

Topological Signal Processing Definitions

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1 Introduction

This document is for me to keep track of the definitions and ideas I have about topological signal processing. This will probably get long, so maybe we can chunk it later. I'm not sure if I want to start all the way with the definition of a topological space. But I think I can start with the ideas of homology and cohomology.

2 Definitions

2.1 Simplicial Homology

The central objects of simplicial homology are simplicial complexes.

- A **k-simplex** is the convex hull of a set of $k+1$ points in some Euclidean space. We can think of a 0-simplex as a vertex. A 1-simplex is an edge, a 2-simplex is a triangle, and so on.
- Note that a k-simplex has $k + 1$ faces, which are the simplices of dimension $k - 1$ that are contained in it.
- A **simplicial complex** is a collection of simplices such that the intersection of any two simplices is either empty or another simplex.
- The **dimension** of a simplicial complex is the maximum dimension of any of the simplices in the complex.

In order to do algebra with simplicial complexes, we need to associate it to algebraic objects that we can manipulate. This is where the chain complex comes in.

- Say X is a k -dimensional simplicial complex. The **chain group** $C_k(X, \mathbb{R})$ is the vector space (over \mathbb{R}) with basis given by the number of k -simplices in X .

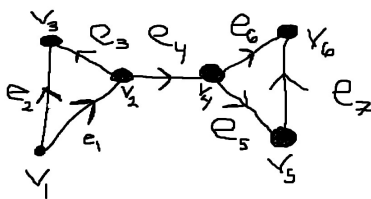


Figure 1: A simplicial complex, with 6 vertices and 7 edges.

Inspecting the image, we see that we have two