Unit 1:

Simulating Neurons

To simulate neural networks, we have to simulate neurons. What computational models do people use for neurons?

By the end of this unit, you will be able to...

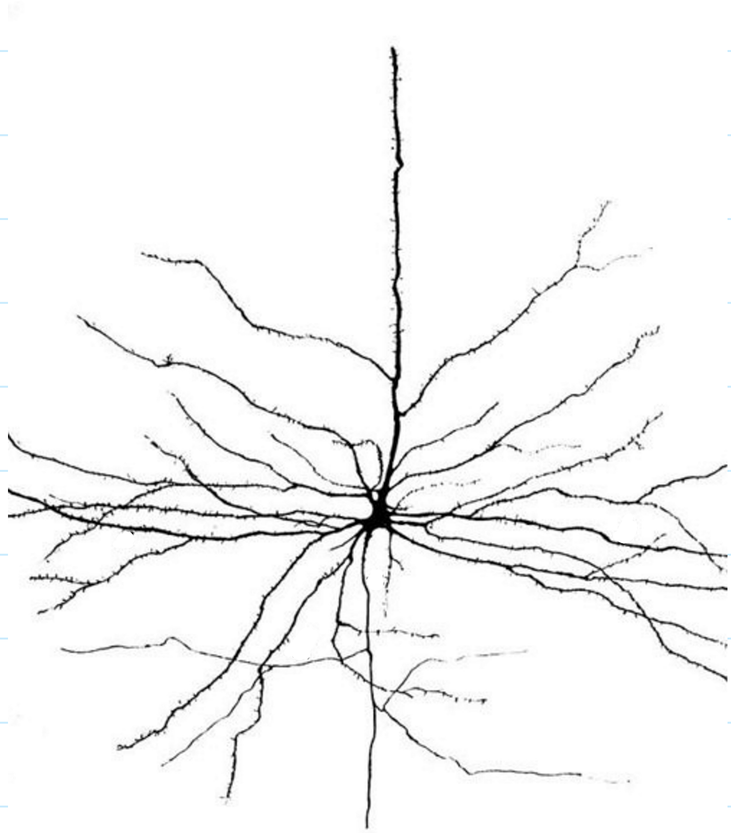
- model the nonlinear nature of how a neuron works, in the form of a famous neuron model.
- describe other, less complicated neuron models, and explain how they differ from other models.
- mathematically formulate the connections between populations of neurons.
- describe how the activity of a spiking neuron can often be summarized using its firing rate.

Neurons

Goal: To see the nonlinear nature of how a neuron works, in the form of a famous neuron model.

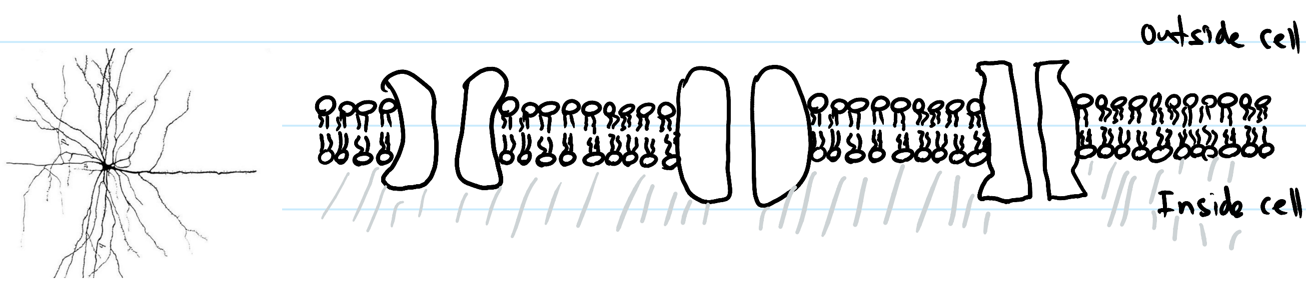
A neuron is a special cell that can send and receive signals from other neurons.

A neuron can be quite long, sending its signal over a long distance; up to <u>50m long</u>! But most are much shorter.



Neuron Membrane Potential

Ions are molecules or atoms in which the number of electrons (-) does not match the number of protons (+), resulting in a net charge. Many ions float around in your cells. The cell's membrane, a lipid bi-layer, stops most ions from crossing. However, ion channels embedded in the cell membrane can allow ions to pass.



Sodium-Potassium Pump exchanges 3 Na+ ions inside the cell for 2 K+ ions outside the cell.

- Causes a higher concentration of Na+ outside the cell, and higher concentration of K+ inside the cell.
- It also creates a net positive charge outside, and thus a net negative charge inside the cell.

This difference in charge across the membrane induces a voltage difference, and is called the

Action Potential

Neurons have a peculiar behaviour: they can produce a spike of electrical activity called

This electrical burst travels along the neuron's to its , where it passes signals to other neurons.

Hodgkin-Huxley Model

Alan Lloyd Hodgkin and Andrew Fielding Huxley received the Nobel Prize in Physiology or Medicine in 1963 for their model of an action potential (spike). Their model is based on the nonlinear interaction between membrane potential (voltage) and the opening and closing of Na+ and K+ ion channels.

Both Na+ and K+ ion channels are voltage-dependent, so their opening and closing changes with the membrane potential.

Let be the membrane potential. A neuron usually keeps a membrane potential of around

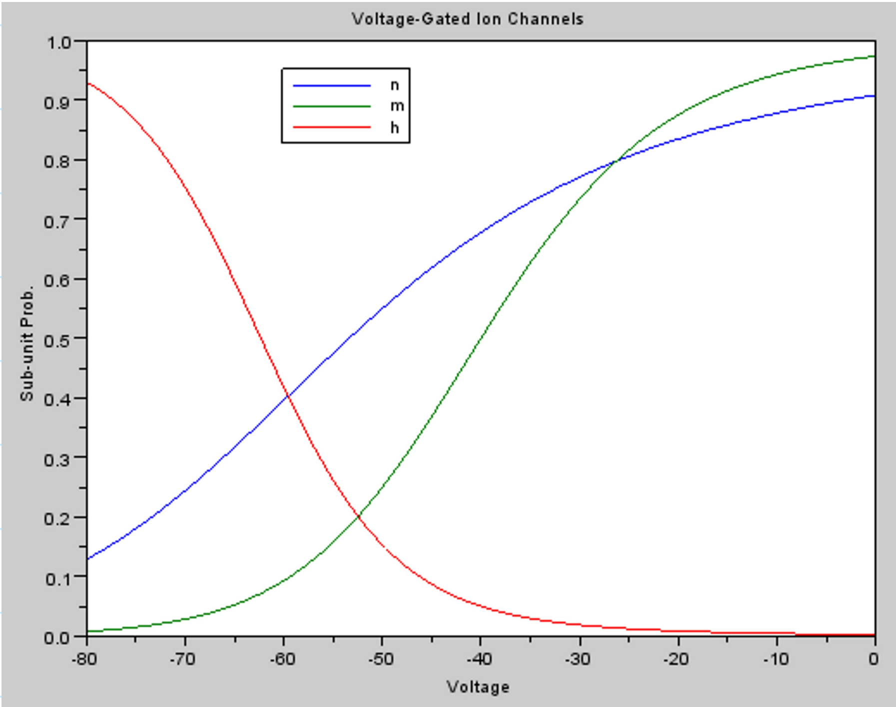
The fraction of K+ channels that are open is , where

$$\frac{dn}{dt} =$$

The fraction of Na+ ion channels open is , where

$$\frac{dm}{dt} =$$

$$\frac{dh}{dt} =$$



These two channels allow ions to flow into/out of the cell, inducing a current… which affects the membrane potential, V.

$$c\frac{dV}{dt} = J_{in} - g_L(V - V_L) - g_{Na}m^3h(V - V_{Na}) - g_K n^4(V - V_K)$$

This system of four differential equations (DEs) governs the dynamics of the membrane potential.

Notice what happens when the input current is:

- negative
- zero
- slightly positive
- very positive

End of L1

(demo HH)

Simpler Neuron Models

Goal: To look at other, less complicated neuron models.

The HH model is already greatly simplified:

- a neuron is treated as a point in space
- conductances are approximated with formulas
- only considers K+, Na+ and generic leak currents
- etc.

But to model a single action potential (spike) takes many time steps of this 4-D system. However, spikes are fairly generic, and it is thought that the *presence* of a spike is more important than its specific shape.

Leaky Integrate-and-Fire (LIF) Model

The leaky integrate-and-fire (LIF) model only considers the sub-threshold membrane potential (voltage), but does NOT model the spike itself. Instead, it simply records when a spike occurs (ie. when the voltage reached the threshold).

$$C\frac{dV}{dt} = J_{in} - g_L(V - V_L)$$

Thus, the voltage can be modelled as

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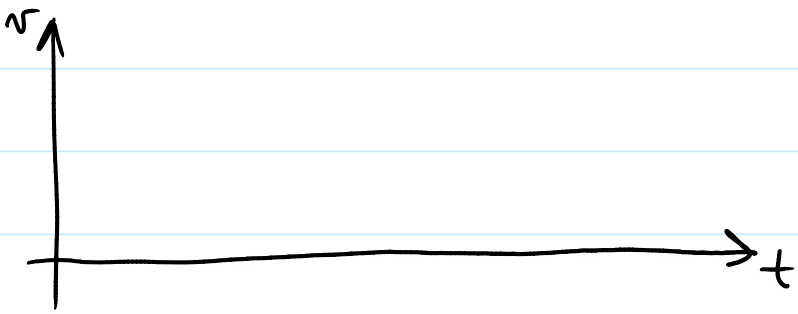
Change of variables:

We integrate the DE for a given input current (or voltage) until v reaches the threshold value of 1.



Then we record a spike at time

After it spikes, it remains dormant during its refractory period, (often just a few milliseconds). Then it can start integrating again.

(demo simple_LIF)

LIF Firing Rate

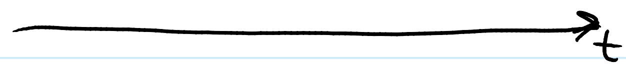
Suppose we hold the input, v_{in}, constant. We can solve the DE analytically between spikes.

Claim: $v(t) = v_{in}\left(1 - e^{-\frac{t}{\tau}}\right)$ is a solution of

$$\tau \frac{dv}{dt} = v_{in} - v, \quad v(0) = 0$$

Proof:

What does the solution look like?



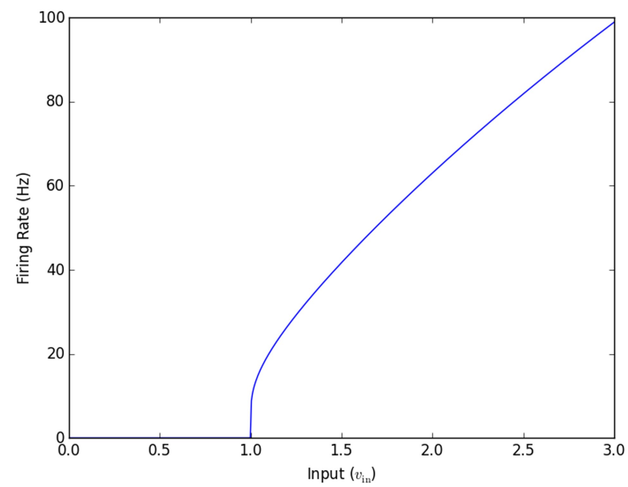
If , then our LIF neuron will spike. At what time will the spike occur (as a function of)?

Thus, the steady-state firing rate for a constant input v_{in} is $\frac{1}{t_{isi}}$

$$G(v_{in}) = \begin{cases} \\ \\ \end{cases}$$

Typical values for cortical neurons:

$$\tau_{ref} = 0.002 \text{ s} \quad (2 \text{ ms})$$
$$\tau_m = 0.02 \text{ s} \quad (20 \text{ ms})$$

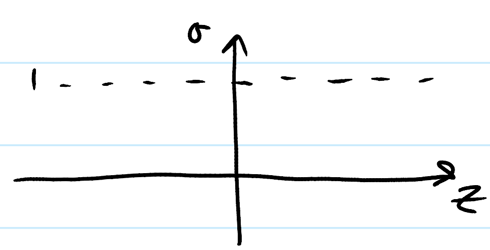


<u>**Sigmoid Neuron**</u>

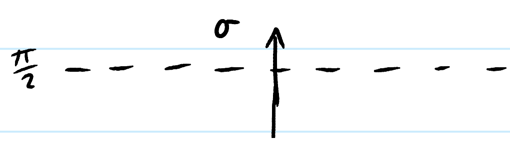
As we've seen, the activity of a neuron is very low, or zero, when the input is low, and the activity goes up and approaches some maximum as the input increases. This general behaviour can be represented by a number of different *activation* functions.

Logistic Curve

$$\sigma(z) =$$



Arctan



$$\sigma(z) =$$

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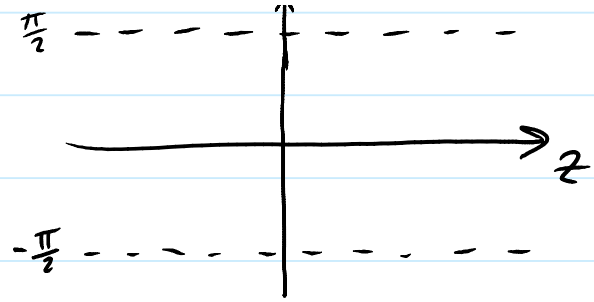
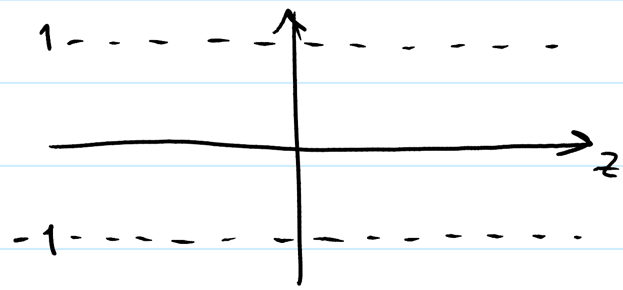


Hyperbolic Tangent

$\sigma(z) =$



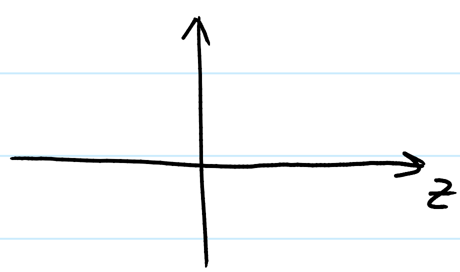
Threshold

$\sigma(z) =$

Rectified Linear Unit (ReLU)

This is just a line that gets capped below at zero.

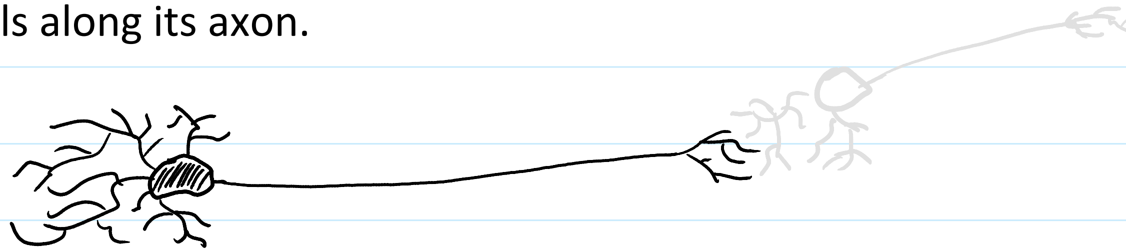
$\mathrm{ReLU}(z) =$



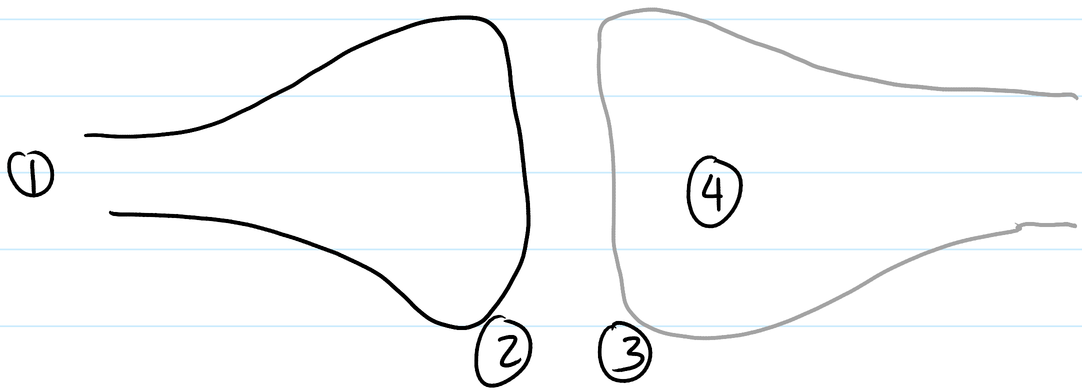
Synapses

Goal: To get an overview of how neurons pass information between them, and how we can model those communication channels.

So far, we've just looked at individual neurons, and how they react to their input. But that input usually comes from other neurons. When a neuron fires an action potential, the wave of electrical activity travels along its axon.



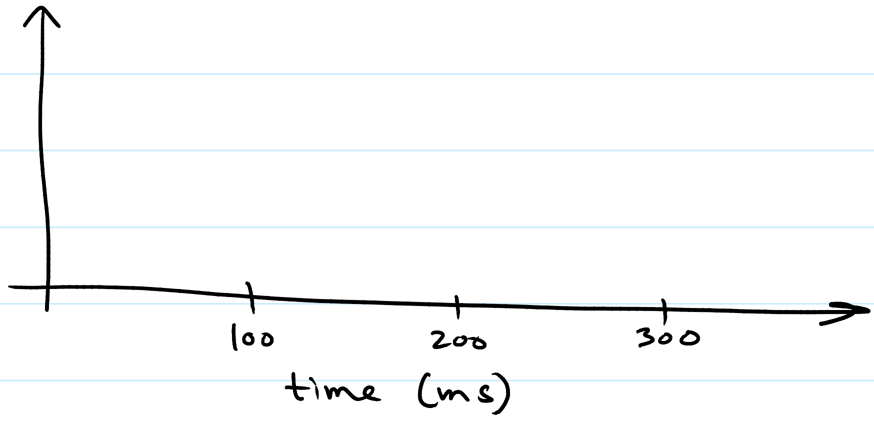
The junction where one neuron communicates with the next neuron is called a **synapse**.



Even though an action potential is very fast, the synaptic processes by which it affects the next neuron takes time. Some synapses are fast (taking just about 10 ms), and some are quite slow (taking over 300 ms). If we represent that time constant using τ_s, then the current entering the post-synaptic neuron can be written

$$h(t) =$$

where k is chosen so that



(see Fig. 5.15 in D&A)

The function $h(t)$ is called a Post-Synaptic Current (PSC), or (in keeping with the ambiguity between current and voltage) Post-Synaptic Potential (PSP).

Multiple spikes form what we call a "spike train", and can be modelled as a sum of Dirac delta functions,

<u>Dirac Delta Function</u>

$\delta(t) = \{$

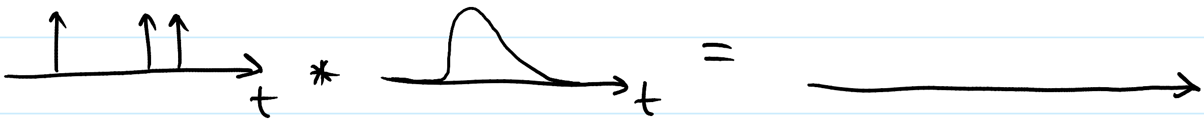
And

$$\int_{-\infty}^{\infty} \delta(t)\, dt$$

And

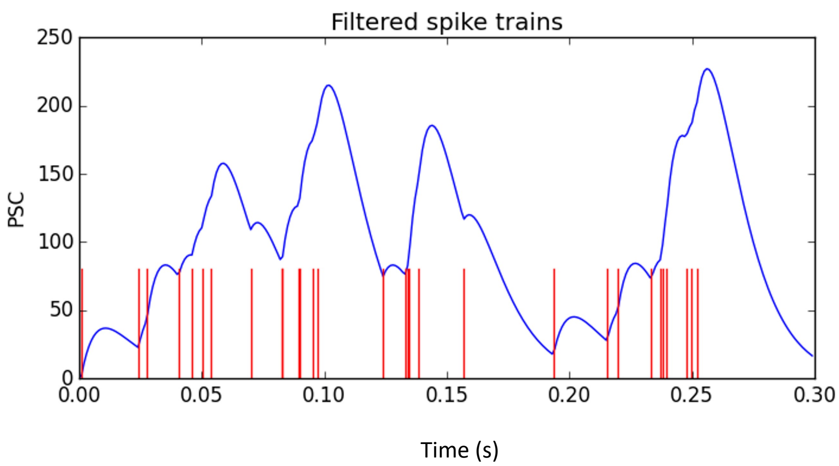
$$\int_{-\infty}^{\infty} f(t)\, \delta(T-t)\, dt$$

How does a spike train influence the post-synaptic neuron?
Answer: You simply add together all the PSCs, one for each spike.
This is actually convolving the spike train with the PSC.



That is,

(demo psc)



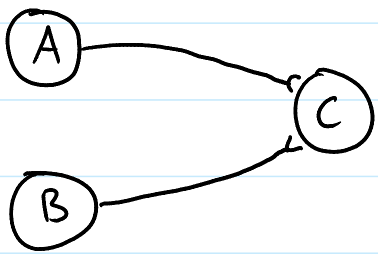
Connection Weight

The total current induced by an action potential onto a particular post-synaptic neuron can vary widely, depending on:

- the number and sizes of the synapses,
- the amount and type of neurotransmitter,
- the number and type of receptors,
- etc.

We can combine all those factors into a single number, the

connection weight. Thus, the total input to a neuron is a weighted sum of filtered spike-trains.

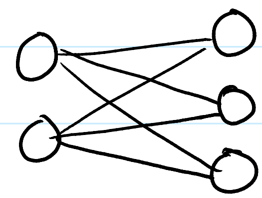


(demo simple_synapse)

Weight Matrices

When we have many pre-synaptic neurons, it is more convenient to use matrix-vector notation to represent the weights and activities.

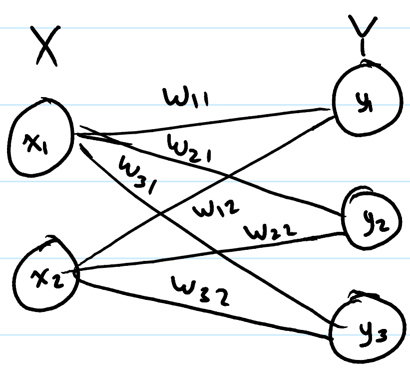
Suppose we have 2 populations, X and Y.

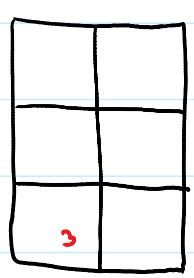


X has nodes

Y has nodes

If every node in X sends its output to every node in Y, then we will have a total of connections, each with its own weight.





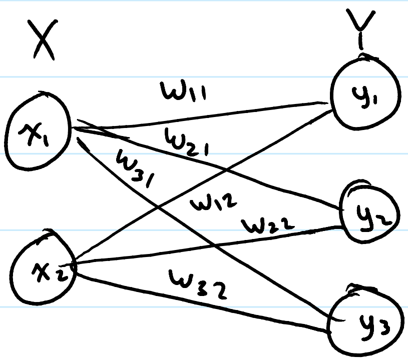
Storing the neuron activities in vectors,

$$\vec{x} = \begin{bmatrix} \\ 2 \end{bmatrix} \qquad \vec{y} = \begin{bmatrix} \\ \\ 3 \end{bmatrix}$$

we can compute the input to the nodes in Y using

Thus,

Another way to represent the biases, $\vec{b}$.



If $n=0$: $h(t) =$

This happens to be the solution of the DE

Proof:

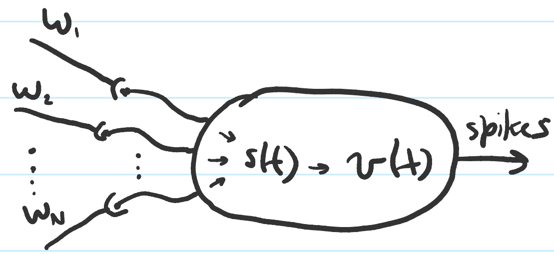
Solving the DE using Euler's Method

Consider the Initial Value Problem (IVP),

$\frac{ds}{} = -s \qquad s(0) = 1$

Consider the Initial Value Problem (IVP),

$$\frac{ds}{dt} = \frac{-s}{\tau} \qquad s(0) = \frac{1}{\tau}$$



Applying Euler's method:

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