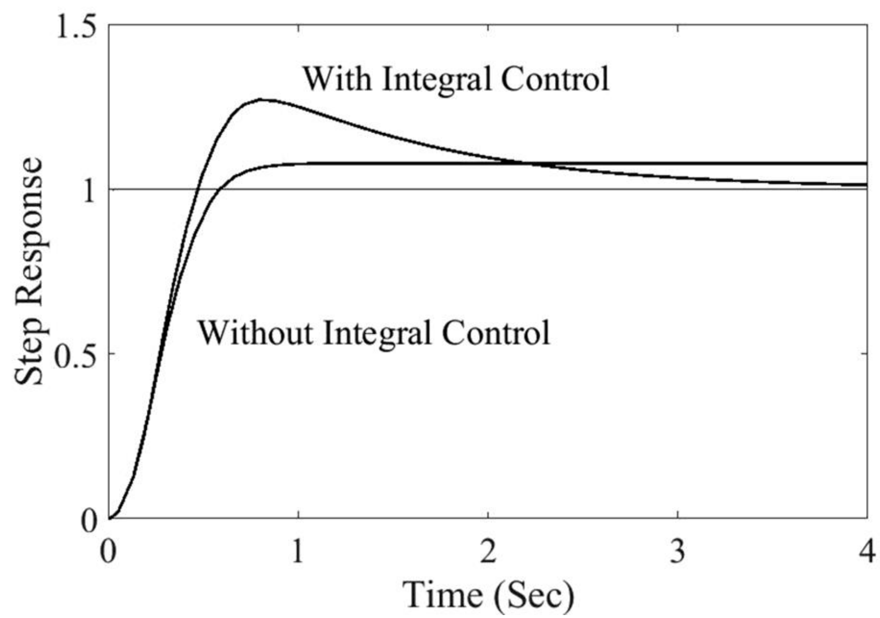
A common design rule-of-thumb (经验法则) for PID control:

1) set $K_I = 0$ and design K_P and K_D, to achieve the desired transient behavior (rise time, settling time, and so forth)

2) choose K_I within the limits to remove the steady-state error.

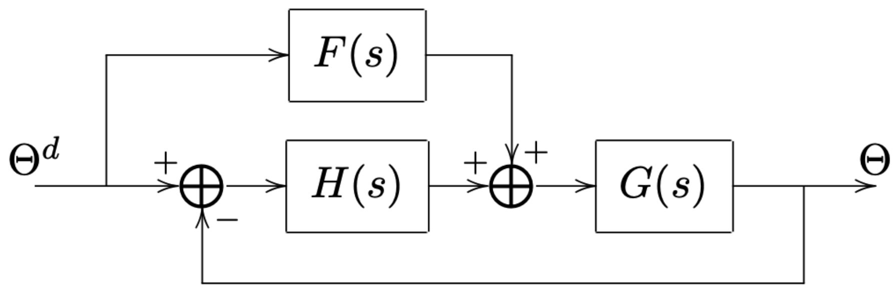
$$K_I < \frac{(B + K_D)K_P}{J}$$

Example. To the previous system we have added an integral control term in the compensator. The step responses are shown in the following figure.



Feedforward Control

The previous analysis was carried out under the assumption that the reference signal and disturbance are constant and is not valid for tracking more general time-varying trajectories such as a cubic polynomial trajectory. Now we introduce the notion of feedforward control as a method to track time-varying trajectories.

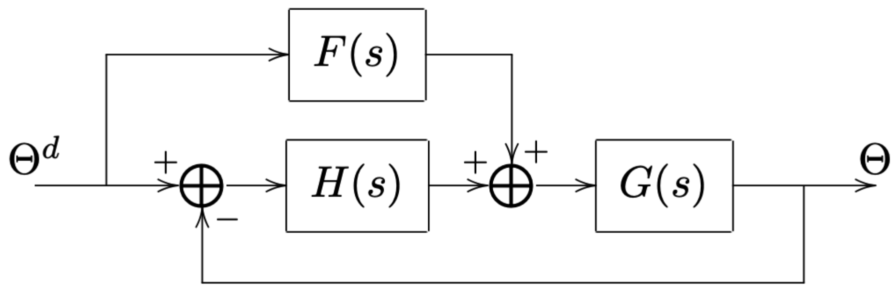


Suppose that $\theta^d(t)$ is a joint space reference trajectory and consider the block diagram, where $G(s)$ represents the forward transfer function of a given system and $H(s)$ is the compensator transfer function.

A feedforward control scheme consists of adding a feedforward path with transfer function $F(s)$. Let each of the three transfer functions be represented as ratios of polynomials

$$G(s) = \frac{q(s)}{p(s)} \qquad H(s) = \frac{c(s)}{d(s)} \qquad F(s) = \frac{a(s)}{b(s)}$$

Feedforward Control



$$G(s) = \frac{q(s)}{p(s)} \qquad H(s) = \frac{c(s)}{d(s)} \qquad F(s) = \frac{a(s)}{b(s)}$$

We assume that $G(s)$ is strictly proper and $H(s)$ is proper(正则). Simple block diagram manipulation shows that the closed-loop transfer function

$$T(s) = \frac{\Theta(s)}{\Theta^d(s)} = \frac{q(s)(c(s)b(s) + a(s)d(s))}{b(s)(p(s)d(s) + q(s)c(s))}$$

The closed-loop characteristic polynomial is $b(s)(p(s)d(s) + q(s)c(s))$.

Therefore, for stability of the closed-loop system, we require that the compensator $H(s)$ and the feedforward transfer function $F(s)$ be chosen so that the polynomials $p(s)d(s) + q(s)c(s)$ and $b(s)$ are Hurwitz.

Feedforward Control

If we choose the feedforward transfer function $F(s)$ equal to $1/G(s)$, the inverse of the forward plant, that is, $a(s) = p(s)$ and $b(s) = q(s)$, then the closed-loop system becomes

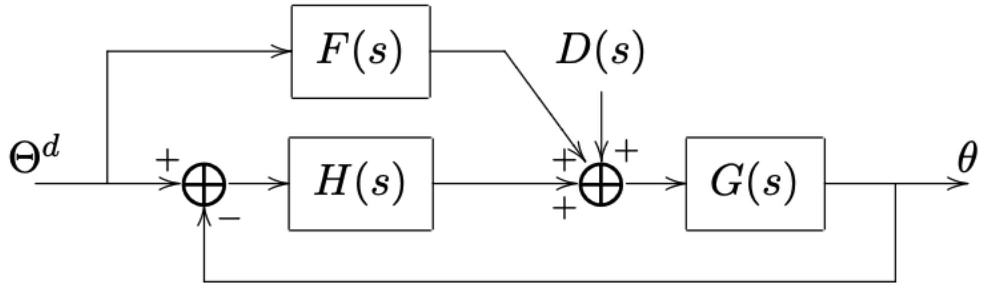
$$q(s)\big(p(s)d(s) + q(s)c(s)\big)\Theta(s) = q(s)\big(p(s)d(s) + q(s)c(s)\big)\Theta^d(s)$$

or, in terms of the tracking error $E(s) = \Theta^d(s) - \Theta(s)$,

$$q(s)\big(p(s)d(s) + q(s)c(s)\big)E(s) = 0$$

Thus, assuming stability, the output $\theta(t)$ will track any reference trajectory $\theta^d(t)$.

Feedforward control with disturbance $D(s)$:

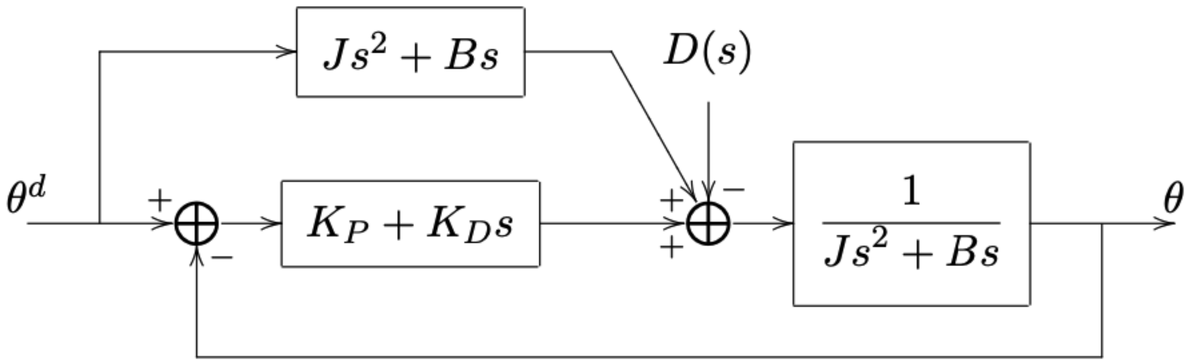


It is easily shown that the tracking error $E(s)$ is given by

$$E(s) = \frac{q(s)d(s)}{p(s)d(s) + q(s)c(s)} D(s)$$

Feedforward Control

Let us apply this idea to the robot control model.



In the time domain the control law can be written as

$$V(t) = J\ddot{\theta}^d(t) + B\dot{\theta}^d + K_D\left(\dot{\theta}^d - \dot{\theta}\right) + K_P\left(\theta^d - \theta\right)$$
$$= f(t) + K_D\dot{e}(t) + K_P e(t)$$

where $f(t)$ is the feedforward signal

$$f(t) = J\ddot{\theta}^d(t) + B\dot{\theta}^d$$

and $e(t)$ is the tracking error $\theta^d(t) - \theta(t)$. Since the forward plant equation is $J\ddot{\theta}(t) + B\dot{\theta}(t) = V(t) - d(t)/r$, the closed-loop error $e(t)$ satisfies the second-order differential equation

$$J\ddot{e}(t) + (B + K_D)\dot{e} + K_P e(t) = d(t)/r$$

Homework

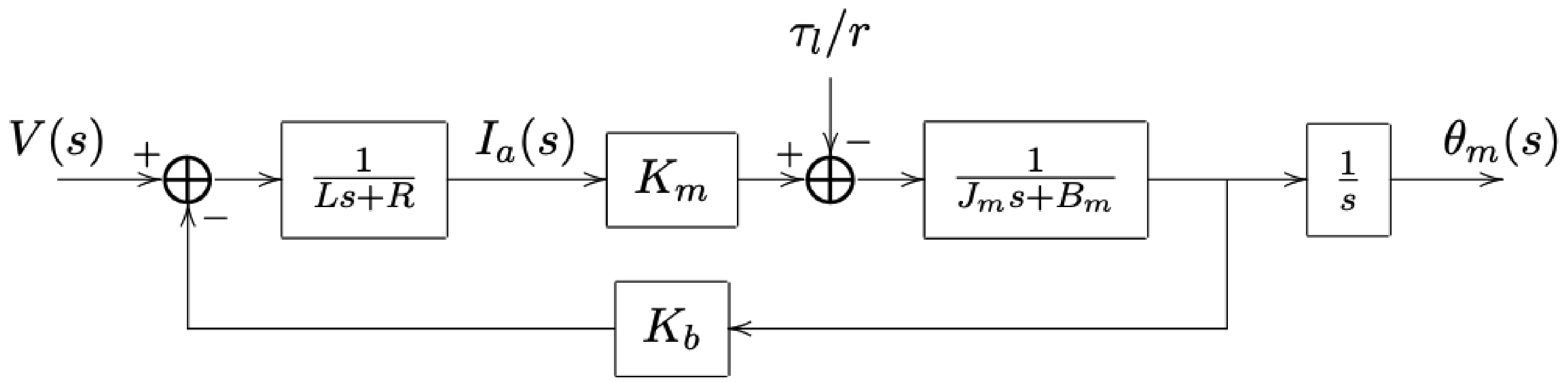
Sakai： http://sakai.sustech.edu.cn

Due date: 21, May

1) Using block diagram reduction techniques derive the transfer functions given by the following two equations:

$$\frac{\Theta_m(s)}{V(s)} = \frac{K_m}{s[(Ls+R)(J_m s + B_m) + K_b K_m]}$$

$$\frac{\Theta_m(s)}{\tau_l(s)} = \frac{-(Ls+R)/r}{s[(Ls+R)(J_m s + B_m) + K_b K_m]}$$

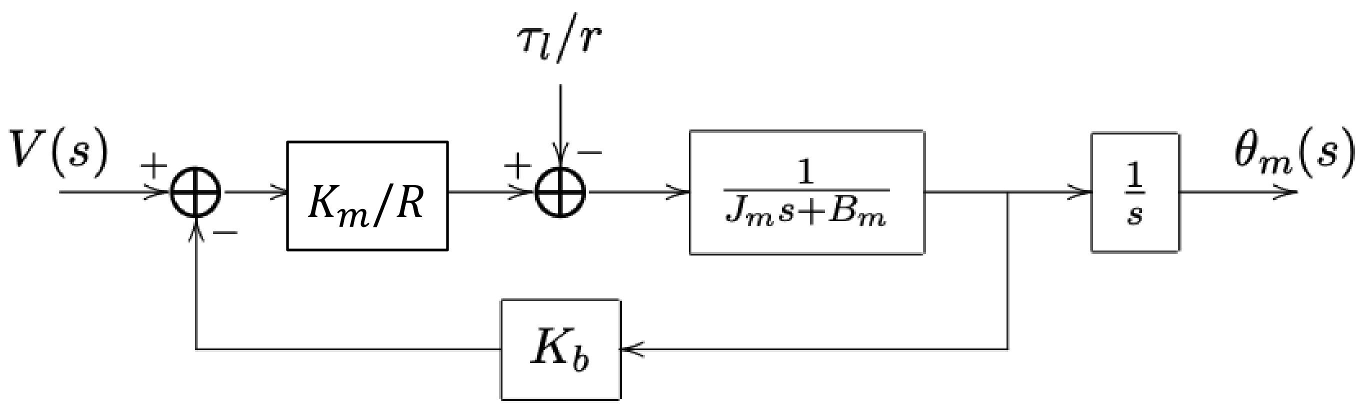


Homework

2) Derive the transfer functions for the reduced-order model given by the following two equations:

$$\frac{\Theta_m(s)}{V(s)} = \frac{K_m/R}{s(J_m s + B_m + K_b K_m/R)}$$

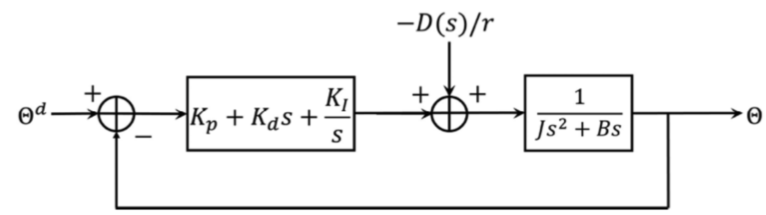
$$\frac{\Theta_m(s)}{\tau_l(s)} = \frac{-1/r}{s(J_m s + B_m + K_b K_m/R)}$$



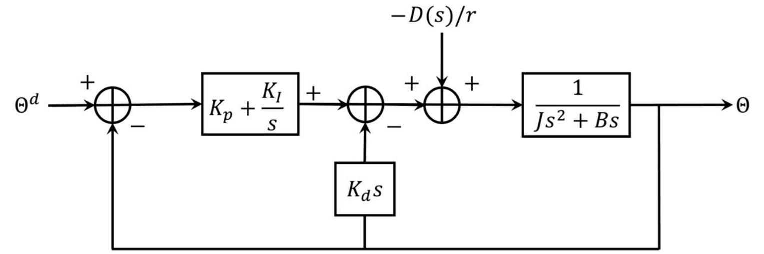
Homework

3) Verify the computation of the closed-loop compensators in the following Table.

With PID Compensator	With PD Compensator
a) $\dfrac{K_D s^2 + K_P s + K_I}{Js^3 + (B+K_D)s^2 + K_P s + K_I}$	d) $\dfrac{K_D s + K_P}{Js^2 + (B+K_D)s + K_P}$
b) $\dfrac{K_P s + K_I}{Js^3 + (B+K_D)s^2 + K_P s + K_I}$	e) $\dfrac{K_P}{Js^2 + (B+K_D)s + K_P}$
c) $\dfrac{-s/r}{Js^3 + (B+K_D)s^2 + K_P s + K_I}$	f) $\dfrac{-1/r}{Js^2 + (B+K_D)s + K_P}$



1-DoF architecture



2-DoF architecture

$$\Theta(s) = G_1(s)\Theta^d(s) + G_2(s)D(s)$$

Closed-loop transfer functions using both the one- and two-degree-of-freedom architectures. a) and d) represent $G_1(s)$ in the one-degree-of-freedom architecture. b) and e) represent $G_1(s)$ in the two-degree-of-freedom architecture. c) and f) represent $G_2(s)$.