南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

Robot Modeling & Control ME331

Section 4: Kinematics III

Chenglong Fu（付成龙）
Dept. of MEE， SUSTech

Outline

- **Review**
  - **Homogeneous Matrix**
    - **Composite Homogeneous Transformation**
    - **Geometric interpretation**
  - **Orientation Representation**
    - **Euler Angle I & II**
    - **Yaw-Pitch-Roll**
- **Denavit-Hartenberg (D-H) Representation**
  - **D-H Convention**
  - **D-H Parameters**

Review

- **Coordinate transformation from $\{B\}$ to $\{A\}$**

$$ {}^{A}r^{P} = {}^{A}R_{B}\,{}^{B}r^{P} + {}^{A}r^{o'} $$

$$ \begin{bmatrix} {}^{A}r^{P} \\ 1 \end{bmatrix} = \begin{bmatrix} {}^{A}R_{B} & {}^{A}r^{o'} \\ 0_{1\times 3} & 1 \end{bmatrix} \begin{bmatrix} {}^{B}r^{P} \\ 1 \end{bmatrix} $$

- **Homogeneous transformation matrix**

$$ {}^{A}T_{B} = \begin{bmatrix} {}^{A}R_{B} & {}^{A}r^{o'} \\ 0_{1\times 3} & 1 \end{bmatrix} = \begin{bmatrix} R_{3\times 3} & P_{3\times 1} \\ 0 & 1 \end{bmatrix} $$

Rotation matrix

Position vector

Scaling

Review

- **Homogeneous Transformation**

  ---Special cases

  1. Translation

$$ {}^{A}T_{B} = \begin{bmatrix} I_{3\times3} & {}^{A}r^{o'} \\ 0_{1\times3} & 1 \end{bmatrix} $$

  2. Rotation

$$ {}^{A}T_{B} = \begin{bmatrix} {}^{A}R_{B} & 0_{3\times1} \\ 0_{1\times3} & 1 \end{bmatrix} $$

Review

- **Composite Homogeneous Transformation Matrix**

  ---Rules:

  – Transformation (rotation/translation) w.r.t (X,Y,Z) (OLD FRAME), using pre-multiplication

  – Transformation (rotation/translation) w.r.t (U,V,W) (NEW FRAME), using post-multiplication

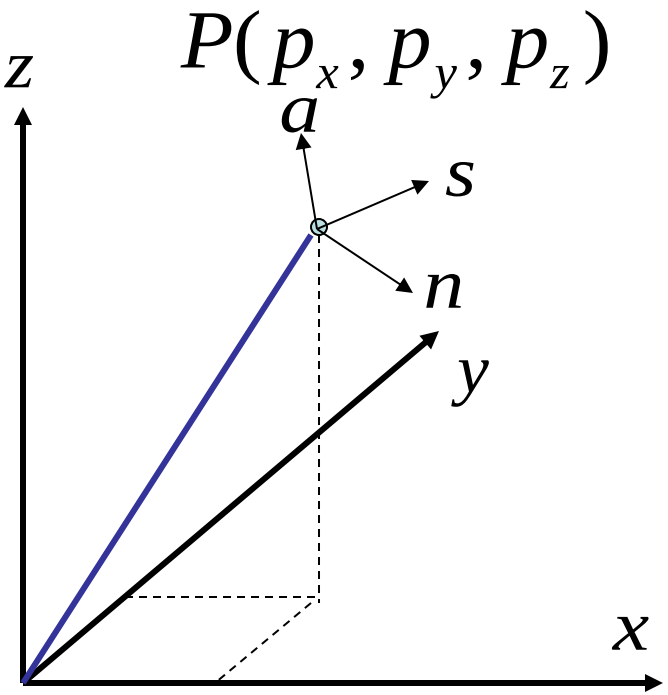
Review

- **Homogeneous Representation**
  - A point in R^3 space

$$P = \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$$ ← Homogeneous coordinate of P w.r.t. OXYZ

  - A frame in R^3 space

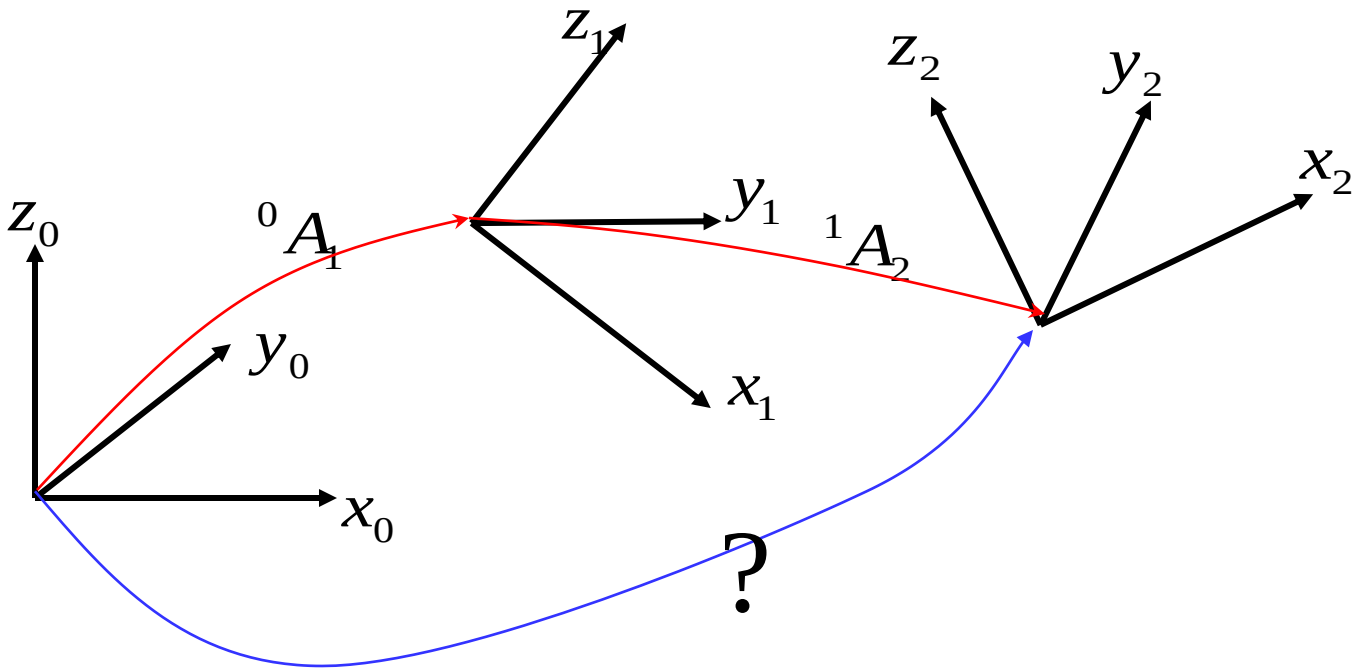
$$F = \begin{bmatrix} n & s & a & P \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Principal axis *n* w.r.t. the reference coordinate system

Review

Composite Homogeneous Transformation Matrix



$${}^{i-1}A_i$$ **Transformation matrix for adjacent coordinate frames**

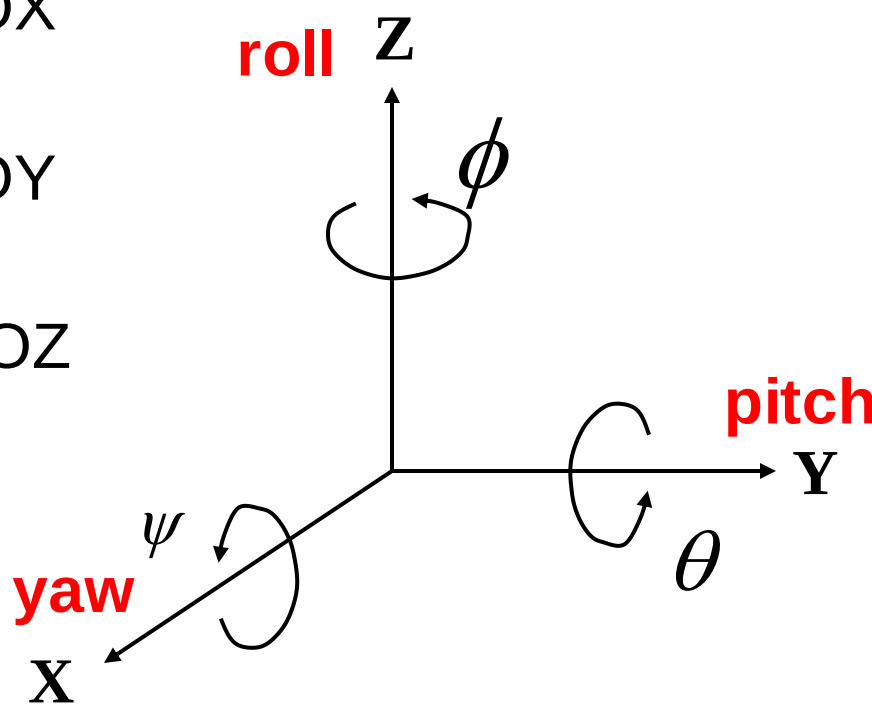
$${}^0A_2 = {}^0A_1\,{}^1A_2$$ **Chain product of successive coordinate transformation matrices**

Orientation Representation

- **Orientation Representation (Euler Angles)**

  ---Description of Yaw, Pitch, Roll

  - A rotation of ψ about the OX axis ($R_{x,\psi}$) -- yaw
  - A rotation of θ about the OY axis ($R_{y,\theta}$) -- pitch
  - A rotation of ϕ about the OZ axis ($R_{z,\phi}$) -- roll



Exercise

- **Geometric Interpretation?**

$$T = \begin{bmatrix} R_{3\times3} & P_{3\times1} \\ 0 & 1 \end{bmatrix}$$

Orientation of OUVW coordinate frame w.r.t. OXYZ frame

Position of the origin of OUVW coordinate frame w.r.t. OXYZ frame

- **Inverse Homogeneous Matrix?**

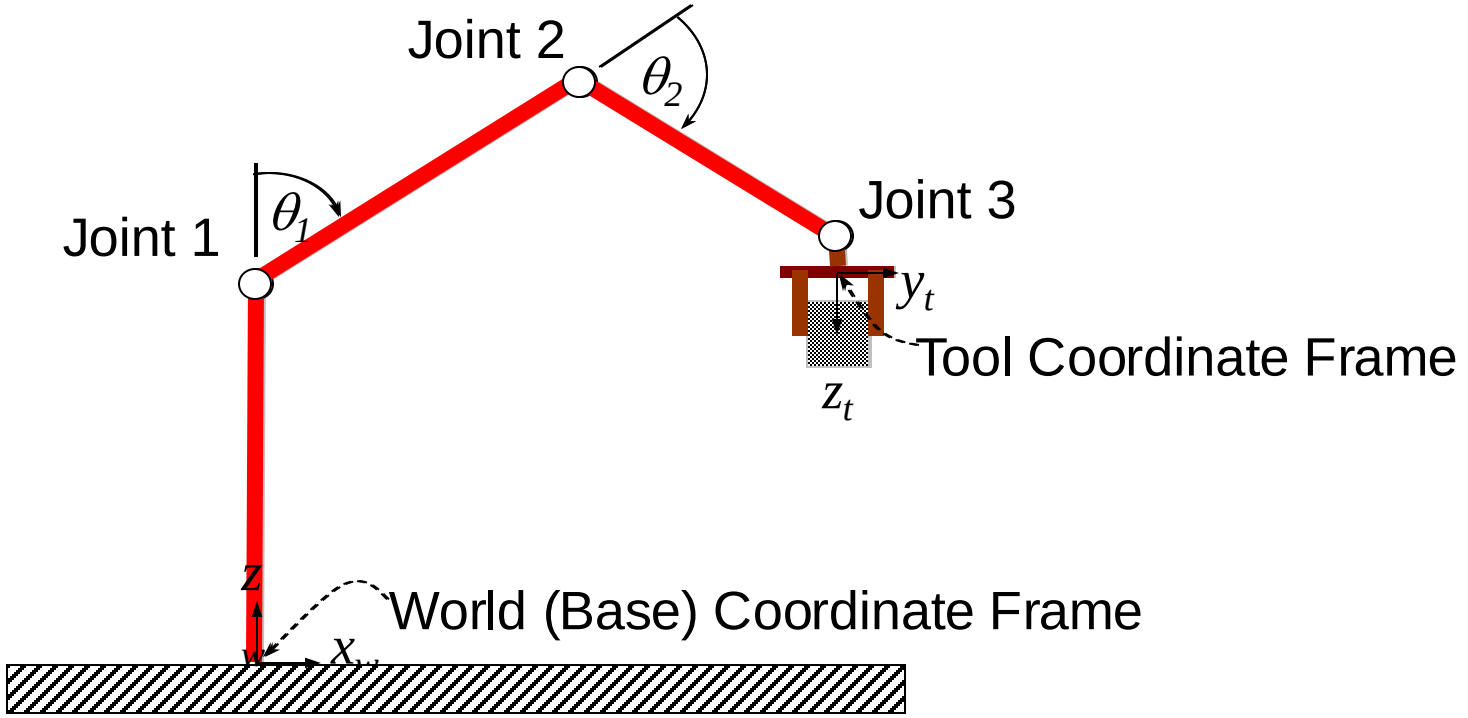
$$T^{-1} = \begin{bmatrix} R^T & -R^T P \\ 0 & 1 \end{bmatrix}$$

Inverse of the rotation submatrix is equivalent to its transpose

Position of the origin of OXYZ reference frame w.r.t. OUVW frame

$$T^{-1}T = \begin{bmatrix} R^T & -R^T P \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R^T R & 0 \\ 0 & 1 \end{bmatrix} = I_{4\times4}$$

Kinematics Model



Link Space

***n* variables**

(θ_1 ... θ_n)

Forward Kinematics

Inverse Kinematics

Tool Space

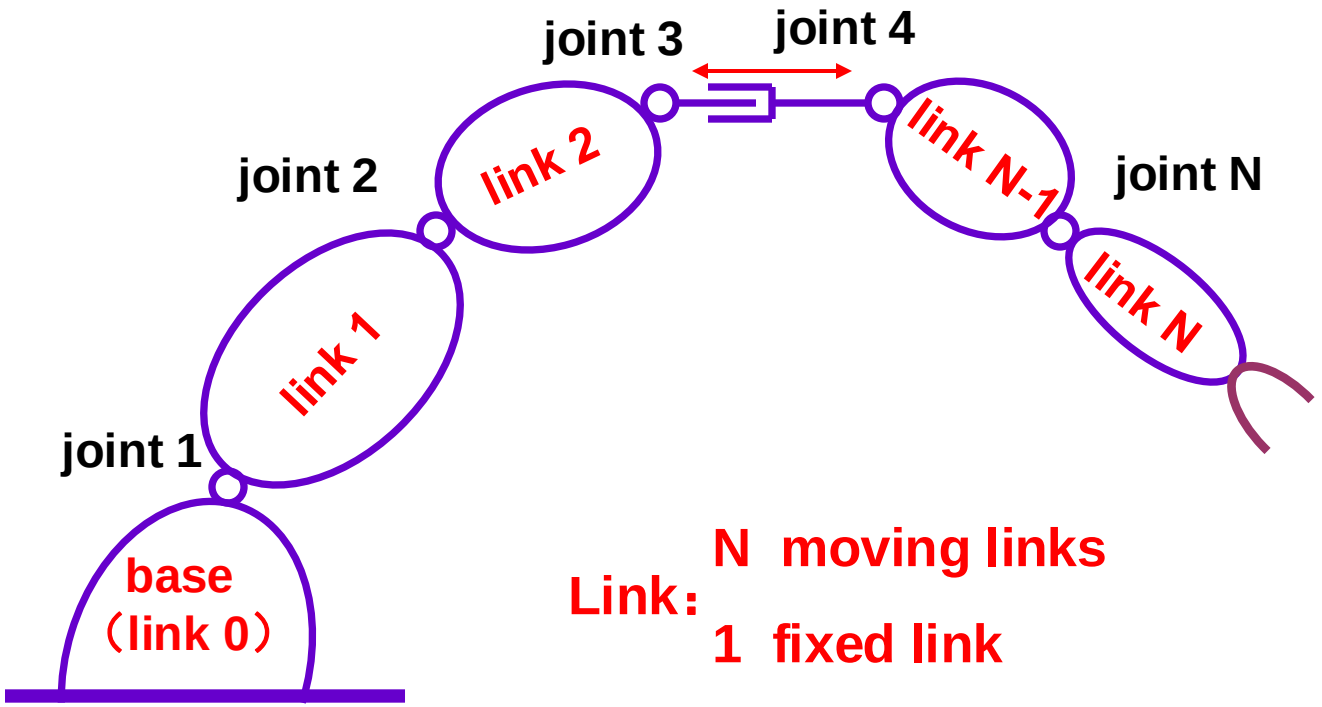
***6* variables**

(x,y,z,θ_x,θ_y,θ_z)

Robot Links and Joints

Robotic Tasks： 1. positioning/orienting

2. force/moment exerted on environment



Link： N moving links

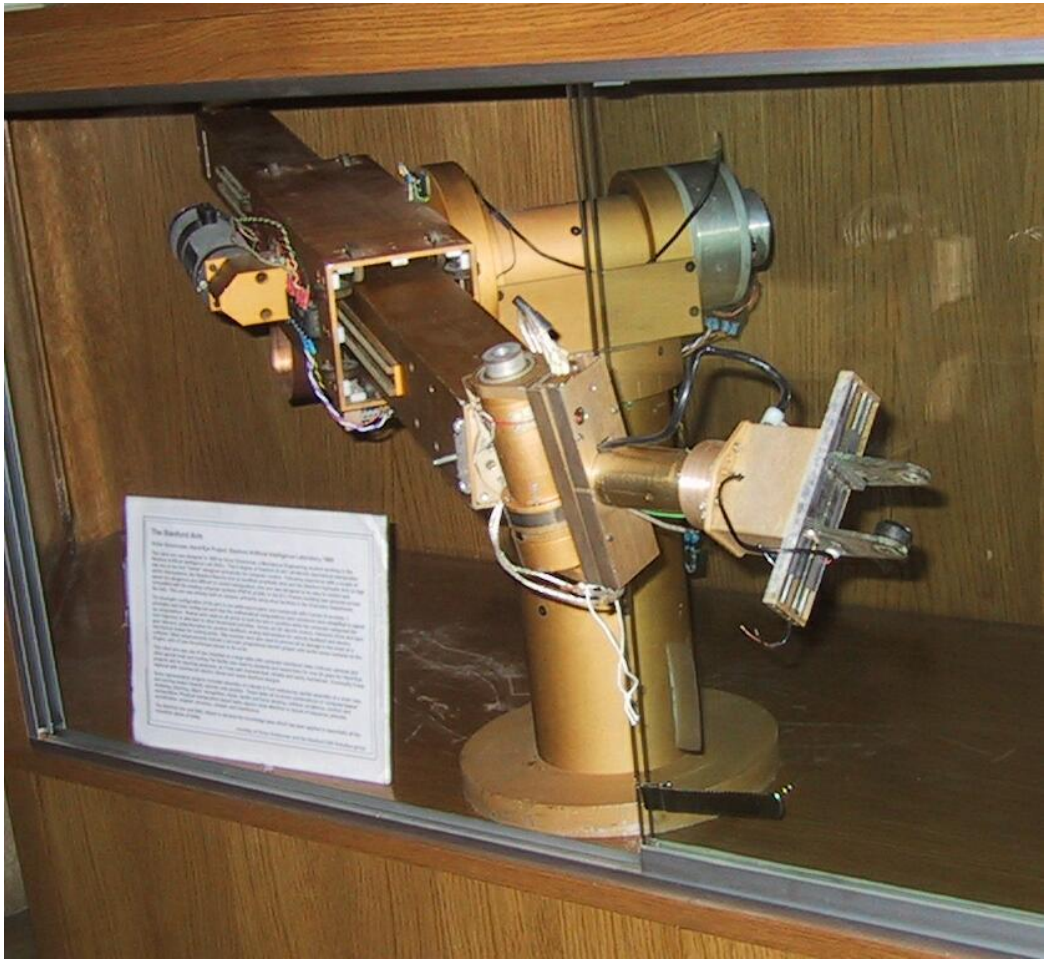
1 fixed link

Joint： connecting mechanism

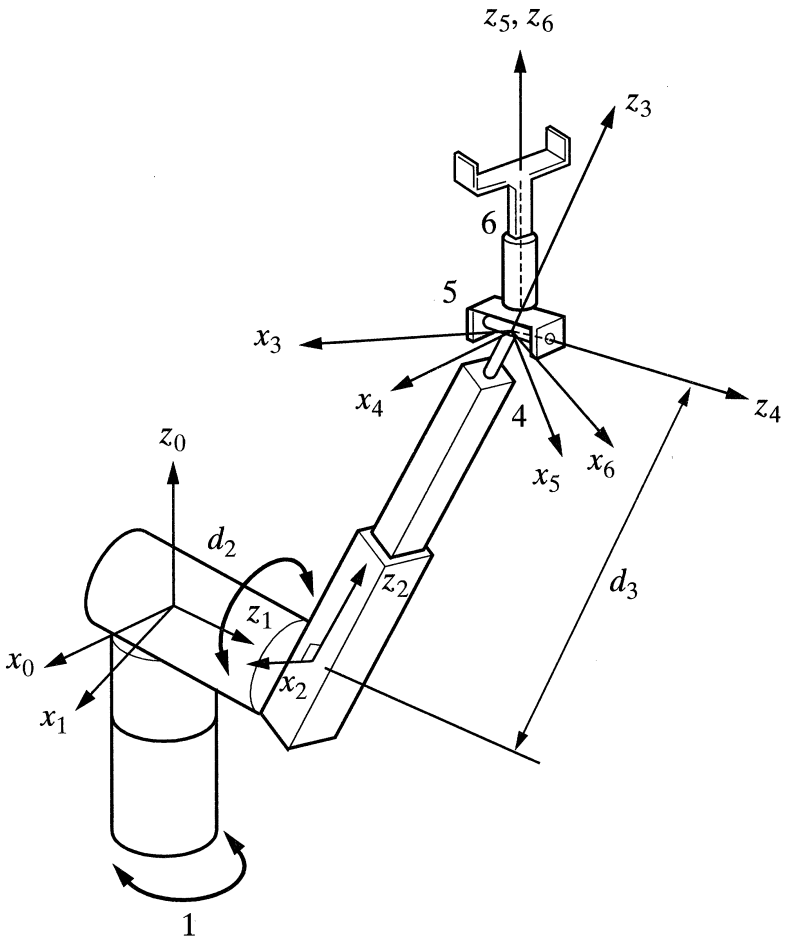
power giving

Denavit-Hartenberg Convention

- **How to establish coordinate systems for a robot?**



The Stanford Arm, 1969



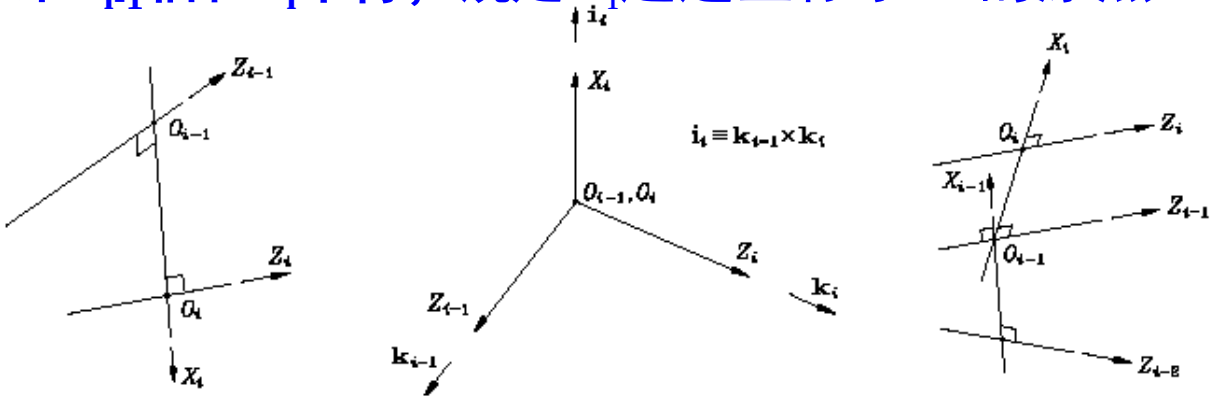
The Stanford Arm

Denavit-Hartenberg Convention

CN

- 连杆从基座连杆到末端执行器依次编号为**0，1，…，*n***
- **建立基座坐标系.** 在基座上建立右手坐标系 (X_0, Y_0, Z_0)，使得 Z_0 指向第1个关节的轴线.
- **确定第*i*个坐标系的Z*i*轴.** 选取Z_i为关节$i+1$的轴线（指向可任选，但通常都将各平行的Z轴均取为相同的指向）.
- **确定第*i*个坐标系的原点.** 选取原点O_i在过Z_{i-1}轴和Z_i轴的公垂线。
- **确定各坐标系的X*i* 轴.** X_i轴为Z_{i-1}和Z_i的公垂线。

★如果Z_{i-1}轴和Z_i平行，规定X_i通过坐标系i-1的原点



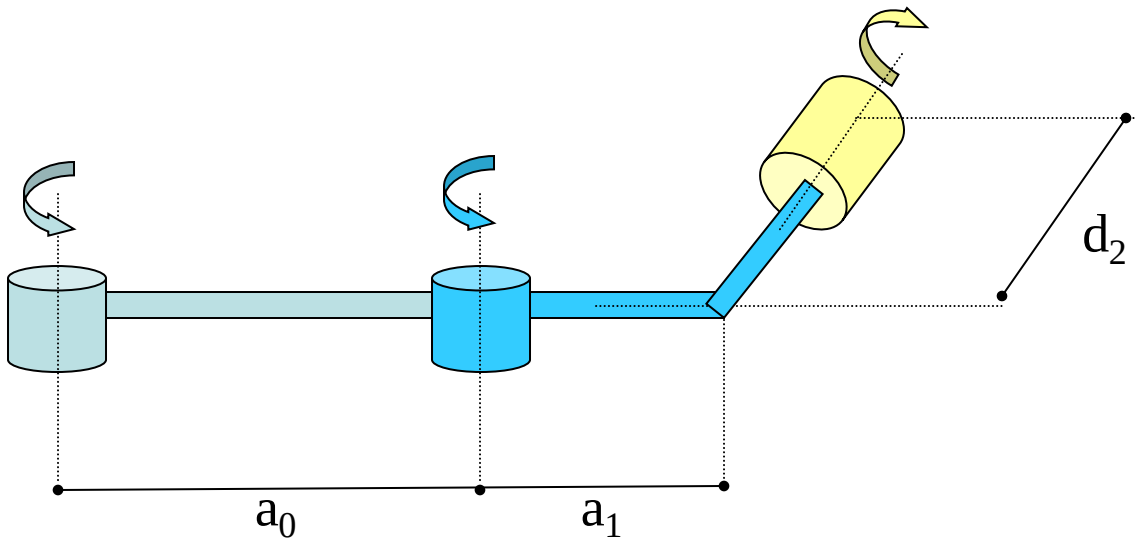
交叉　　相交　　平行

Denavit-Hartenberg Convention

- Number the joints from 1 to n starting with the base and ending with the end-effector. **EN**
- *Establish the base coordinate system.* Establish a right-handed orthonormal coordinate system (X_0, Y_0, Z_0) at the supporting base with Z_0 axis lying along the axis of motion of joint 1.
- *Establish joint axis.* Align the Z_i with the axis of motion (rotary or sliding) of joint i+1.
- *Establish the origin of the i-th coordinate system.* Locate the origin of the i-th coordinate at the intersection of the Z_i & Z_{i-1} or at the intersection of common normal between the Z_i & Z_{i-1} axes and the Z_i axis.
- *Establish X_i axis.* Establish $X_i = \pm(Z_{i-1} \times Z_i) / \|Z_{i-1} \times Z_i\|$ or along the common normal between the Z_{i-1} & Z_i axes when they are parallel.
- *Establish Y_i axis.* Assign $Y_i = +(Z_i \times X_i) / \|Z_i \times X_i\|$ to complete the right-handed coordinate system.
- Find the link and joint parameters

Example 1

- **Assign link coordinate frames for 3R Joints**

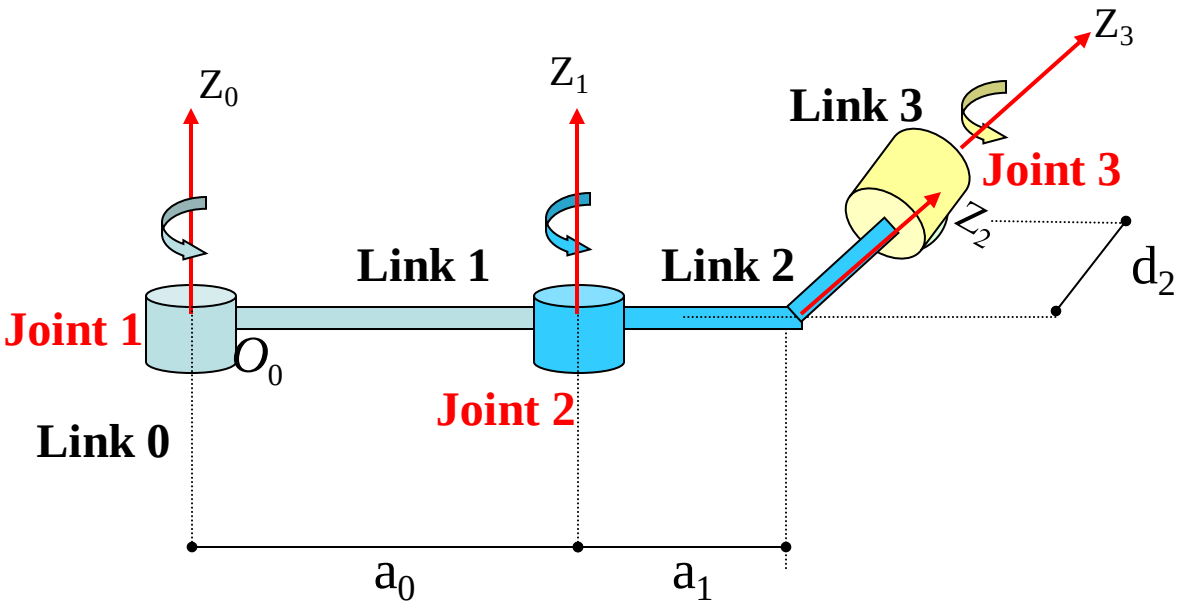


Example 1

- **Number the links and the joints**
- **Establish joint axis Z_i:**
  - **Align the Z_i with the axis of motion of joint *i*+1.**

★ 指向可任选，但通常都将各平行的Z轴取相同的指向

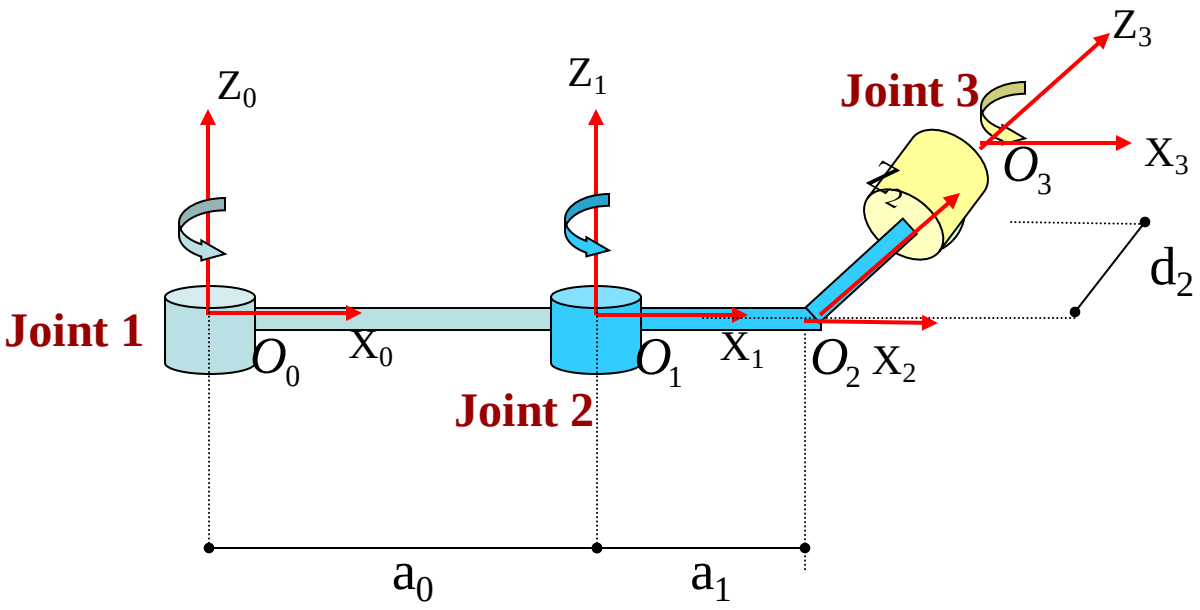
★ 杆n的远端没有关节n+1关节，可选Z_n 轴与Z_{n-1} 轴重合；



Example 1

CN

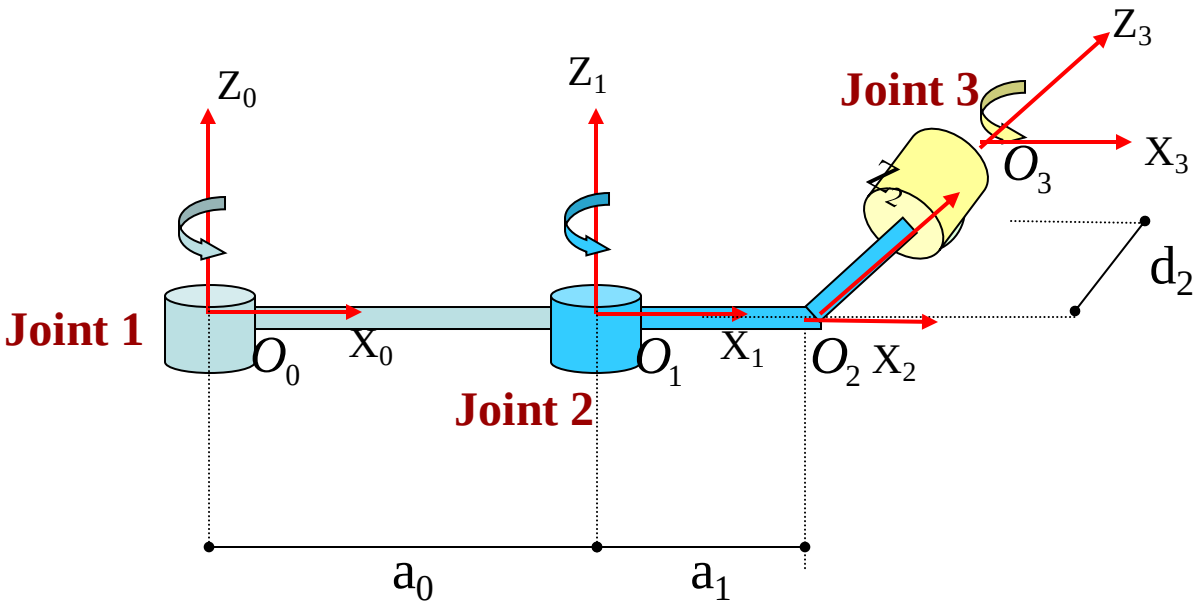
- 选取原点O_i轴在过Z_{i-1}轴和Z_i轴的公垂线上（即O_i为此公垂线与Z_i轴的交点）
- 选取X_i轴沿过Z_{i-1}轴和Z_i轴的公垂线，方向从Z_{i-1}指向Z_i；（如果Z_{i-1}与Z_i平行， X_i通过坐标系i-1的原点）



Example 1

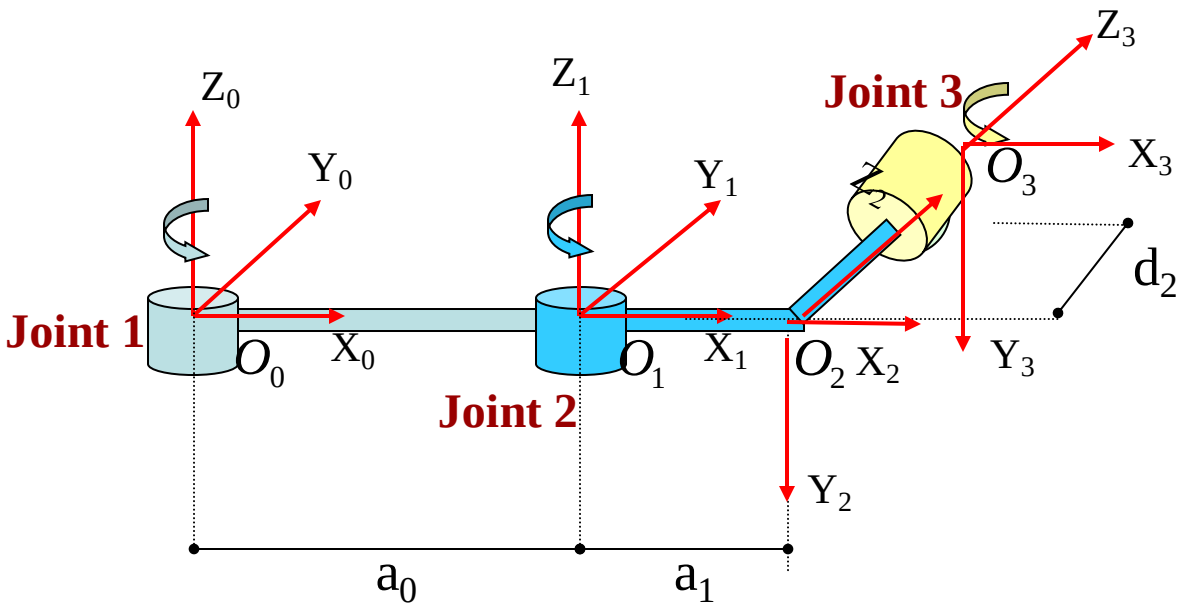
EN

- Locate the origin of the ith coordinate at the intersection of the Z_i & Z_{i-1} or at the intersection of common normal between the Z_i & Z_{i-1} axes and the Z_i axis.
- the X_i axis lies along the common normal from the Z_{i-1} axis to the Z_i axis $X_i = \pm(Z_{i-1} \times Z_i) / \|Z_{i-1} \times Z_i\|$, (if Z_{i-1} is parallel to Z_i, then X_i is specified arbitrarily, subject only to X_i being perpendicular to Z_i);



Example 1

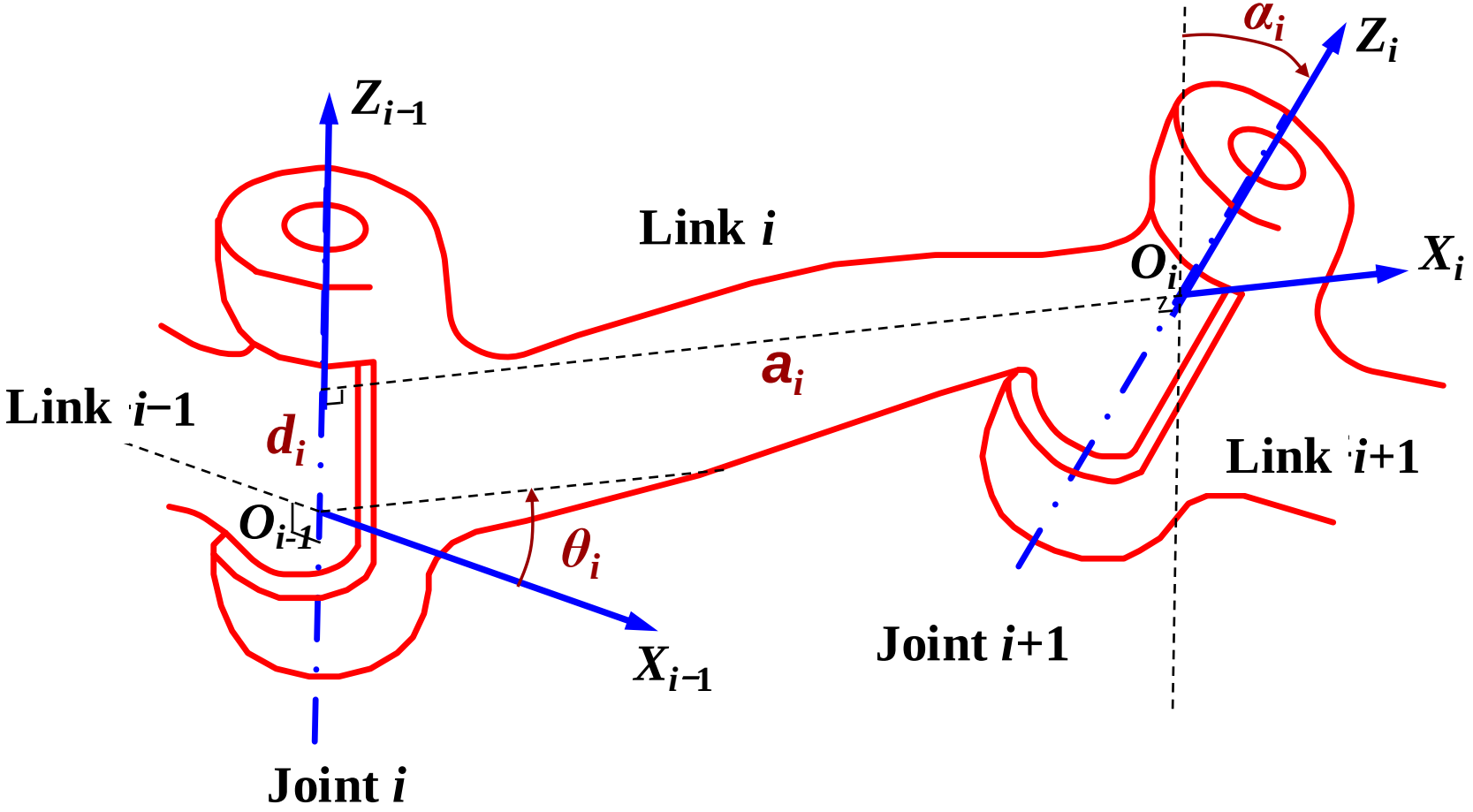
– **Assign** $Y_i = +(Z_i \times X_i) / \|Z_i \times X_i\|$ **to complete the right-handed coordinate system.**



D-H Parameters

- D-H Parameters (Link and Joint Parameters)

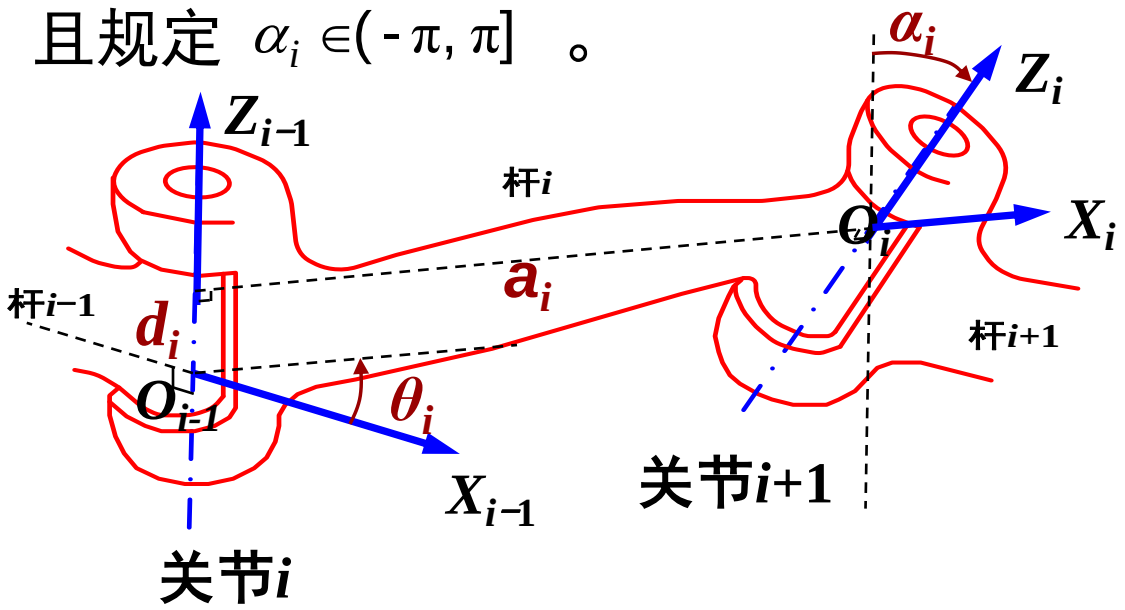
Link length a_i *Link twist* α_i Joint distance d_i Joint angle θ_i



D-H Parameters

CN

- **关节转角** θ_i：定义为从X_{i-1}轴到X_i轴的转角，绕Z_{i-1}轴正向转动为正，且规定 $\theta_i \in (-\pi, \pi]$ 。
- **关节距离** d_i: 定义为从X_{i-1} 轴到X_i 轴的距离，沿Z_{i-1} 轴的指向为正；当关节i 是移动关节时，d_i 为关节变量。
- **杆件长度** a_i: 定义为从Z_{i-1} 轴到Z_i 的距离，沿X_i轴指向为正。
- **杆件扭角** α_i: 定义为从Z_{i-1} 轴到Z_i 的转角，绕X_i轴正向转动为正，且规定 $\alpha_i \in (-\pi, \pi]$ 。



Link and Joint Parameters EN

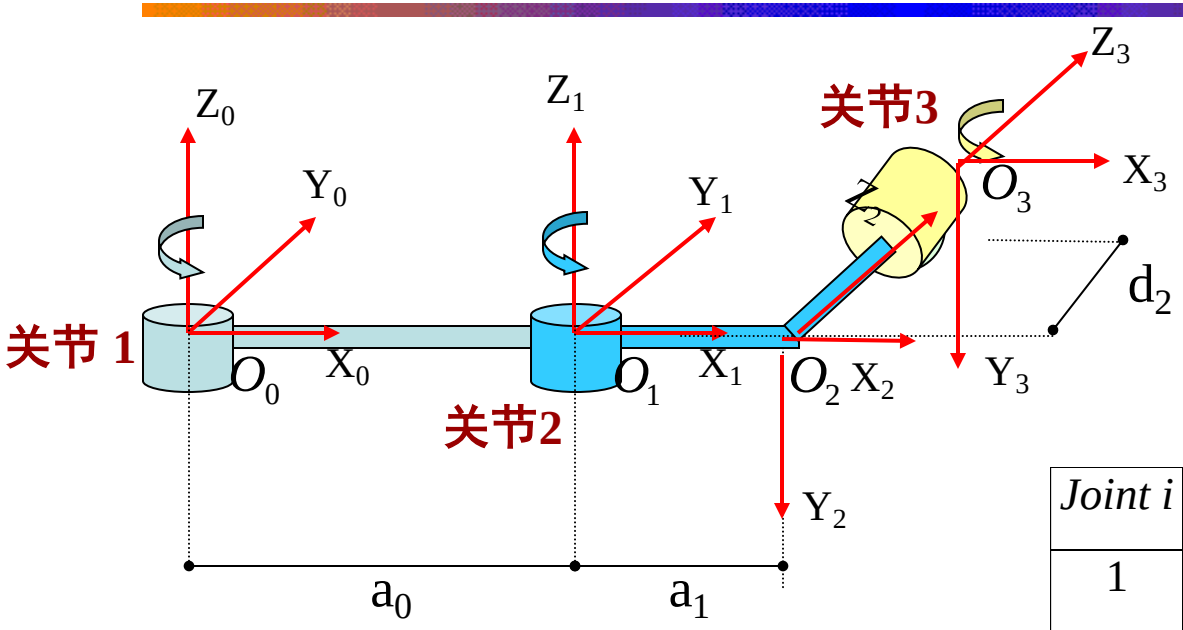
- *Joint angle* θ_i: the angle of rotation from the X_{i-1} axis to the X_i axis about the Z_{i-1} axis. It is the joint variable if joint i is rotary.

- *Joint distance* d_i: the distance from the origin of the (i-1) coordinate system to the intersection of the Z_{i-1} axis and the X_i axis along the Z_{i-1} axis. It is the joint variable if joint i is prismatic.

- *Link length* a_i: the distance from the intersection of the Z_{i-1} axis and the X_i axis to the origin of the ith coordinate system along the X_i axis.

- *Link twist angle* α_i: the angle of rotation from the Z_{i-1} axis to the Z_i axis about the X_i axis.

Example 1: D-H Parameters CN



α_i：从Z_{i-1}轴到Z_i轴的转角，X_i轴为正向

a_i：从Z_{i-1}轴到Z_i轴的距离，X_i轴为正向

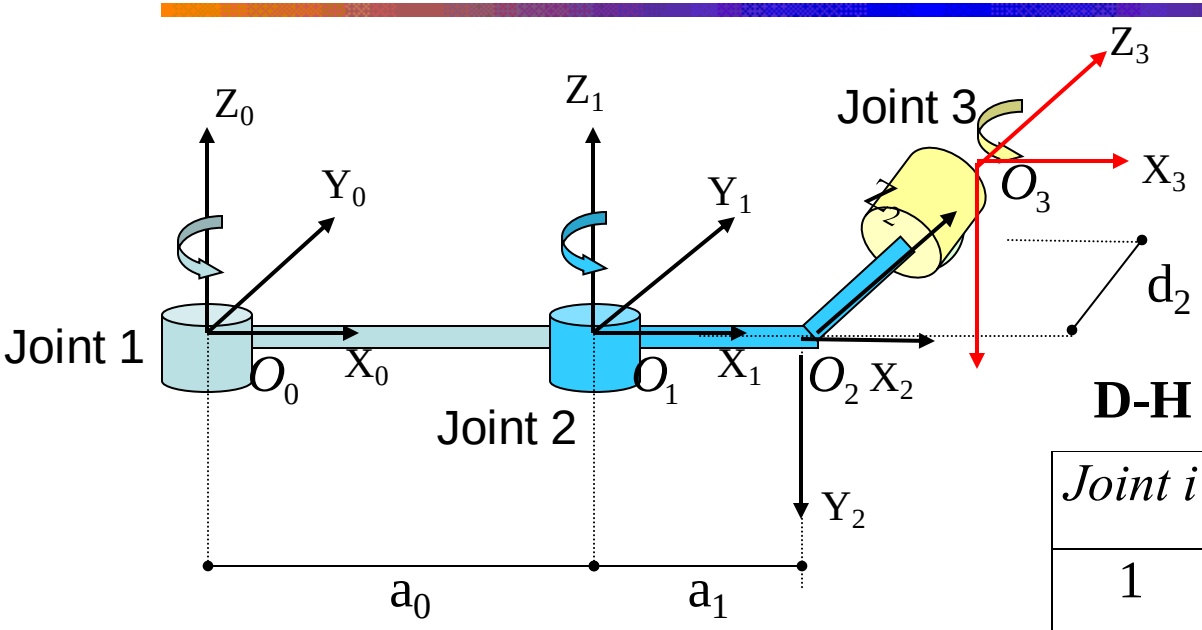
d_i：从X_{i-1}轴到X_i轴的距离，Z_{i-1}轴为正向

θ_i：从X_{i-1}轴到X_i轴的转角，Z_{i-1}轴为正向

D-H 杆件参数表

Joint i	α_i	a_i	d_i	θ_i
1	0	a_0	0	θ_1
2	-90	a_1	0	θ_2
3	0	0	d_2	θ_3

Example 1: D-H Parameters EN



D-H Link Parameter Table

Joint i	a_i	α_i	d_i	q_i
1	0	a_0	0	q_1
2	-90	a_1	0	q_2
3	0	0	d_2	q_3

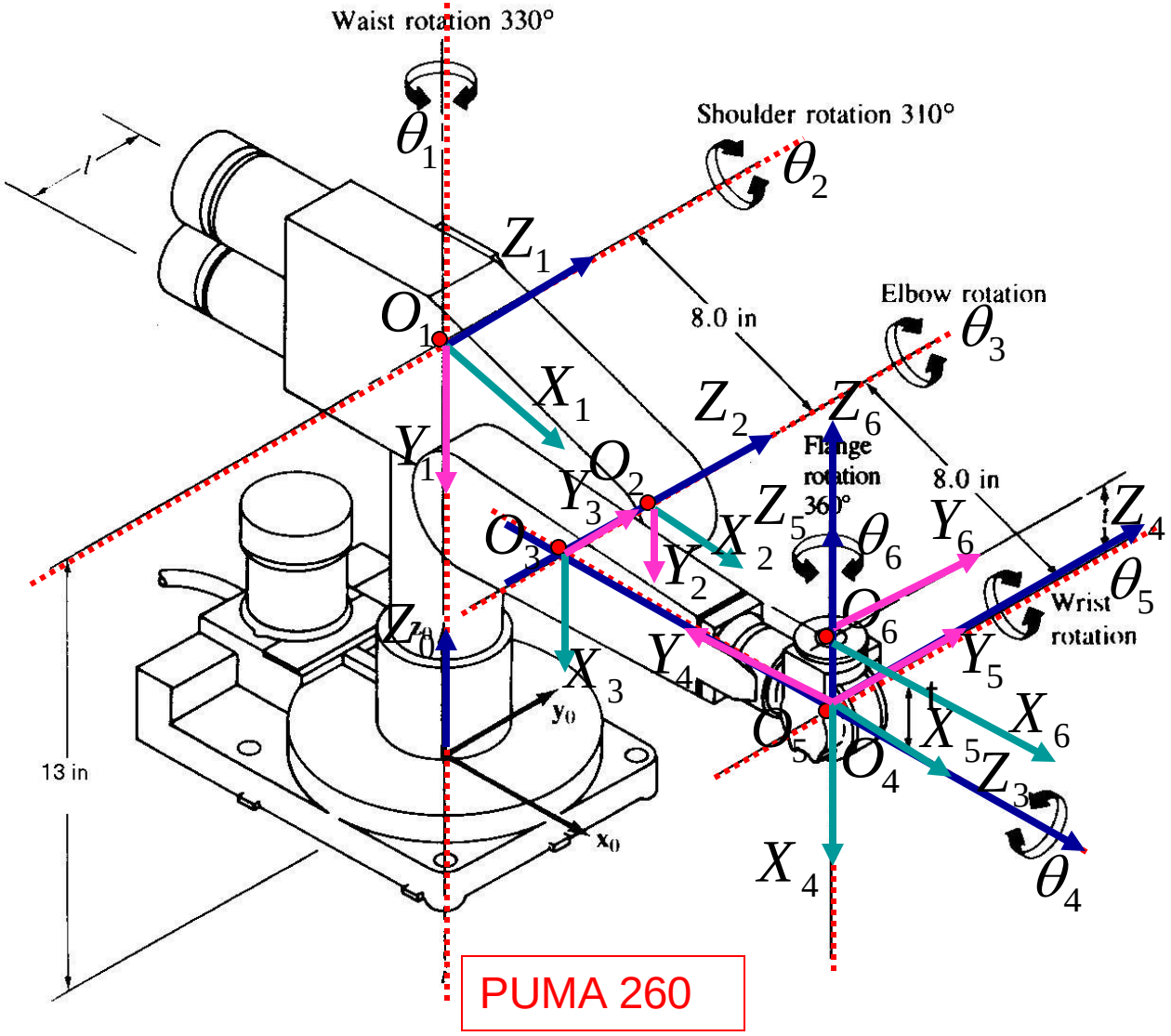
α_i : rotation angle from Z_{i-1} to Z_i about X_i

a_i : distance from intersection of Z_{i-1} & X_i to origin of i coordinate along X_i

d_i : distance from origin of (i-1) coordinate to intersection of Z_{i-1} & X_i along Z_{i-1}

θ_i : rotation angle from X_{i-1} to X_i about Z_{i-1}

Example 2: PUMA 260



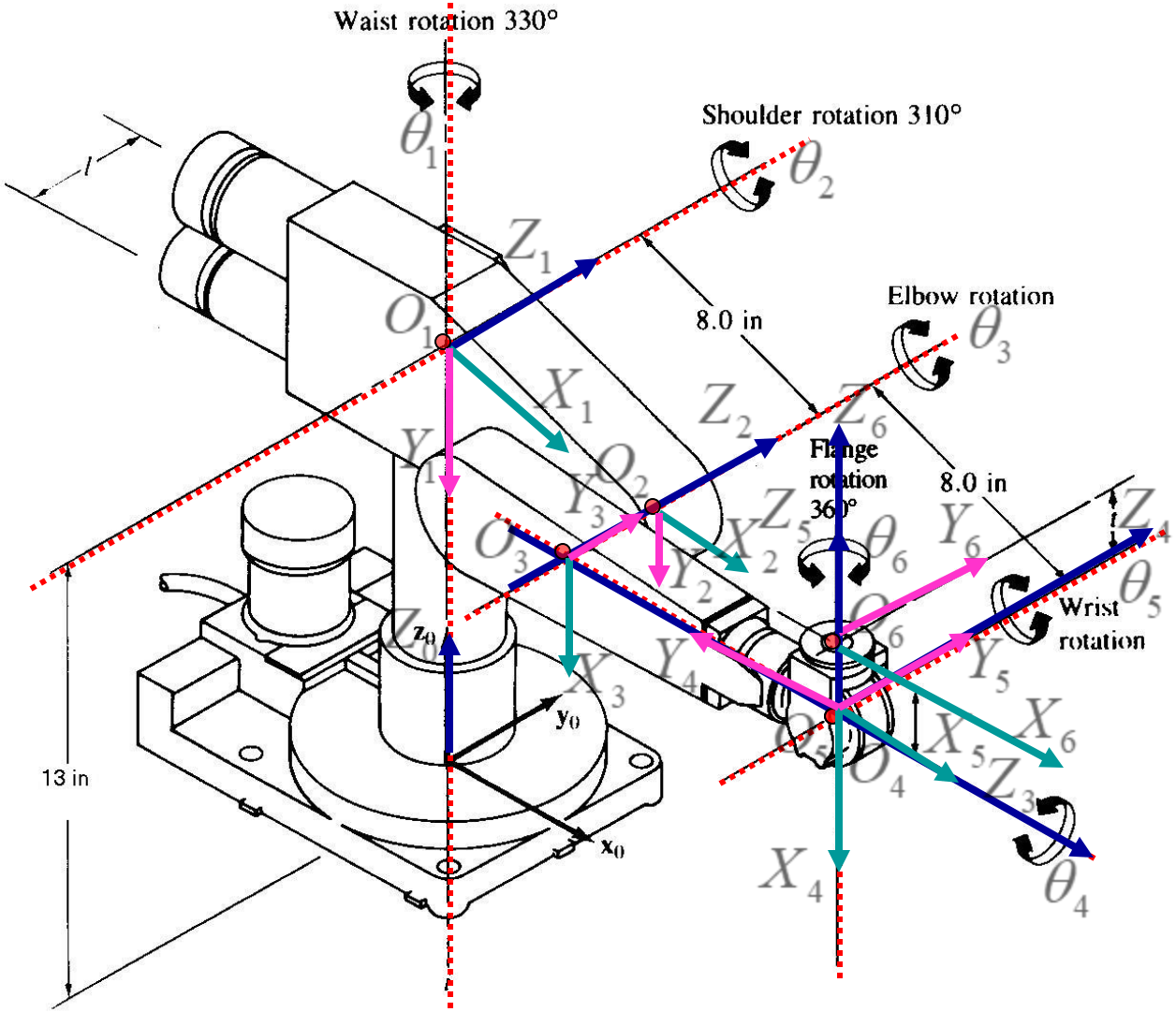
1. Number the joints
2. Establish base frame
3. Establish joint axis Z_i
4. Locate origin, (intersect. of Z_i & Z_{i-1}) OR (intersect of common normal & Z_i)
5. Establish X_i,Y_i

$$X_i = \pm(Z_{i-1} \times Z_i) / \left\| Z_{i-1} \times Z_i \right\|$$

$$Y_i = +(Z_i \times X_i) / \left\| Z_i \times X_i \right\|$$

Example 2: D-H Parameters

CN



J	θ_i	α_i	a_i	d_i
1	θ_1	-90	0	13
2	θ_2	0	8	0
3	θ_3	90	0	-l
4	θ_4	-90	0	8
5	θ_5	90	0	0
6	θ_6	0	0	t

θ_i：从X_{i-1}轴到X_i轴的转角

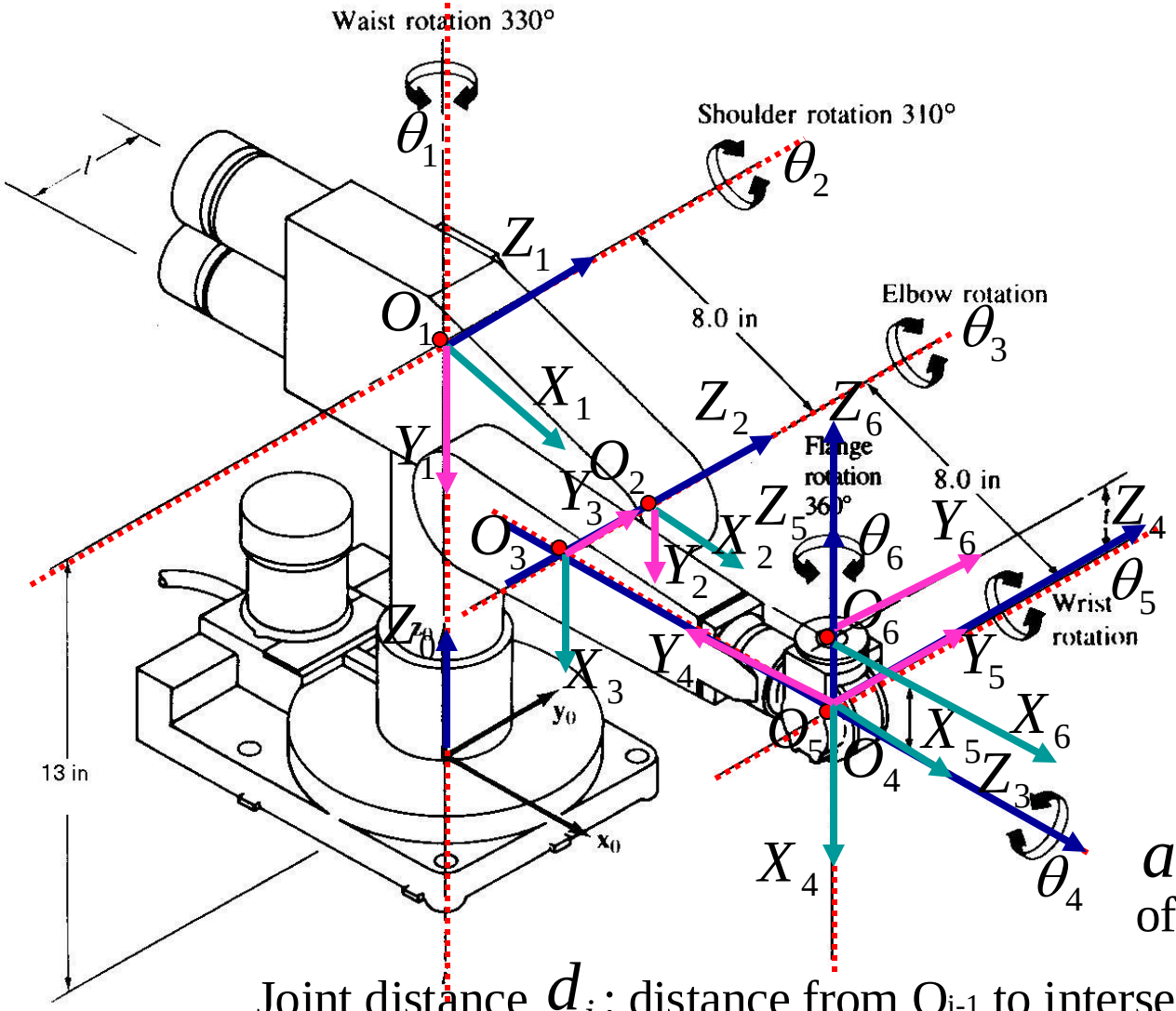
α_i：从Z_{i-1}轴到Z_i轴的转角

a_i：从Z_{i-1}轴到Z_i轴的距离

d_i：从X_{i-1}轴到X_i轴的距离

Link Parameters

EN



J	θ_i	α_i	a_i	d_i
1	θ_1	-90	0	13
2	θ_2	0	8	0
3	θ_3	90	0	-l
4	θ_4	-90	0	8
5	θ_5	90	0	0
6	θ_6	0	0	t

θ_i : angle from X_{i-1} to X_i about Z_{i-1}

α_i : angle from Z_{i-1} to Z_i about X_i

a_i : distance from intersection of Z_{i-1} & X_i to O_i along X_i

Joint distance d_i : distance from O_{i-1} to intersection of Z_{i-1} & X_i along Z_{i-1}

Summary

Denavit-Hartenberg (D-H) Representation

- D-H Convention
- D-H Parameters

Homework 4

Homework 4 is posted at http://bb.sustech.edu.cn

Due date: March 10, 2025

Next class: Forward Kinematics (Wednesday, March 5)

作业要求（Requirements）：

1. 文件格式为以自己学号姓名作业序号命名的pdf文件；

（File name: YourSID_YourName_04.pdf）

2. 作业里也写上自己的姓名和学号。

（Write your name and SID in the homework）