Test 1

$n = 2; a = 1; b = 3$

$x_0 = 1; x_1 = 2; x_2 = 3$

$$l_0 = \prod_{k=0;k\neq 0}^{2} \frac{x - x_k}{x_0 - x_k} = \frac{x - x_1}{x_0 - x_1} * \frac{x - x_2}{x_0 - x_2} = \frac{x - 2}{-1} * \frac{x - 3}{-2} = \frac{1}{2}x^2 - \frac{5}{2}x + 3$$

$l_0(x_0) = l_0(1) = 1; l_0(x_1) = l_0(2) = 0; l_0(x_2) = l_0(2) = 0;$

$$l_1 = \prod_{k=0;k\neq 1}^{2} \frac{x - x_k}{x_1 - x_k} = \frac{x - x_0}{x_1 - x_0} * \frac{x - x_1}{x_0 - x_2} = \frac{x - 1}{1} * \frac{x - 3}{-1} = -x^2 + 4x - 3$$

$l_1(x_0) = l_1(1) = 0; l_1(x_1) = l_1(2) = 1; l_1(x_2) = l_1(2) = 0;$

$$l_2 = \prod_{k=0;k\neq 2}^{2} \frac{x - x_k}{x_2 - x_k} = \frac{x - x_0}{x_2 - x_0} * \frac{x - x_2}{x_0 - x_1} = \frac{x - 1}{2} * \frac{x - 2}{1} = \frac{1}{2}x^2 - \frac{3}{2}x + 1$$

$l_2(x_0) = l_2(1) = 0; l_2(x_1) = l_2(2) = 0; l_2(x_2) = l_2(2) = 1;$

$1*$

$A_0 = 1; A_1 = 4; A_2 = 9$

$$L(x) = \sum_{i=0}^{2} A_i * l_i(x) = 1(\frac{1}{2}x^2 - \frac{5}{2}x + 3) + 4(x^2 + 4x - 3) + 9(\frac{1}{2}x^2 - \frac{3}{2}x + 1) = x^2$$

$2*$

$A_0 = 2; A_1 = 1; A_2 = 3$

$$L(x) = \sum_{i=0}^{2} A_i * l_i(x) = 2(\frac{1}{2}x^2 - \frac{5}{2}x + 3) + 1(x^2 + 4x - 3) + 3(\frac{1}{2}x^2 - \frac{3}{2}x + 1) = \frac{3}{2}x^2 - \frac{11}{2}x + 6$$

Test 2

$n = 1; a = 1; b = 2$

$x_0 = 1; x_1 = 2$

$$l_0 = \prod_{k=0; k \neq 0}^{1} \frac{x - x_k}{x_0 - x_k} = \frac{x - x_1}{x_0 - x_1} = \frac{x - 2}{-1} = -x + 2$$

$l_0(x_0) = l_0(1) = 1; l_0(x_1) = l_0(2) = 0$

$$l_0 = \prod_{k=0; k \neq 1}^{1} \frac{x - x_k}{x_1 - x_k} = \frac{x - x_0}{x_1 - x_0} = \frac{x - 1}{1} = x - 1$$

$l_1(x_0) = l_1(1) = 0; l_1(x_1) = l_1(2) = 1$

1*

$A_0 = f(x_0) = f(1) = 1; A_1 = f(x_1) = f(2) = 2$

$$L(x) = \sum_{i=0}^{1} A_i * l_i(x) = 1(-x + 2) + 2(x - 1) = x$$

2*

$A_0 = f(x_0) = f(1) = 0; A_1 = f(x_1) = f(2) = 0$

$$L(x) = \sum_{i=0}^{1} A_i * l_i(x) = 0(-x + 2) + 0(x - 1) = 0$$

3*

$A_0 = f(x_0) = f(1) = 3; A_1 = f(x_1) = f(2) = 1$

$$L(x) = \sum_{i=0}^{1} A_i * l_i(x) = 3(-x + 2) + 1(x - 1) = -2x + 5$$

Test 3

$n = 3; a = 0; b = 6$

$x_0 = 0; x_1 = 2; x_2 = 4; x_3 = 6$

$$l_0 = \prod_{k=0;k\neq0}^{3} \frac{x - x_k}{x_0 - x_k} = \frac{x - x_1}{x_0 - x_1} * \frac{x - x_2}{x_0 - x_2} * \frac{x - x_3}{x_0 - x_3} =$$

$$= \frac{x - 2}{-2} * \frac{x - 4}{-4} * \frac{x - 6}{-6} = -\frac{1}{48}x^3 + \frac{1}{4}x^2 - \frac{44}{48}x + 1$$

$$l_1 = \prod_{k=0;k\neq1}^{3} \frac{x - x_k}{x_1 - x_k} = \frac{x - x_0}{x_1 - x_0} * \frac{x - x_2}{x_1 - x_2} * \frac{x - x_3}{x_1 - x_3} =$$

$$= \frac{x - 0}{2} * \frac{x - 4}{-2} * \frac{x - 6}{-4} = \frac{1}{16}x^3 - \frac{5}{8}x^2 + \frac{3}{2}x$$

$$l_2 = \prod_{k=0;k\neq2}^{3} \frac{x - x_k}{x_2 - x_k} = \frac{x - x_0}{x_2 - x_0} * \frac{x - x_1}{x_2 - x_1} * \frac{x - x_3}{x_2 - x_3} =$$

$$= \frac{x - 0}{4} * \frac{x - 2}{2} * \frac{x - 6}{-2} = -\frac{1}{16}x^3 + \frac{1}{2}x^2 + \frac{3}{4}x$$

$$l_3 = \prod_{k=0;k\neq3}^{3} \frac{x - x_k}{x_3 - x_k} = \frac{x - x_0}{x_3 - x_0} * \frac{x - x_1}{x_3 - x_1} * \frac{x - x_2}{x_3 - x_2} =$$

$$= \frac{x - 0}{6} * \frac{x - 2}{4} * \frac{x - 6}{2} = \frac{1}{48}x^3 - \frac{1}{6}x^2 + \frac{1}{4}x$$

$1*$

$A_0 = f(x_0) = f(0) = 0; A_1 = f(x_1) = f(2) = 0;$

$A_2 = f(x_2) = f(4) = 0; A_3 = f(x_3) = f(6) = 0;$

$$L(x) = \sum_{i=0}^{3} A_i * l_i(x) = 0$$