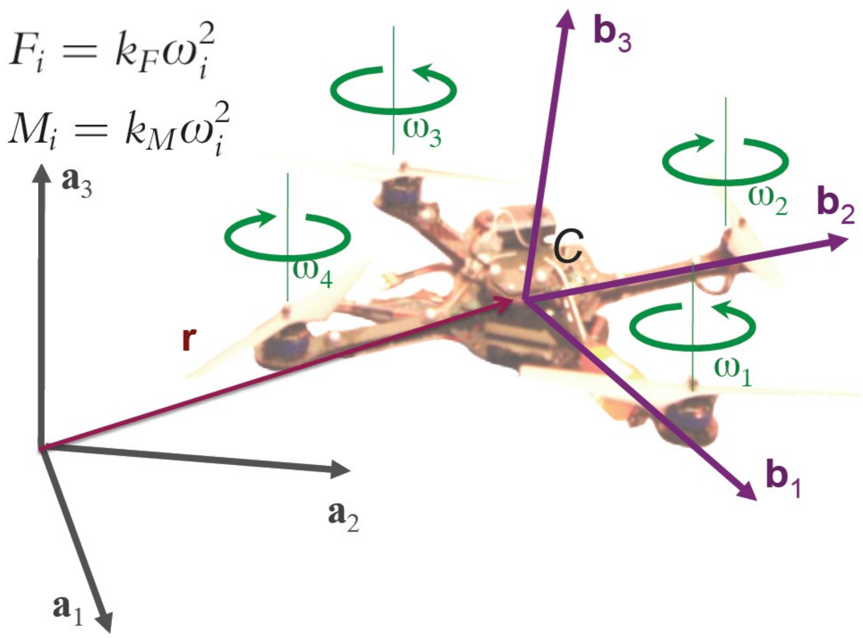
Lecture 2C.1

Formulation

Now that we've looked at positions and velocities, it's time to study the dynamics of a quadrotor. Again, we have two coordinate systems: one attached to the moving robot, and the other is the inertial coordinate system.

The body fixed coordinate system is described by the set of unit vectors b_1, b_2, and b_3. Similarly, the unit vectors a_1, a_2, and a_3 describe a coordinate system that's fixed to the inertial frame.



The robot has four rotors, each of which is independently actuated. **r** is the position vector of the centre of mass, and we know expressions for the thrust that the motors produce on the airframe:

$$F_i = k_F \omega_i^2$$

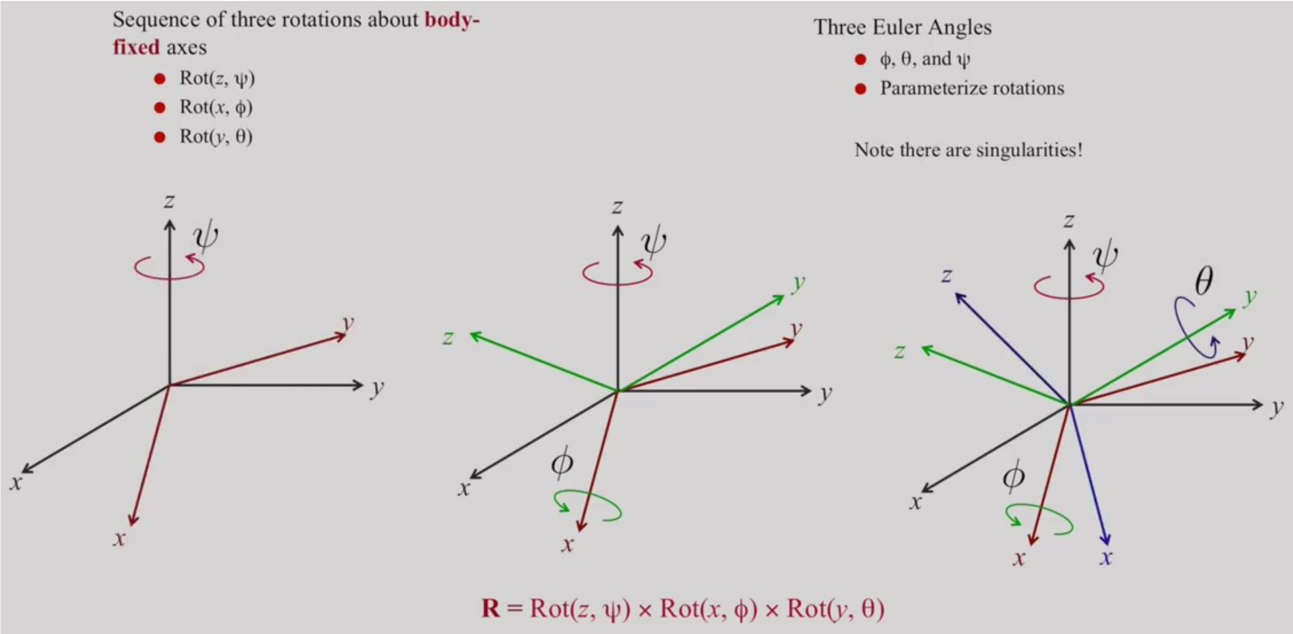
We also know the expressions for the reaction moments:

$$M_i = k_M \omega_i^2$$

Both of these are proportional to the square of the angular speeds of the rotors.

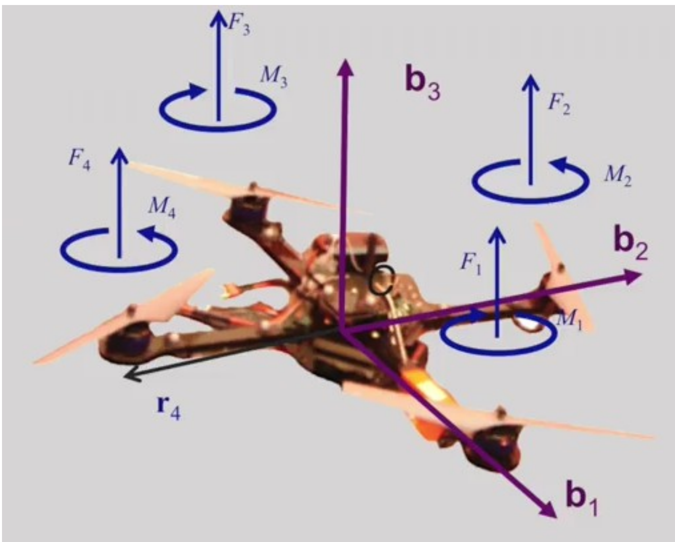
We've seen how Euler Angles are used to represent rotations. We will use the Z-X-Y convention, the first rotation being about the z-axis to φ, the second one about the x-axis, and the third about the y-axis. The first axis is yaw, the second is roll, and the third is pitch.

Again just to remind ourselves, this is the Z-X-Y convention:



The first rotation is about the z-axis through φ, the second is about the x-axis through ϕ, this is the roll angle, and finally, the pitch about the y-axis through θ. Of course, as we have seen, there are singularities. Singularities occur when the roll angle, $\phi = 0$. Also, even when the angle $\phi \neq 0$, we can have two sets of Euler angles for every rotation.

Let's look at the external forces and moments that act on the airframe. We have four thrusts: F_1, F_2, F_3, and F_4, and then four moments, M_1, M_2, M_3 and M_4.



The sum of the forces is obtained by adding up the thrust vectors and the gravity vector:

$$F = F_1 + F_2 + F_3 + F_4 + mg\bar{a}_3$$

The sum of the moments is obtained by adding up the reaction moments, as well as the moments of the thrust forces:

$$M = (r_1 \times F_1) + (r_2 \times F_2) + (r_3 \times F_3) + (r_4 \times F_4) + M_1 + M_2 + M_3 + M_4$$

Remember, these are vector computations. You have to add 3x1 vectors in each of these equations.

To predict the net acceleration, we have to write down the equations of motion. These come from Newton and Euler and they're called the Newton-Euler Equations.