Sources of error

R.W. Oldford

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We need to consider what general sources might contibute to error (besides calculational/floating point errors).

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> *Imagine the following hypothetical medical situation where you, the patient, having been diagnosed with a form of cancer are trying to make a choice between two different treatments available. The treatments are (a) Surgery and (b) Radiation. The decision as to which treatment you will take is entirely yours.*
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- What is the population $\mathcal{P}$? What are its units?
- How about variate(s)? What is the kind of variate(s)?
- What population attribute is of interest?
- What role is played by the question asked?

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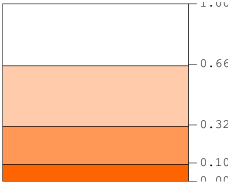
In all cases, the historical information presented was **identical**.

After the historical information was presented, each group was instructed:

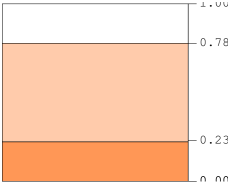
> *Based on this information, you must choose one of the two treatments. Circle one of the following as your answer:*
>
> ***(a) Surgery*** ***(b) Radiation***

Surgery or radiation: Groups 1 and 2 pictures presented.

In each diagram below the area of the horizontal strip is the probability of the outcome which labels the strip.



(a) Surgery



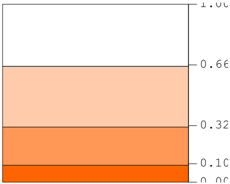
(b) Radiation

200 patients diagnosed with cancer

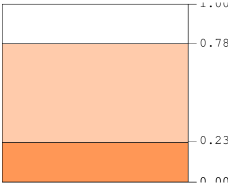
100 receive (a) surgery, 100 (b) radiation treatment.

Surgery or radiation: Group 1 was told

In each diagram below the area of the horizontal strip is the probability of the outcome which labels the strip.



(a) Surgery

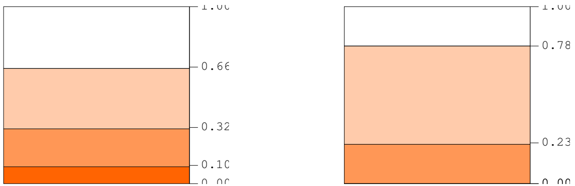


(b) Radiation

Figure 1: 200 patients diagnosed with cancer – 100 receive (a) surgery, 100 (b) radiation treatment.
From bottom to top the categories are y_1= "Does not survive treatment", y_2 = "Survives treatment, but only to one year, y_3= "Survives more than one but fewer than five years" and y_4 = "Survives at least 5 years".
The area (or equivalently the height) of each shaded rectangle matches the proportion of the 100 which are in that category.
The shading matches the category across the two figures and for radiation the bottom most category, y_1, is absent because all survive radiation treatment.

Surgery or radiation: Group 2 was told

In each diagram below the area of the horizontal strip is the probability of the outcome which labels the strip.



(a) Surgery (b) Radiation

Figure 2: 200 patients diagnosed with cancer – 100 receive (a) surgery, 100 (b) radiation treatment.
From bottom to top the categories are y_1= "Die during treatment", y_2 = "Die by the end of the first year, y_3= "Die by the end of five years" and y_4 = "Survives at least 5 years".
The area (or equivalently the height) of each shaded rectangle matches the proportion of the 100 which are in that category.
The shading matches the category across the two figures and for radiation the bottom most category, y_1, is absent because no one died during radiation treatment.

Surgery or radiation: Groups 3 and 4

Groups 3 and 4 were presented the historical information as text with numbers.

Group 3:

> ***(a) Surgery****: Of 100 people having surgery 90 live through the post-operative period, 68 are alive at the end of the first year, and 34 are alive at the end of five years.*
>
> ***(b) Radiation therapy****: Of 100 people having radiation therapy, all live through the treatment, 77 are alive at the end of one year, and 22 are alive at the end of five years.*

Surgery or radiation: Groups 3 and 4

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> ***(b) Radiation therapy****: Of 100 people having radiation therapy, all live through the treatment, 77 are alive at the end of one year, and 22 are alive at the end of five years.*

Group 4:

> ***Surgery****: Of 100 people having surgery 10 die during surgery or the post-operative period, 32 die by the end of the first year, and 66 die by the end of five years.*
>
> ***Radiation therapy****: Of 100 people having radiation therapy, none die during treatment, 23 die by the end of one year, and 78 die by the end of five years.*

Surgery or radiation: results

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3. the average height that people think the tallest redwood cpuld be, in metres.

Questions:

- what is a population unit here?
- what is the population of interest?

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   Circle one:

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Giant redwoods: Results

Data:

```
redwoods <- read.csv(path_concat(dataDirectory, "redwood.csv"))
# Last two rows
tail(redwoods, n = 2)
```

```
##     A more guess
## 37 50   no    35
## 38 50  yes   100
```

```
# Number A = 50
A_50 <- redwoods$A == 50
sum(A_50)
```

```
## [1] 19
```

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# Number A = 100
A_100 <- redwoods$A == 100
sum(A_100)
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```
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A_100 <- redwoods$A == 100
sum(A_100)
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Proportions:

```
said_yes <- redwoods$more == "yes"
# Proportion think tallest is greater than 50 metres
round(sum(A_50 & said_yes)/sum(A_50), 2)
```

```
## [1] 0.84
```

```
# Proportion think tallest is greater than 100 metres
round(sum(A_100 & said_yes)/sum(A_100), 2)
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Giant redwoods: Results

Average tallest heights:

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mean(redwoods$guess[A_100])
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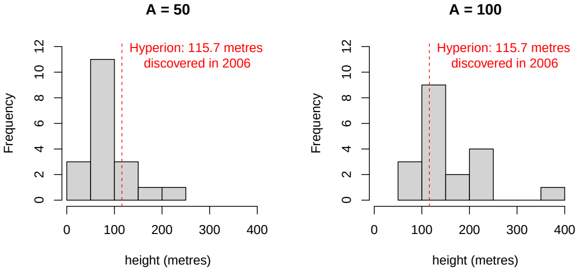
Histogram of tallest heights:



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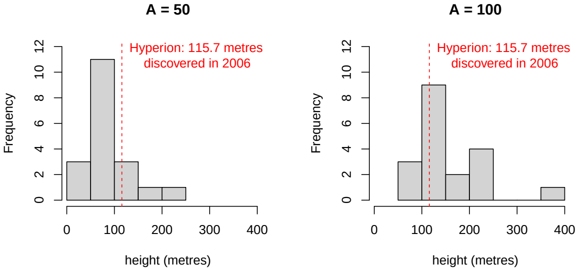
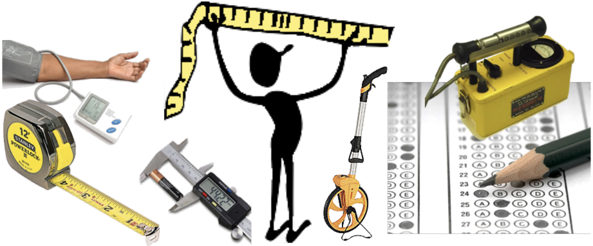


What's going on?

Source of error: Measurement



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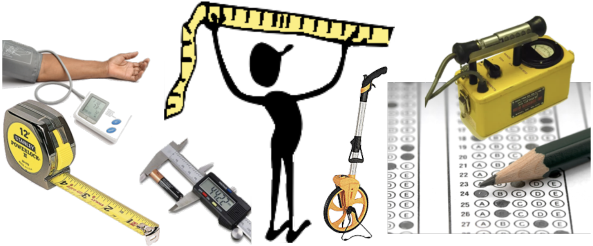

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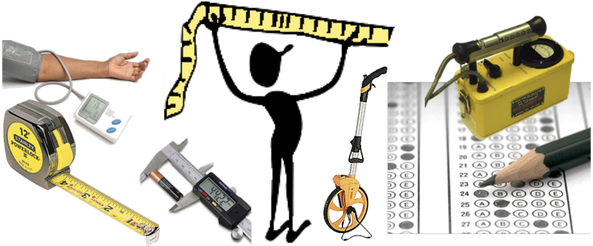
Examples:

- guessing the height of the tallest known redwood in metres

Source of error: Measurement



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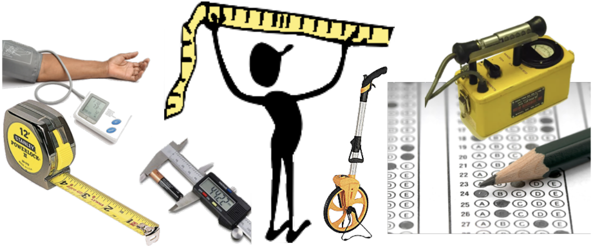

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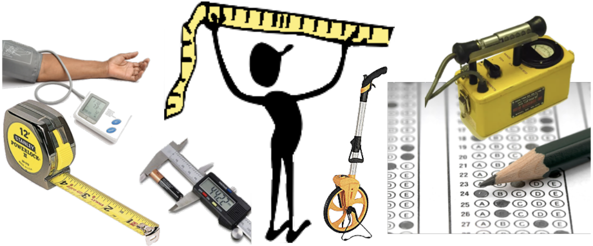

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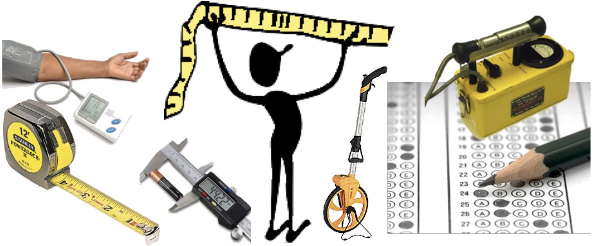

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Note however that we have two measuring systems, one where the idea that the greatest height might be 50 metres was first planted, and one where the idea that the greatest height might be 100 metres was first planted.

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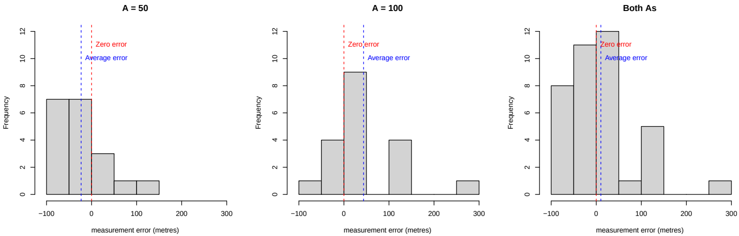
Histograms of these measurement errors for each group and then combined



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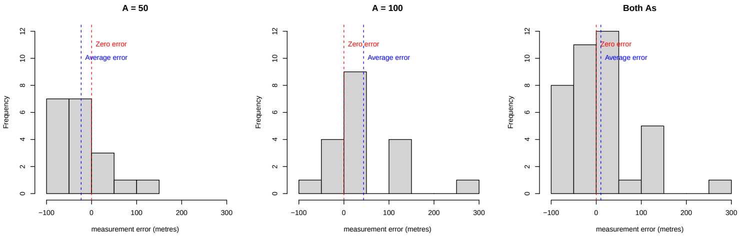

The average of all possible measurement errors is called the **measuring bias** given by

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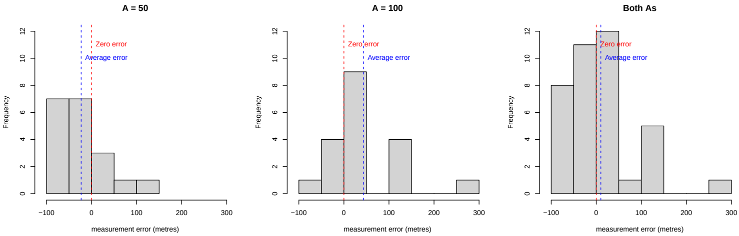

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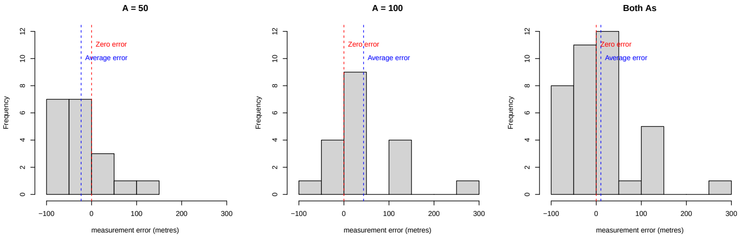

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Estimates of the measuring bias for each of the first, second, and then combined measuring systems are respectively, -23.2, 43.7, and 10.2 metres.

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Expressing these as estimated **standard deviations** of the measuring systems (i.e. by taking the square roots) gives quantities on the same scale as the errors, namely 55.3, 79.9, and 75.8 metres.

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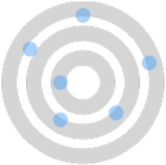

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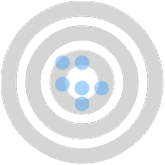

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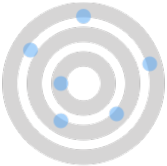

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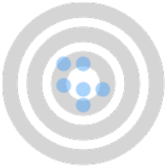

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There is often a trade off between variability and bias. Most practitioners prefer to improve (i.e. lower) the variability of a measuring system, then, afterwards, to reduce the bias.

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- the data on people's guesses for the height of the tallest known redwood.

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Target populations, study populations, and samples

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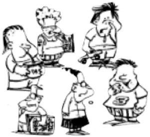

Sample
$\mathcal{S}$

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Target populations, study populations, and samples

Or looking at how we might draw conclusions:

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Study Population
$\mathcal{P}_{Study}$

Sample
$\mathcal{S}$

Target populations, study populations, and samples

Or worse, and fairly common in medical studies:

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Separation like this allows us to focus on where the error sources might be and what might be done about them.

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- the study error could be large even when we could argue that $\mathcal{P}_{Target}$ should be very much like $\mathcal{P}_{Study}$, again depending on the attribute (e.g. $a(\mathcal{P}) = max_{u \in \mathcal{P}} X(u)$ and $\mathcal{P}_{Study} \subset \mathcal{P}_{Target}$).

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E.g. When $\mathcal{P}_{Target}$ consists of humans, having $\mathcal{P}_{Study} \subset \mathcal{P}_{Target}$ could be unethical.

Sample selection – the blocks competition

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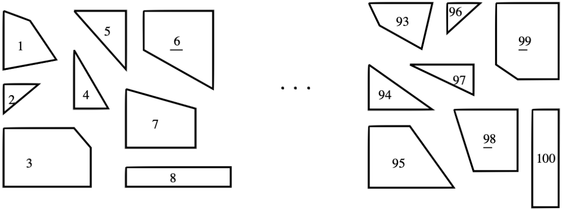
The blocks are of of uniform thickness and density (all blocks were cut from the same opaque plastic sheet of about 5mm thickness), but have different shapes as such as shown below:



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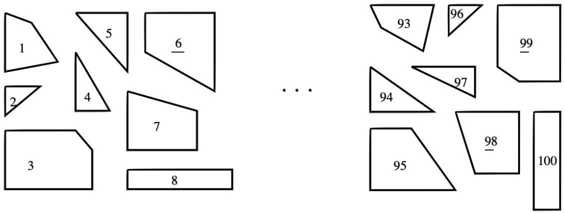

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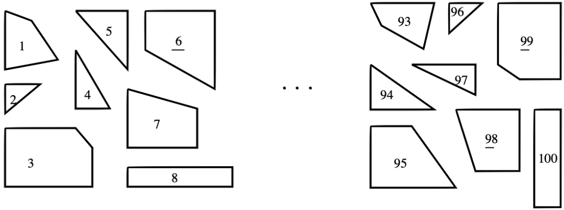

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We want a sample $\mathcal{S} \subset \mathcal{P}_{Study}$ of $n = 10$ blocks selected from the 100, whose average weight is (nearly) the same as the average weight of all 100.

That is, we would like a sample with zero (or at least small in absolute value) **sample error** $a(\mathcal{S}) - a(\mathcal{P}_{Study})$.

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- Hand in your card when your sample selection is complete.

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If we were to select m different samples from $\mathcal{C}$ with

$$Pr(\mathcal{S} = \mathcal{S}_i) = \frac{k_i}{N_{\mathcal{C}}}$$

then both sampling bias and variability could be estimated from the m values of $a(\mathcal{S})$. (Replace $N_{\mathcal{C}}$ by $m-1$ in the variability estimate.)

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Inductive inference

We draw conclusions about $a(\mathcal{P}_{Target})$ from an attribute $a_{\approx}(\mathcal{S}) = \widehat{a}(\mathcal{S})$, being based on measurements of units in some sample $\mathcal{S}$. A case needs to be made for the validity of each step along **the inductive path**:

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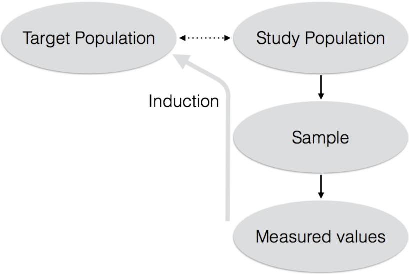

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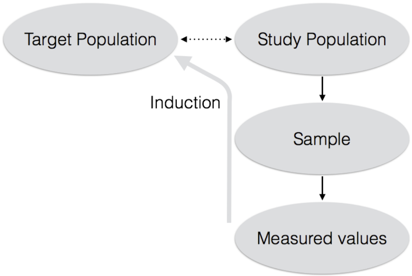

The overall **inductive error** separates into three conceptually distinct sources:

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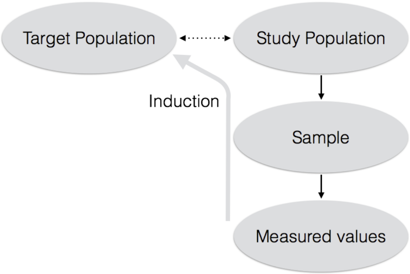

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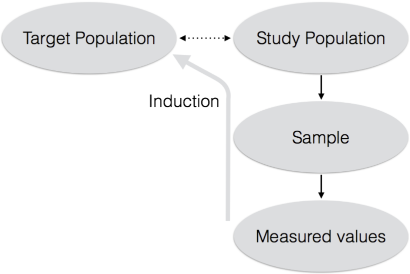

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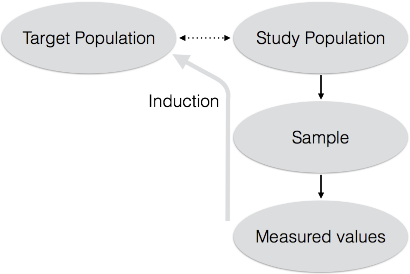

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Example - Visible minorities in Canada 2006

Recall the `minority` data from loon.data.

Questions:

- What population attribute(s) are of interest?

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This one is a little harder.

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The October 31 issue of *Literary Digest* announced that Landon would be the winner with 57.1% of the vote and 370 electoral votes.

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Example - November 3, 1936 US Presidential Election

Results:

Poll Actual

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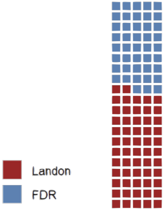

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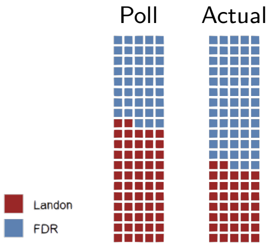

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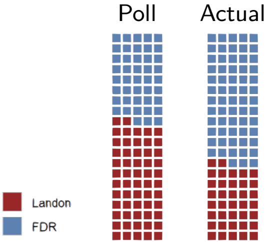

- Study Error?

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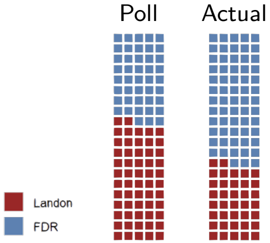

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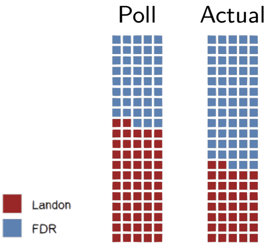

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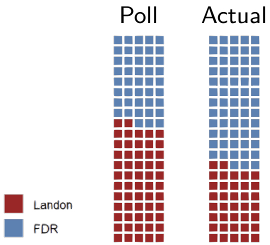

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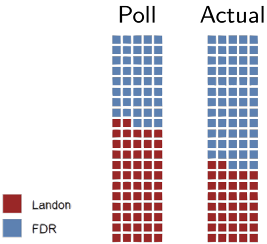

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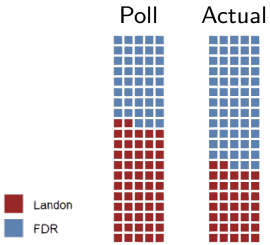

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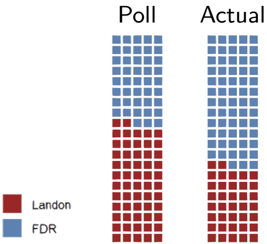

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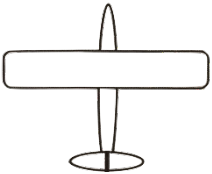
Example - World War II US Bombers

During the Second World War, US Statistician Abraham Wald was trying to determine where USAF bombers should have armour added to them in order to reduce the number of airplanes lost.



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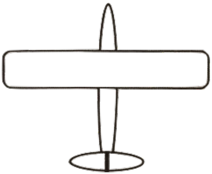

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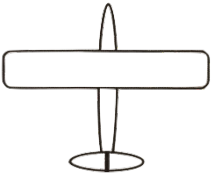

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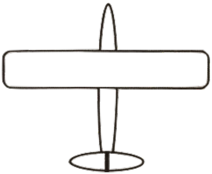

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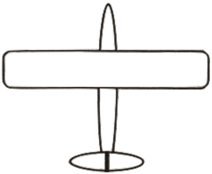

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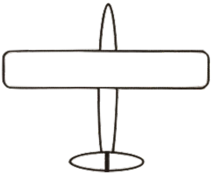

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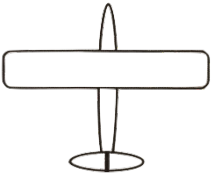

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Target population the planes that were shot down.

Study population the planes that actually return.

Sample returning planes that are accessible and measured.

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Example - World War II US Bombers

Wald's solution:

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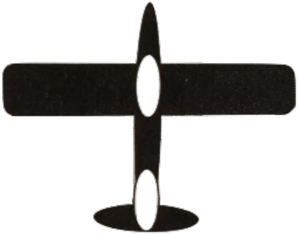

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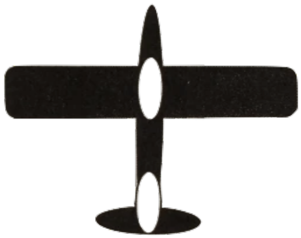

Dark spots show holes on all returning planes in the sample. Clearly in error for the corresponding target population attribute.

Because they didn't return, they must have holes where these did not. Take advantage of known study error and add extra armour on these areas.

"Wald and his wife died when the Air India plane in which they were travelling crashed in the Nilgiri Mountains, in southern India, while on an extensive lecture tour at the invitation of the Indian government." – Wikipedia

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Sources of error?

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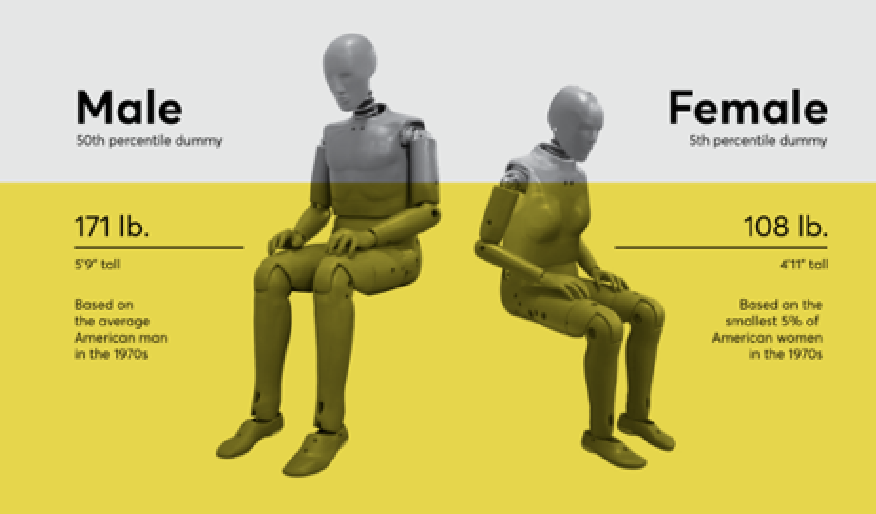

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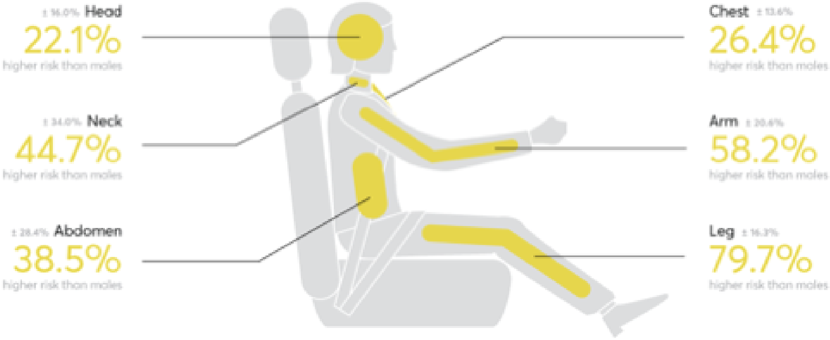

Male versus Female crash tests

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Is "machine learning fairness" ethical?