Lecture 2A.2

Rotations

Let’s explore the properties of rotation matrices. To remind you, rotation matrices are 3x3 matrices that have the properties of orthogonality, and the fact that the determinant equals +1.

$$SO(3) = \{R \in \Re^{3x3} \; | R^T R = RR^T = I, \det R = 1\}$$

Matrices of these kinds constitute the **special Orthogonal** group. We use SO(3) to refer to this group. It's the *special Orthogonal group in three dimensions*.

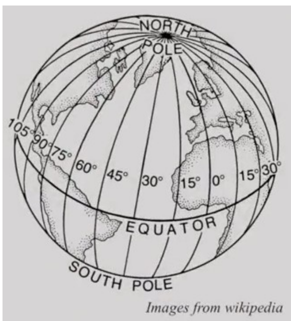
Rotation matrices are a great way of describing rotations, but they're not intuitively very obvious. So sometimes we prefer to use coordinates to describe SO(3).

Coordinates for SO(3):

- Rotation Matrices
- Euler angles
- Axis-angle parameterisation
- Exponential Coordinates
- Quaternions

In addition to **rotation matrices**, we'll discuss a set of angles called **Euler angles** that are often used to describe rotations. We also look at **parameterization** that explicitly describes the **axis of rotation** and the **angle of rotation**. It's also possible to use **exponential coordinates**, although we this isn’t discussed in this course, and finally, it's possible to use **quaternions** (again, we don't discuss this in this class).

Let's begin by looking at coordinates on an object that we're familiar with: the sphere. In fact, let's consider the coordinates that we might use to describe a location on the Earth's surface.



We know that coordinates on the Earth's surface are described using latitudes and longitudes, but are these descriptors unique? In other words, given any point on the

Earth's surface, is there a unique combination of latitudes and longitudes that describe that point?

In fact, it turns out that the answer to this question is no. The poles each have unique latitudes, but their longitudes are not well defined. In fact, the North and South Poles can be described by any longitude. It's only the latitude that's well defined at these points on the surface of the Earth.

We want a coordinate chart in which every point on the Earth's surface maps to a pair of coordinates and that these coordinates are unique. Since this is hard to do, what we generally do is we use a collection of coordinates or a collection of charts. So, for example, we can agree to use a different nomenclature to describe the area around the poles as we approach the North or South Pole.

If we have a set of coordinates, there should be a one-to-one map between the Earth's surface and the coordinates.

One question we might want to ask is what's the minimum number of charts we need to cover the Earth's surface? Remember the chart has to lend itself to a coordinate system that's one to one. It turns out that the answer to this question is two.

But the more important question is what's the minimum number of charts we need to cover the entire rotation group, SO(3)?