Experimental Results: Logic and hypothesis testing

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48 marks

In this question, we will explore the data collected in class.

You will need to download the data `class_data.csv` from the assignment website and save it somewhere. Supposing the data to have been saved in a directory `dataDirectory`, read it into R as

```r
# Usual helper for paths
path_concat <- function(path1, path2, sep="/") {
                        paste(path1, path2, sep = sep)
}

data <- read.csv(file = path_concat('./dataDirectory', "class_data.csv"))
```

Having loaded the data, you might have a look at its contents using any of the standard functions:

```r
# just print it
data
# view its contents as a spreadsheet (in RStudio)
View(data)
# or, look at its data structure
str(data)
```

You will find that it is a `data.frame` with variables

```r
names(data)
```

```
## [1] "random_digit"  "student_digit" "green_card1"   "green_card2"
## [5] "red_card1"     "red_card2"
```

Each row contains one student's answer to the questions associated with these names.

Here we will only consider the results of the two hypothesis tests using the cards.

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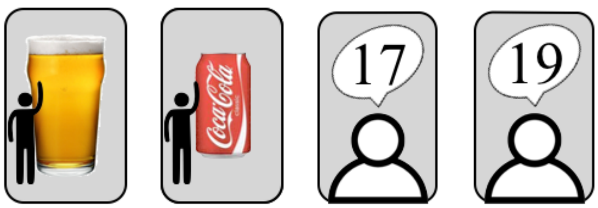
1. Recall the experimental scenario associated with the `green` card:

   - Patrons in a licensed restaurant may drink alcohol legally only if they are 19 years old or older.
   - Each patron has a card on showing the beverage they are drinking on one side and their age in years on the other.
   - We want to test the hypothesis

     H: No patron is illegally drinking alcohol.
   - Cards of four of the patrons were selected and placed on a table showing only one side of each card.
   - The four cards were:



(a) (b) (c) (d)

   - To test the hypothesis, each person had five seconds to choose two of these cards to turn over.
   - The variables `green_card1` and `green_card2` record the first and second choices made by that person.

   a. *(4 marks)* What could each card potentially reveal about the hypothesis were it turned over?

      - Beer: If this card is chosen, we can know the age of a patron who have alcoholic drink. If the age is smaller than 19, we know the hypothesis is wrong
      - Coke: If this card is chosen, no judgement can be revealed because all patrons can drink coke.
      - Age17: If this card is chosen, we can know the type of drink this patron drink. If the drink is beer, we know the hypothesis is wrong.
      - Age19: If this card is chosed, no judgement can be revealed because all patrons over 19 can drink both coke and beer.

   b. *(1 mark)* Use your answers above to justify why (a) and (c) are the correct cards to turn over.

      - If we choose card B, we know the age of a patron who drinks coke. Since everyone can drink coke, knowing their age is not helpful in judging the hypythesis.
      - If we choose card D, we know what kind of drink a patron over 19 years choose. Since all patrons over 19 can drink coke and beer legally, knowing what the patron is not helpful.

   c. *(1 mark)* Suppose a person simply selected two different cards at random (i.e. each available card has an equal probability of being selected).

      Explain why the probability that they would select both correct cards is $\frac{1}{6}$?

      - The probability of select cards A and C is: choose 2 from 4(choose A and C from {A,B,C,D})

$$\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{24}{4} = 6$$

      - There are 6 ways of selecting 2 cards. the probability of selecting both cards correct is 1/6.

   d. *(2 marks)* Which card was most often selected first? Was it correct?

      Which card was most often selected second? How many times? And was it correct?

      Show your code.

```r
# get the green_card from the data, address column by name
green_1<- data$green_card1
green_2 <-data$green_card2

most_selected_card <- function(cards){
  card_table <-table(cards)
  card_name <-card_table[which.max(card_table)]
  return(card_name)
}

print("the first selected card is:")
```

```
## [1] "the first selected card is:"
```

```r
first_card <-most_selected_card(green_1)
print(first_card)
```

```
##  a
## 26
```

```r
print("the second selected card is:")
```

```
## [1] "the second selected card is:"
```

```r
second_card <-most_selected_card(green_2)
print(second_card)
```

```
##  c
## 23
```

```
    - The first most often selected is (a), card beer, which is correct.
    - The second most often selected is (c), card age17, which is correct.

e. For each green card answer, construct a logical vector containing `TRUE` when the answer is correct a

    i. *(1 mark)* Show your code for creating `correct_g1` and `correct_g2`.
```

```r
green_1 <- data$green_card1
green_2 <- data$green_card2

correct_g1 <- green_1 == "a" | green_1 == "c"
correct_g2 <- green_2 == "a" | green_2 == "c"
print(correct_g1)
```

```
##  [1]  TRUE  TRUE FALSE  TRUE  TRUE FALSE FALSE  TRUE  TRUE  TRUE  TRUE  TRUE
## [13]  TRUE FALSE  TRUE  TRUE  TRUE FALSE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [25]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [37]  TRUE  TRUE  TRUE  TRUE FALSE  TRUE
```

```r
print(correct_g2)
```

```
##  [1] FALSE  TRUE  TRUE FALSE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE FALSE  TRUE
## [13] FALSE  TRUE  TRUE FALSE  TRUE  TRUE  TRUE FALSE  TRUE  TRUE  TRUE  TRUE
## [25] FALSE  TRUE FALSE FALSE  TRUE  TRUE  TRUE  TRUE FALSE FALSE FALSE  TRUE
## [37] FALSE  TRUE FALSE  TRUE FALSE  TRUE
```

```r
print(length(which(correct_g1==TRUE)))
```

```
## [1] 36
```

```r
print(length(which(correct_g2==TRUE)))
```

```
## [1] 27
```

```
    ii. *(2 marks)* Determine the proportion of people who had a correct first selection.  How does this

        Show your code.
```

```r
percent_g1 <- length(which(correct_g1==TRUE))/length(correct_g1)
percent_g2 <- length(which(correct_g2==TRUE))/length(correct_g2)
print(percent_g1)
```

```
## [1] 0.8571429
```

```r
print(percent_g2)
```

```
## [1] 0.6428571
```

```
        The proportion of people who had a correct first selection is 0.8571429.
        If the first selection was random, the proportion should be 0.25.
        The proportion of people actually choose the correct first selection is higher that that of firs

    iii. *(2 marks)* Determine the proportion of people who got *both* selections correct.  How does thi

            Show your code.
```

```r
correct_g_both = correct_g1 & correct_g2
percent_g_both <- length(which(correct_g_both==TRUE))/length(correct_g_both)
print(percent_g_both)
```

```
## [1] 0.5238095
```

```
        - Theis proportion is higher than taht of making selection randomly, which is 1/6
    iv. *(2 marks)* Now consider only those who got the first card right.  Determine what proportion of

        How does this compare to the probability of selecting the second correctly at random,  *given* t

        Show your code.
```

```r
correct_g_both = correct_g1 & correct_g2

percent_g1 <- length(which(correct_g1==TRUE))/length(correct_g1)
percent_g_both <- length(which(correct_g_both==TRUE))/length(correct_g_both)

percent_g_both_when_1 =  percent_g_both/percent_g1
print(percent_g_both_when_1)
```

```
## [1] 0.6111111
```

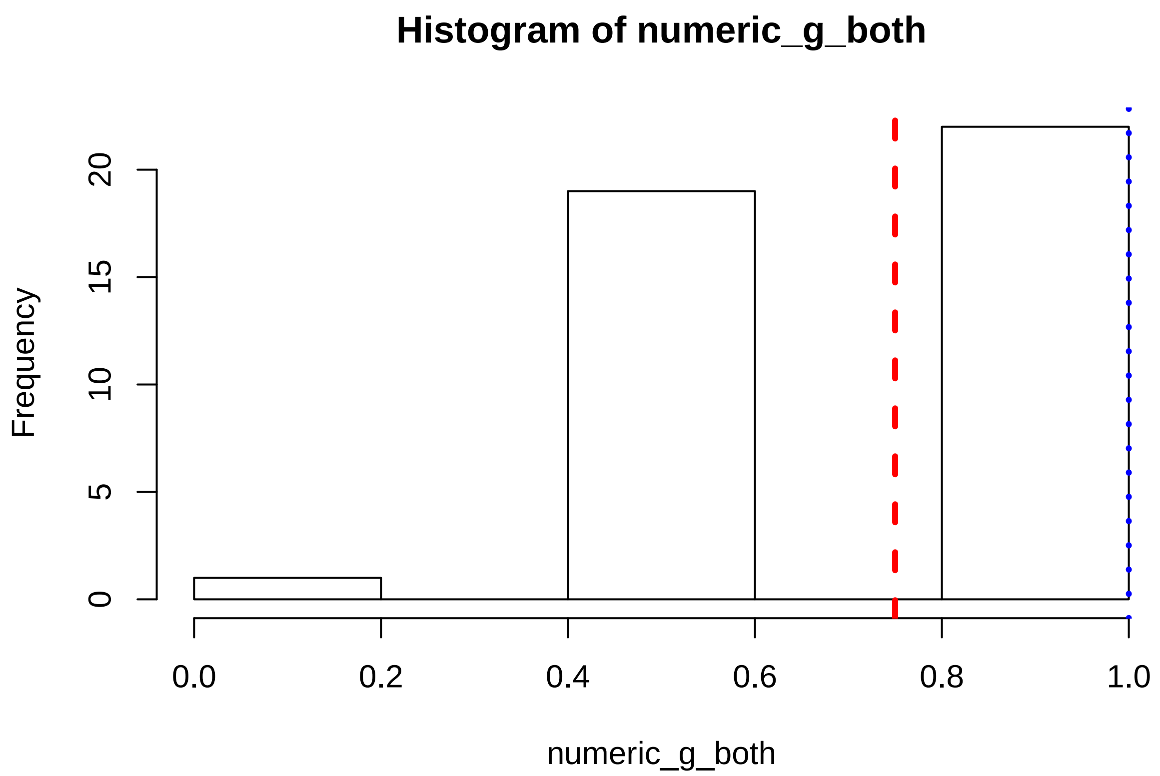
```
        - Theis proportion is higher than taht of making selection randomly, which is 1/3.
        - To make a correct selection when first is right, player has to choose the other correct card i



    v. *(2 marks)* Construct a new numeric vector `prop_correct_green`, from the logical vectors you hav

        Using `hist()`, draw a histogram of these proportions.
```

```
Add a red dashed vertical line using `abline()` to the plot at the average proportion correct. U

Add a blue dotted vertical line to the plot at the median proportion correct.  Use `lwd = 3, lty

Show your code and the plot.
```

```r
# if correct, the proportion is 0.5, add the proportions for green_card1 and green_card2
numeric_g1 <- replace(correct_g1, correct_g1==TRUE, 0.5)
numeric_g1 <- replace(numeric_g1, numeric_g1==FALSE, 0)
numeric_g2 <- replace(correct_g2, correct_g2==TRUE, 0.5)
numeric_g2 <- replace(numeric_g2, numeric_g2==FALSE, 0)

numeric_g_both <- numeric_g1+numeric_g2
print(numeric_g_both)
```

```
##  [1] 0.5 1.0 0.5 0.5 1.0 0.5 0.5 1.0 1.0 1.0 0.5 1.0 0.5 0.5 1.0 0.5 1.0 0.5 1.0
## [20] 0.5 1.0 1.0 1.0 1.0 0.5 1.0 0.5 0.5 1.0 1.0 1.0 1.0 0.5 0.5 0.5 1.0 0.5 1.0
## [39] 0.5 1.0 0.0 1.0
```

```r
hist(numeric_g_both)
abline(v = mean(numeric_g_both), col = "red", lwd = 3, lty = 2)
abline(v = median(numeric_g_both), col = "blue", lwd = 3, lty = 3)
```



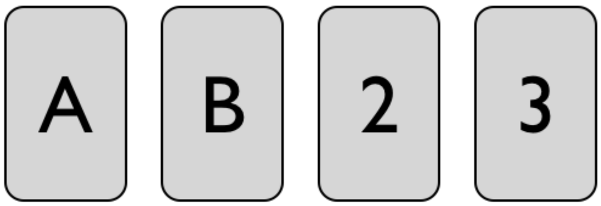

```
vi. *(2 marks)* Comment on how well the people did overall.  What is the basis for your conclusion
```

---

2. Recall the experimental scenario associated with the `red` card:

   - Have a special deck with 52 cards.
   - Each card an upper case letter on one side and a positive non-zero integer on the other.
   - We want to test the hypothesis

     H: Cards with a vowel on one side will have an even number on the other.
   - Have four cards selected and placed on a table showing only one side of each card.
   - The four cards were:



(a) (b) (c) (d)

   - To test the hypothesis, each person had five seconds to choose two of these cards to turn over.
   - The variables `red_card1` and `red_card2` record the first and second choices made by that person.

   a. *(4 marks)* What could each card potentially reveal about the hypothesis were it turned over?

      - Card A: Turing over the card 3 can potentially revel the hypothesis because if the number on the other side is odd, then the hypothesis is wrong.
      - Card B: Tuning over the card B cannot revel anything about hypothesis becasue consonant can have both odd and even number on the other side.
      - Card 2: Tuning over the card 2 cannot revel anything about hypothesis becasue an even number can have both consonant and voewl on the other side.
      - Card 3: Turing over the card 3 can potentially revel the hypothesis because if the letter on the other side is vowel, then the hypothesis is wrong.

   b. *(1 mark)* Use your answers above to justify why (a) and (d) are the correct cards to turn over.

      - If we choose B: Since B is not a vowel, a consonant can have both odd and even number on the other side. Card B is not the right choice as we cannot revel anything abou the hypothesis.
      - If we choose 2: Since 2 is even number, an even number can have both consonant and vowel on the other side. Card 2 is not the right choice as we cannot revel anything abou the hypothesis.
      - If we choose A: Since A is a vowel, a vowel can only have even number on the other side. If the number is odd, we know the hypothesis is wrong.
      - If we choose 3: Since 3 is an odd number, a odd number can only have consonant on the other side. If the letter is a vowel, then we know the hypothesis is wrong.
      - In conclusion, (a) and (d) are the correct cards to turn over.

   c. *(2 marks)* Which card was most often selected first? Was it correct?

      Which card was most often selected second? How many times? And was it correct?

      Show your code.

```
# get the green_card from the data, address column by name
red_1<- data$red_card1
red_2 <-data$red_card2

most_selected_card <- function(cards){
  card_table <-table(cards)
```

```r
      card_name <-card_table[which.max(card_table)]
      return(card_name)
    }

    print("the first selected card is:")
```

```
## [1] "the first selected card is:"
```

```r
    first_card <-most_selected_card(red_1)
    print(first_card)
```

```
##  a
## 37
```

```r
    print("the second selected card is:")
```

```
## [1] "the second selected card is:"
```

```r
    second_card <-most_selected_card(red_2)
    print(second_card)
```

```
##  c
## 25
```

    - The first most often selected is (a), the card A, which is correct. It was chosen 37 times.
    - The second most often selected is (c), the card 2, which is incorrect. It was chosen 25 times.

e. For each red card answer, construct a logical vector containing `TRUE` when the answer is correct and

    i. *(1 mark)* Show your code for creating `correct_r1` and `correct_r2`.

```r
correct_r1 <- red_1 == "a" | red_1 == "d"
correct_r2 <- red_2 == "a" | red_2 == "d"
print(correct_r1)
```

```
##  [1]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE FALSE
## [13]  TRUE  TRUE FALSE FALSE  TRUE  TRUE  TRUE FALSE  TRUE  TRUE  TRUE  TRUE
## [25]  TRUE  TRUE FALSE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
## [37]  TRUE  TRUE  TRUE  TRUE  TRUE  TRUE
```

```r
print(correct_r2)
```

```
##  [1] FALSE FALSE FALSE FALSE  TRUE FALSE FALSE FALSE  TRUE FALSE FALSE  TRUE
## [13] FALSE FALSE  TRUE FALSE FALSE FALSE FALSE  TRUE  TRUE  TRUE FALSE FALSE
## [25] FALSE  TRUE  TRUE  TRUE  TRUE FALSE  TRUE FALSE FALSE FALSE  TRUE FALSE
## [37] FALSE FALSE  TRUE  TRUE FALSE FALSE
```

    ii. *(2 marks)* Determine the proportion of people who had a correct first selection. How does thi

        Show your code.

```r
percent_r1 <- length(which(correct_r1==TRUE))/length(correct_r1)
percent_r2 <- length(which(correct_r2==TRUE))/length(correct_r2)
print(percent_r1)
```

```
## [1] 0.8809524
```

```r
print(percent_r2)
```

```
## [1] 0.3571429
```

```
    * The proportion of people who had a correct first selection is 0.8809524. It is higher than 0.
    * The proportion of people who had a correct second selection is 0.3571429


iii. *(2 marks)* Determine the proportion of people who got *both* selections correct.  How does th

        Show your code.
```

```r
correct_r_both = correct_r1 & correct_r2
percent_r_both <- length(which(correct_r_both==TRUE))/length(correct_r_both)
print(percent_r_both)
```

```
## [1] 0.2619048
```

```
- The proportion of people who got both selections correct is 0.2619048.
- It is higher than 1/6 which is the proportion of people who got both selections correct randomly.


iv. *(2 marks)* Now consider only those who got the first card right.  Determine what proportion of

    How does this compare to the probability of selecting the second correctly at random,  *given*

    Show your code.
```

```r
correct_r_both = correct_r1 & correct_r2

percent_r1 <- length(which(correct_r1==TRUE))/length(correct_r1)
percent_r_both <- length(which(correct_r_both==TRUE))/length(correct_r_both)

percent_r_both_when_1 =  percent_r_both/percent_r1
print(percent_r_both_when_1)
```

```
## [1] 0.2972973
```

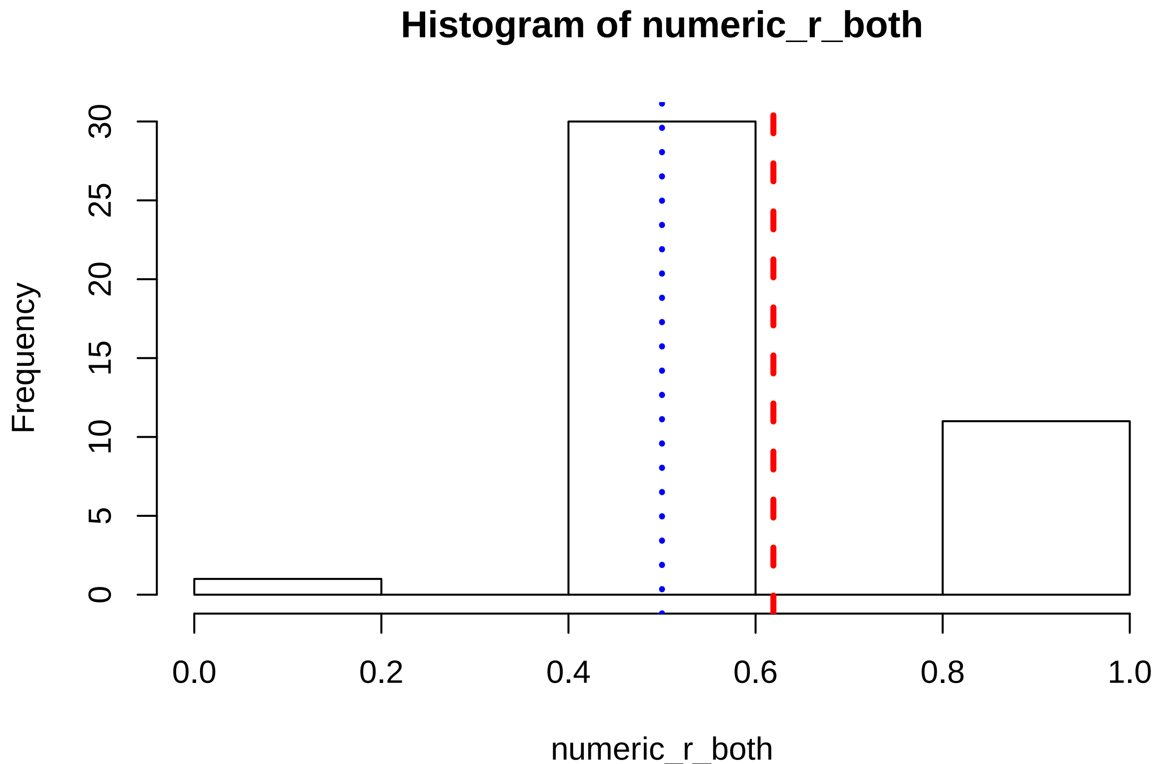
```
    - The proportion of these *also* got the second card right when they got first card correct is
    - It is less than 1/3, which is probability of selecting the second correctly at random.


v. *(2 marks)* Construct a new numeric vector `prop_correct_red`, from the logical vectors you have

    Using `hist()`, draw a histogram of these proportions.

    Add a red dashed vertical line using `abline()` to the plot at the average proportion correct.

    Add a blue dotted vertical line to the plot at the median proportion correct.  Use `lwd = 3, lt

    Show your code and the plot.
```

```r
# if correct, the proportion is 0.5, add the proportions for green_card1 and green_card2
numeric_r1 <- replace(correct_r1, correct_r1==TRUE, 0.5)
numeric_r1 <- replace(numeric_r1, numeric_r1==FALSE, 0)
numeric_r2 <- replace(correct_r2, correct_r2==TRUE, 0.5)
numeric_r2 <- replace(numeric_r2, numeric_r2==FALSE, 0)

numeric_r_both <- numeric_r1+numeric_r2
```

```r
print(numeric_r_both)
```

```
##  [1] 0.5 0.5 0.5 0.5 1.0 0.5 0.5 0.5 1.0 0.5 0.5 0.5 0.5 0.5 0.5 0.0 0.5 0.5 0.5
## [20] 0.5 1.0 1.0 0.5 0.5 0.5 1.0 0.5 1.0 1.0 0.5 1.0 0.5 0.5 0.5 1.0 0.5 0.5 0.5
## [39] 1.0 1.0 0.5 0.5
```

```r
hist(numeric_r_both)
abline(v = mean(numeric_r_both), col = "red", lwd = 3, lty = 2)
abline(v = median(numeric_r_both), col = "blue", lwd = 3, lty = 3)
```




```
vi. *(2 marks)* Comment on how well the people did overall.  What is the basis for your conclusion
```

3. This question compares the results of the two separate hypothesis tests studied in the previous two questions.

   a. *(2 marks)* Explain why the two tests and hypotheses were *logically identical*.

      - In first test, if we take the patron can drink alcohol legally as event P, the patron can is over 19 as event Q
      - We have the logic that P is sufficient condition of Q. And Q is necessary condition of P
      - In second test, if we take the letter is vowel as event P, the number is even as event Q.
      - we have the logic that P is sufficient condition of Q. And Q is necessary condition of P

   b. *(2 marks)* In what sense are the tests of hypothesis conducted in the first two questions like a statistical test of significance?

   c. Suppose that in these experiments, the two cards were chosen at random (with equal probability of selection). Let P be the proportion that any individual might get correct when selecting at random.

      i. *(3 marks)* Determine the value of each of the following probabilities (show your work):

         - $Pr(P = 0)$ Selecting the only two correct cards from four cards....

$$\binom{4}{2} = 6$$

$$Pr(P = 0) = \frac{1}{6}$$

         - $Pr(P = 0.5)$ Selecting a correct card then a wrong card OR selecting a wrong card then a correct card

$$Pr(P = 0.5) = \frac{2}{4} \cdot \frac{2}{3} + \frac{2}{4} \cdot \frac{2}{3} = \frac{2}{3}$$

         - $Pr(P = 1)$ Selecting the only two wrong cards from four cards....

$$\binom{4}{2} = 6$$

$$Pr(P = 1) = \frac{1}{6}$$

      ii. *(1 mark)* Hence determine $E(P)$.

$$E(P) = \sum_{x=0,0.5,1} P(x) \cdot Pr(P = x) = 0 \cdot \frac{1}{6} + 0.5 \cdot \frac{2}{3} + 1 \cdot \frac{1}{6} = \frac{1}{2}$$

      iii. *(1 mark)* How does the observed average proportion correct compare with $E(P)$ for each of the hypotheses tested? That is, which is closer to random and which is farther away?

```
* The observed average for red card is closer to 0.5, which means that is more closer to rando

* The observed average for greem card is farther away from the random(away from 0.5).
```

   d. Based on all analyses in all of the above questions:

      i. *(2 marks)* Which of the two hypotheses were found to be more easily tested by the persons involved? Give your reasoning.

         - the hypothesis about green card is more easily tested by person because the proportin of selecting both card correct is higher than that for the hypothesis about red card.

ii. *(2 marks)* What differences between the two set ups might account for the difference in performance?

- The first set up are easier and the second are more difficult for people to make decisions. This is because almost everyone have been checked ID in bars to get a alcohol drink. Therefore we have idea about what should be done to check the first hypothesis.