Lecture 1B.1

Basic Mechanics

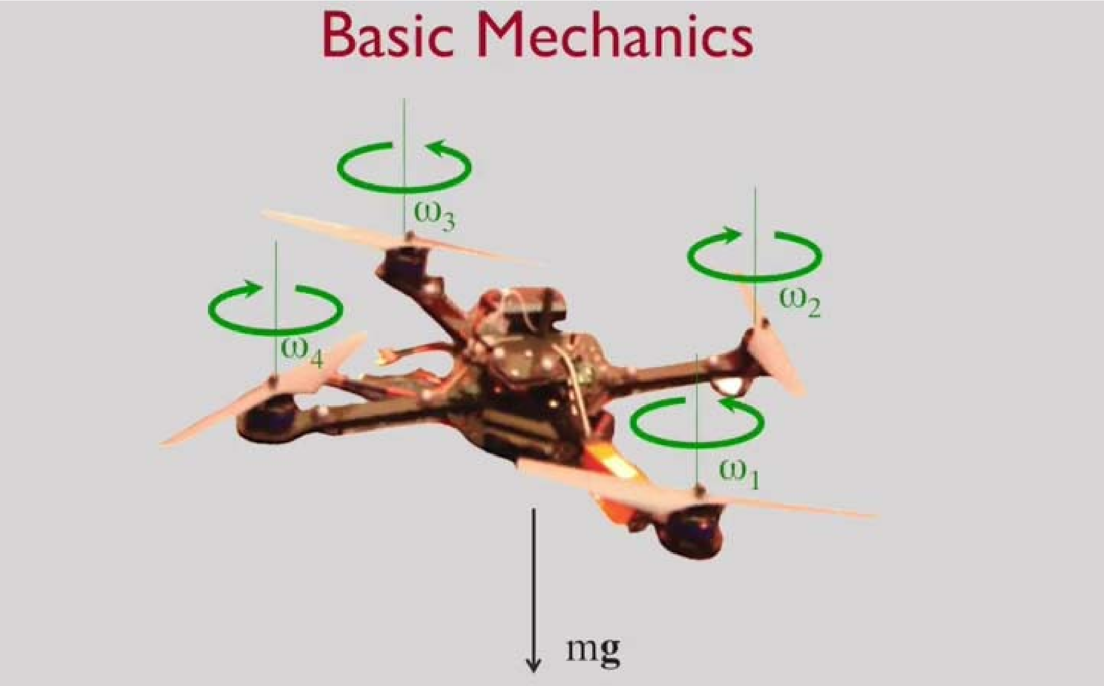
This module is going to explore how quad rotors work. We'll look at the basic mechanics and draw some conclusions about how to design quad rotors.

The lectures will cover the following topics:

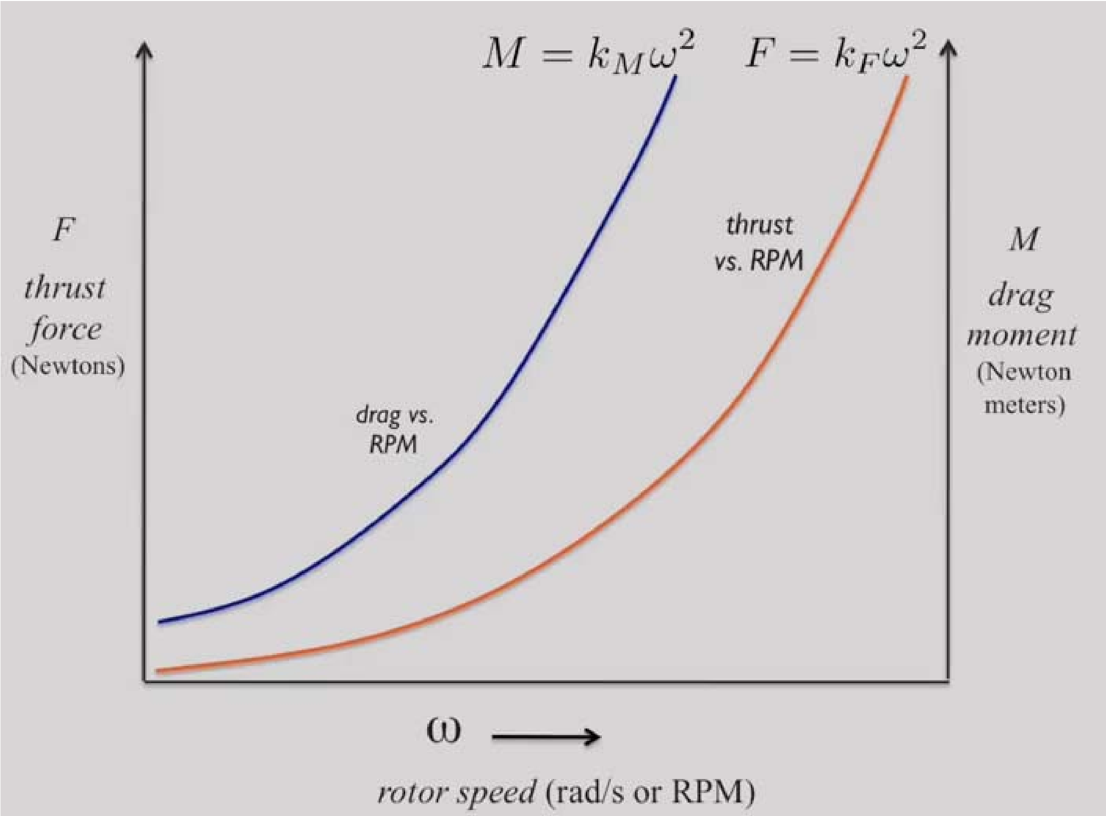
- Basic mechanics
- Control
- Design considerations
- Agility
- Component selection
- Effects of size

Let's start with the basic mechanics.

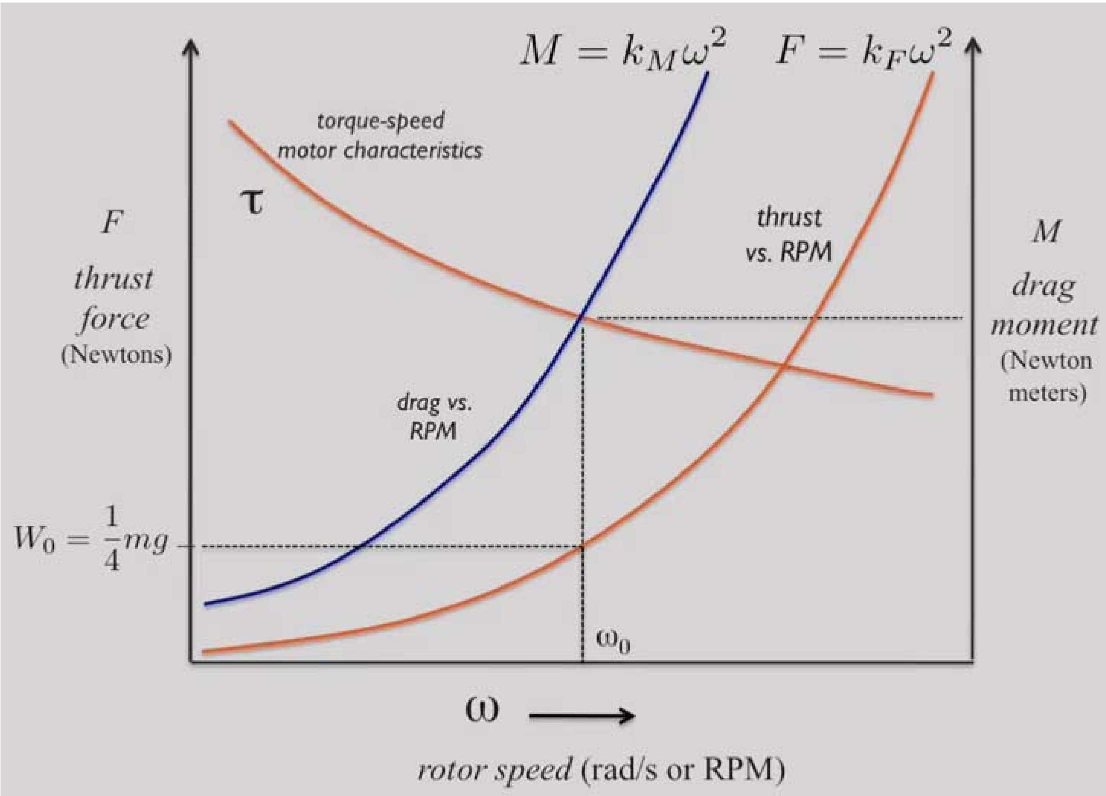
A quad rotor has four rotors that support the vehicle's weight. Each rotor spins and generates the thrust:



If you plot the thrust, or the thrust force, against the RPMs of the motors or their angular velocity, you'll find that the relationship is approximately quadratic. Every time a rotor spins, there's also a drag that the rotor has to overcome. That drag moment is also quadratic:

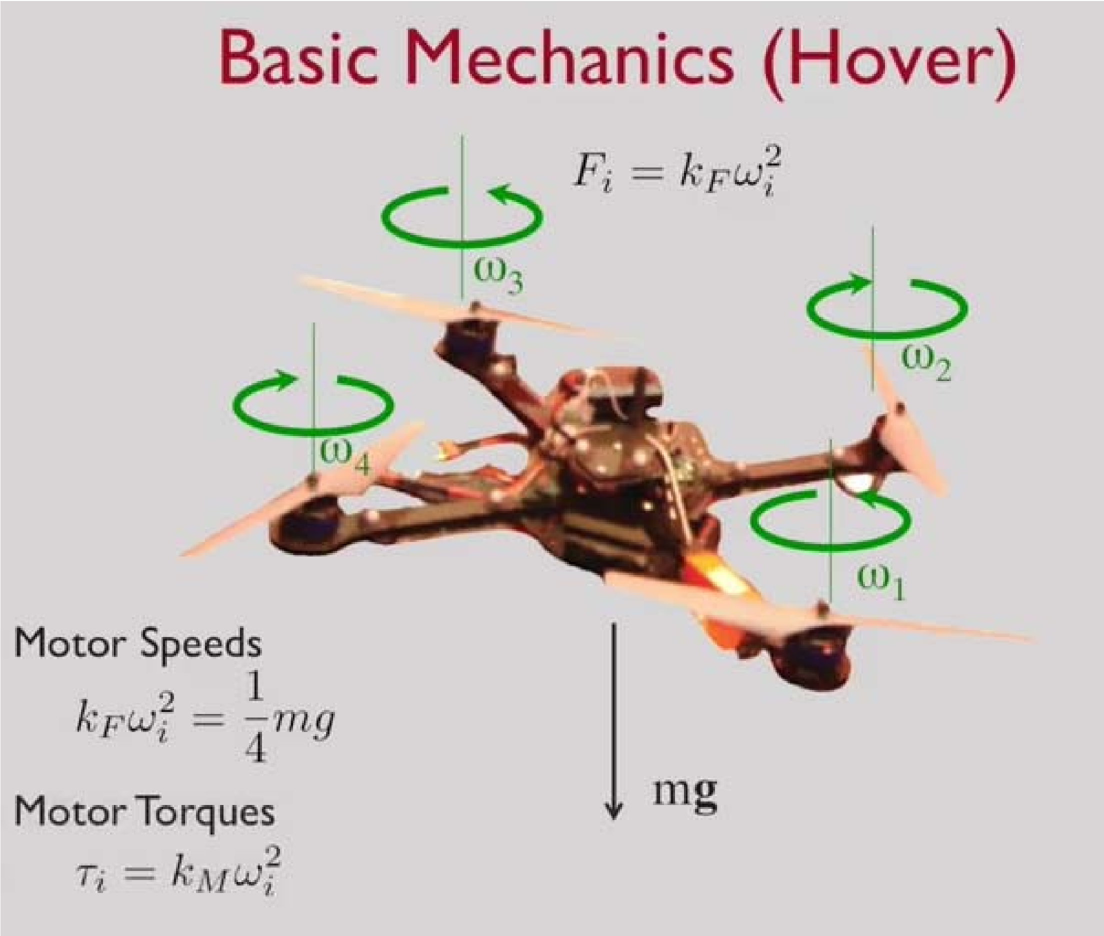


In the case of a quad rotor, every rotor has to support roughly one fourth of the weight in equilibrium. So, by looking at the thrust forces vs. rpm curve, you can determine the speed that'll be required to produce one fourth the weight. This gives you the operating speed, ω_0.



Of course, that operating speed produces a drag moment, and every rotor has to overcome the drag moment. You have to select the size of the motor, so that they can produce the torque needed to overcome this drag moment.

So when the robot is hovering, the rotor speeds, ω_i, compensate for the weight, mg. Using the weight you can determine the basic operating speed for every rotor. And that in turn tells you what torque you need to apply at every motor:



The equations are fairly simple if you assume that you know the constant of proportionality between the force and the square of the RPM, k_F, and the constant of proportionality between the drag moment and the square of the RPM, k_M.

$$F_i = k_F \omega_i^2$$

$$M_i = k_M \omega_i^2$$

The resultant force can be calculated quite easily. It's the sum of the four thrusts and the gravity force.

$$F = F_1 + F_2 + F_3 + F_4 - mg$$

Also, if you know where the centre of mass is, you can quickly calculate moments about the centre of mass. Then the total moment is obtained by calculating the moments due to the forces exerted by the rotors and the reactions due to the rotors spinning in counter-clockwise or clockwise directions (these reactions are moments, and add to the net moment).

If the distance from the centre of mass to the motor is r_i, and the reaction due to the motor is M_i, the resultant moment is given by:

$$M = r_1F_1 + r_2F_2 + r_3F_3 + r_4F_4 + M_1 + M_2 + M_3 + M_4$$

In equilibrium, the resultant force is obviously zero, and the resulting moment is also zero.

But what happens when the resultant forces and moments are non-zero? In this case, you'll get a net acceleration.

To keep things simple let's first consider an acceleration in the vertical direction.

As we have seen, when every motor thrust is the same, and their sum equals the weight of the quadrotor, the vehicle will hover. If you increase the motor speeds, but keep all 4 equal, then the robot accelerates upwards. If you decrease the motor speeds, keeping them all equal, obviously the robot will accelerate down.

The combination of motor thrusts and the weight determines which way the robot accelerates.

$$\sum_{i=1}^{4} k_F \omega_i^2 + mg = ma$$