$Exploring \ sample \ attributes$

 $Transformations\ and\ model\ summaries$

R.W. Oldford



By 2014, Facebook was the most used social network averaging about 1.28 billion monthly active users.



By 2014, Facebook was the most used social network averaging about 1.28 billion monthly active users.

A cosmetics company had been using a Facebook page they created to reach their customers as well as potential customers.



By 2014, Facebook was the most used social network averaging about 1.28 billion monthly active users.

A cosmetics company had been using a Facebook page they created to reach their customers as well as potential customers.

Likely, the page had been active for a few years, when they decided to investigate the effectiveness of the various postings they had made on their page.



By 2014, Facebook was the most used social network averaging about 1.28 billion monthly active users.

A cosmetics company had been using a Facebook page they created to reach their customers as well as potential customers.

Likely, the page had been active for a few years, when they decided to investigate the effectiveness of the various postings they had made on their page.

To get some idea, they decided to collect all of the posts they published in 2014 (January 1 to December 31) together with different features recorded on each post.



By 2014, Facebook was the most used social network averaging about 1.28 billion monthly active users.

A cosmetics company had been using a Facebook page they created to reach their customers as well as potential customers.

Likely, the page had been active for a few years, when they decided to investigate the effectiveness of the various postings they had made on their page.

To get some idea, they decided to collect all of the posts they published in 2014 (January 1 to December 31) together with different features recorded on each post.

Questions:

▶ What is a unit here?



By 2014, Facebook was the most used social network averaging about 1.28 billion monthly active users.

A cosmetics company had been using a Facebook page they created to reach their customers as well as potential customers.

Likely, the page had been active for a few years, when they decided to investigate the effectiveness of the various postings they had made on their page.

To get some idea, they decided to collect all of the posts they published in 2014 (January 1 to December 31) together with different features recorded on each post.

Questions:

- ▶ What is a unit here?
- What is the target population?



By 2014, Facebook was the most used social network averaging about 1.28 billion monthly active users.

A cosmetics company had been using a Facebook page they created to reach their customers as well as potential customers.

Likely, the page had been active for a few years, when they decided to investigate the effectiveness of the various postings they had made on their page.

To get some idea, they decided to collect all of the posts they published in 2014 (January 1 to December 31) together with different features recorded on each post.

Questions:

- ▶ What is a unit here?
- What is the target population?
- The study population?



By 2014, Facebook was the most used social network averaging about 1.28 billion monthly active users.

A cosmetics company had been using a Facebook page they created to reach their customers as well as potential customers.

Likely, the page had been active for a few years, when they decided to investigate the effectiveness of the various postings they had made on their page.

To get some idea, they decided to collect all of the posts they published in 2014 (January 1 to December 31) together with different features recorded on each post.

Questions:

- ▶ What is a unit here?
- What is the target population?
- ▶ The study population?
- ▶ The sample?

Source:

S. Moro, P. Rita and B. Vala (2016). "Predicting social media performance metrics and evaluation of the impactation on brand building: A data mining approach". *Journal of Business Research*, 69, pp. 3341-3351.

A post on the company's Facebook page is a unit

The **target population** is the set of all such posts (*including all posts in the near/foreseeable future*)

The **study population** is the set of all of the posts that the company had made *to date*.

The sample is the set of posts in the year.



A post on the company's Facebook page is a unit

The **target population** is the set of all such posts (*including all posts in the near/foreseeable future*)

The **study population** is the set of all of the posts that the company had made *to date*.

The **sample** is the set of posts in the year. There were a total of 790 such posts.



A post on the company's Facebook page is a unit

The **target population** is the set of all such posts (*including all posts in the near/foreseeable future*)

The **study population** is the set of all of the posts that the company had made *to date*.

The **sample** is the set of posts in the year. There were a total of 790 such posts. Unfortunately, of these only 500 are available to us.



A post on the company's Facebook page is a unit

The **target population** is the set of all such posts (*including all posts in the near/foreseeable future*)

The **study population** is the set of all of the posts that the company had made *to date*.

The **sample** is the set of posts in the year. There were a total of 790 such posts. Unfortunately, of these only 500 are available to us. So **our sample** is only of size 500, taken from the 790 in 2014. We don't know how these were chosen.



A post on the company's Facebook page is a unit

The **target population** is the set of all such posts (*including all posts in the near/foreseeable future*)

The **study population** is the set of all of the posts that the company had made *to date*.

The **sample** is the set of posts in the year. There were a total of 790 such posts. Unfortunately, of these only 500 are available to us. So **our sample** is only of size 500, taken from the 790 in 2014. We don't know how these were chosen.

The company recorded 19 variates on each post.



A post on the company's Facebook page is a unit

The **target population** is the set of all such posts (*including all posts in the near/foreseeable future*)

The **study population** is the set of all of the posts that the company had made *to date*.

The **sample** is the set of posts in the year. There were a total of 790 such posts. Unfortunately, of these only 500 are available to us. So **our sample** is only of size 500, taken from the 790 in 2014. We don't know how these were chosen.

The company recorded 19 variates on each post. Only 13 of these are recorded in our dataset.



A post on the company's Facebook page is a unit

The **target population** is the set of all such posts (*including all posts in the near/foreseeable future*)

The **study population** is the set of all of the posts that the company had made *to date*.

The **sample** is the set of posts in the year. There were a total of 790 such posts. Unfortunately, of these only 500 are available to us. So **our sample** is only of size 500, taken from the 790 in 2014. We don't know how these were chosen.

The company recorded 19 variates on each post. Only 13 of these are recorded in our dataset.

Don't really have any population attributes (yet).





For each post, values of the following 13 variates were recorded:

▶ share: the total (lifetime) number of times the post was shared



- ▶ share: the total (lifetime) number of times the post was shared
- ▶ like: the total (lifetime) number of times the post "liked"



For each post, values of the following 13 variates were recorded:

▶ share: the total (lifetime) number of times the post was shared

▶ like: the total (lifetime) number of times the post "liked"

comment: the total (lifetime) number of comments attached to the post



- ▶ share: the total (lifetime) number of times the post was shared
- like: the total (lifetime) number of times the post "liked"
- comment: the total (lifetime) number of comments attached to the post
- ▶ All.interactions: the sum of share, like, and comment



- ▶ share: the total (lifetime) number of times the post was shared
- like: the total (lifetime) number of times the post "liked"
- comment: the total (lifetime) number of comments attached to the post
- ▶ All.interactions: the sum of share, like, and comment
- ▶ Page.likes: the number of "likes" for the facebook page at the original time of the posting



- ▶ share: the total (lifetime) number of times the post was shared
- like: the total (lifetime) number of times the post "liked"
- comment: the total (lifetime) number of comments attached to the post
- ▶ All.interactions: the sum of share, like, and comment
- ▶ Page.likes: the number of "likes" for the facebook page at the original time of the posting
- ▶ Impressions: the total (lifetime) number of times the post has been displayed, whether the post is clicked or not. The same post may be seen by a facebook user several times (e.g. via a page update in their News Feed once, whenever a friend shares it, etc.).



- share: the total (lifetime) number of times the post was shared
- like: the total (lifetime) number of times the post "liked"
- comment: the total (lifetime) number of comments attached to the post
- ▶ All.interactions: the sum of share, like, and comment
- ▶ Page.likes: the number of "likes" for the facebook page at the original time of the posting
- ▶ Impressions: the total (lifetime) number of times the post has been displayed, whether the post is clicked or not. The same post may be seen by a facebook user several times (e.g. via a page update in their News Feed once, whenever a friend shares it, etc.).
- ▶ Impressions.when.page.liked: the total (lifetime) number of times the post has been displayed to someone who has "liked" the page



- ▶ share: the total (lifetime) number of times the post was shared
- ▶ like: the total (lifetime) number of times the post "liked"
- comment: the total (lifetime) number of comments attached to the post
- ▶ All.interactions: the sum of share, like, and comment
- ▶ Page.likes: the number of "likes" for the facebook page at the original time of the posting
- ▶ Impressions: the total (lifetime) number of times the post has been displayed, whether the post is clicked or not. The same post may be seen by a facebook user several times (e.g. via a page update in their News Feed once, whenever a friend shares it, etc.).
- Impressions.when.page.liked: the total (lifetime) number of times the post has been displayed to someone who has "liked" the page
- ▶ Post.Hour: the hour of the day at the original time of the posting (0-23)



- share: the total (lifetime) number of times the post was shared
- ▶ like: the total (lifetime) number of times the post "liked"
- comment: the total (lifetime) number of comments attached to the post
- ▶ All.interactions: the sum of share, like, and comment
- Page.likes: the number of "likes" for the facebook page at the original time of the posting
- ▶ Impressions: the total (lifetime) number of times the post has been displayed, whether the post is clicked or not. The same post may be seen by a facebook user several times (e.g. via a page update in their News Feed once, whenever a friend shares it, etc.).
- ▶ Impressions.when.page.liked: the total (lifetime) number of times the post has been displayed to someone who has "liked" the page
- ▶ Post.Hour: the hour of the day at the original time of the posting (0-23)
- Post.Weekday: the day of the week at the original time of the posting (1-7) beginning with Sunday



- ▶ share: the total (lifetime) number of times the post was shared
- ▶ like: the total (lifetime) number of times the post "liked"
- comment: the total (lifetime) number of comments attached to the post
- ▶ All.interactions: the sum of share, like, and comment
- Page.likes: the number of "likes" for the facebook page at the original time of the posting
- ▶ Impressions: the total (lifetime) number of times the post has been displayed, whether the post is clicked or not. The same post may be seen by a facebook user several times (e.g. via a page update in their News Feed once, whenever a friend shares it, etc.).
- ▶ Impressions.when.page.liked: the total (lifetime) number of times the post has been displayed to someone who has "liked" the page
- Post.Hour: the hour of the day at the original time of the posting (0-23)
- Post.Weekday: the day of the week at the original time of the posting (1-7) beginning with Sunday
- ▶ Post.Month: the month of the year at the original time of the posting (1-12)



- ▶ share: the total (lifetime) number of times the post was shared
- ▶ like: the total (lifetime) number of times the post "liked"
- comment: the total (lifetime) number of comments attached to the post
- All.interactions: the sum of share, like, and comment
- Page.likes: the number of "likes" for the facebook page at the original time of the posting
- ▶ Impressions: the total (lifetime) number of times the post has been displayed, whether the post is clicked or not. The same post may be seen by a facebook user several times (e.g. via a page update in their News Feed once, whenever a friend shares it, etc.).
- ▶ Impressions.when.page.liked: the total (lifetime) number of times the post has been displayed to someone who has "liked" the page
- Post.Hour: the hour of the day at the original time of the posting (0-23)
- Post.Weekday: the day of the week at the original time of the posting (1-7) beginning with Sunday
- ▶ Post.Month: the month of the year at the original time of the posting (1-12)
- Category: the category of the post (as determined by two separate human reviewers according to the campaign associated with the post), one of Action (special offers and contests), Product (direct advertisement, explicit brand content), or Inspiration (non-explicit brand related content)



- share: the total (lifetime) number of times the post was shared
- ▶ like: the total (lifetime) number of times the post "liked"
- comment: the total (lifetime) number of comments attached to the post
- ▶ All.interactions: the sum of share, like, and comment
- Page.likes: the number of "likes" for the facebook page at the original time of the posting
- ▶ Impressions: the total (lifetime) number of times the post has been displayed, whether the post is clicked or not. The same post may be seen by a facebook user several times (e.g. via a page update in their News Feed once, whenever a friend shares it, etc.).
- ▶ Impressions.when.page.liked: the total (lifetime) number of times the post has been displayed to someone who has "liked" the page
- Post.Hour: the hour of the day at the original time of the posting (0-23)
- Post.Weekday: the day of the week at the original time of the posting (1-7) beginning with Sunday
- ▶ Post.Month: the month of the year at the original time of the posting (1-12)
- Category: the category of the post (as determined by two separate human reviewers according to the campaign associated with the post), one of Action (special offers and contests), Product (direct advertisement, explicit brand content), or Inspiration (non-explicit brand related content)
- > Type: the type of content of the post, one of Link, Photo, Status, or Video



- ▶ share: the total (lifetime) number of times the post was shared
- ▶ like: the total (lifetime) number of times the post "liked"
- comment: the total (lifetime) number of comments attached to the post
- ▶ All.interactions: the sum of share, like, and comment
- Page.likes: the number of "likes" for the facebook page at the original time of the posting
- Impressions: the total (lifetime) number of times the post has been displayed, whether the post is clicked or not. The same post may be seen by a facebook user several times (e.g. via a page update in their News Feed once, whenever a friend shares it, etc.).
- ▶ Impressions.when.page.liked: the total (lifetime) number of times the post has been displayed to someone who has "liked" the page
- ▶ Post.Hour: the hour of the day at the original time of the posting (0-23)
- Post.Weekday: the day of the week at the original time of the posting (1-7) beginning with Sunday
- ▶ Post.Month: the month of the year at the original time of the posting (1-12)
- Category: the category of the post (as determined by two separate human reviewers according to the campaign associated with the post), one of Action (special offers and contests), Product (direct advertisement, explicit brand content), or Inspiration (non-explicit brand related content)
- ▶ Type: the type of content of the post, one of Link, Photo, Status, or Video
- Paid: 1 if the company paid Facebook for advertising, 0 otherwise



It can be useful to separate variates into two different groups according to whether we are (perhaps only temporarily) thinking of them as **response** variates or as **explanatory** variates.



It can be useful to separate variates into two different groups according to whether we are (perhaps only temporarily) thinking of them as **response** variates or as **explanatory** variates.

Here the company is curious about the effect of each posting so any variate that relates to the characteristics of the posting itself could be explanatory, whereas those that might measure the reaction of a viewer to the posting would be possible response variates.



It can be useful to separate variates into two different groups according to whether we are (perhaps only temporarily) thinking of them as **response** variates or as **explanatory** variates.

Here the company is curious about the effect of each posting so any variate that relates to the characteristics of the posting itself could be explanatory, whereas those that might measure the reaction of a viewer to the posting would be possible response variates.

Note that oftentimes, but not always, explanatory variates are in our control (i.e. we can either *select units* that have specific values or *assign values* to units) and response variates are not.



It can be useful to separate variates into two different groups according to whether we are (perhaps only temporarily) thinking of them as **response** variates or as **explanatory** variates.

Here the company is curious about the effect of each posting so any variate that relates to the characteristics of the posting itself could be explanatory, whereas those that might measure the reaction of a viewer to the posting would be possible response variates.

Note that oftentimes, but not always, explanatory variates are in our control (i.e. we can either *select units* that have specific values or *assign values* to units) and response variates are not.

For example, explanatory variates could include those related to the time of the post (Post.hour, Post.day,Post.month), the Type of the post (one of Link, Photo, Status, or Video), the Category of post (one of Action, Product, or Inspiration), and whether the post was Paid advertising. Any of these could have their value assigned to the post. An explanatory variate whose value could have been somewhat selected would be the number of "likes" the Facebook page had accrued at the time of the post.



It can be useful to separate variates into two different groups according to whether we are (perhaps only temporarily) thinking of them as **response** variates or as **explanatory** variates.

Here the company is curious about the effect of each posting so any variate that relates to the characteristics of the posting itself could be explanatory, whereas those that might measure the reaction of a viewer to the posting would be possible response variates.

Note that oftentimes, but not always, explanatory variates are in our control (i.e. we can either *select units* that have specific values or *assign values* to units) and response variates are not.

For example, explanatory variates could include those related to the time of the post (Post.hour, Post.day,Post.month), the Type of the post (one of Link, Photo, Status, or Video), the Category of post (one of Action, Product, or Inspiration), and whether the post was Paid advertising. Any of these could have their value assigned to the post. An explanatory variate whose value could have been somewhat selected would be the number of "likes" the Facebook page had accrued at the time of the post.

Response variates would then include the number of times the post was shared, liked, commented on, the number of times the post was displayed (Impressions), and the number of times to someone who indicated they had liked the page (Impressions.when.page.liked).



It can be useful to separate variates into two different groups according to whether we are (perhaps only temporarily) thinking of them as **response** variates or as **explanatory** variates.

Here the company is curious about the effect of each posting so any variate that relates to the characteristics of the posting itself could be explanatory, whereas those that might measure the reaction of a viewer to the posting would be possible response variates.

Note that oftentimes, but not always, explanatory variates are in our control (i.e. we can either *select units* that have specific values or *assign values* to units) and response variates are not.

For example, explanatory variates could include those related to the time of the post (Post.hour, Post.day,Post.month), the Type of the post (one of Link, Photo, Status, or Video), the Category of post (one of Action, Product, or Inspiration), and whether the post was Paid advertising. Any of these could have their value assigned to the post. An explanatory variate whose value could have been somewhat selected would be the number of "likes" the Facebook page had accrued at the time of the post.

Response variates would then include the number of times the post was shared, liked, commented on, the number of times the post was displayed (Impressions), and the number of times to someone who indicated they had liked the page (Impressions.when.page.liked).

Sometimes, as with Page.likes, Impressions, Impressions.when.page.liked, variates mightwaternoo considered as response variates for one purpose and as explantory ones for another.

The data

The first and last few rows of the data:

```
file <- path_concat(dataDirectory, "facebook.csv")
facebook <- read.csv(file)
head(facebook, n = 4)</pre>
```

```
##
     All.interactions share like comment Impressions.when.page.liked Impressions
## 1
                   100
                          17
                               79
                                                                  3078
                                                                               5091
## 2
                  164
                          29
                              130
                                                                 11710
                                                                              19057
## 3
                   80
                         14
                               66
                                        0
                                                                  2812
                                                                               4373
## 4
                 1777
                         147 1572
                                       58
                                                                 61027
                                                                              87991
     Paid Post. Hour Post. Weekday Post. Month
                                                            Type Page.likes
##
                                                Category
## 1
                                          12
                                                  Product
                                                           Photo
                                                                      139441
## 2
        0
                 10
                                          12
                                                  Product Status
                                                                     139441
## 3
                  3
                                          12 Inspiration Photo 139441
## 4
                                          12
                 10
                                                  Product Photo
                                                                     139441
tail(facebook, n = 4)
```

```
##
       All.interactions share like comment Impressions.when.page.liked Impressions
## 497
                      75
                            22
                                  53
                                                                      3961
                                                                                   6229
                                           0
## 498
                     115
                            18
                                  93
                                                                      4742
                                                                                   7216
## 499
                     136
                            38
                                  91
                                                                      4534
                                                                                   7564
## 500
                     119
                            28
                                  91
                                                                      3861
                                                                                   7292
       Paid Post. Hour Post. Weekday Post. Month
                                                    Category Type Page.likes
##
## 497
                     8
                                                     Product Photo
                                                                         81370
## 498
                                   5
                                                      Action Photo
                                                                         81370
## 499
                    11
                                                 Inspiration Photo
                                                                         81370
                                                                                     WATERLOO
## 500
                                                     Product Photo
                                                                         81370
         NΑ
```

The data - summary

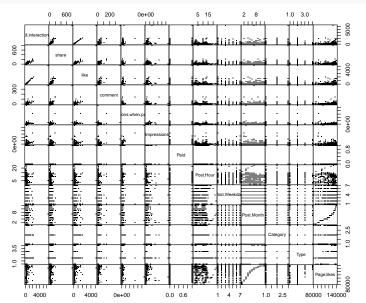
summary(facebook)

```
All.interactions
                        share
                                          like
                                                         comment
   Min.
         .
              0.0
                    Min. : 0.00
                                     Min. :
                                               0.0
                                                      Min.
                                                            : 0.000
   1st Qu.: 71.0
                    1st Qu.: 10.00
                                    1st Qu.: 56.5
                                                      1st Qu.:
                                                              1.000
   Median : 123.5
                    Median : 19.00
                                    Median : 101.0
                                                      Median :
                                                               3.000
   Mean
          : 212.1
                    Mean
                           : 27.27
                                     Mean
                                            : 177.9
                                                           : 7.482
                                                      Mean
   3rd Qu.: 228.5
                    3rd Qu.: 32.25
                                     3rd Qu.: 187.5
                                                      3rd Qu.: 7.000
##
   Max.
          :6334.0
                    Max.
                           :790.00
                                     Max.
                                            :5172.0
                                                      Max.
                                                             :372.000
                    NA's
                           :4
                                     NA's
##
                                          : 1
   Impressions.when.page.liked Impressions
                                                      Paid
                                                                   Post.Hour
   Min. :
                                          570
                                                       :0.0000
                                                                 Min. : 1.00
##
               567
                               Min.
                                    :
                                                Min.
   1st Qu.:
              3970
                                          5695
                                                                 1st Qu.: 3.00
##
                               1st Qu.:
                                                1st Qu.:0.0000
                                                Median :0.0000
   Median :
              6256
                               Median :
                                          9051
                                                                 Median: 9.00
         : 16766
                                         29586
                                                       :0.2786
                                                                      : 7.84
   Mean
                               Mean
                                                Mean
                                                                 Mean
   3rd Qu.: 14860
                               3rd Qu.:
                                         22086
                                                3rd Qu.:1.0000
                                                                 3rd Qu.:11.00
##
   Max.
          :1107833
                               Max.
                                      :1110282
                                                 Max.
                                                       :1.0000
                                                                 Max.
                                                                        :23.00
##
                                                 NA's
                                                        :1
                    Post.Month
##
    Post.Weekday
                                          Category
                                                        Type
                                                  Link : 22
   Min.
         :1.00
                  Min.
                       : 1.000
                                   Action
                                              :215
   1st Qu.:2.00
                  1st Qu.: 4.000
                                   Inspiration: 155 Photo: 426
   Median :4.00
                  Median : 7.000
                                   Product
                                              :130 Status: 45
   Mean
         :4.15
                  Mean : 7.038
                                                     Video: 7
   3rd Qu.:6.00
                  3rd Qu.:10.000
   Max. :7.00
                  Max. :12.000
##
##
##
     Page.likes
   Min. : 81370
   1st Qu.:112676
   Median :129600
   Mean
          :123194
   3rd Qu.: 136393
##
   Max.
          :139441
##
```



The data - plot summary

plot(facebook, gap = 0, pch =".")





The data - interactive plot summaries

```
library(loon)
findCatVars <- function(data) {
    isCatVar <- sapply(names(data),
                       FUN = function(name) {is.factor(data[.name])})
    catVars <- names(data)[isCatVar]
    catVars
l_barPlots <- function(data, linkingGroup, together = TRUE){</pre>
    if(missing(linkingGroup)) {
        linkingGroup <- deparse(substitute(data)) # use the data frame name
        7
    catVars <- findCatVars(data)
    if (together) {
        parent <- tktoplevel()
        tktitle(parent) <- "Counts for factors"
        nrows <- floor(sgrt(length(catVars)))
        ncols <- ceiling(sqrt(length(catVars)))</pre>
        row <- 0
        col <- 0
        for (var in catVars) {
            barplot <- 1 hist(data[,var],
                               linkingGroup = linkingGroup,
                               title = var.
                               xlabel = var.
                               parent = parent)
            if (col >= ncols){
                row <- row + 1
                col <- 0}
            tkgrid(barplot, row = row, column = col, sticky = "nesw")
            col <- col + 1}
        # Continued next slide
        # ...
```

RLOO

The data - interactive plot summaries

Can now call 1_barPlots() on our data set:

```
bp <- 1_barPlots(facebook)</pre>
```

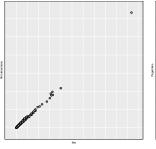
And a pairs plot

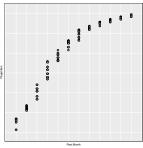


From the plots there are some obvious fairly strong relationships.



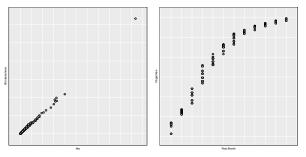
From the plots there are some obvious fairly strong relationships. Notably,







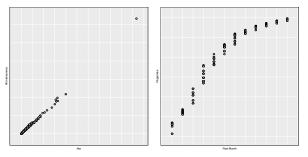
From the plots there are some obvious fairly strong relationships. Notably,



But then, ${\tt All.interactions} = {\tt sum}$ of share, like, and comment which appears to be dominated by like.



From the plots there are some obvious fairly strong relationships. Notably,

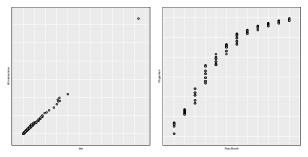


But then, ${\tt All.interactions} = {\tt sum}$ of share, like, and comment which appears to be dominated by like.

Also, Page.likes are total Facebook page likes at the time of posting. The number of Page.likes is necessarily non-decreasing in time.



From the plots there are some obvious fairly strong relationships. Notably,



But then, ${\tt All.interactions} = {\tt sum}$ of share, like, and comment which appears to be dominated by like.

Also, Page.likes are total Facebook page likes at the time of posting. The number of Page.likes is necessarily non-decreasing in time.

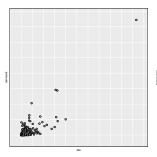
In both cases, it would seem that there is a very strong relation between the variates in each pair. One or the other of each pair might be discarded.

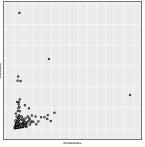
WATERLOO

For many other plots there also appears to be a relationship, though fairly weak.



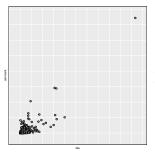
For many other plots there also appears to be a relationship, though fairly weak. For example,

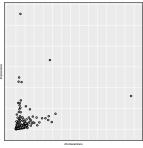






For many other plots there also appears to be a relationship, though fairly weak. For example,







$Transforming\ variates$

All of the response variates appear to have severely skewed distributions (e.g. share, like, etc.). We might consider a monotonic (and order preserving) transformation such as the logarithm.



Transforming variates

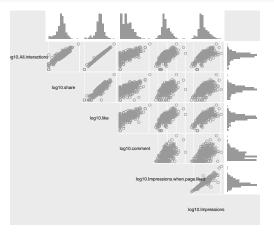
All of the response variates appear to have severely skewed distributions (e.g. share, like, etc.). We might consider a monotonic (and order preserving) transformation such as the logarithm.

Note that zero occurs in the counts for All.interactions, share, like, and comment, so we add a small amount first (+1) to all of these.

```
##
    log10.All.interactions log10.share log10.like log10.comment
## 1
                 2.004321 1.255273 1.903090
                                                  0.6989700
## 2
                 2.217484 1.477121 2.117271
                                                  0.7781513
                 1.908485 1.176091 1.826075 0.0000000
## 3
## 4
                 3.249932 2.170262 3.196729 1.7708520
## 5
                 2.595496 1.698970 2.513218 1.3010300
## 6
                 2.271842
                            1.531479
                                      2.184691
                                                  0.3010300
    log10.Impressions.when.page.liked log10.Impressions
##
## 1
                           3.488269
                                           3.706803
## 2
                           4.068557
                                           4.280055
## 3
                           3.449015
                                           3.640779
## 4
                           4.785522
                                           4.944438
## 5
                           3.794349
                                           4.133347
## 6
                           4.205042
                                           4.319085
```



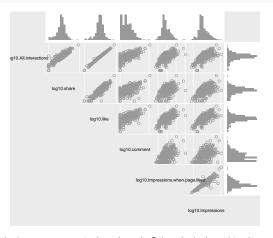
Transforming variates



Some histograms look more symmetric, less skewed. Others look skewed in the opposite direction.



Transforming variates

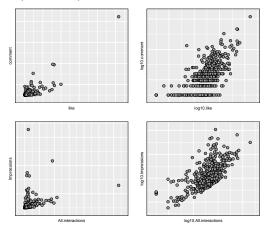


Some histograms look more symmetric, less skewed. Others look skewed in the opposite direction.

Similarly, some plots look more like straight lines; others look like they are now spread out a lotwiterlook the left instead of the right.

$Transforming\ variates$

For example, compare these plots:



Is there some way of not going too far?



When x > 0, one possible "family" of transformations is the power family.

For x > 0, we can write this family (over the power α) as

$$T_{\alpha}(x) = \begin{cases} ax^{\alpha} + b & (\alpha \neq 0) \\ c\log(x) + d & (\alpha = 0) \end{cases}$$

where a,b,c,d and α are real numbers with a>0 when $\alpha>0$, a<0 when $\alpha<0$, and c>0 when $\alpha=0$. The choices of a,b,c and d are somewhat arbitrary otherwise.

Since location and scale are not important in determining the **shape** of the distribution, we might settle on particular choices which are more mathematically convenient. For example,

$$T_{\alpha}(x) = \frac{x^{\alpha} - 1}{\alpha} \quad \forall \ \alpha$$

which requires no separate equation for the x=0 case, since $\lim_{\alpha\to 0} T_{\alpha}(x) = \log(x)$.



We could have different values of α , say α_x and α_y , respectively for x and y points in a scatterplot.



We could have different values of α , say α_x and α_y , respectively for x and y points in a scatterplot. In loon , we can create an interactive tool that allows us to view a scatterplot of pairs $(T_{\alpha_x}(x), T_{\alpha_y}(y))$ as we vary α_x and α_y .



We could have different values of α , say α_x and α_y , respectively for x and y points in a scatterplot. In loon , we can create an interactive tool that allows us to view a scatterplot of pairs $(T_{\alpha_x}(x), T_{\alpha_y}(y))$ as we vary α_x and α_y .

The code looks like this:

```
power_xy <- function(x, y=NULL, xlab=NULL, ylab=NULL,</pre>
                      linkingGroup.
                      linkingKey, from = -5, to = 5,
                      ...) {
    if (is.null(xlab)) {xlab <- "x"}</pre>
    if (is.null(ylab)) {ylab <- "y"}</pre>
    if (missing(linkingKey)) {linkingKey <- paste0(seq(0, length(x) -1))}
    if (missing(linkingGroup)) {linkingGroup <- departe(substitute(x))}
    powerfun <- function(x, alpha) {
        if (alpha == 0) log(x) else (x^alpha-1)/alpha}
    scale01 <- function(x) {
        minx <- min(x, na.rm = TRUE)
        maxx \leftarrow max(x, na.rm = TRUE)
        (x-minx)/(maxx - minx) + 1
    tt <- tktoplevel()
    xs < - scale01(x)
    vs <- scale01(v)
    # CONTINUED ON NEXT SLIDE
```

```
# CONTINUED FROM PREVIOUS
# scatterplot
p <- 1 plot(x=xs, y=ys, xlabel=xlab, ylabel=ylab,
            linkingGroup = linkingGroup,
             linkingKey = linkingKey,
             parent=tt, ...)
# Save the x and u values from p
xy_p \leftarrow data.frame(x = p["x"], y = p["y"])
fit \leftarrow lm(y \sim x, data = xy_p)
# layer fit
xrng <- range(xv p$x)
vhat <- predict(fit, data.frame(x = xrng))</pre>
line <- 1_layer_line(p, x = xrng, y = yhat, linewidth = 3, index = "end")
# histogram on each
h_x <- l_hist(xs, xlabel=xlab, yshows="density",
               linkingGroup = linkingGroup, linkingKey = linkingKey)
h v <- 1 hist(vs. xlabel=vlab, vshows="density",
               linkingGroup = linkingGroup, linkingKey = linkingKey,
               swapAxes = TRUE)
# save the original histogram values
h \times vals \leftarrow h \times ["x"]
h_y_vals \leftarrow h_y["x"]
# CONTINUED ON NEXT SLIDE
```



```
# CONTINUED FROM PREVIOUS SLIDE
# Set up power transformations
alpha x <- tclVar('1')
alpha v <- tclVar('1')
# Create two slider scales for the two powers
# Reverse to and from because scales are vertical
sx <- tkscale(tt, orient='vertical', label='power x', variable=alpha x,</pre>
              from=to, to=from, resolution=0.1)
sy <- tkscale(tt, orient='vertical', label='power y', variable=alpha_y,</pre>
              from=to, to=from, resolution=0.1)
# pack the sliders into the same window as the scatterplot
tkpack(sy, sx, fill='y', side='right')
tkpack(p, fill='both', expand=TRUE)
# CONTINUED on NEXT SLIDE
```



$The \ family \ of \ power \ transformations$

```
# CONTINUED FROM PREVIOUS SLIDE
# the update function, called when sliders move
update <- function(...) {
    ## powers
    alphax <- as.numeric(tclvalue(alpha x))</pre>
    alphay <- as.numeric(tclvalue(alpha_y))</pre>
    ## labels
    xlabel <- if (alphax==0) {</pre>
        paste0("log(",xlab,")")
    } else {
        if (alphax==1) {
            xlab
        } else {
             paste0(xlab, "^", alphax)
        }
    ylabel <- if (alphay==0) {</pre>
        paste0("log(", vlab, ")")
    } else {
        if (alphay==1) {
            ylab
        } else {
             paste0(ylab, "^", alphay)
```

```
# CONTINUED FROM PREVIOUS SLIDE
# new plot x and u
p_xnew <- scale01(powerfun(xy_p$x, alphax))</pre>
p_xnew_range <- range(p_xnew)</pre>
p ynew <- scale01(powerfun(xy p$y, alphay))
# update the fitted line values
fit.temp <- lm(p ynew ~ p xnew)
yhat <- predict(fit.temp, data.frame(p_xnew = p_xnew_range))</pre>
1 configure(line, y = yhat, x = p_xnew_range)
# update plot
l_configure(p,
            x = p_xnew,
            y = p_ynew,
            xlabel = xlabel,
            vlabel = vlabel
1 scaleto world(p)
# CONTINUED on NEXT SLIDE
```



```
# CONTINUED FROM PREVIOUS SLIDE
    # update the histograms
    binwidthx <- h_x['binwidth']
    binwidthy <- h y['binwidth']
    h_x_vals_new <- scale01(powerfun(h_x_vals, alphax))
    h y vals new <- scale01(powerfun(h y vals, alphay))
    1 configure(h x, x = h x vals new,
                binwidth = binwidthx.
                #origin = originx,
                vshows="density".
                xlabel= xlabel)
    l_configure(h_y, x = h_y_vals_new,
                binwidth = binwidthy,
                #origin = originy,
                vshows="density",
                xlabel= vlabel)
    l scaleto plot(h x)
   1 scaleto plot(h v)
# end of update function
# attach the update to the sliders
tkconfigure(sx, command=update)
tkconfigure(sy, command=update)
invisible(p)
```



We can now try this function out in loon .

```
# Need to get a version of facebook and if necessary remove nas with na.omit()
# First add 1 where necessary
facebook1 <- facebook
facebook1[,c("All.interactions", "share",
             "like", "comment")] <- 1 + facebook[, c("All.interactions", "share",
                                                      "like", "comment")]
# facebook1 <- na.omit(facebook1) # USE THIS
p_MonthPageLikes <- with(facebook1,</pre>
                         power xy(x=Post.Month, y=Page.likes,
                                   xlab="month of post", ylab="number page likes",
                                   title = "Page likes vs month",
                                   linkingGroup = "facebook",
                                   itemLabel = paste0("Category: ", Category, "\n",
                                                      "Type: ", Type, "\n",
                                                      "Paid: ".
                                                      c("no", "yes")[Paid]),
                                   showItemLabels = TRUE))
```



Try this function out on the "like" and "share" variates. We need a wider range of powers.



Tukey's ladder of power transformations

```
A convenient mnemonic ....
Tukey's "ladder" of power transformations
... -2, -1, -1/2, -1/3, 0, 1/3, 1/2, 1, 2, ...
 <--- down up --> ^start
  Or, as a ladder:
                Power
                      <-- raw data (start)
                1/2
                 O <-- logarithm
               -1/2
                Etc.
```

John Tukey suggested imagining that the powers were arranged in a "ladder" with the smallest WATERLOO powers on the bottom and the largest on the top.

Bump rule 1 for making densities (histograms) more symmetric.

The location of the density's bump (mode) tells you which way to "move" on the ladder.



Bump rule 1 for making densities (histograms) more symmetric.

The location of the density's bump (mode) tells you which way to "move" on the ladder.

if it is concentrated on "lower" values,



Bump rule 1 for making densities (histograms) more symmetric.

The location of the density's bump (mode) tells you which way to "move" on the ladder.

- if it is concentrated on "lower" values, then move the power "lower" on the ladder
 - i.e. move down the ladder
- if it is concentrated on "higher" values,



Bump rule 1 for making densities (histograms) more symmetric.

The location of the density's bump (mode) tells you which way to "move" on the ladder.

- if it is concentrated on "lower" values, then move the power "lower" on the ladder
 - i.e. move down the ladder
- if it is concentrated on "higher" values, then move the power "higher" on the ladder
 - ▶ i.e. move up the ladder



Bump rule 1 for making densities (histograms) more symmetric.

The location of the density's bump (mode) tells you which way to "move" on the ladder.

- if it is concentrated on "lower" values, then move the power "lower" on the ladder
 - i.e. move down the ladder
- if it is concentrated on "higher" values, then move the power "higher" on the ladder
 - i.e. move up the ladder

Bump rule 2 for straightening scatterplots.

Bump is now the direction of the curve of the scatterplot,



Bump rule 1 for making densities (histograms) more symmetric.

The location of the density's bump (mode) tells you which way to "move" on the ladder.

- if it is concentrated on "lower" values, then move the power "lower" on the ladder
 - i.e. move down the ladder
- if it is concentrated on "higher" values, then move the power "higher" on the ladder
 - i.e. move up the ladder

Bump rule 2 for straightening scatterplots.

Bump is now the direction of the curve of the scatterplot,

Bump points to a direction on the x axis AND on a (possibly different) direction on the y axis.



Tukey's ladder of power transformations - Bump rules

Bump rule 1 for making densities (histograms) more symmetric.

The location of the density's bump (mode) tells you which way to "move" on the ladder.

- if it is concentrated on "lower" values, then move the power "lower" on the ladder
 - i.e. move down the ladder
- if it is concentrated on "higher" values, then move the power "higher" on the ladder
 - i.e. move up the ladder

Bump rule 2 for straightening scatterplots.

Bump is now the direction of the curve of the scatterplot,

Bump points to a direction on the x axis AND on a (possibly different) direction on the y axis.

▶ Bump pointing up (or down) on the x axis means move up (or down) on α_x ladder.



Tukey's ladder of power transformations - Bump rules

Bump rule 1 for making densities (histograms) more symmetric.

The location of the density's bump (mode) tells you which way to "move" on the ladder.

- if it is concentrated on "lower" values, then move the power "lower" on the ladder
 - i.e. move down the ladder
- if it is concentrated on "higher" values, then move the power "higher" on the ladder
 - i.e. move up the ladder

Bump rule 2 for straightening scatterplots.

Bump is now the direction of the curve of the scatterplot,

Bump points to a direction on the x axis AND on a (possibly different) direction on the y axis.

- ▶ Bump pointing up (or down) on the x axis means move up (or down) on α_x ladder.

Tukey's ladder of power transformations - Bump rules

Bump rule 1 for making densities (histograms) more symmetric.

The location of the density's bump (mode) tells you which way to "move" on the ladder.

- if it is concentrated on "lower" values, then move the power "lower" on the ladder
 - i.e. move down the ladder
- if it is concentrated on "higher" values, then move the power "higher" on the ladder
 - i.e. move up the ladder

Bump rule 2 for straightening scatterplots.

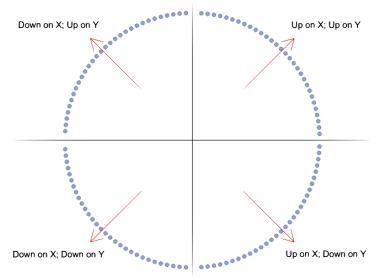
Bump is now the direction of the curve of the scatterplot,

Bump points to a direction on the x axis AND on a (possibly different) direction on the y axis.

- ▶ Bump pointing up (or down) on the x axis means move up (or down) on α_x ladder.

Tukey's ladder of power transformations - Bump rules

Monotonic curved relations will look like one of the four quadrants



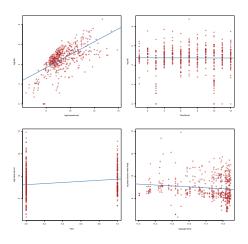


Consider, for example, the plots and fitted lines produced by the code below.

```
fit1 <- lm(log(like + 1) ~ log(Impressions), data = facebook)
fit2 <- lm(log(All.interactions +1) ~ Post.Month, data = facebook)
fit3 <- lm(log(Impressions) ~ Paid, data = facebook)
fit4 <- lm(log(Impressions.when.page.liked) ~ log(Page.likes + 1), data = facebook)
savePar <- par(mfrow=c(2,2))</pre>
colour <- adjustcolor("firebrick", 0.5)</pre>
with (facebook.
     { plot(log(Impressions), log(like + 1), ylab = "log(like)", pch=19, col=colour)
       abline(fit1, col="steelblue", lwd=2)
       plot(Post.Month, log(All.interactions +1), vlab = "log(all)", pch=19, col=colour)
       abline(fit2, col="steelblue", lwd=2)
       plot(Paid, log(Impressions), pch=19, col=colour)
       abline(fit3, col="steelblue", lwd=2)
       plot(log(Page.likes + 1), log(Impressions.when.page.liked),
            ylab = "log(impressions when liked)", xlab = "log(page likes)",
            pch=19, col=colour)
       abline(fit4, col="steelblue", lwd=2)
par(savePar)
```



Results are:





First fit results:

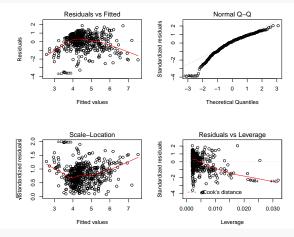
```
summary(fit1)
```

```
##
## Call:
## lm(formula = log(like + 1) ~ log(Impressions), data = facebook)
##
## Residuals:
##
    Min
               10 Median
                              30
                                    Max
## -3.5784 -0.4542 0.1555 0.5969 1.8605
##
## Coefficients:
##
                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                 -1.58284 0.32893 -4.812 1.98e-06 ***
## log(Impressions) 0.65416 0.03478 18.808 < 2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9135 on 497 degrees of freedom
    (1 observation deleted due to missingness)
## Multiple R-squared: 0.4158, Adjusted R-squared: 0.4146
## F-statistic: 353.7 on 1 and 497 DF. p-value: < 2.2e-16
```



First fit results:

```
oldPar <- par(mfrow=c(2,2))
plot(fit1)</pre>
```





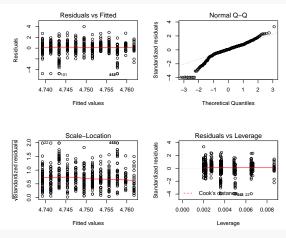
Second fit results:

```
summary(fit2)
##
## Call:
## lm(formula = log(All.interactions + 1) ~ Post.Month, data = facebook)
##
## Residuals:
## Min 10 Median
                                    Max
                             30
## -4.7577 -0.4743 0.0746 0.6856 4.0044
##
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 4.763830 0.126806 37.568 <2e-16 ***
## Post Month -0.002049 0.016309 -0.126 0.9
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.205 on 498 degrees of freedom
## Multiple R-squared: 3.17e-05, Adjusted R-squared: -0.001976
## F-statistic: 0.01579 on 1 and 498 DF, p-value: 0.9001
```



Second fit results:

```
oldPar <- par(mfrow=c(2,2))
plot(fit2)</pre>
```



par(oldPar)



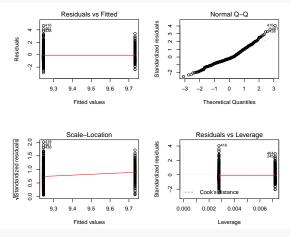
Third fit results:

```
summary(fit3)
##
## Call:
## lm(formula = log(Impressions) ~ Paid, data = facebook)
##
## Residuals:
## Min
               10 Median 30
                                    Max
## -2.9002 -0.7373 -0.2215 0.6549 4.6743
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 9.24586 0.06114 151.236 < 2e-16 ***
               0.48681 0.11583 4.203 3.13e-05 ***
## Paid
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.16 on 497 degrees of freedom
    (1 observation deleted due to missingness)
## Multiple R-squared: 0.03432, Adjusted R-squared: 0.03238
## F-statistic: 17.66 on 1 and 497 DF. p-value: 3.128e-05
```



Third fit results:

```
oldPar <- par(mfrow=c(2,2))
plot(fit3)</pre>
```



par(oldPar)



Fourth fit results:

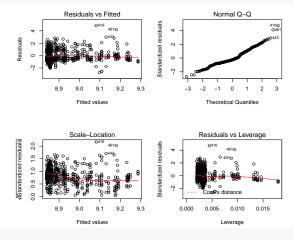
```
summary(fit4)
```

```
##
## Call:
## lm(formula = log(Impressions.when.page.liked) ~ log(Page.likes +
##
      1), data = facebook)
##
## Residuals:
##
      Min
              10 Median 30
                                    Max
## -2.7574 -0.6522 -0.2636 0.6733 4.8359
##
## Coefficients:
##
                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                     18.6926
                                 3.7743 4.953 1e-06 ***
## log(Page.likes + 1) -0.8319 0.3222 -2.582 0.0101 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.033 on 498 degrees of freedom
## Multiple R-squared: 0.01321, Adjusted R-squared: 0.01122
## F-statistic: 6.664 on 1 and 498 DF, p-value: 0.01012
```



Fourth fit results:

```
oldPar <- par(mfrow=c(2,2))
plot(fit4)</pre>
```



par(oldPar)



How about:

```
fit5 <- lm(I(Page.likes^3) ~ sqrt(Post.Month), data = facebook)</pre>
```



##

```
How about:
fit5 <- lm(I(Page.likes^3) ~ sqrt(Post.Month), data = facebook)
summary(fit5)
##
## Call:
## lm(formula = I(Page.likes^3) ~ sqrt(Post.Month), data = facebook)
##
## Residuals:
##
         Min
                     10
                           Median
                                                   Max
## -2.757e+14 -1.000e+14 5.535e+12 9.590e+13 2.475e+14
##
## Coefficients:
##
                     Estimate Std. Error t value Pr(>|t|)
## (Intercept) -4.510e+14 2.039e+13 -22.12 <2e-16 ***
## sqrt(Post.Month) 9.424e+14 7.687e+12 122.60 <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 1.186e+14 on 498 degrees of freedom
Multiple R-squared: 0.9679, Adjusted R-squared: 0.9679
F-statistic: 1.503e+04 on 1 and 498 DF, p-value: < 2.2e-16</pre>



Fifth fit results:

Transformed Page.likes and Post.Month

