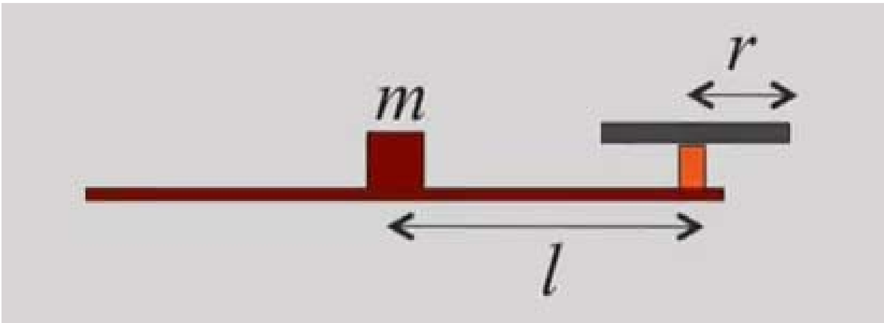
Effects of Size

Now we will examine the effects of sizing the platform. What does it mean if we have a larger platform? Well clearly it becomes bigger and heavier, so the thrust-to-weight ratio changes.

What happens if we have a smaller platform? Does the thrust-to-weight ratio improve?

There are a few things to consider. There are the mass, the inertia of the platform, the maximum amount of thrust it can exert, and the maximum moment it can generate.



Let's look at the characteristic length, l, which is roughly half the vehicle diameter. The mass of the vehicle will scale as the cube of this characteristic length, and the moments-of-inertia scale as the fifth power of the characteristic length.

$$m \sim l^3 \qquad I \sim l^5$$

This is because mass scales with the volume, and volume scales as l cubed. Moments of inertia go as mass times length squared, and therefore it's scaled as l to the fifth.

If we consider the total thrust applied by the rotors, this scales as the area spanned by a single rotor. It also scales as the square of the blade tip speed. So if omega is the rotor speed, and r is the rotor radius, then the product of omega and r gives you the blade tip speed and the thrust scales as velocity squared.

$$F \sim \pi r^2 \times (\omega r)^2$$

In short, the thrust scales as r squared times v squared, where v now is the blade tip speed.

$$F \sim r^2 v^2$$

Let's now look at the moment that can be generated by a vehicle like this. Clearly, if we apply a thrust, F, on each rotor, the resulting moment scales as force times length.

So if F is the thrust and l is the characteristic length then the moment scales as f times l:

$$M \sim Fl$$

Let's assume that the rotor size scales with the characteristic length. This is reasonable since this is a geometric constraint. In this case the thrust scales as l^2 times v^2,.and the moment scales as l^3 times v^2.

$$F \sim l^2v^2 \qquad\qquad M \sim l^3v^2$$

Now, we can calculate maximum acceleration by taking the total thrust and dividing it by the mass, and the maximum angular acceleration is given by the total moment divided by the moment of inertia:

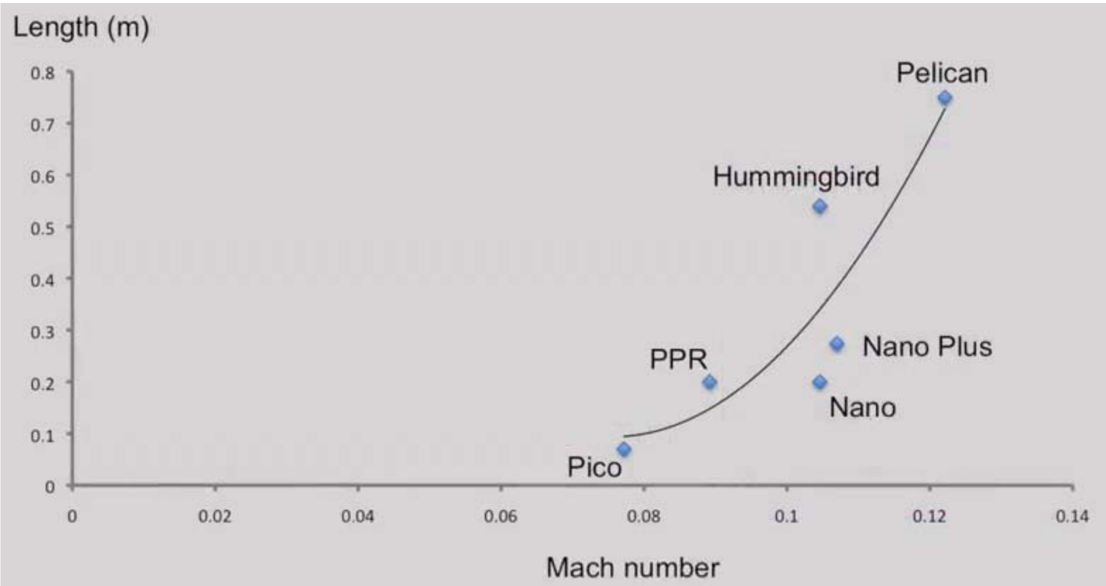
$$a \sim \frac{F}{m} \qquad\qquad \alpha \sim \frac{M}{I}$$

If we substitute the appropriate scaling rules (mass scaling as l^3 and inertia as l^5) we see that the maximum accelerations, linear and angular, go as v^2 over l and v^2 over l^2 respectively:

$$a \sim \frac{v^2}{l} \qquad\qquad \alpha \sim \frac{v^2}{l^2}$$

How does the blade tip speed, v, scale with respect to the characteristic length? It turns out that there are a couple of ways of looking at it. Scaling experiments in the lab have found that the blade tip speed scales as the square-root of the characteristic length:

$$t \sim \sqrt{l}$$



This is generally true at the scales that used in the laboratory, but these are smaller vehicles, and this may not hold for much larger platforms like commercial helicopters.

This paradigm is often called Froude scaling.

- Froude scaling

$$v \sim \sqrt{l}$$

$$F \sim l^3$$

$$\boxed{a \sim 1, \ \alpha \sim \frac{1}{l}}$$

In contrast, in aerodynamics there's a different paradigm called Mach scaling.

- Mach scaling

$$v \sim 1$$

$$F \sim l^2$$

$$\boxed{a \sim \frac{1}{l}, \ \alpha \sim \frac{1}{l^2}}$$

Froude scaling suggests that the blade tip-speed scales as the square root of length, while Mach scaling suggests that the blade tip-speed is roughly constant, independent of length.

Calculating the maximum thrust with these assumptions shows that it scales as l^3 with Froude scaling and as l^2 with Mach scaling. Further, if we also calculate the maximum acceleration and maximum angular acceleration we find that angular acceleration increases as we scale down the size of the platform in both cases. This, in fact, results in greater agility.

So, a key idea is that the smaller we make the vehicle, the larger the acceleration we can produce in the angular direction, which, in turn, allows greater agility.