南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

Robot Modeling & Control ME331

Section 18： Control II

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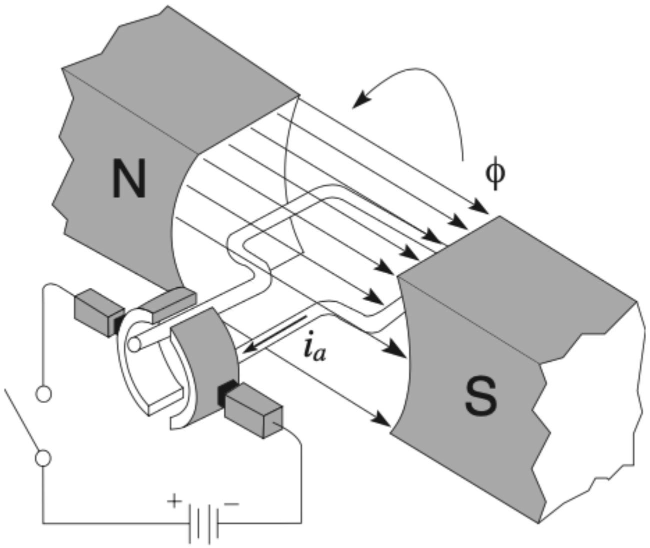
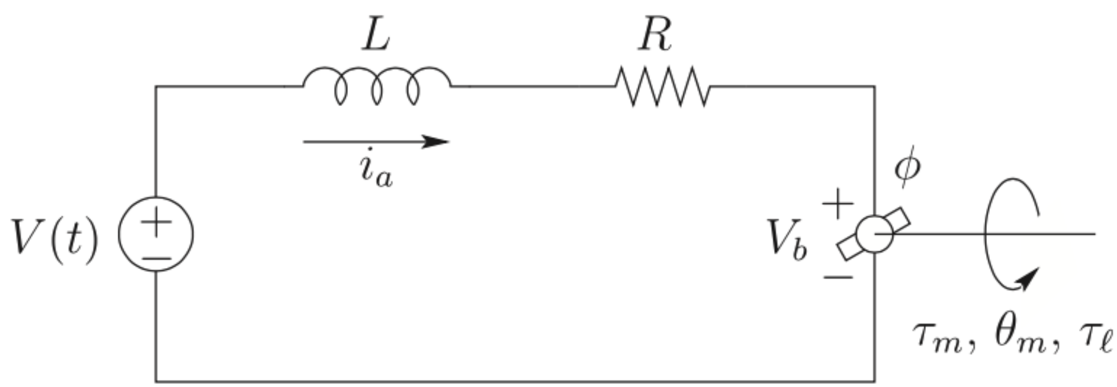
Outline

- Review
  - Actuator Dynamics + Load Dynamics
  - Independent Joint Control（PID +Feedforward）
- Control of Manipulators
  - Multivariable Control
    1. Computed Torque Control
    2. Cartesian-based Control
  - Force Control
    1. Direct Force Control
    2. Impedence Control

Review: Actuator Dynamics



Principle of operation of a permanent magnet DC motor.

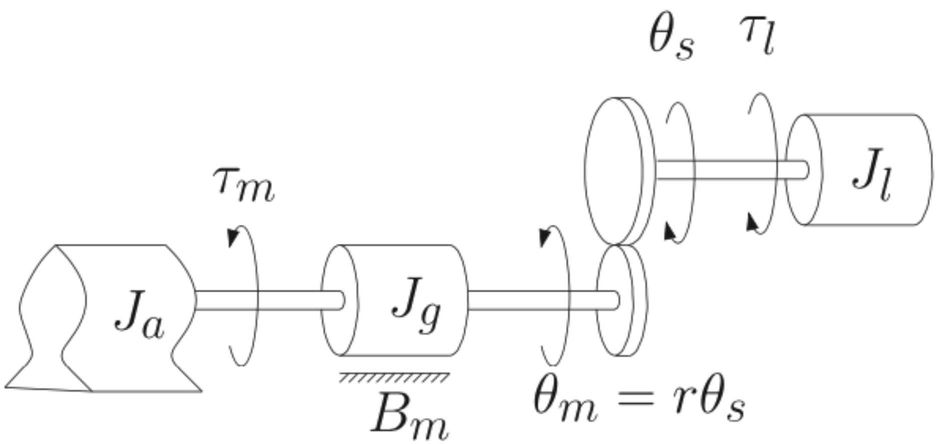


Circuit diagram for an armature controlled DC motor. The rotor windings have an effective inductance *L* and effective resistance *R*. The applied voltage *V* is the control input.

The differential equation for the armature current is then

$$\left.\begin{aligned} L\frac{di_a}{dt} + Ri_a &= V - V_b \\ \tau_m &= K_1\phi i_a = K_m i_a \\ V_b &= K_2\phi\omega_m = K_b\omega_m \end{aligned}\right\} \quad \tau_m = \frac{K_m}{R}\left(V - K_b\omega_m - L\frac{di_a}{dt}\right)$$

Review: Load Dynamics



J_a = actuator inertia
J_g = gear inertia
J_l = load inertia
B_m = the coefficient of motor friction
r = the gear ratio ($r \gg 1$)

We set $J_m = J_a + J_g$ the sum of the actuator and gear inertias.

In terms of the motor angle θ_m, the equation of motion of this system is then

$$J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} = \tau_m - \tau_l/r$$
$$= K_m i_a - \tau_l/r$$

$\Rightarrow$ $(J_m s^2 + B_m s)\Theta_m(s) = K_m I_a(s) - \tau_l(s)/r$

$$L\frac{di_a}{dt} + Ri_a = V - V_b$$

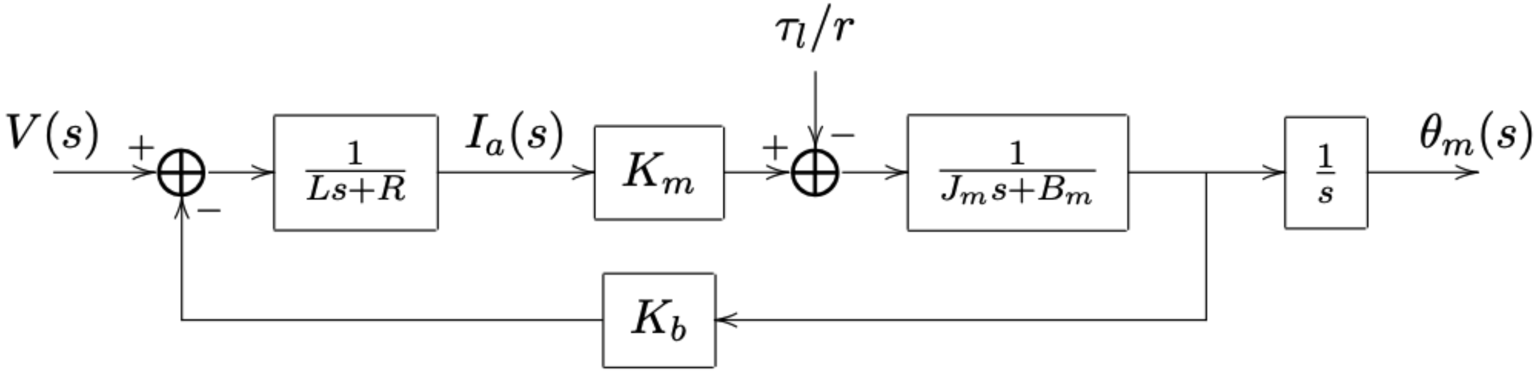
$$V_b = K_b\omega_m = K_b\frac{d\theta_m}{dt}$$

$\Rightarrow$ $(Ls + R)I_a(s) = V(s) - K_b s\Theta_m(s)$

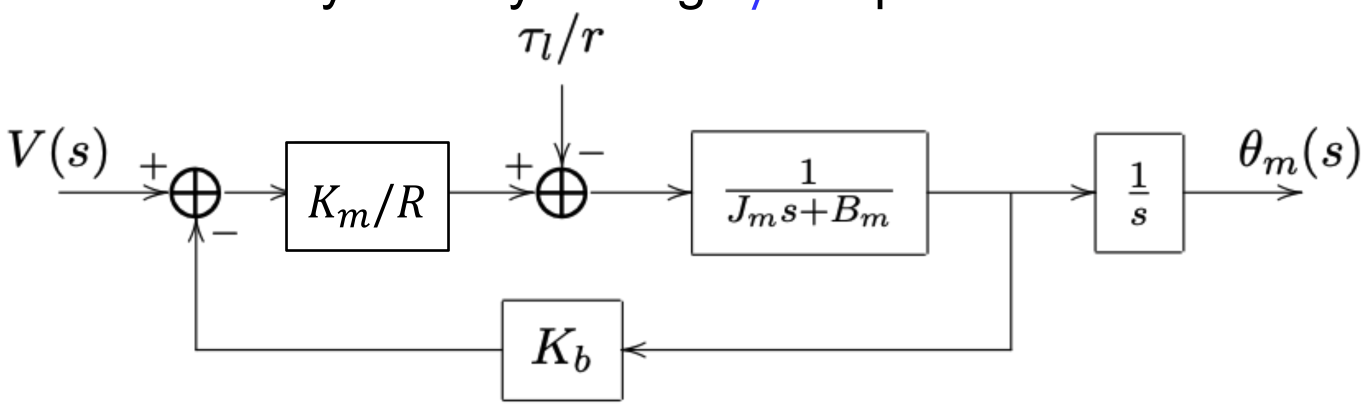
Review: Load Dynamics

$$(J_m s^2 + B_m s)\Theta_m(s) = K_m I_a(s) - \tau_l(s)/r$$

$$(Ls + R)I_a(s) = V(s) - K_b s\Theta_m(s)$$



The reduced-order system by setting L/R equal to zero



$$J_m\ddot{\theta}_m(t) + \left(B_m + \frac{K_b K_m}{R}\right)\dot{\theta}_m(t) = \left(\frac{K_m}{R}\right)V(t) - \frac{\tau_l(t)}{r}$$

Review

- **Independent Joint Model**

$$J_{m_k}\ddot{\theta}_{m_k} + \left(B_{m_k} + K_{b_k}K_{m_k}/R_k\right)\dot{\theta}_{m_k} = \left(K_{m_k}/R_k\right)V_k - \frac{\tau_k}{r_k} \quad (1)$$

Setting $J_{eff_k} := J_{m_k}; \quad B_{eff_k} := B_{m_k} + \frac{K_{b_k}K_{m_k}}{R_k}; \quad u_k := \left(K_{m_k}/R_k\right)V_k$

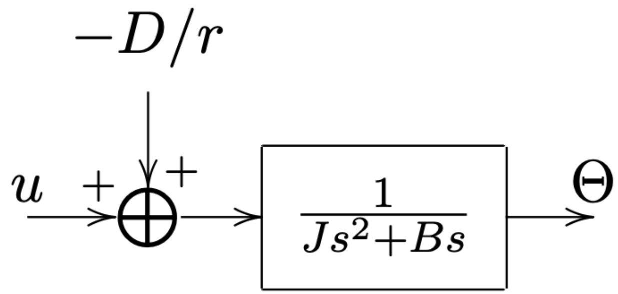
We write Equation (1) as

$$J_{eff_k}\ddot{\theta}_{m_k} + B_{eff_k}\dot{\theta}_{m_k} = u_k - \frac{d_k}{r_k}$$

where d_k is treated as a disturbance and defined by

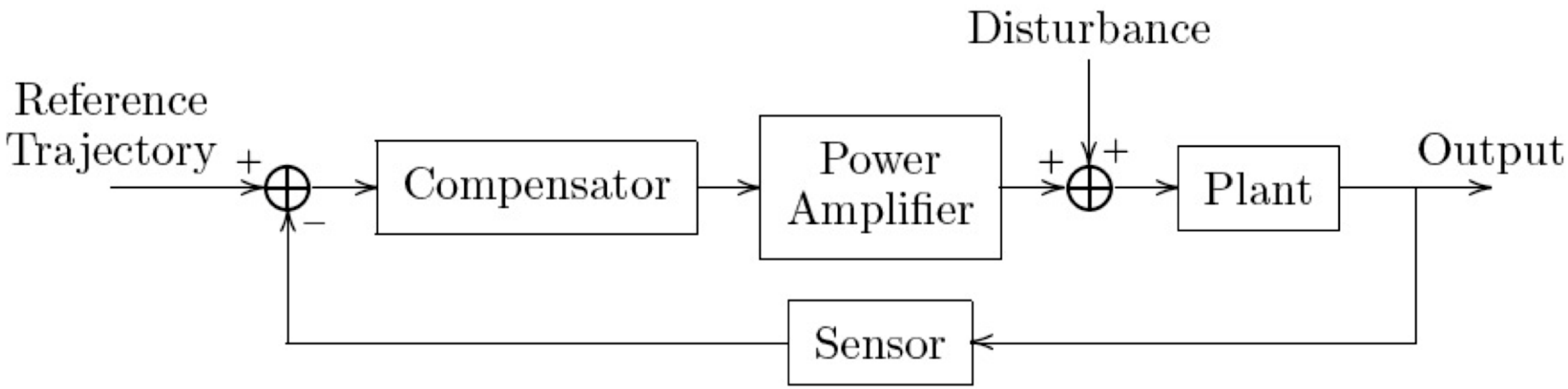
$$d_k = \sum_{j=1}^{n} d_{jk}(q)\ddot{q}_j + \sum_{i,j=1}^{n} c_{ijk}(q)\dot{q}_i\dot{q}_j + g_k(q)$$

Block diagram of the simplified:
(open-loop system.)



Review

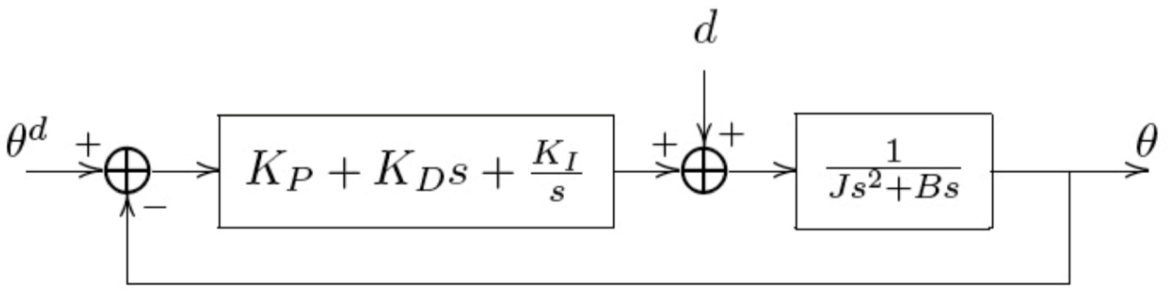
- **Independent Joint Control**
  - Each Axis → SISO
  - Coupling effect → disturbance
  - Objectives: tracking and disturbance rejection



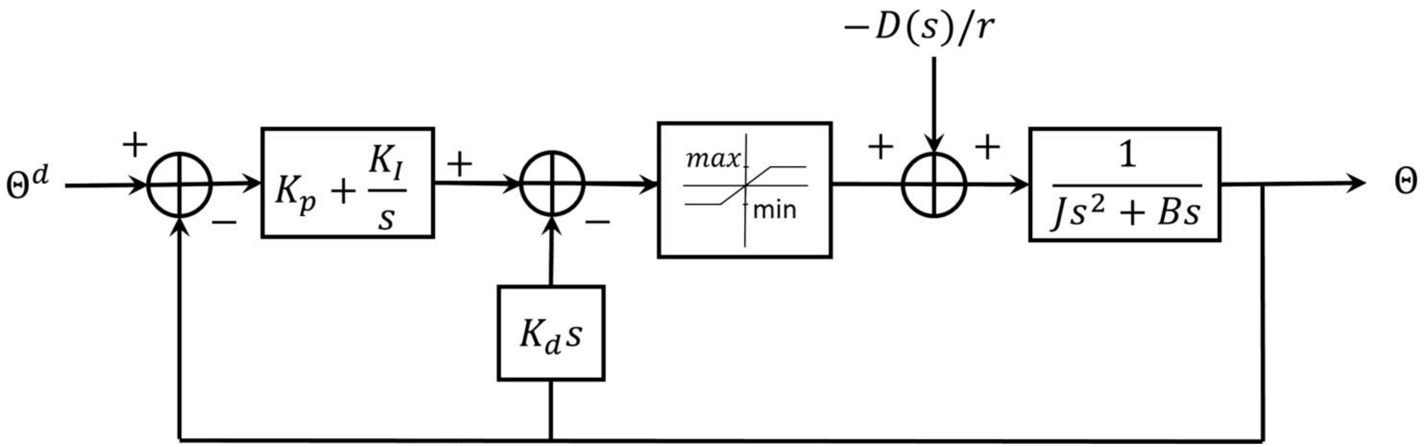
Basic structure of a feedback control system. The compensator measures the "error" between a "reference" and a measured "output" and produces signals to the plant that are designed to drive the error to zero despite the presence of disturbances.

Review

- **Independent Joint Control**
  - **PID controller**



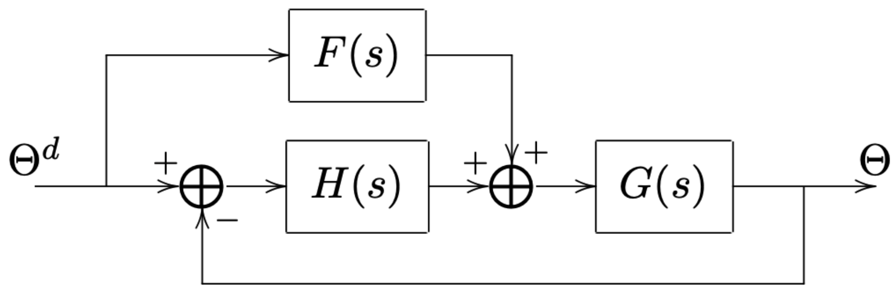
One-degree-of-freedom PID controller



Two-degree-of-freedom PID controller with input saturation

Feedforward Control

The previous analysis was carried out under the assumption that the reference signal and disturbance are constant and is not valid for tracking more general time-varying trajectories such as a cubic polynomial trajectory. Now we introduce the notion of feedforward control as a method to track time-varying trajectories.

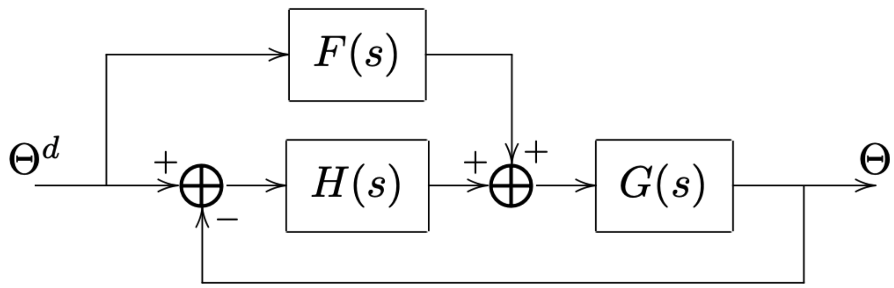


Suppose that $\theta^d(t)$ is a joint space reference trajectory and consider the block diagram, where $G(s)$ represents the forward transfer function of a given system and $H(s)$ is the compensator transfer function.

A feedforward control scheme consists of adding a feedforward path with transfer function $F(s)$. Let each of the three transfer functions be represented as ratios of polynomials

$$G(s) = \frac{q(s)}{p(s)} \qquad H(s) = \frac{c(s)}{d(s)} \qquad F(s) = \frac{a(s)}{b(s)}$$

Feedforward Control



$$G(s) = \frac{q(s)}{p(s)} \qquad H(s) = \frac{c(s)}{d(s)} \qquad F(s) = \frac{a(s)}{b(s)}$$

We assume that $G(s)$ is strictly proper and $H(s)$ is proper(正则). Simple block diagram manipulation shows that the closed-loop transfer function

$$T(s) = \frac{\Theta(s)}{\Theta^d(s)} = \frac{q(s)(c(s)b(s) + a(s)d(s))}{b(s)(p(s)d(s) + q(s)c(s))}$$

The closed-loop characteristic polynomial is $b(s)(p(s)d(s) + q(s)c(s))$. Therefore, for stability of the closed-loop system, we require that the compensator $H(s)$ and the feedforward transfer function $F(s)$ be chosen so that the polynomials $p(s)d(s) + q(s)c(s)$ and $b(s)$ are Hurwitz.

Feedforward Control

If we choose the feedforward transfer function $F(s)$ equal to $1/G(s)$, the inverse of the forward plant, that is, $a(s) = p(s)$ and $b(s) = q(s)$, then the closed-loop system becomes

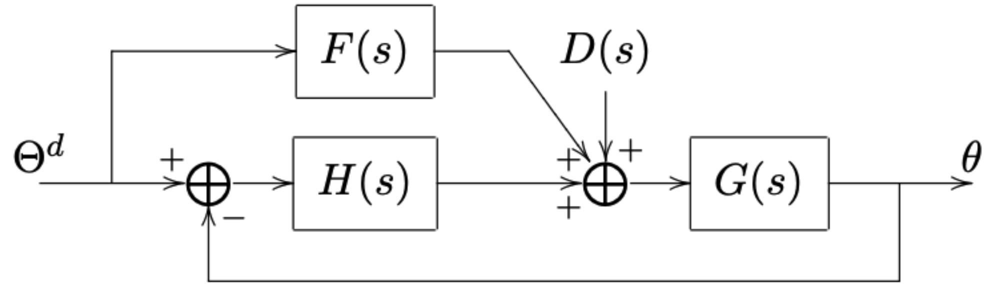
$$q(s)\big(p(s)d(s) + q(s)c(s)\big)\Theta(s) = q(s)\big(p(s)d(s) + q(s)c(s)\big)\Theta^d(s)$$

or, in terms of the tracking error $E(s) = \Theta^d(s) - \Theta(s)$,

$$q(s)\big(p(s)d(s) + q(s)c(s)\big)E(s) = 0$$

Thus, assuming stability, the output $\theta(t)$ will track any reference trajectory $\theta^d(t)$.

Feedforward control with disturbance $D(s)$:

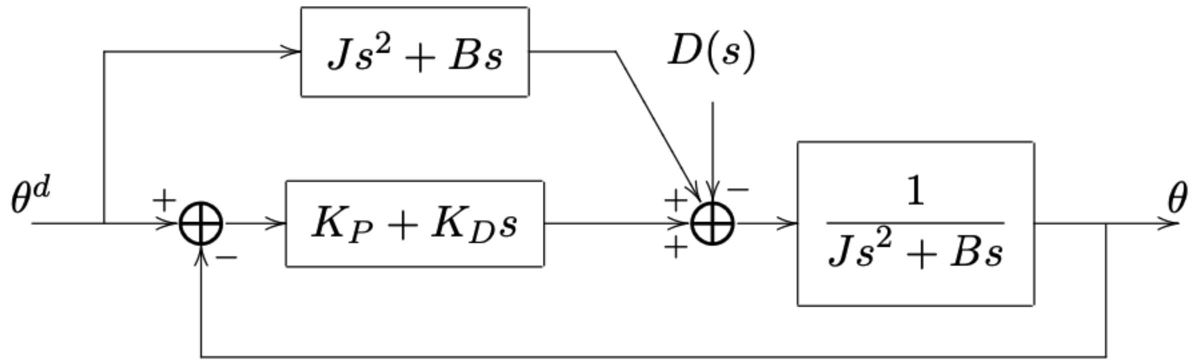


It is easily shown that the tracking error $E(s)$ is given by

$$E(s) = \frac{q(s)d(s)}{p(s)d(s) + q(s)c(s)} D(s)$$

Feedforward Control

Let us apply this idea to the robot control model.



In the time domain the control law can be written as

$$V(t) = J\ddot{\theta}^d(t) + B\dot{\theta}^d + K_D\left(\dot{\theta}^d - \dot{\theta}\right) + K_P\left(\theta^d - \theta\right)$$
$$= f(t) + K_D\dot{e}(t) + K_P e(t)$$

where $f(t)$ is the feedforward signal

$$f(t) = J\ddot{\theta}^d(t) + B\dot{\theta}^d$$

and $e(t)$ is the tracking error $\theta^d(t) - \theta(t)$. Since the forward plant equation is $J\ddot{\theta}(t) + B\dot{\theta}(t) = V(t) - d(t)/r$, the closed-loop error $e(t)$ satisfies the second-order differential equation

$$J\ddot{e}(t) + (B + K_D)\dot{e} + K_P e(t) = d(t)/r$$

Independent Joint Control

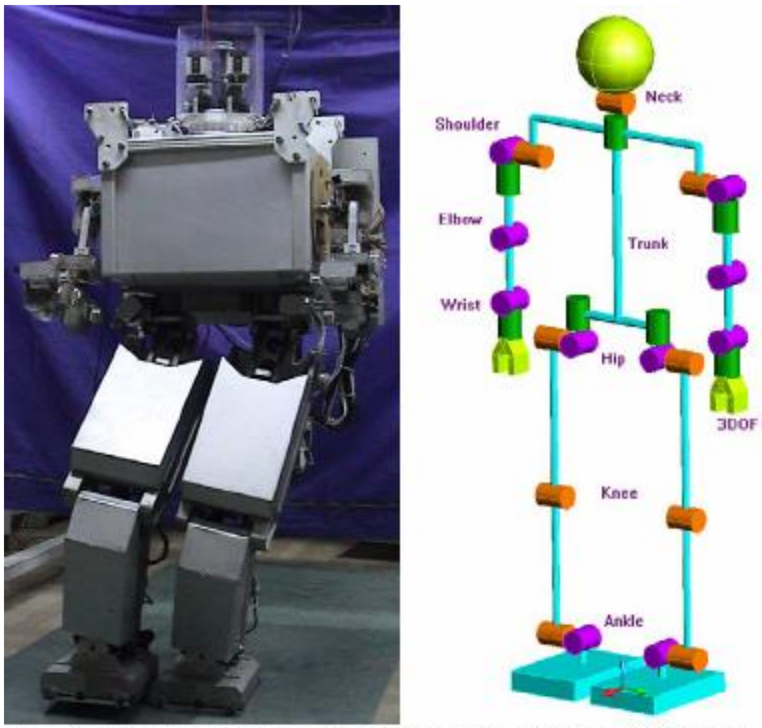


Fig. 1 THBIP-I humanoid robot and its DOF configuration

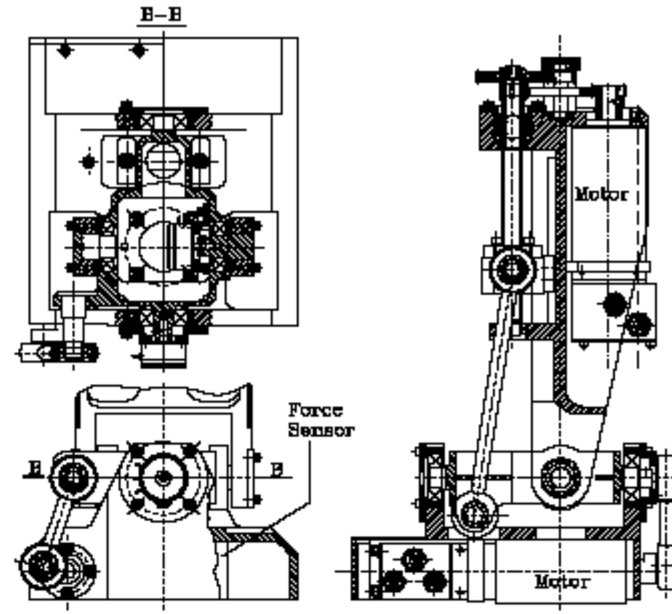
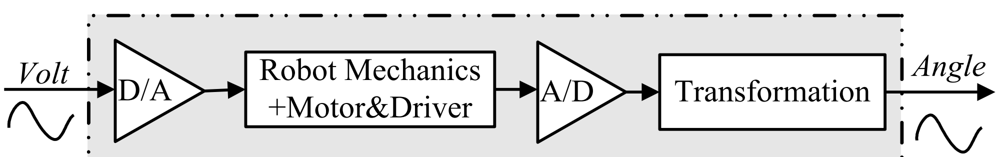
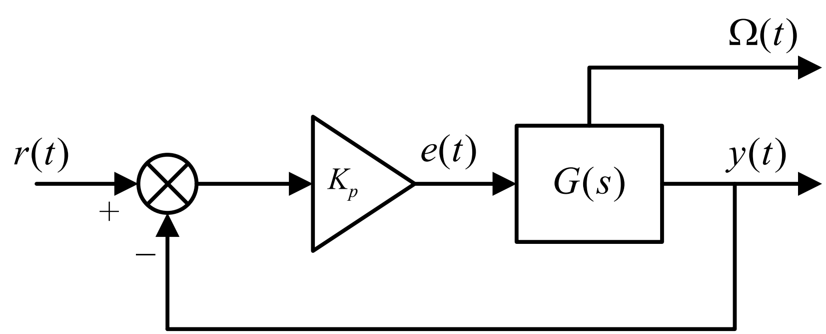


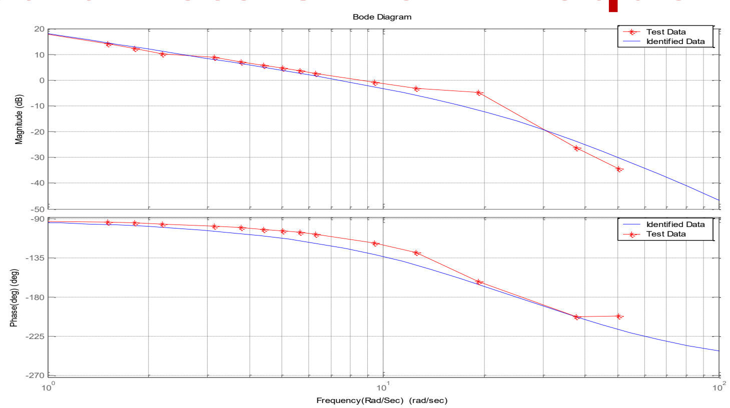
Fig. 2 Mechanism of the 2 DOF ankle joints



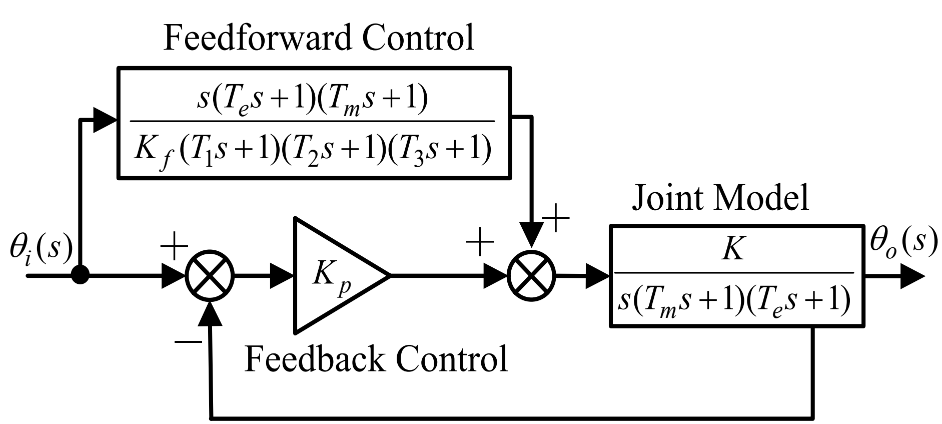


Independent Joint Control

- **Determine the structure of TF** $G(s) = \dfrac{K_p}{s(T_e s+1)(T_m s+1)}$
- **Identify TF parameters from frequency response test**



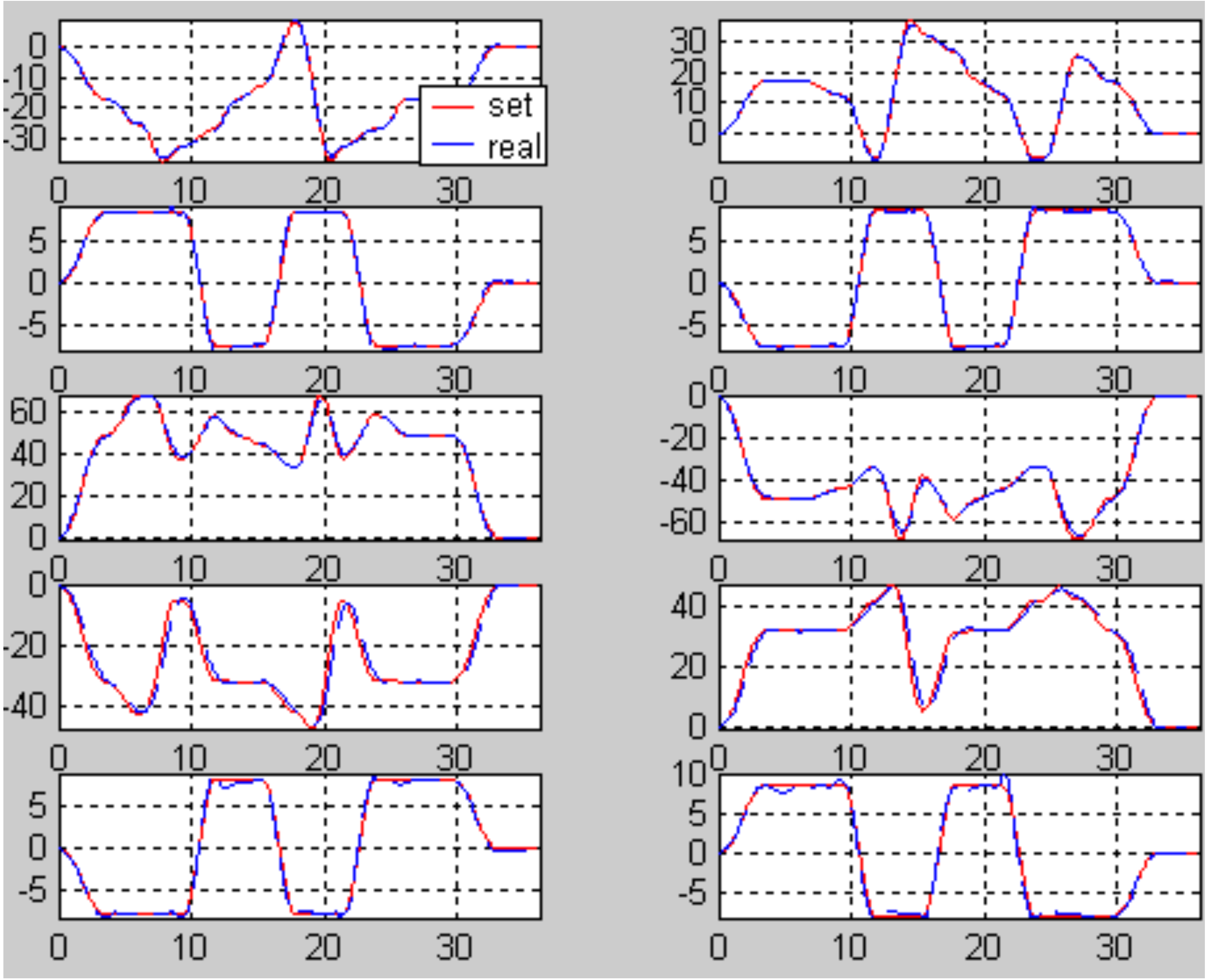
- **Design controller（Feedforward control scheme）**



Independent Joint Control

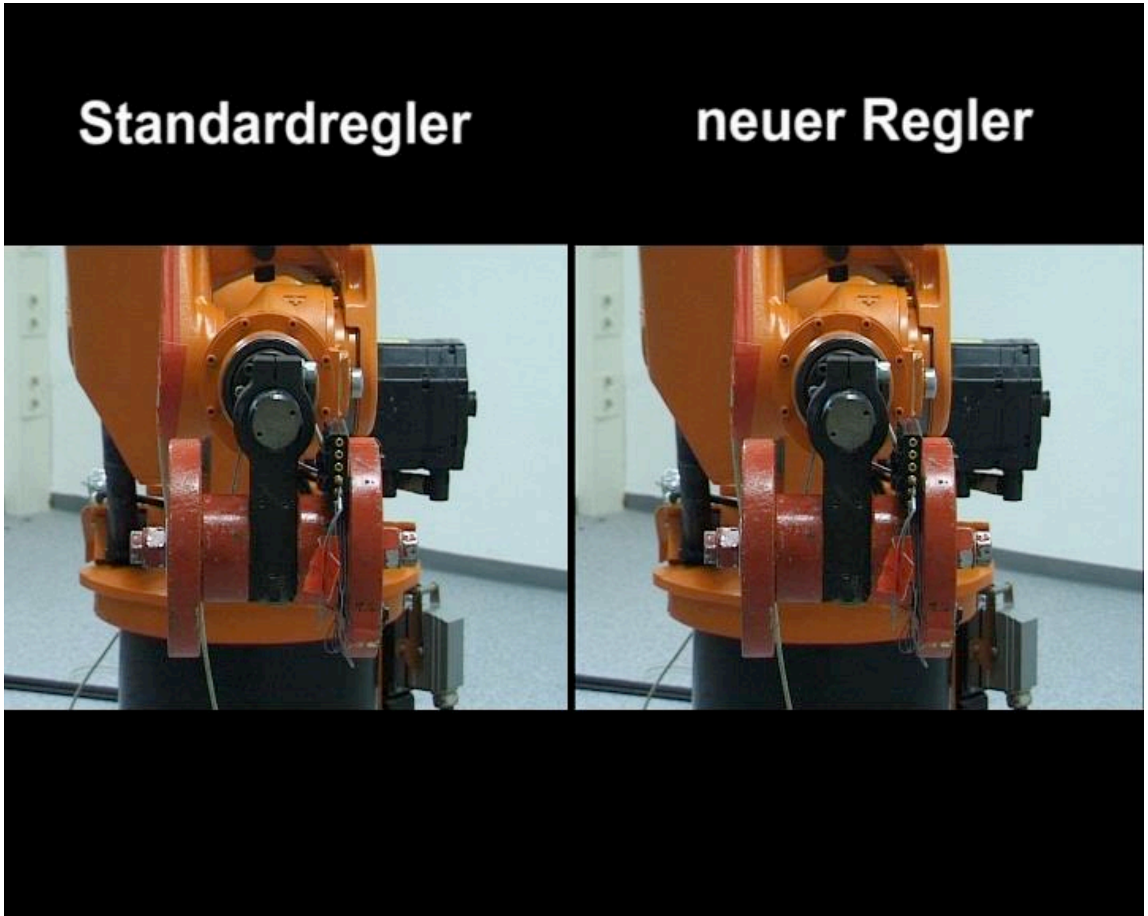
- **Walking Experiment**

Independent Joint Control



Benefits of model-based control

- Trajectory tracking task: comparison between PID and advanced controller



Multivariable Control

given the robot dynamic model

$$\boxed{B(q)\ddot{q} + \underbrace{n(q,\dot{q})}_{c(q,\dot{q}) + g(q) + \text{ friction model}} = u}$$

and a twice-differentiable desired trajectory for $t \in [0,T]$

$$q_d(t) \;\rightarrow\; \dot{q}_d(t), \ddot{q}_d(t)$$

applying the feedforward torque in nominal conditions

$$\boxed{u_d = B(q_d)\ddot{q}_d + n(q_d, \dot{q}_d)}$$

yields exact reproduction of the desired motion

- provided that $q(0)=q_d(0)$, $\dot{q}(0)=\dot{q}_d(0)$ (initial matched state)

Multivariable Control

In practice ...

a number of differences from the nominal condition

- initial state is "not matched" to the desired trajectory $q_d(t)$
- disturbances on the actuators, truncation errors on data, ...
- inaccurate knowledge of robot dynamic parameters (link masses, inertias, center of mass positions)
- unknown value of the carried payload
- presence of unmodeled dynamics (complex friction phenomena, transmission elasticity, ...)

Multivariable Control

Introducing feedback

$$\hat{u}_d = \hat{B}(q_d)\ddot{q}_d + \hat{n}(q_d,\dot{q}_d)$$

with $\hat{B}$, $\hat{n}$ estimates of terms (or coefficients) in the dynamic model

note: $\hat{u}_d$ can be computed off line [e.g., by $\widehat{NE}_\alpha(q_d,\dot{q}_d,\ddot{q}_d)$]

feedback is introduced to make the control scheme more robust

different possible implementations depending on amount of computational load share

- OFF LINE (⇔ open loop)
- ON LINE (⇔ closed loop)

two-step control design:
1. compensation (feedforward) or cancellation (feedback) of nonlinearities
2. synthesis of a linear control law stabilizing the trajectory error to zero

Multivariable Control

- **Computed Torque Control**
  - Robot system:

$$\boldsymbol{D}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{G}(\boldsymbol{q}) = \boldsymbol{\tau}$$

  - Controller:

$$\boldsymbol{\tau} = \boldsymbol{D}(\boldsymbol{q})\left[\ddot{\boldsymbol{q}}^{\mathrm{d}} + \boldsymbol{k}_{\mathrm{v}}\left(\dot{\boldsymbol{q}}^{\mathrm{d}} - \dot{\boldsymbol{q}}\right) + \boldsymbol{k}_{\mathrm{p}}\left(\boldsymbol{q}^{\mathrm{d}} - \boldsymbol{q}\right)\right] + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{G}(\boldsymbol{q})$$

$$(\ddot{q}^d - \ddot{q}) + k_v(\dot{q}^d - \dot{q}) + k_p(q^d - q) = 0$$

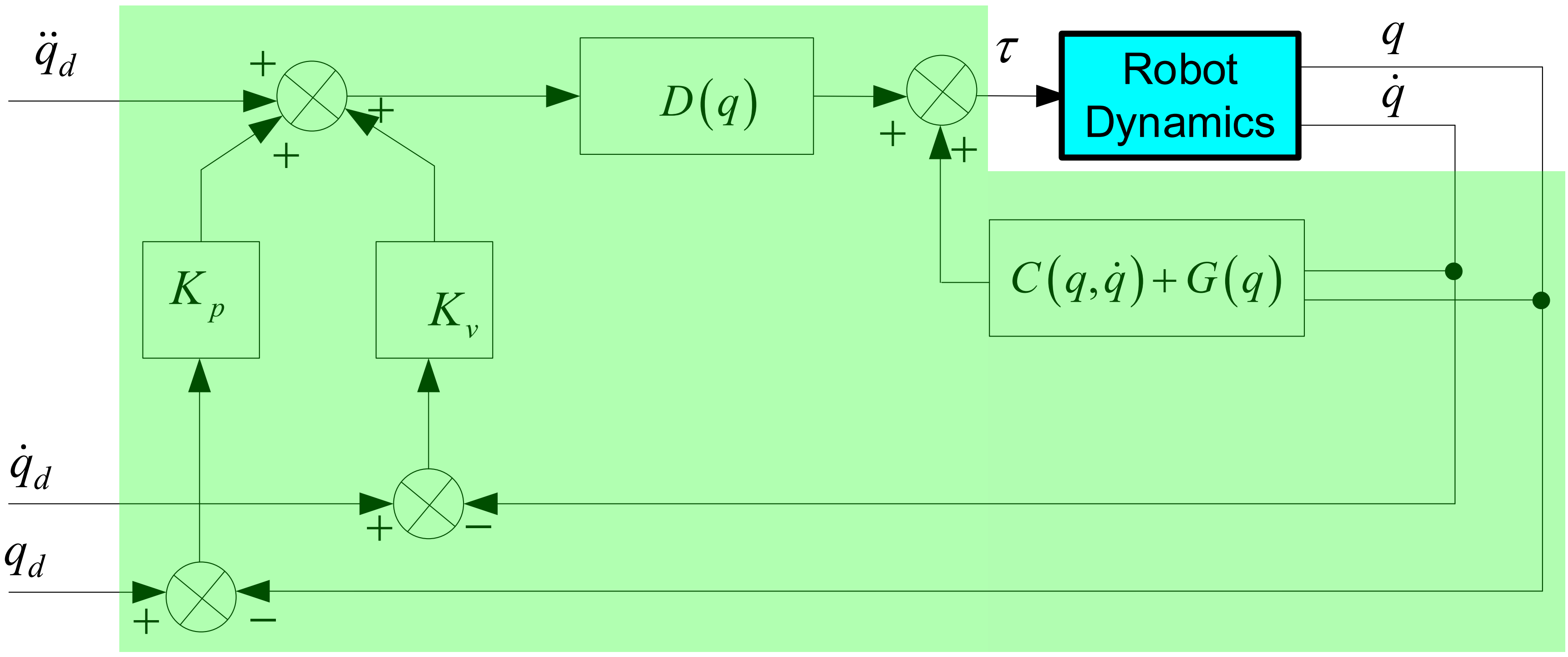
Error dynamics $\quad \ddot{e} + k_v\dot{e} + k_p e = 0$

Advantage: compensated for the dynamic effects

Condition: robot dynamic model is known

Computed Torque Control

$$\boldsymbol{\tau} = \boldsymbol{D}(\boldsymbol{q})\left[\ddot{\boldsymbol{q}}^{\mathrm{d}} + \boldsymbol{k}_{\mathrm{v}}(\dot{\boldsymbol{q}}^{\mathrm{d}} - \dot{\boldsymbol{q}}) + \boldsymbol{k}_{\mathrm{p}}(\boldsymbol{q}^{\mathrm{d}} - \boldsymbol{q})\right] + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{G}(\boldsymbol{q})$$



Error dynamics $\ddot{e} + k_v\dot{e} + k_p e = 0$

How to choose *K*p, *K*v to make the system stable?

Computed Torque Control

- **Review of linear system**
  - State space equation $\dot{x} = Ax + Bu$ **(Equ. 1)**
  - Example:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u \end{cases} \Longrightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

  - The eigenvalue of *A* matrix is the root of characteristic equation

$$|\lambda I - A| = 0 \qquad |\lambda I - A| = \begin{vmatrix} \lambda & -1 \\ 0 & \lambda \end{vmatrix} = \lambda^2 = 0$$

  - Asymptotic stable $\Longleftrightarrow$ λ have negative real part

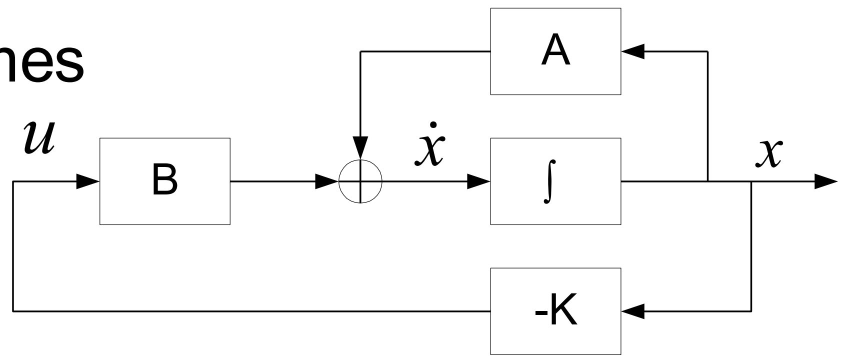
Computed Torque Control

– Find a state feedback control $u = -K \cdot x$ such that the closed loop system is asymptotically stable

$$u = \begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \text{(Equ. 2)}$$

– Closed loop system becomes

$$\dot{x} = (A - BK)x$$



– Chose K, such that all eigenvalues of A'=(A-BK) have negative real parts

$$\left|\lambda I - A'\right| = \begin{vmatrix} \lambda & -1 \\ k_1 & \lambda + k_2 \end{vmatrix} = \lambda^2 + k_2 \lambda + k_1 = 0$$

Computed Torque Control

How to chose Kp, Kv to make the system stable?

Error dynamics $\ddot{e} + k_v \dot{e} + k_p e = 0$

Define states: $x_1 = e$, $x_2 = \dot{e}$ $\Rightarrow$ $\dot{x}_1 = x_2$, $\dot{x}_2 = -k_v x_2 - k_p x_1$

In matrix form: $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_p & -k_v \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = AX$$

Characteristic equation: $$|\lambda I - A| = \begin{vmatrix} \lambda & -1 \\ k_p & \lambda + k_v \end{vmatrix} = \lambda^2 + k_v \lambda + k_p = 0$$

The eigenvalue of A matrix is: $$\lambda_{1,2} = \frac{-k_v \pm \sqrt{k_v{}^2 - 4k_p}}{2}$$

Condition: λ have negative real part $\Rightarrow$

One of a selections: $k_v > 0$, $k_p > k_v^2 / 4$

Multivariable Control

Cartesian-based Control

- **Review—Feedback linearization**

Nonlinear system

$$\dot{X} = f(x) + G(x)U$$

$$U = [-G^{-1}(x)f(x) + G^{-1}(x)V]$$

$$\dot{X} = V$$

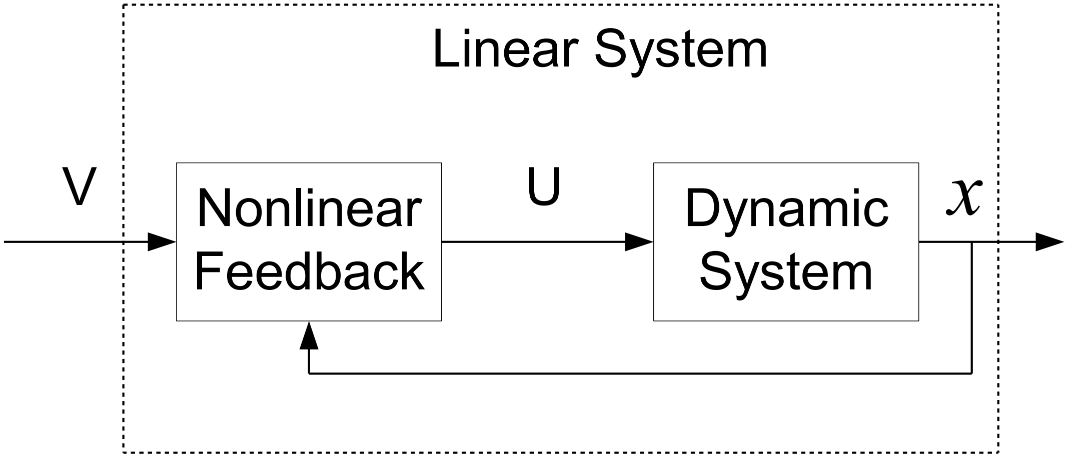
Example:

Original system:

$$\ddot{x} + \cos x = U$$

Nonlinear feedback:

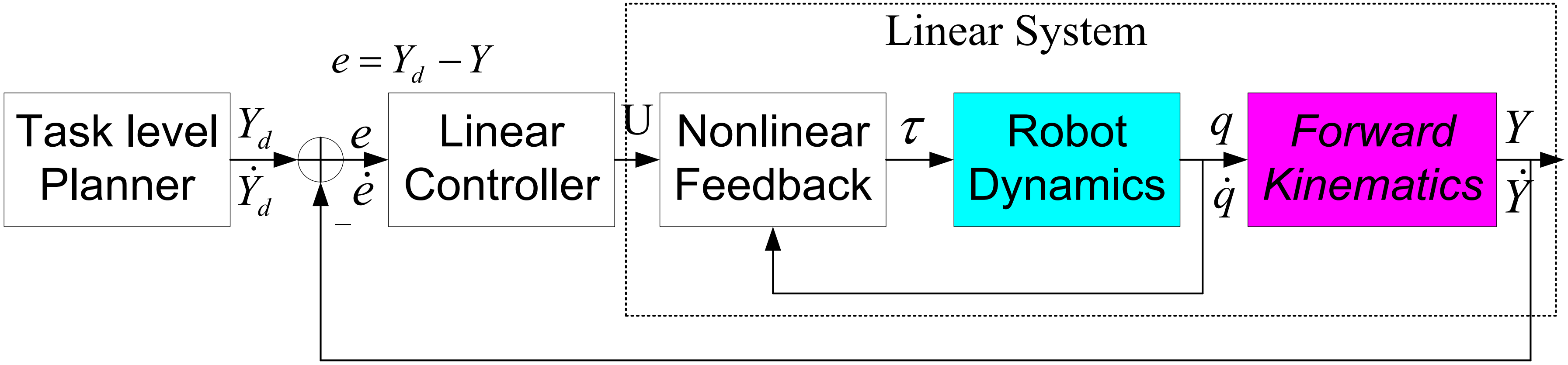
$$U = \cos x + V$$



Linear system: $\ddot{x} = V$

Cartesian-based Control

- **Non-linear Feedback Control**



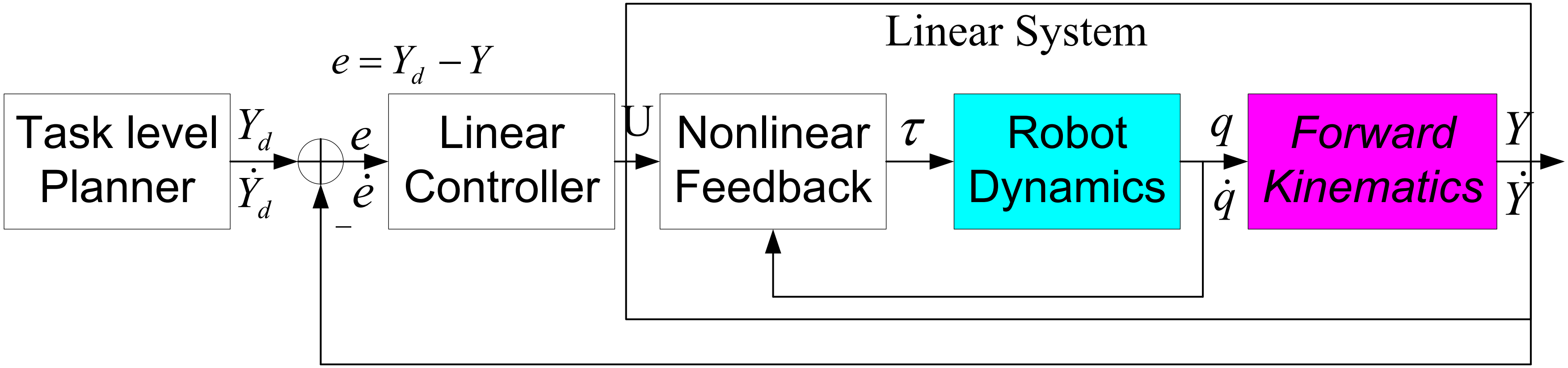
Robot System: $$\begin{cases} D(q)\ddot{q} + C(q,\dot{q}) + G(q) = \tau \\ Y = h(q) \end{cases}$$

Jacobian: $\dot{Y} = \dfrac{d}{dq}[h(q)]\cdot\dot{q} = J\dot{q} \Longrightarrow \ddot{Y} = \dot{J}\dot{q} + J\ddot{q} \Longrightarrow \ddot{q} = J^{-1}(\ddot{Y} - \dot{J}\dot{q})$

$$D(q)J^{-1}(\ddot{Y} - \dot{J}\dot{q}) + C(q,\dot{q}) + G(q) = \tau$$

Cartesian-based Control

- **Non-linear Feedback Control**



Design the nonlinear feedback controller as:

$$\tau = D(q)J^{-1}\left(U - \dot{J}\dot{q}\right) + C(q,\dot{q}) + G(q)$$

Then the linearized dynamic model:

$$D(q)J^{-1}\ddot{Y} = D(q)J^{-1}U \quad \Rightarrow \quad \ddot{Y} = U$$

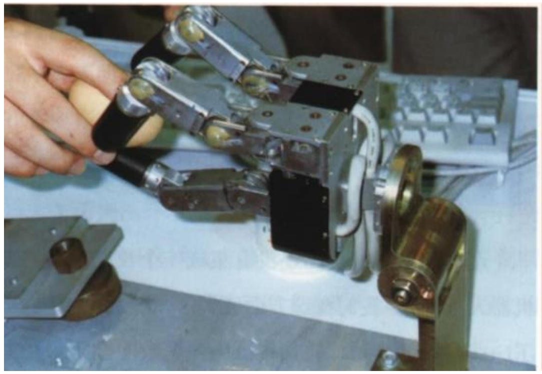
Design the linear controller: $U = \ddot{Y}_d + k_v(\dot{Y}_d - \dot{Y}) + k_p(Y_d - Y)$

Error dynamic equation: $\ddot{e} + k_v\dot{e} + k_p e = 0$

Force Control

Pure position control works like this:

1. You command the robot's joint and thus arm to a particular position.
2. The robot tries to reach that position, no matter what forces are applied against it within its environment.



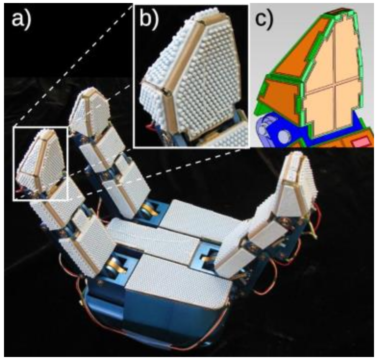
Force control provides an alternative

1. Applying a controlled force to an object
2. Dealing with geometric uncertainty in assembly
3. Improving the force feedback in bilateral teleoperation
4. Avoiding high forces applied to the environment for safety reasons.

Force Control

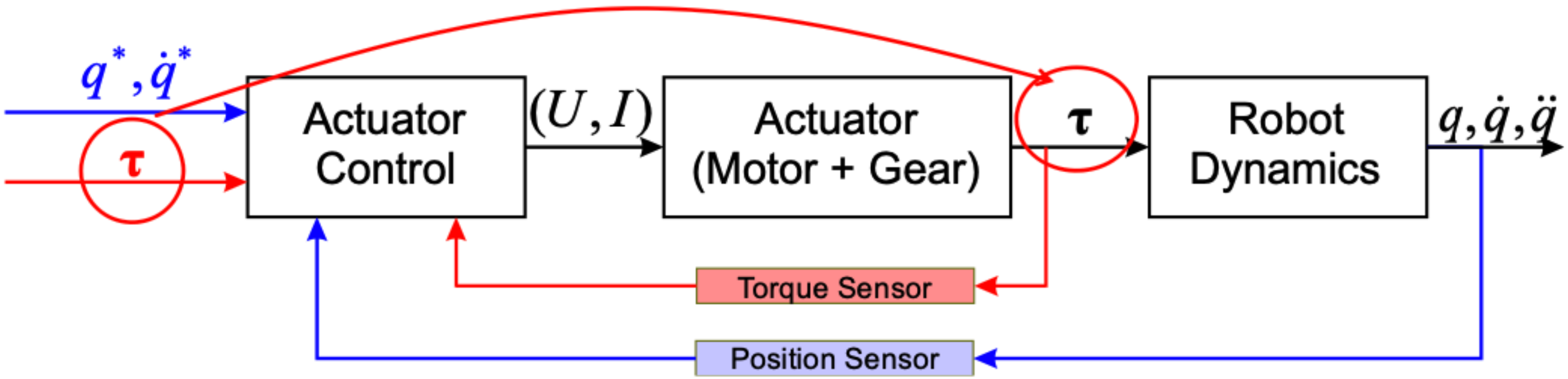
Force and Torque Sensing:

1. Wrist force sensors
2. Joint torque sensors
3. Tactile or hand sensors



Types of Force Control:

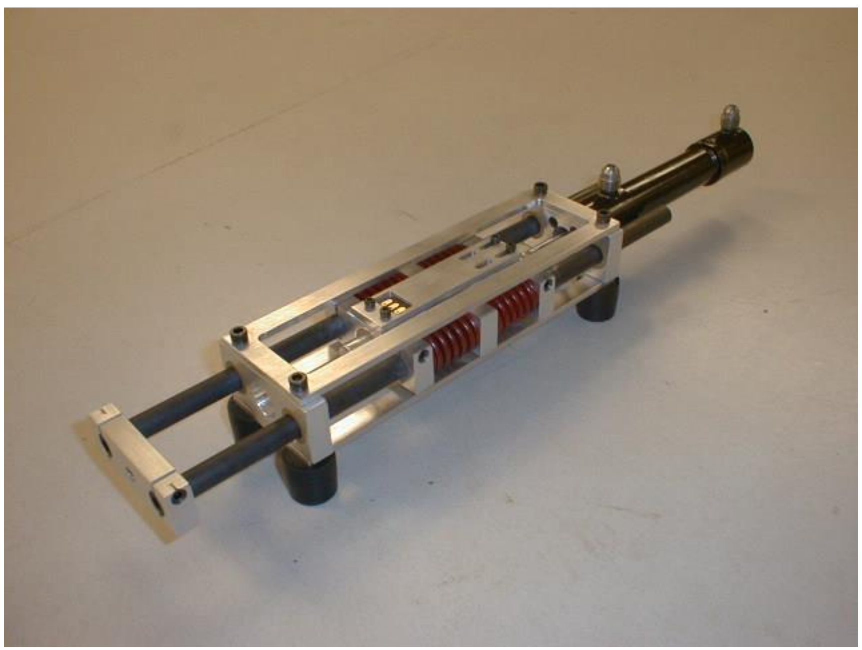
1. Direct force control

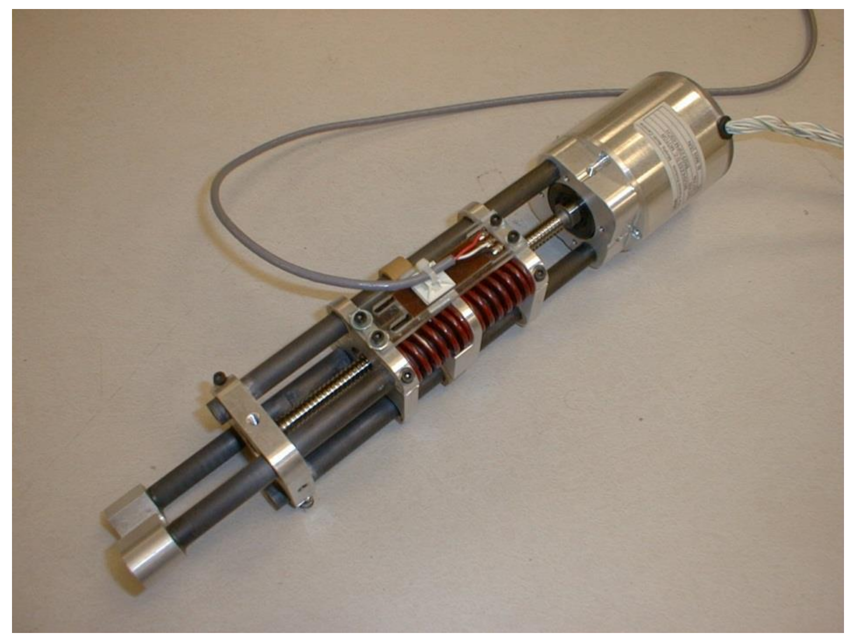


Joint Torque Control of a Robot Arm

Force Control

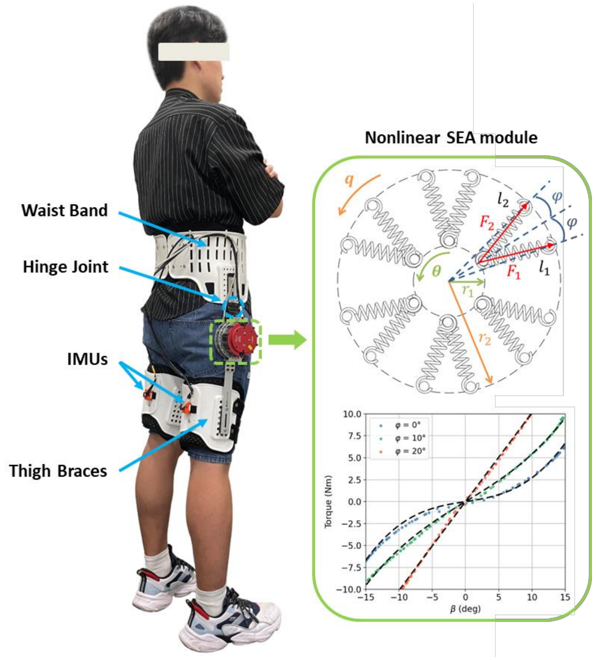
SEA （Series Elastic Actuator）

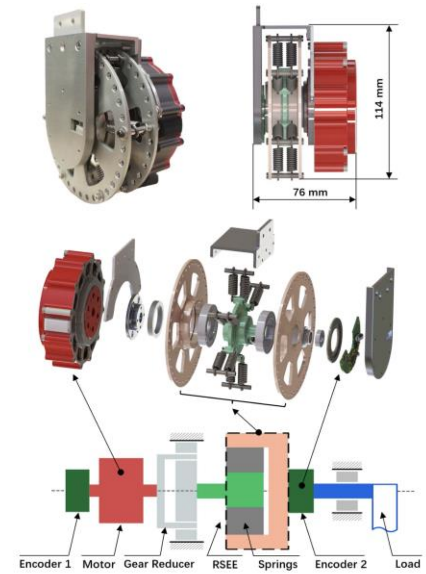


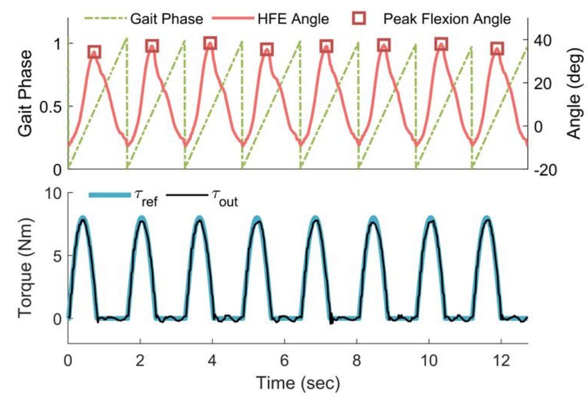


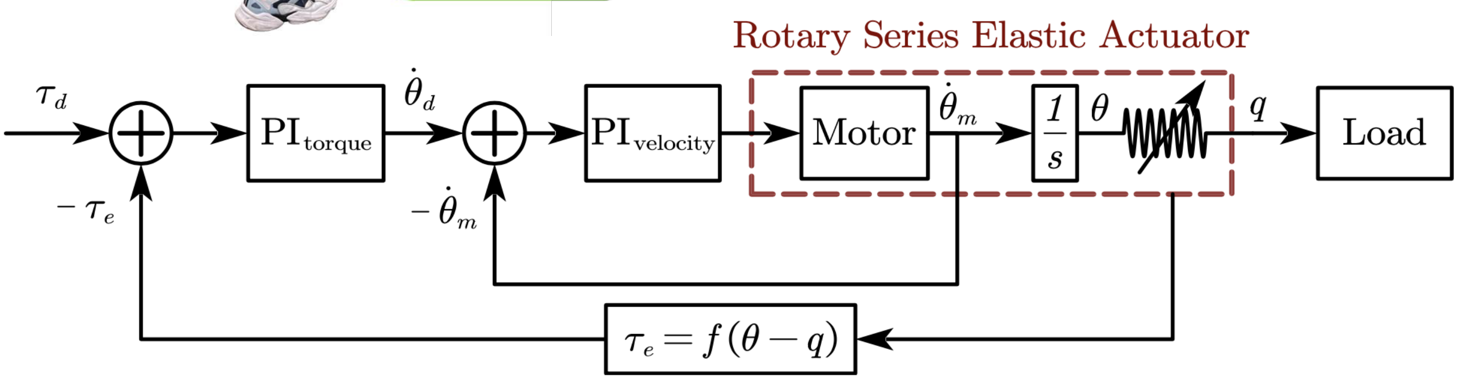
Force Control

1. Direct force control







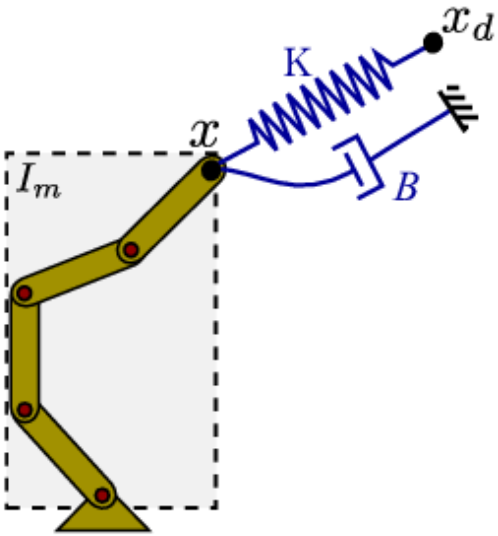


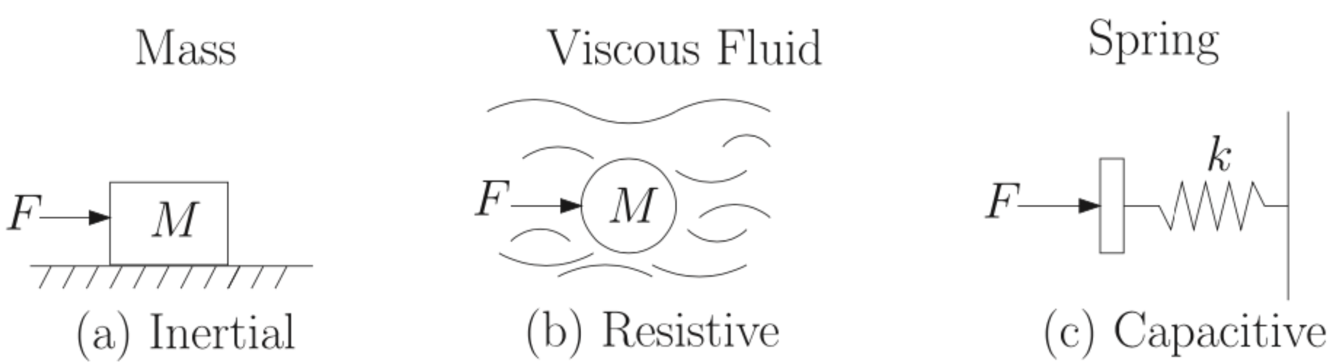
Y. Qian, S, Han, Y. Wang, H. Yu, C. Fu. Toward Improving Actuation Transparency and Safety of a Hip Exoskeleton with a Novel Nonlinear Series Elastic Actuator. **IEEE/ASME Transactions on Mechatronics**, 28(1): 417-428, 2023.

Force Control

2. Impedance control

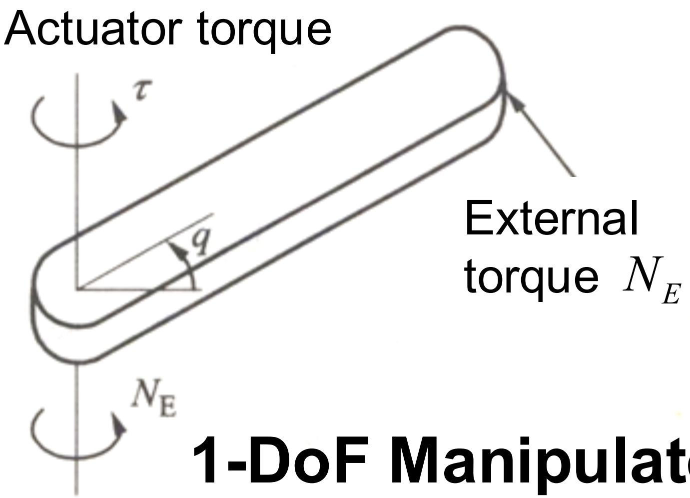
Impedance control applies a virtual mass-spring-damper between the target position and the actual position of the robot.





Force Control

2. Impedance control



1-DoF Manipulator

Plant model： $m\ddot{q} + d\dot{q} = \tau + N_E$

Impedance model：

$$m_d\ddot{q} - d_d\dot{q} + k_d(q - q_d) = N_E$$

（1）External torque N_E is detectable

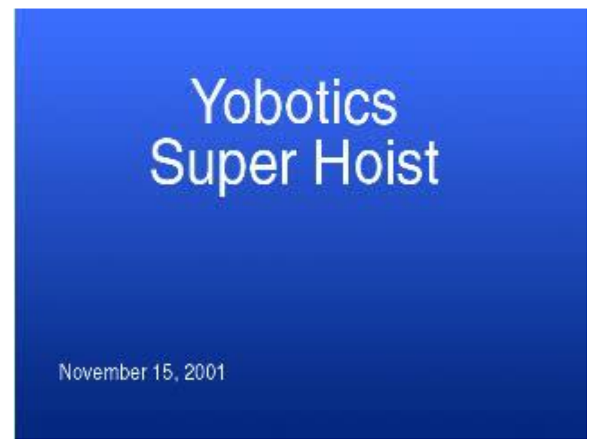
$$\tau = \left(d - \frac{m}{m_d}d_d\right)\dot{q} + \frac{m}{m_d}k_d(q_d - q) + \frac{m}{m_d}d_d\dot{q} - \left(1 - \frac{m}{m_d}\right)N_E$$

（2）Joint acceleration is detectable

$$\tau = (m - m_d)\ddot{q} + (d - d_d)\dot{q} + k_d(q_d - q) + d_d\dot{q}$$

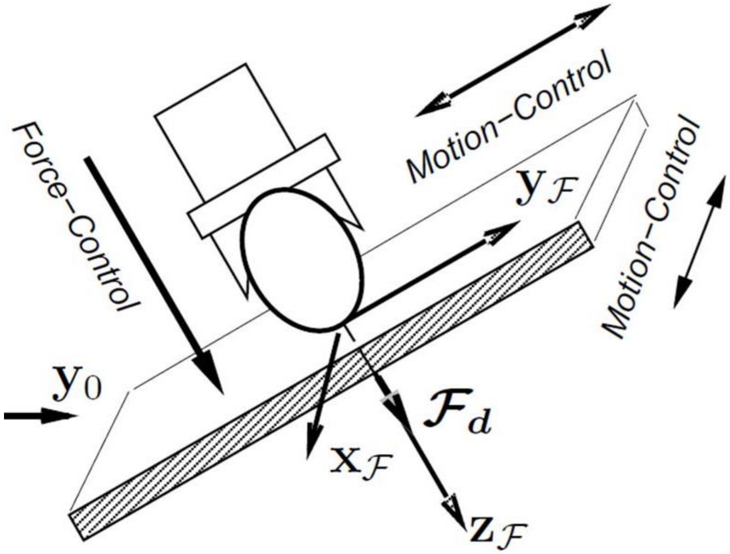
Force Control

SEA （Series Elastic Actuator）



Force Control

2. Hybrid force/position control



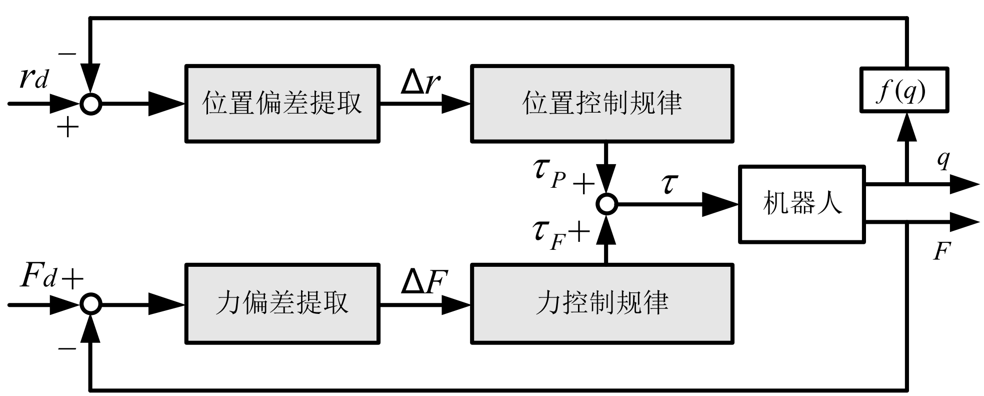
[Basic idea]

It separates all 6 axes of the task (3 force and 3 torque) and applies either a motion based control or a force based control onto each of the axes.

[Position error] $\Delta r = \{e_P^T(r_d - r)\}e_P, r = f(q)$

[Force error] $\Delta F = \{e_F^T(F_d - F)\}e_F$

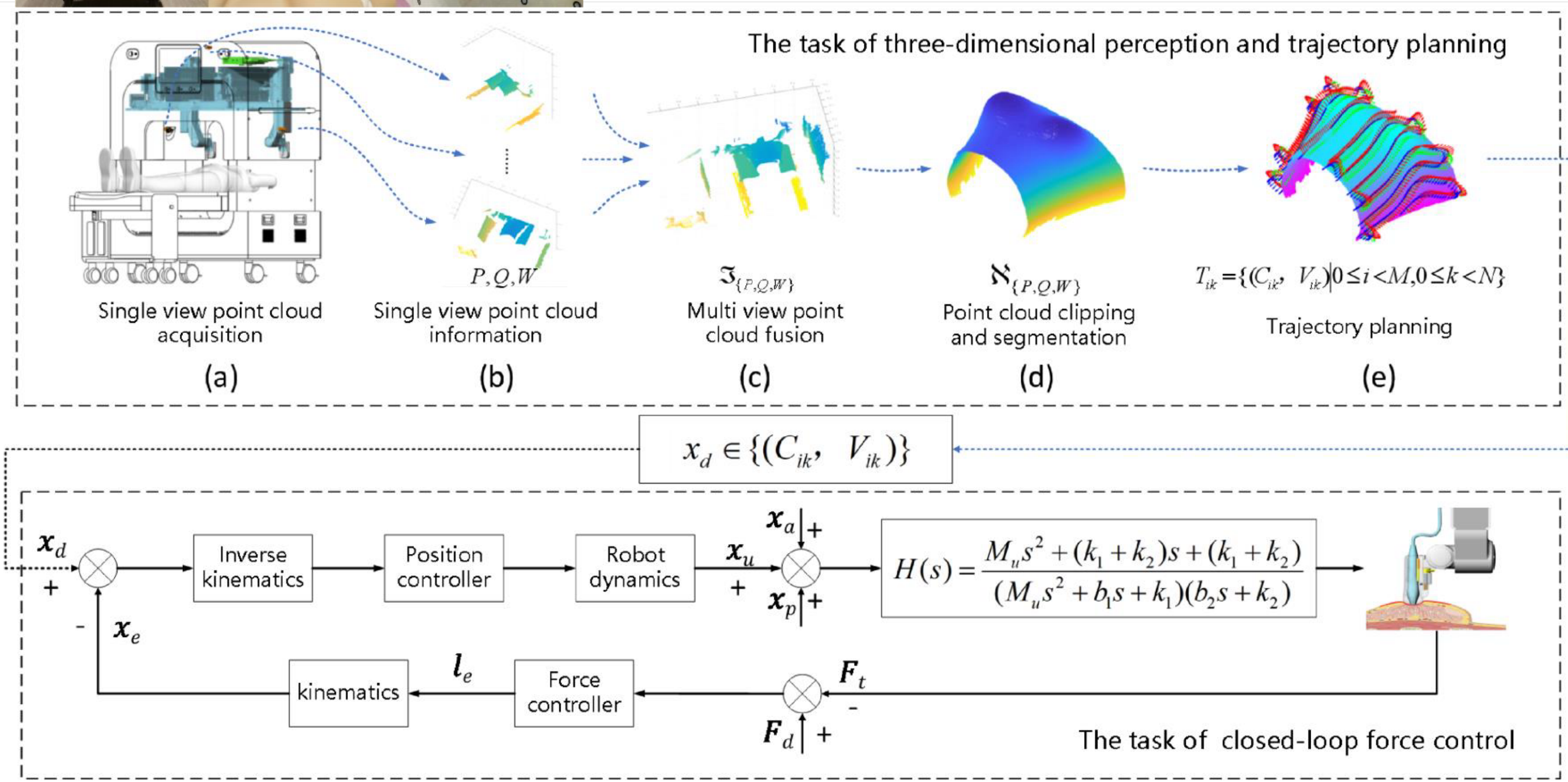
Constrained 2-DoF manipulator



Force Control

2. Hybrid force/position control

J. Tan, B. Li, Y. Li, B. Li, X. Chen, J. Wu, B. Luo, Y. Leng, Y. Rong, C. Fu. A Flexible and Fully Autonomous Breast Ultrasound Scanning System. **IEEE Transactions on Automation Science and Engineering**, 20(3): 1920-1933, 2023.



Homework 18

Classify the following robot tasks as motion control, force control, hybrid motion–force control, impedance control, or some combination. Justify your answer.

(a) Tightening a screw with a screwdriver.

(b) Pushing a box along the floor.

(c) Pouring a glass of water.

(d) Shaking hands with a human.

(e) Throwing a baseball to hit a target.

(f) Shoveling snow.

(g) Digging a hole.

(h) Giving a back massage.

(i) Vacuuming the floor.

(j) Carrying a tray of glasses.

Final Project Presentation

Date: May 28, 2025

1. **8 minutes for Group Presentation + 3 minutes for Q&A;**
2. **The presentation needs to clarify the specific contribution of each group member to the project;**
3. **PPT, WORD report and Use podium desktop PC (make sure video can be played), or your own laptop (HDMI interface);**
4. **Other attachments (programs, data, videos, etc.). Please submit them in BB system before June 11, 2025 (submission as a group ZIP file).**