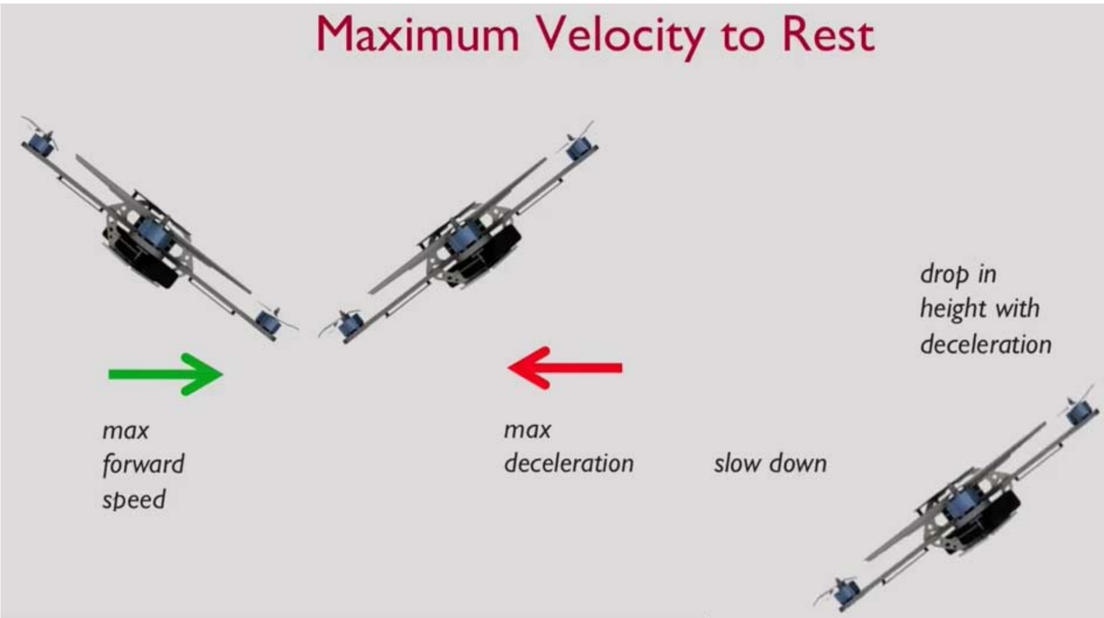
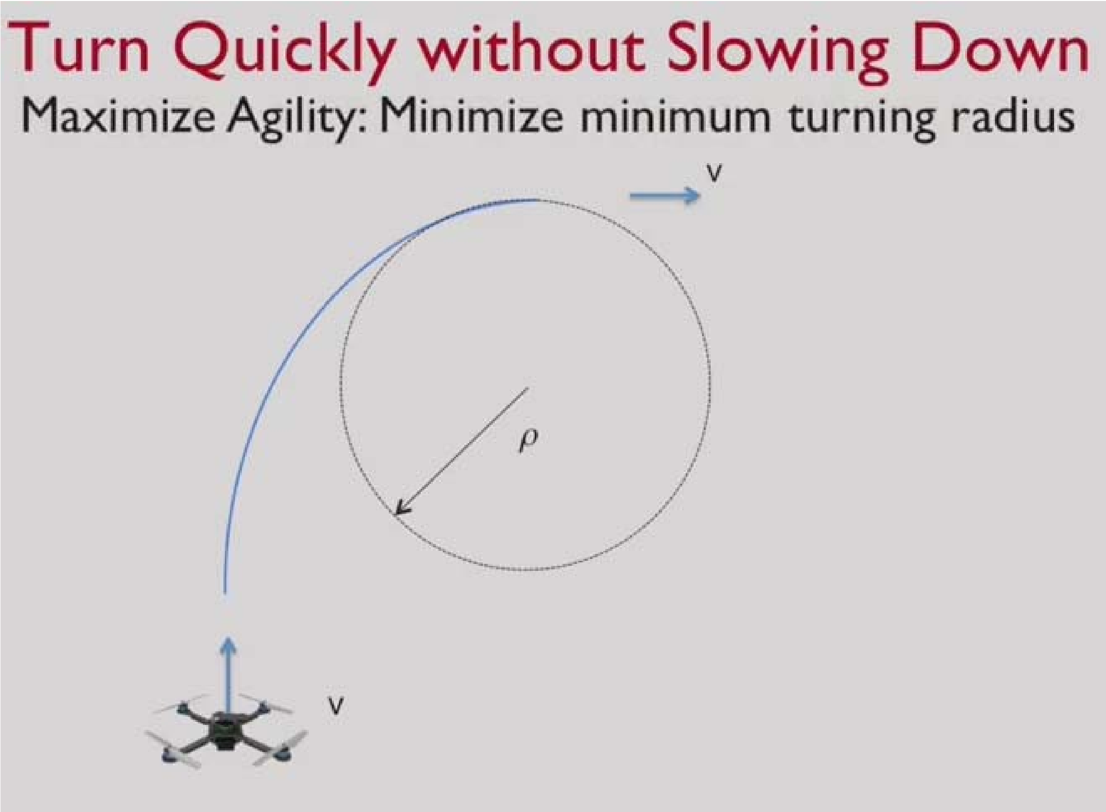
Agility and Manoeuvrability

Let’s start thinking about how to create agile robots. Agile robots are robots that can start from a position of rest, accelerate quickly to a maximum speed, stop when it sees obstacles, and then accelerate again to a maximum speed.

So how does a robot stop from a configuration where it is going at maximum forward speed? The robot will be pitching forward when it's moving at maximum speed. To bring it to a position of rest, we have to pitch it backward, reversing the direction of thrust so that we get a deceleration. This means that the robot will be pitching back at an aggressive angle. This will cause the robot to slow down, but as a result, the thrust vector which now points in a direction other than vertical. This will also cause the robot to lose height since the component of thrust in the vertical direction will now be less than the weight.

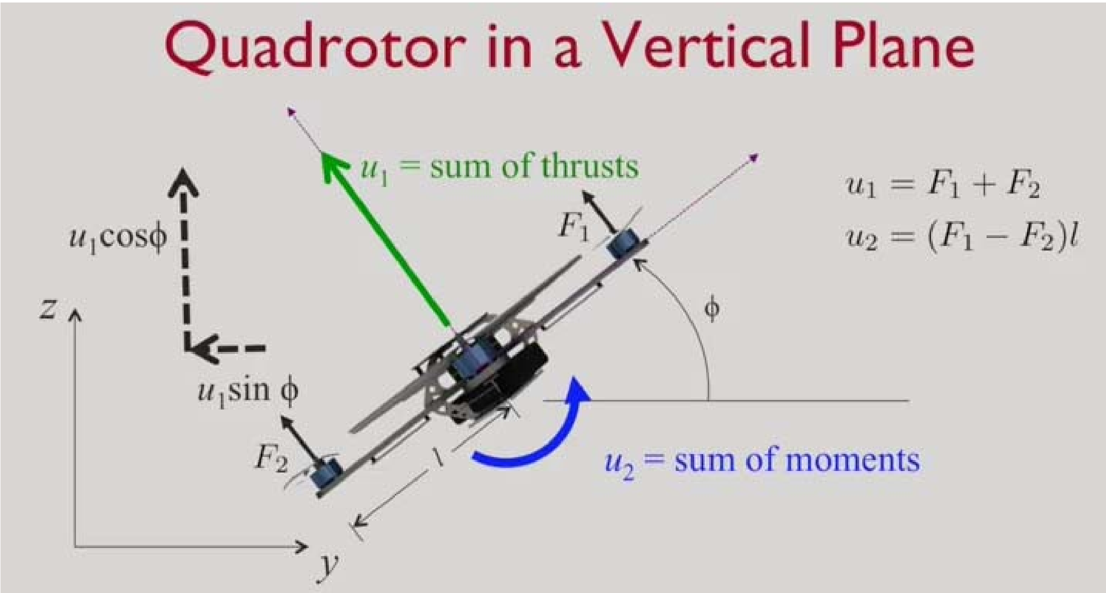


To maximise agility, we really want to minimise the stopping distance. The other factor that defines agility is the robot's ability to turn quickly. Consider a robot flying forward at maximum speed and then turning as quickly as possible. What we'd like to do here is to minimize the turning radius, ρ.



In both cases, stopping from maximum speed and turning at maximum speed, it is actually sufficient to consider a fairly simple model of a quadrotor.

Here is a diagram of a vehicle in the vertical plane:



The propellers generate a thrust, and the sum of these two thrusts (actually 4 thrusts for a quadrotor), is the vector u_1. The vector u_1 now has two components, one in the horizontal direction and the other in the vertical direction.

The difference of the thrusts contributes to the moments and that's u_2.

The equations of motion in the plane are three equations that describe how the components of the thrust, u_1, and how the turning moment, u_2, accelerate the robot in the y-z plane, and also turn the robot in the direction of the pitch angle ϕ.

$$\begin{array}{l} \text{linear acceleration, } a \left\{ \begin{array}{l} \\ \\ \end{array} \right. \\ \text{angular acceleration, } \alpha \end{array} \begin{bmatrix} \ddot{y} \\ \ddot{z} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 \\ -g \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{1}{m}\sin\phi & 0 \\ \frac{1}{m}\cos\phi & 0 \\ 0 & \frac{1}{I_{xx}} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Again, we have two accelerations, one linear, denoted by a, with components in the y and the z directions and one angular, denoted by α, which obviously has only one component, which is the rotation in the plane.

The two key challenges for agility are to be able to accelerate quickly and to pitch and roll quickly. To accelerate quickly, we need to maximise acceleration, denoted by a_{max}. In order to roll and pitch quickly we need to maximise α_{max}. The first quantity is the linear acceleration. The second quantity is the angular acceleration.

Agility

Two key ideas

- Accelerate quickly

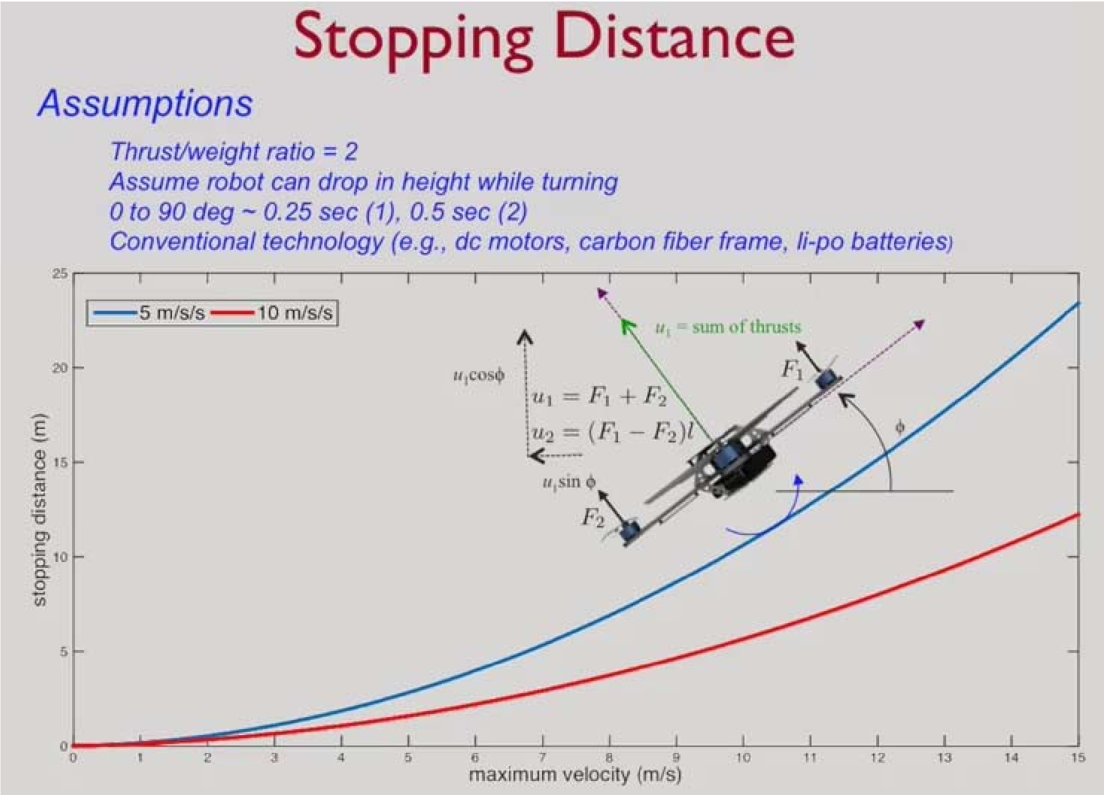
  $\text{maximize } \underbrace{a_{max}}_{\text{linear acceleration}}$ $\qquad \boxed{\text{maximize } \frac{u_{1,max}}{W}}$

- Roll/pitch quickly

  $\text{maximize } \underbrace{\alpha_{max}}_{\text{angular acceleration}}$ $\qquad \boxed{\text{maximize } \frac{u_{2,max}}{I_{xx}}}$

To maximise the first quantity, we must maximise the ratio of u_1 to W. In other words, divide the maximum thrust by the weight, and maximise that ratio. The second quantity can be maximised by taking u_2, which is the turning moment, and maximise that divided by the moment of inertia along the x-axis, I_{xx}.

One of the things we can do is to calculate the stopping distance for different rates of acceleration. Here we show two curves. One at 5 meters per second squared, (roughly half the acceleration due to gravity) and the other at 10 meters per second squared, (roughly equal to the acceleration due to gravity). In both cases, we've essentially used a dynamic simulation to create a graph of the stopping distance with respect to the maximum velocity:



As the robot velocity increases, the stopping distance increases. Clearly, the faster the robot can accelerate or decelerate the smaller the stopping distance. These are two curves we have generated to give you a flavour of what it means to maximise the agility of a robot. We want to be able to stop quickly if the vehicle sees an unexpected obstacle, and this is critical to manoeuvrability.