Experimental Results: Logic and hypothesis testing

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48 marks

In this question, we will explore the data collected in class.

You will need to download the data `class_data.csv` from the assignment website and save it somewhere. Supposing the data to have been saved in a directory `dataDirectory`, read it into R as

```
# Usual helper for paths
path_concat <- function(path1, path2, sep="/") {
                       paste(path1, path2, sep = sep)
}

data <- read.csv(file = path_concat(dataDirectory, "class_data.csv"))
```

Having loaded the data, you might have a look at its contents using any of the standard functions:

```
# just print it
data
# view its contents as a spreadsheet (in RStudio)
View(data)
# or, look at its data structure
str(data)
```

You will find that it is a `data.frame` with variables

```
names(data)
```

```
## [1] "random_digit"  "student_digit" "green_card1"   "green_card2"
## [5] "red_card1"     "red_card2"
```

Each row contains one student's answer to the questions associated with these names.

Here we will only consider the results of the two hypothesis tests using the cards.

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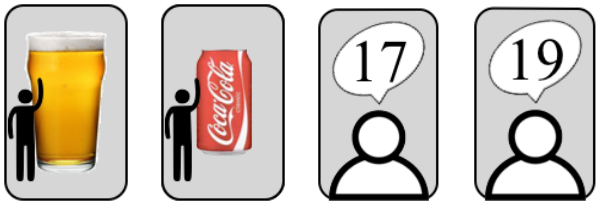
1. Recall the experimental scenario associated with the `green` card:
   - Patrons in a licensed restaurant may drink alcohol legally only if they are 19 years old or older.
   - Each patron has a card on showing the beverage they are drinking on one side and their age in years on the other.
   - We want to test the hypothesis

     H: No patron is illegally drinking alcohol.
   - Cards of four of the patrons were selected and placed on a table showing only one side of each card.
   - The four cards were:



(a) (b) (c) (d)

   - To test the hypothesis, each person had five seconds to choose two of these cards to turn over.
   - The variables `green_card1` and `green_card2` record the first and second choices made by that person.

   a. *(4 marks)* What could each card potentially reveal about the hypothesis were it turned over?

   b. *(1 mark)* Use your answers above to justify why (a) and (c) are the correct cards to turn over.

   c. *(1 mark)* Suppose a person simply selected two different cards at random (i.e. each available card has an equal probability of being selected).

      Explain why the probability that they would select both correct cards is $\frac{1}{6}$?

   d. *(2 marks)* Which card was most often selected first? Was it correct?

      Which card was most often selected second? How many times? And was it correct?

      Show your code.

   e. For each green card answer, construct a logical vector containing `TRUE` when the answer is correct and `FALSE` when it is not. Assign these vectors to `correct_g1` and `correct_g2` respectively. Use these logical vectors to answer the following:

      i. *(1 mark)* Show your code for creating `correct_g1` and `correct_g2`.

      ii. *(2 marks)* Determine the proportion of people who had a correct first selection. How does this compare to the probability of getting it correct if the first selection was a random one?

      Show your code.

      iii. *(2 marks)* Determine the proportion of people who got *both* selections correct. How does this compare to the probability of getting both correct if the selections were made at random?

      Show your code.

      iv. *(2 marks)* Now consider only those who got the first card right. Determine what proportion of these *also* got the second card right.

      How does this compare to the probability of selecting the second correctly at random, *given* the first was selected correctly?

      Show your code.

v. *(2 marks)* Construct a new numeric vector `prop_correct_green`, from the logical vectors you have created, which records the proportion (i.e. 0, 0.5, or 1) of correct cards identified by each person.

   Using `hist()`, draw a histogram of these proportions.

   Add a red dashed vertical line using `abline()` to the plot at the average proportion correct. Use `lwd = 3, lty = 2`.

   Add a blue dotted vertical line to the plot at the median proportion correct. Use `lwd = 3, lty = 3`.

   Show your code and the plot.

vi. *(2 marks)* Comment on how well the people did overall. What is the basis for your conclusions?

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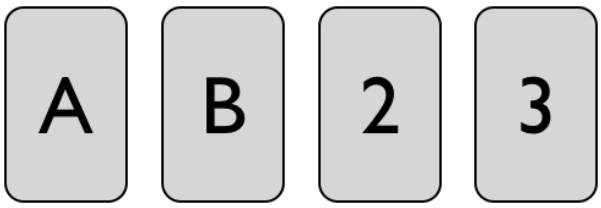
2. Recall the experimental scenario associated with the `red` card:
   - Have a special deck with 52 cards.
   - Each card an upper case letter on one side and a positive non-zero integer on the other.
   - We want to test the hypothesis

     H: Cards with a vowel on one side will have an even number on the other.
   - Have four cards selected and placed on a table showing only one side of each card.
   - The four cards were:



(a) (b) (c) (d)

   - To test the hypothesis, each person had five seconds to choose two of these cards to turn over.
   - The variables `red_card1` and `red_card2` record the first and second choices made by that person.

   a. *(4 marks)* What could each card potentially reveal about the hypothesis were it turned over?

   b. *(1 mark)* Use your answers above to justify why (a) and (d) are the correct cards to turn over.

   c. *(2 marks)* Which card was most often selected first? Was it correct?

      Which card was most often selected second? How many times? And was it correct?

      Show your code.

   d. For each red card answer, construct a logical vector containing `TRUE` when the answer is correct and `FALSE` when it is not. Assign these vectors to `correct_r1` and `correct_r2` respectively. Use these logical vectors to answer the following:

      i. *(1 mark)* Show your code for creating `correct_r1` and `correct_r2`.

      ii. *(2 marks)* Determine the proportion of people who had a correct first selection. How does this compare to the probability of getting it correct if the first selection was a random one?

      Show your code.

      iii. *(2 marks)* Determine the proportion of people who got *both* selections correct. How does this compare to the probability of getting both correct if the selections were made at random?

      Show your code.

      iv. *(2 marks)* Now consider only those who got the first card right. Determine what proportion of these *also* got the second card right.

      How does this compare to the probability of selecting the second correctly at random, *given* the first was selected correctly?

      Show your code.

      v. *(2 marks)* Construct a new numeric vector `prop_correct_red`, from the logical vectors you have created, which records the proportion (i.e. 0, 0.5, or 1) of correct cards identified by each person.

      Using `hist()`, draw a histogram of these proportions.

      Add a red dashed vertical line using `abline()` to the plot at the average proportion correct. Use `lwd = 3, lty = 2`.

Add a blue dotted vertical line to the plot at the median proportion correct. Use `lwd = 3, lty = 3`.

Show your code and the plot.

vi. *(2 marks)* Comment on how well the people did overall. What is the basis for your conclusions?

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3. This question compares the results of the two separate hypothesis tests studied in the previous two questions.

   a. *(2 marks)* Explain why the two tests and hypotheses were *logically identical*.

   b. *(2 marks)* In what sense are the tests of hypothesis conducted in the first two questions like a statistical test of significance?

   c. Suppose that in these experiments, the two cards were chosen at random (with equal probability of selection). Let P be the proportion that any individual might get correct when selecting at random.

      i. *(3 marks)* Determine the value of each of the following probabilities (show your work):

         - $Pr(P = 0)$
         - $Pr(P = 0.5)$
         - $Pr(P = 1)$

      ii. *(1 mark)* Hence determine $E(P)$.

      iii. *(1 mark)* How does the observed average proportion correct compare with $E(P)$ for each of the hypotheses tested? That is, which is closer to random and which is farther away?

   d. Based on all analyses in all of the above questions:

      i. *(2 marks)* Which of the two hypotheses were found to be more easily tested by the persons involved? Give your reasoning.

      ii. *(2 marks)* What differences between the two set ups might account for the difference in performance?