南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

Robot Modeling & Control ME331

Section 5： Kinematics IV

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Outline

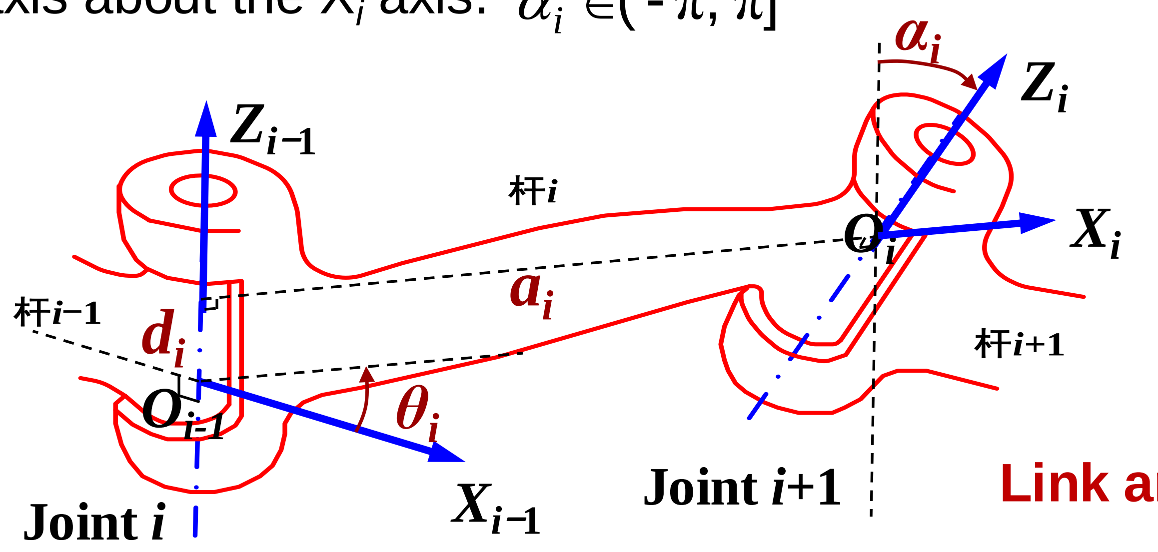
- Review
  - Kinematics
  - Denavit-Hartenberg (D-H) Convention
    - Establish coordinate systems
    - Find the link and joint parameters
- Homogeneous Transformation with D-H Conv.
- Forward Kinematics Equations
- Inverse Kinematics

Denavit-Hartenberg Convention

- Number the joints from 1 to n starting with the base and ending with the end-effector.
- *Establish the base coordinate system.* Establish a right-handed orthonormal coordinate system (X_0, Y_0, Z_0) at the supporting base with Z_0 axis lying along the axis of motion of joint 1.
- *Establish joint axis.* Align the Z_i with the axis of motion (rotary or sliding) of joint i+1.
- *Establish the origin of the i-th coordinate system.* Locate the origin of the i-th coordinate at the intersection of the Z_i & Z_{i-1} or at the intersection of common normal between the Z_i & Z_{i-1} axes and the Z_i axis.
- *Establish X_i axis.* Establish $X_i = \pm(Z_{i-1} \times Z_i)/\|Z_{i-1} \times Z_i\|$ or along the common normal between the Z_{i-1} & Z_i axes when they are parallel.
- *Establish Y_i axis.* Assign $Y_i = +(Z_i \times X_i)/\|Z_i \times X_i\|$ to complete the right-handed coordinate system.
- Find the link and joint parameters

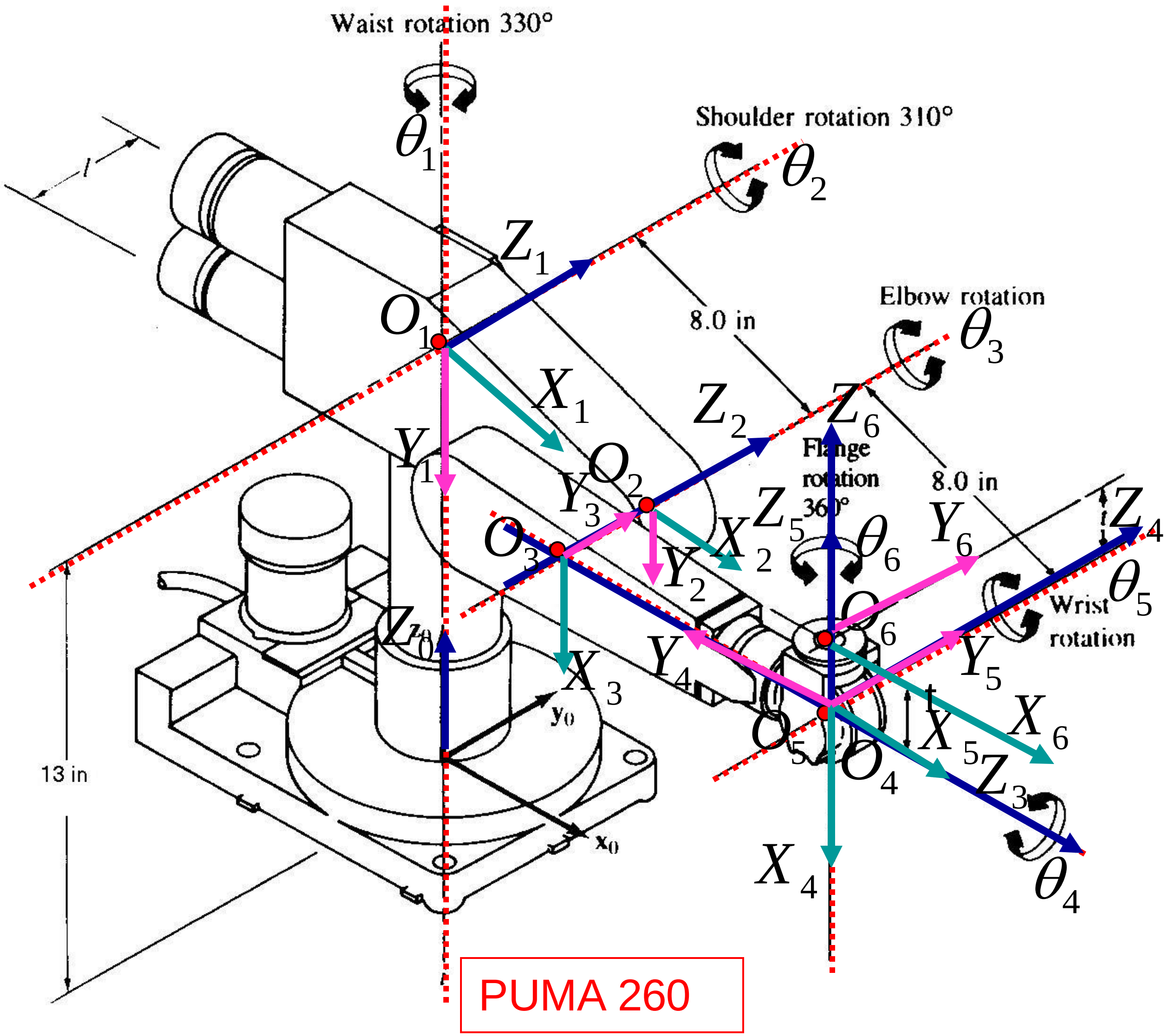
Link and Joint Parameters

- *Joint angle* θ_i : the angle of rotation from the X_{i-1} axis to the X_i axis about the Z_{i-1} axis. It is the joint variable if joint i is rotary. $\theta_i \in (-\pi, \pi]$
- *Joint distance* d_i : the distance from the origin of the (i-1) coordinate system to the intersection of the Z_{i-1} axis and the X_i axis along the Z_{i-1} axis. It is the joint variable if joint *i* is prismatic.
- *Link length* a_i : the distance from the intersection of the Z_{i-1} axis and the X_i axis to the origin of the *i*-th coordinate system along the X_i axis.
- *Link twist angle* α_i : the angle of rotation from the Z_{i-1} axis to the Z_i axis about the X_i axis. $\alpha_i \in (-\pi, \pi]$



Link and Joint Parameters

Example : PUMA 260

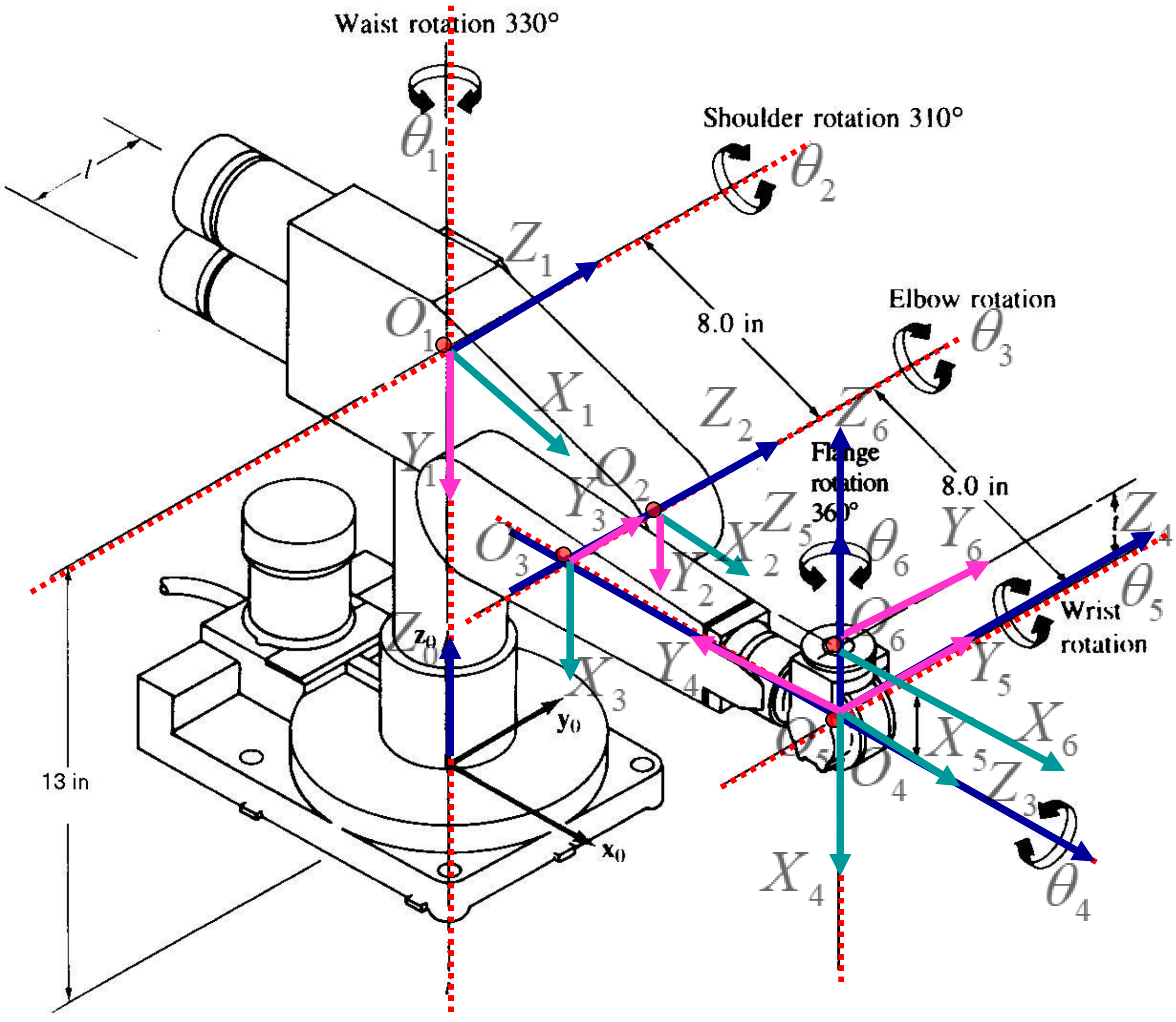


1. **Number the joints**
2. **Establish base frame**
3. **Establish joint axis Z_i**
4. **Locate origin, (intersect. of Z_i & Z_{i-1}) OR (intersect of common normal & Z_i)**
5. **Establish X_i,Y_i**

$$X_i = \pm(Z_{i-1} \times Z_i) / \left\| Z_{i-1} \times Z_i \right\|$$

$$Y_i = +(Z_i \times X_i) / \left\| Z_i \times X_i \right\|$$

Link Parameters



J	θ_i	α_i	a_i	d_i
1	θ_1	-90	0	13
2	θ_2	0	8	0
3	θ_3	90	0	$-l$
4	θ_4	-90	0	8
5	θ_5	90	0	0
6	θ_6	0	0	t

θ_i: 从X_{i-1}轴到X_i轴的转角, Z_{i-1}

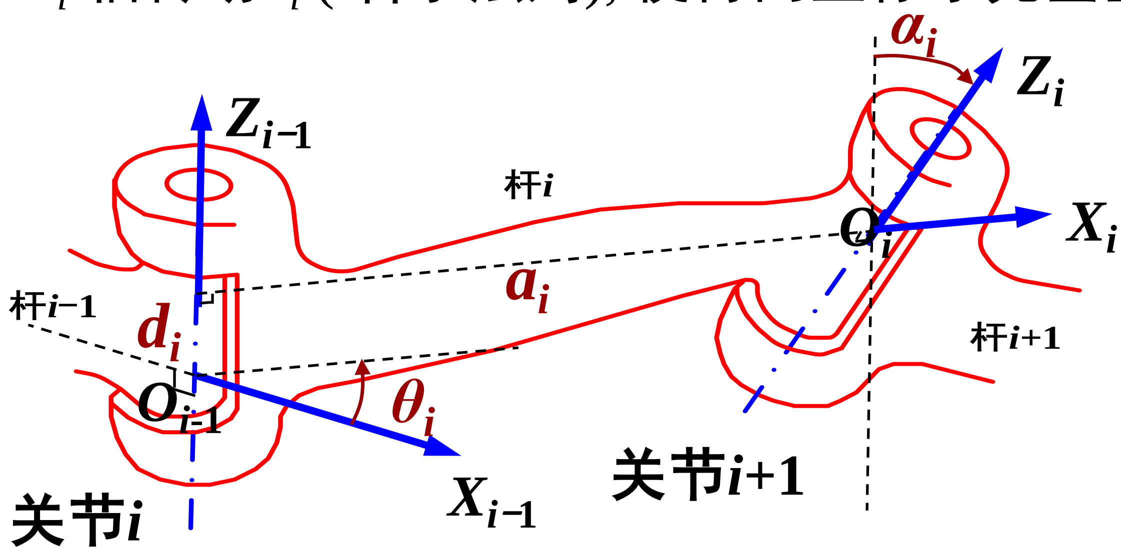
α_i : 从Z_{i-1}轴到Z_i轴的转角, X_i

a_i : 从Z_{i-1}轴到Z_i轴的距离, X_i

d_i : 从X_{i-1}轴到X_i轴的距离, Z_{i-1}

Transformation between *i-1* and *i*

- **relate the *i*-th coordinate frame to the (*i*-1)-th coordinate frame:**
  - 绕 $Z_{i\text{-}1}$ 轴转动 θ_i，使得 $X_{i\text{-}1}$ 轴与 X_i 轴平行；
  - 沿 $Z_{i\text{-}1}$ 轴移动 d_i， 使得$X_{i\text{-}1}$ 轴与 X_i 轴指向相同；
  - 沿 X_i 轴移动 a_i，使得 $O_{i\text{-}1}$ 和 O_i 重合， $X_{i\text{-}1}$ 轴与 X_i 轴重合；
  - 绕 X_i 轴转动α_i (右手法则), 使得两坐标系完全重合。



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Transformation between *i-1* and *i*

- Four successive elementary transformations are required to relate the *i*-th coordinate frame to the (*i*-1)-th coordinate frame:
  - Rotate about the Z_{i-1} axis an angle of θ_i to align the X_{i-1} axis with the X_i axis.
  - Translate along the Z_{i-1} axis a distance of d_i, to bring X_{i-1} and X_i axes into coincidence.
  - Translate along the X_i axis a distance of a_i to bring the two origins O_{i-1} and O_i as well as the X axis into coincidence.
  - Rotate about the X_i axis an angle of α_i (in the right-handed sense), to bring the two coordinates into coincidence.

Transformation between *i-1* and *i*

- **D-H transformation matrix for adjacent coordinate frames, *i* and *i*-1.**
  - **Rotate$_{Z_{i\text{-}1}}$(θ_i); Move$_{Z_{i\text{-}1}}$(d$_i$); Move$_{X_i}$(a_i); Rotate$_{X_i}$(α_i)**
  - The position and orientation of the ***i*** th frame coordinate can be expressed in the (***i***-*1*) th frame by the following homogeneous transformation matrix:

Reference coordinate

$$T_i^{i-1} = T(z_{i-1}, d_i) \quad T(z_{i-1}, \theta_i) \quad T(x_i, a_i) \quad T(x_i, \alpha_i)$$

Target coordinate

$$= \begin{bmatrix} C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Kinematic Equations

- **Forward Kinematics**
  - **Given joint variables**
  - **Find End-effector position & orientation**

$$q = (q_1, q_2, \cdots q_n)$$

$$\Downarrow$$

$$Y = (x, y, z, \phi, \theta, \psi)$$

- **Homogeneous matrix** T_n^0
  - **specifies the location of the *i* th coordinate frame w.r.t. the base coordinate system**
  - **chain product of successive coordinate transformation matrices of** T_i^{i-1}

$$T_n^0 = T_1^0 \, T_2^1 \cdots T_n^{n-1}$$

Position vector

Orientation matrix

$$= \begin{bmatrix} \boxed{R_n^0} & \boxed{P_n^0} \\ \mathbf{0} & 1 \end{bmatrix} = \begin{bmatrix} n & s & a & P_n^0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

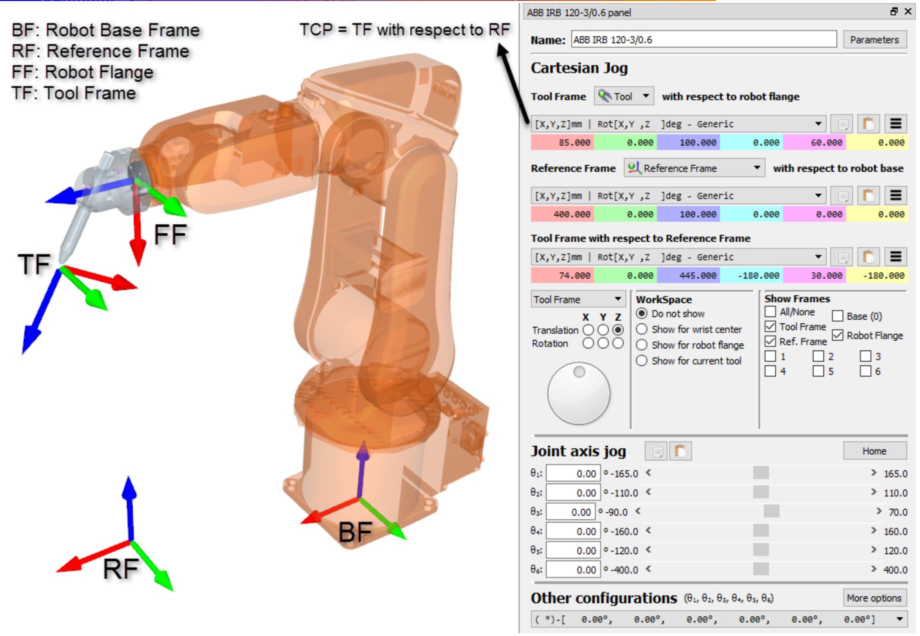
Kinematic Equations

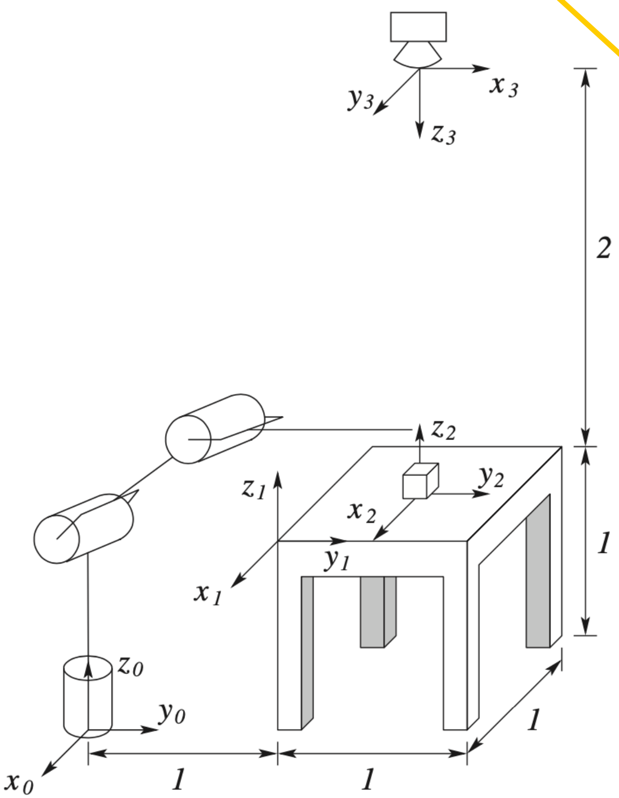
- **Other representations**
  - **TF with respect to RF**

$$T_{TF}^{RF} = \boxed{T_{BF}^{RF}} \cdot \boxed{T_{FF}^{BF}} \cdot \boxed{T_{TF}^{FF}}$$

T_n^0

Constant Matrix





Find the homogeneous transformations relating each of these frames to the base frame $o_0x_0y_0z_0$. Find the homogeneous transformation relating the frame $o_2x_2y_2z_2$ to the camera frame $o_3x_3y_3z_3$.

Kinematic Equations

- **Other representations**
  - **Yaw-Pitch-Roll representation for orientation**

$$T = R_{z,\phi} R_{y,\theta} R_{x,\psi}$$

$$= \begin{bmatrix} C\phi & -S\phi & 0 & 0 \\ S\phi & C\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta & 0 & S\theta & 0 \\ 0 & 1 & 0 & 0 \\ -S\theta & 0 & C\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\psi & -S\psi & 0 \\ 0 & S\psi & C\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C\phi C\vartheta & C\phi S\theta S\psi - S\phi C\psi & C\phi S\theta C\psi + S\phi S\psi & 0 \\ S\phi C\theta & S\phi S\theta S\psi + C\phi C\psi & S\phi S\theta C\psi - C\phi S\psi & 0 \\ -S\theta & C\theta S\psi & C\theta C\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Representing forward kinematics

- **Given: Homogeneous matrix**

$$T = \begin{bmatrix} \boxed{\begin{matrix} n_x & s_x & a_x \\ n_y & s_y & a_y \\ n_z & s_z & a_z \end{matrix}} & \boxed{\begin{matrix} p_x \\ p_y \\ p_z \end{matrix}} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & 1 \end{bmatrix}$$

- **Find: Forward kinematics**

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix} \Rightarrow \begin{bmatrix} \boxed{\begin{matrix} p_x \\ p_y \\ p_z \end{matrix}} \\ \boxed{\begin{matrix} \phi \\ \theta \\ \varphi \end{matrix}} \end{bmatrix}$$

Representing forward kinematics

- **Yaw-Pitch-Roll representation for orientation**

$$T_n^0 = \begin{bmatrix} C\phi C\vartheta & C\phi S\theta S\psi - S\phi C\psi & C\phi S\theta C\psi + S\phi S\psi & p_x \\ S\phi C\theta & S\phi S\theta S\psi + C\phi C\psi & S\phi S\theta C\psi - C\phi S\psi & p_y \\ -S\theta & C\theta S\psi & C\theta C\psi & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Homogeneous matrix according to forward kinematics**

$$T_n^0 = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\theta = \sin^{-1}(-n_z)$$

$$\psi = \cos^{-1}\left(\frac{a_z}{\cos\theta}\right)$$

$$\phi = \cos^{-1}\left(\frac{n_x}{\cos\theta}\right)$$

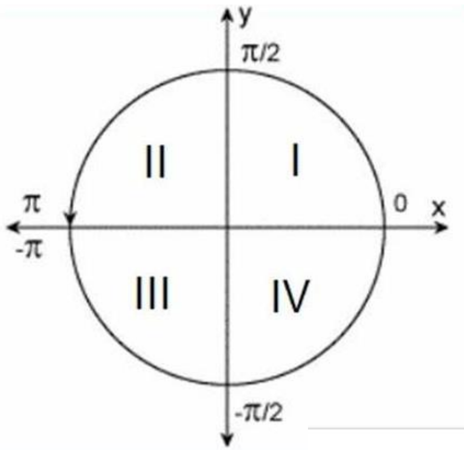
Problem? **Solution is inconsistent and ill-conditioned!!**

1. The sign of an angle cannot be determined by $\cos^{-1}()$ function.
2. The accuracy uniformity of $\sin^{-1}()$ and $\cos^{-1}()$ functions is poor.

Four-quadrant arctangent atan2(y,x)

Compute arc tangent with two parameters

- Returns the principal value of the arc tangent of y/x, expressed in radians.
- To compute the value, the function takes into account the sign of both arguments in order to determine the quadrant.



$$\theta = a\tan 2(y,x) = \begin{cases} 0^\circ \le \theta \le 90^\circ & \textit{for} \quad +x \quad \textit{and} \quad +y \\ 90^\circ \le \theta \le 180^\circ & \textit{for} \quad -x \quad \textit{and} \quad +y \\ -180^\circ \le \theta \le -90^\circ & \textit{for} \quad -x \quad \textit{and} \quad -y \\ -90^\circ \le \theta \le 0^\circ & \textit{for} \quad +x \quad \textit{and} \quad -y \end{cases}$$

$$\operatorname{atan2}(y,x) = \begin{cases} \arctan\left(\frac{y}{x}\right) & x > 0 \\ \arctan\left(\frac{y}{x}\right) + \pi & y \ge 0, x < 0 \\ \arctan\left(\frac{y}{x}\right) - \pi & y < 0, x < 0 \\ +\frac{\pi}{2} & y > 0, x = 0 \\ -\frac{\pi}{2} & y < 0, x = 0 \\ \text{undefined} & y = 0, x = 0 \end{cases}$$

Yaw-Pitch-Roll Representation

Homogeneous matrix according to forward kinematics

=

Yaw-Pitch-Roll representation for orientation

$$T = R_{z,\phi} R_{y,\theta} R_{x,\psi}$$

$$\begin{bmatrix} n_x & s_x & a_x & 0 \\ n_y & s_y & a_y & 0 \\ n_z & s_z & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \underline{\begin{bmatrix} C\phi & -S\phi & 0 & 0 \\ S\phi & C\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} C\theta & 0 & S\theta & 0 \\ 0 & 1 & 0 & 0 \\ -S\theta & 0 & C\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\psi & -S\psi & 0 \\ 0 & S\psi & C\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_{z,\phi}^{-1} T = R_{y,\theta} R_{x,\psi}$$

Yaw-Pitch-Roll Representation

$$\begin{bmatrix} C\phi & S\phi & 0 & 0 \\ -S\phi & C\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x & s_x & a_x & 0 \\ n_y & s_y & a_y & 0 \\ n_z & s_z & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta & 0 & S\theta & 0 \\ 0 & 1 & 0 & 0 \\ -S\theta & 0 & C\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\psi & -S\psi & 0 \\ 0 & S\psi & C\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Compare LHS and RHS of Equation A, we have:

[2,1] $-\sin\phi \cdot n_x + \cos\phi \cdot n_y = 0$ $\Rightarrow$ $\phi = a\tan 2(n_y, n_x)$

[1,1] **[3,1]** $\begin{cases} \cos\phi \cdot n_x + \sin\phi \cdot n_y = \cos\theta \\ n_z = -\sin\theta \end{cases}$ $\Rightarrow$ $\theta = a\tan 2(-n_z, \cos\phi \cdot n_z + \sin\phi \cdot n_y)$

[2,2] **[2,3]** $\begin{cases} -\sin\phi \cdot s_x + \cos\phi \cdot s_y = \cos\psi \\ -\sin\phi \cdot a_x + \cos\phi \cdot a_y = -\sin\psi \end{cases}$

Paul *et al.* (1981)

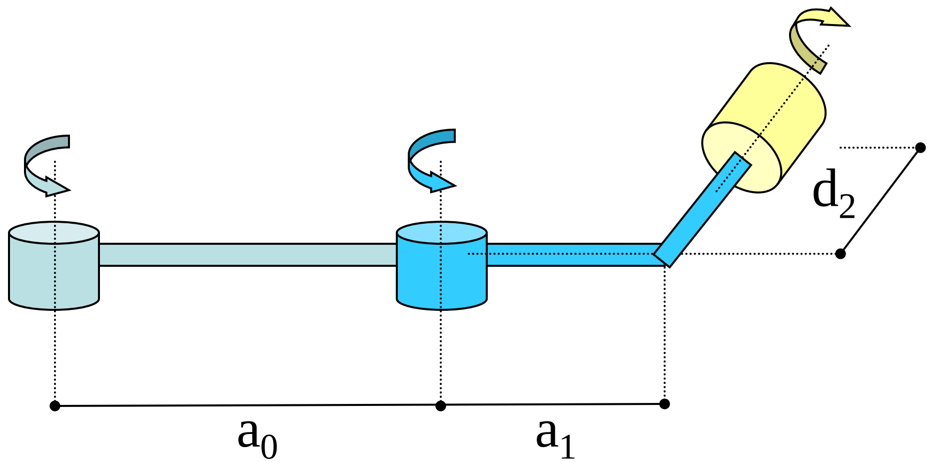
$\Downarrow$

$$\psi = a\tan 2(\sin\phi \cdot a_x - \cos\phi \cdot a_y, -\sin\phi \cdot s_x + \cos\phi \cdot s_y)$$

Forward Kinematics

- **Steps to derive kinematics model:**
  - Assign D-H coordinates frames
  - Find link parameters
  - Transformation matrices of adjacent joints
  - Calculate kinematics matrix
  - When necessary, Euler angle representation

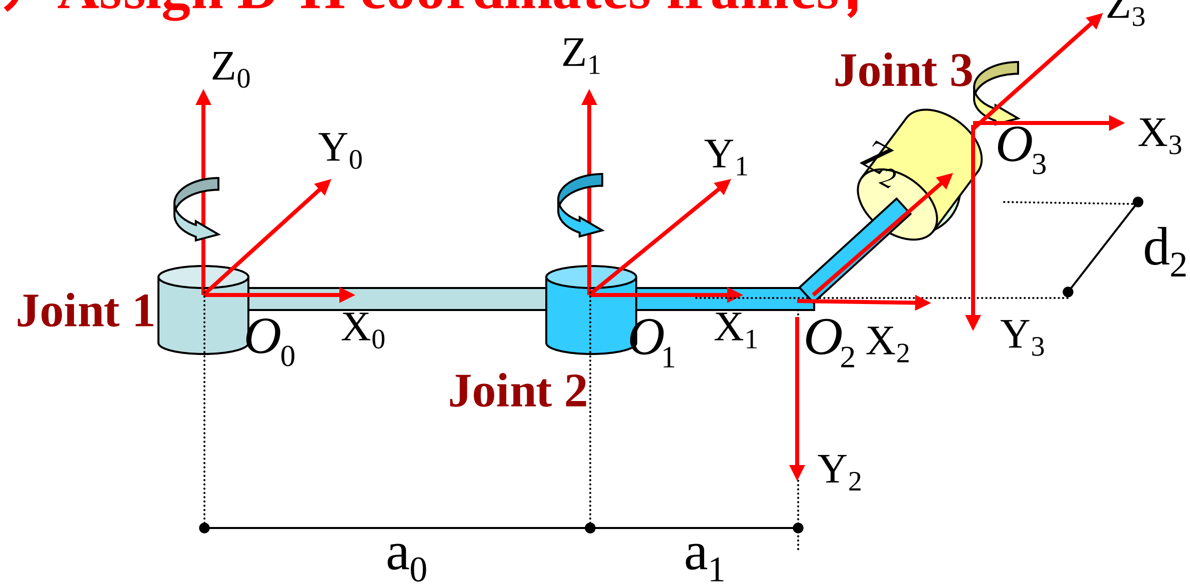
Example: 3R Robot



2）Find D-H parameters；

Joint i	α_i	a_i	d_i	θ_i
1	0	a_0	0	θ_1
2	-90	a_1	0	θ_2
3	0	0	d_2	θ_3

1）Assign D-H coordinates frames；



Example: 3R Robot

Joint i	α_i	a_i	d_i	θ_i
1	0	a_0	0	θ_1
2	-90	a_1	0	θ_2
3	0	0	d_2	θ_3

$$T_1^0 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & a_0\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & a_0\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = \begin{bmatrix} \cos\theta_2 & 0 & -\sin\theta_2 & a_1\cos\theta_2 \\ \sin\theta_2 & 0 & \cos\theta_2 & a_1\sin\theta_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3）Transformation matrices of adjacent joints；

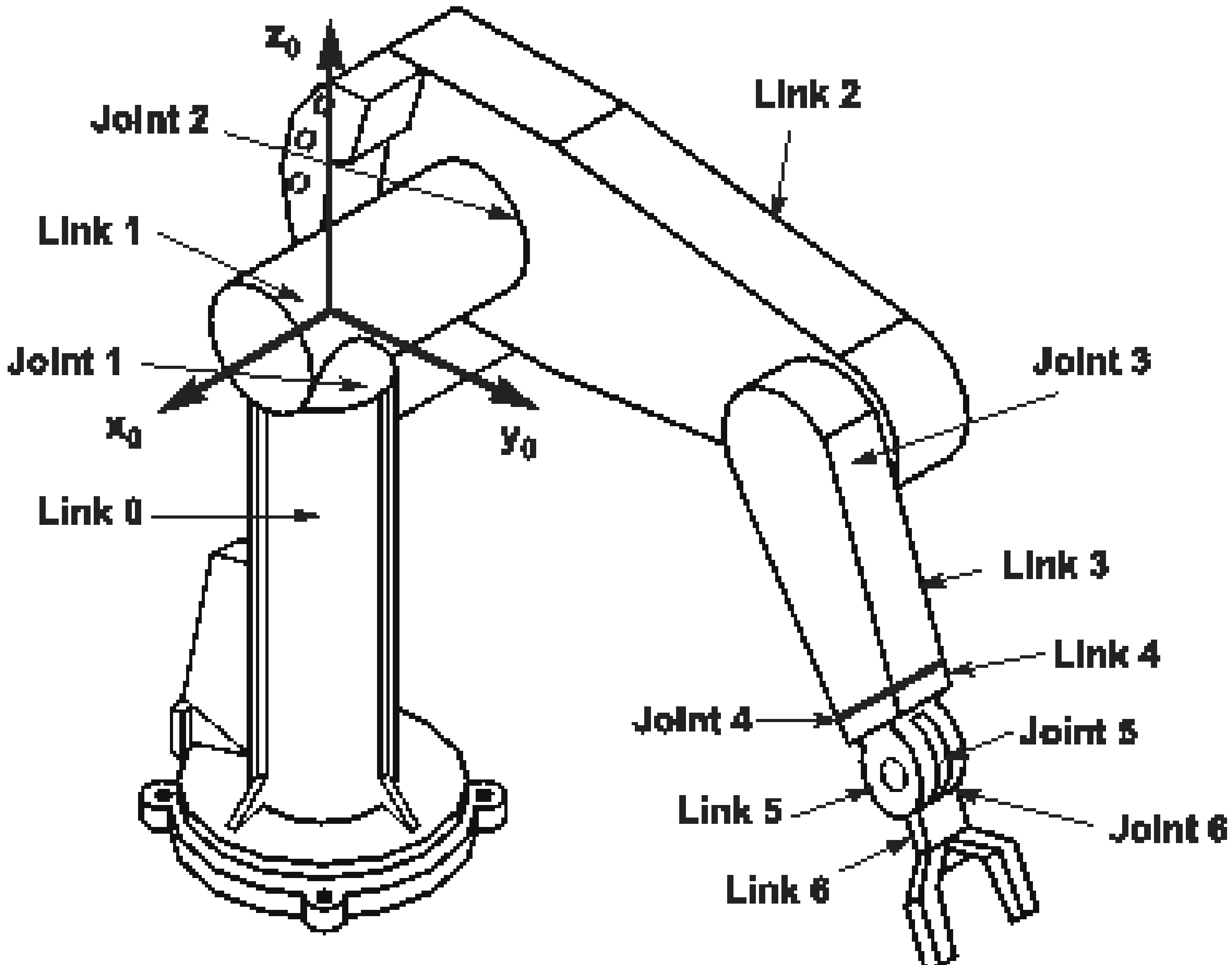
$$T_i^{i-1} = \begin{bmatrix} C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4）Calculate kinematics matrix

$$T_3^0 = (T_1^0)(T_2^1)\ (T_3^2)$$

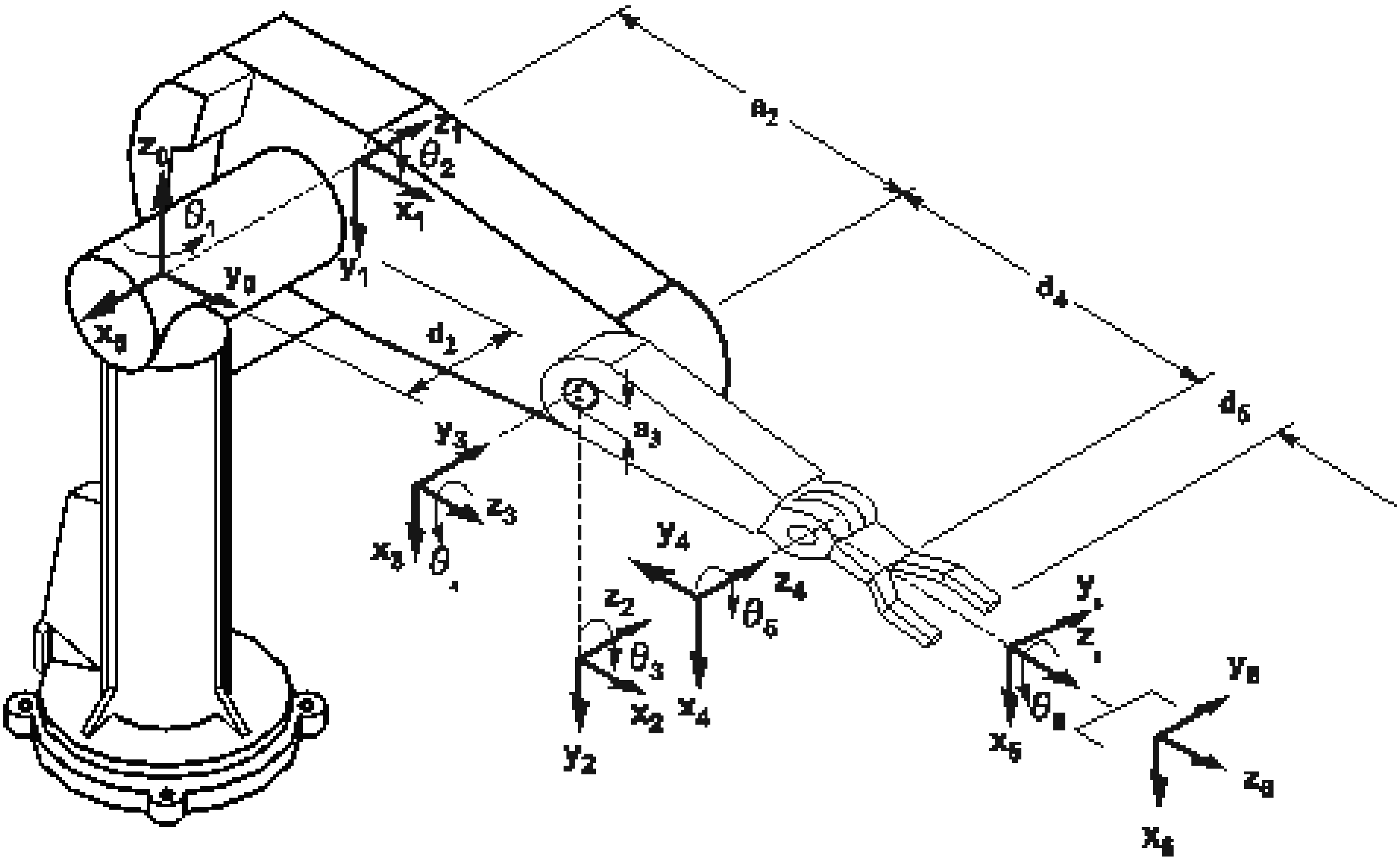
Example: Puma 560

- **Derive forward kinematics model of Puma 560**



Example: Puma 560

1）Assign D-H coordinates frames；



Example: Puma 560

2）Find D-H parameters

PUMA 560 robot arm D-H parameters

Joint i	θ_i	α_i	$a_i(mm)$	$d_i(mm)$
1	θ_1	-90	0	0
2	θ_2	0	431.8	149.09
3	θ_3	90	-20.32	0
4	θ_4	-90	0	433.07
5	θ_5	90	0	0
6	θ_6	0	0	56.25

Example: Puma 560

3）Transformation matrices of adjacent joints；

$$ {}^{0}T_{1} = \begin{pmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad {}^{1}T_{2} = \begin{pmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{2}T_{3} = \begin{pmatrix} \cos\theta_3 & 0 & \sin\theta_3 & a_3\cos\theta_3 \\ \sin\theta_3 & 0 & -\cos\theta_3 & a_3\sin\theta_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad {}^{3}T_{4} = \begin{pmatrix} \cos\theta_4 & 0 & -\sin\theta_4 & 0 \\ \sin\theta_4 & 0 & \cos\theta_4 & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{4}T_{5} = \begin{pmatrix} \cos\theta_5 & 0 & \sin\theta_5 & 0 \\ \sin\theta_5 & 0 & -\cos\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad {}^{5}T_{6} = \begin{pmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Example: Puma 560

4）Calculate Kinematics Matrix

$$
{}^0\boldsymbol{T}_6 = {}^0\boldsymbol{T}_1\, {}^1\boldsymbol{T}_2\, {}^2\boldsymbol{T}_3\, {}^3\boldsymbol{T}_4\, {}^4\boldsymbol{T}_5\, {}^5\boldsymbol{T}_6 = \begin{pmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
\begin{aligned}
n_x &= c_1(c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6) - s_1(s_4c_5c_6 + c_4s_6) \\
n_y &= s_1(c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6) + c_1(s_4c_5c_6 + c_4s_6) \\
n_z &= -s_{23}(c_4c_5c_6 - s_4s_6) - c_{23}s_5c_6
\end{aligned}
$$

$$
\begin{aligned}
s_x &= c_1(-c_{23}(c_4c_5s_6 + s_4c_6) + s_{23}s_5s_6) - s_1(-s_4c_5s_6 + c_4c_6) \\
s_y &= s_1(-c_{23}(c_4c_5s_6 + s_4c_6) + s_{23}s_5s_6) + c_1(-s_4c_5s_6 + c_4c_6) \\
s_z &= s_{23}(c_4c_5s_6 + s_4c_6) - c_{23}s_5s_6
\end{aligned}
$$

$$
\begin{aligned}
a_x &= c_1(c_{23}c_4s_5 + s_{23}c_5) - s_1s_4s_5 \\
a_y &= s_1(c_{23}c_4s_5 + s_{23}c_5) + c_1s_4s_5 \\
a_z &= -s_{23}c_4s_5 + c_{23}c_5)
\end{aligned}
$$

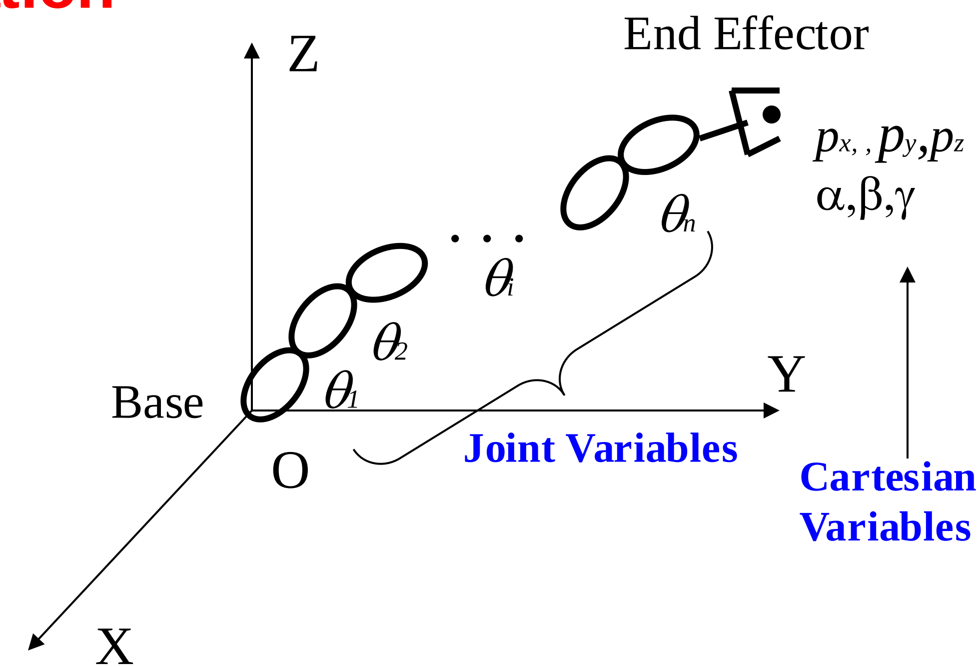
$$
\begin{aligned}
p_x &= c_1(d_6(c_{23}c_4s_5 + s_{23}c_5) + s_{23}d_4 + a_3c_{23} + a_2c_2) - s_1(d_6s_4s_5 + d_2) \\
p_y &= s_1(d_6(c_{23}c_4s_5 + s_{23}c_5) + s_{23}d_4 + a_3c_{23} + a_2c_2) + c_1(d_6s_4s_5 + d_2) \\
p_z &= d_6(c_{23}c_5 - s_{23}c_4s_5) + c_{23}d_4 - a_3s_{23} - a_2s_2
\end{aligned}
$$

Inverse Kinematics

1）Given a desired position & orientation of the end-effector

2）Find the joint variables which can bring the robot the desired configuration

$$\boldsymbol{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \cdot \\ \cdot \\ \cdot \\ \theta_n \end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix} p_x \\ p_y \\ p_z \\ \alpha \\ \beta \\ \gamma \end{bmatrix}$$

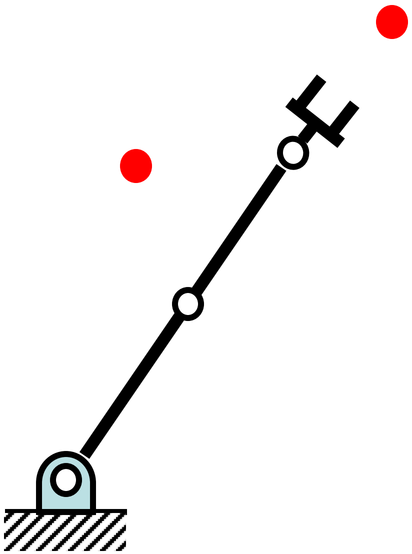


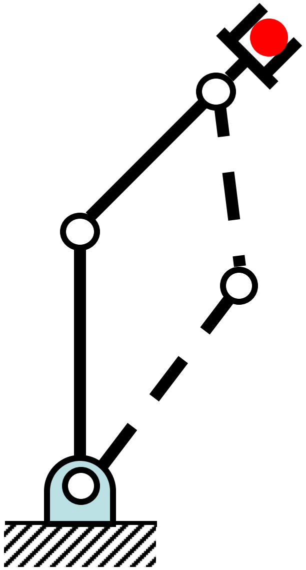
(Joint) $\boldsymbol{\theta} \longleftarrow \mathbf{x}$ (Cartesian)

Inverse Kinematics

Inverse Kinematics

- More difficult than FK
  - May not have a solution
  - Systematic closed-form solution in general is not available
  - Solution not unique
    - Redundant robot
    - Elbow-up/elbow-down configuration
  - Robot dependent





Inverse Kinematics

- **Transformation Matrix**

$$\begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = T_1^0 T_2^1 T_3^2 T_4^3 T_5^4 T_6^5 = T(\theta) \Rightarrow \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix}$$

Special cases make the closed-form arm solution possible:

1. **Three adjacent joint axes intersecting (PUMA， Stanford)**
2. **Three adjacent joint axes parallel to one another (ASEA， MINIMOVER)**

Summary

D-H Representation

- D-H Convention
- D-H Parameters

Forward Kinematics

- Steps to derive kinematics model
- Yaw-Pitch-Roll Representation
  — Four-quadrant arctangent atan2(y,x)

Inverse Kinematics

- Systematic closed-form solution in general is not available
- Solution not unique

Homework 5

Homework 5 is posted at http://bb.sustech.edu.cn

Due date: March 12, 2025

Next class: Inverse Kinematics (Monday, March 10)

作业要求（Requirements）：

1. **文件格式为以自己学号姓名作业序号命名的pdf文件；**

   （File name: YourSID_ YourName_04.pdf）

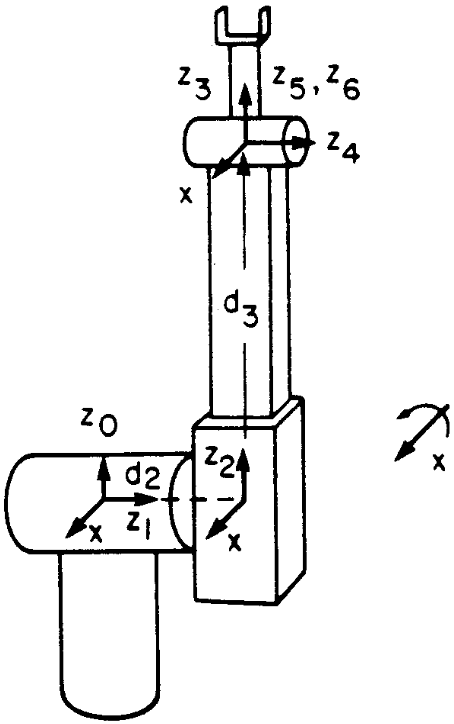
2. **作业里也写上自己的姓名和学号。**

   （Write your name and SID in the homework）

Homework 5

Problem 1: Given the Stanford arm as following figure, where d_2=0.1m

1. Find the forward kinematic model for the arm and represent it in homogeneous matrix form. (Suggestion: the matrix chain product can be done either by hand or using **Matlab symbolic toolbox** if you have the software, simplify the results)
2. Represent the orientation with Yaw-Pitch-Roll angels（use atan2 function）.



Problem 2: As show in the following figure ,find the homogeneous transformations relating each of these frames to the base frame $o_0x_0y_0z_0$. Find the homogeneous transformation relating the frame $o_2x_2y_2z_2$ to the camera frame $o_3x_3y_3z_3$.