PHY2049 Formulas: Sheet 1

Chapter 21 – 23 (Electric forces and fields, Gauss' law)

Coulomb's Law $\mathbf{F}=\frac{kQq}{r^2}\hat{\mathbf{r}}\equiv\frac{Qq}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}$ (point charge) $\hat{\mathbf{r}}$ = unit vector from Q to q.

Electric field $\mathbf{F}=q\mathbf{E}$ (general) $\mathbf{E}=\frac{kQ}{r^2}\hat{\mathbf{r}}\equiv\frac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}$ (single point charge Q)

$\mathbf{E}=\sum_i\frac{kq_i}{r_i^2}\hat{\mathbf{r}}_i$ or $\mathbf{E}=\int\frac{kdq}{r^2}\hat{\mathbf{r}}$ (point charges or continuous)

Electric dipole $U=-\mathbf{p}\cdot\mathbf{E}=-pE\cos\theta$ $\vec{\tau}=\mathbf{p}\times\mathbf{E}$ $\tau=pE\sin\theta$

Gauss' law $\Phi_E=\sum_i\mathbf{E}_i\cdot\mathbf{A}_i=\oint_S\mathbf{E}\cdot d\mathbf{A}=\frac{Q_{\text{encl}}}{\varepsilon_0}$ $\Phi_E=$ "electric flux"

Chapter 24 – 25 (Electric potential, capacitors)

Work $W=\int_i^f\mathbf{F}\cdot d\mathbf{s}=K_f-K_i=\frac{1}{2}mv_f^2-\frac{1}{2}mv_i^2$

Conservative force $U_f-U_i=-\int_i^f\mathbf{F}\cdot d\mathbf{s}\rightarrow U_i+K_i=U_f+K_f$ (energy conservation)

Electric potential $V=\frac{U}{q}$ (general) $V=\frac{kQ}{r}\equiv\frac{Q}{4\pi\varepsilon_0 r}$ (point charge Q)

Potential difference $\Delta V\equiv V_f-V_i=-\mathbf{E}\cdot\left(\mathbf{x}_f-\mathbf{x}_i\right)$ (E constant) $V_f-V_i=-\int_i^f\mathbf{E}\cdot d\mathbf{s}$ (E variable)

E field from V $E_x=-\frac{\partial V}{\partial x}$ $E_y=-\frac{\partial V}{\partial y}$ $E_z=-\frac{\partial V}{\partial z}$

Capacitors $C=\frac{\varepsilon_0 A}{d}$ (flat plates) $C=\frac{2\pi\kappa\varepsilon_0 L}{\ln\left(b/a\right)}$ (cyl shell) $C=4\pi\kappa\varepsilon_0\frac{ab}{b-a}$ (sph shell)

Capacitors (cont) $q=CV$ $E\rightarrow\frac{E}{\kappa}$ $C\rightarrow\kappa C$ $C_{\text{eq}}=C_1+C_2$ (parallel) $\frac{1}{C_{\text{eq}}}=\frac{1}{C_1}+\frac{1}{C_2}$ (series)

Energy $U_E=\frac{1}{2}CV^2=\frac{q^2}{2C}$ (energy) $u_E=\frac{1}{2}\varepsilon_0 E^2$ (energy density)

Chapter 26 – 27 (Electric current, circuits)

Current $i\equiv\frac{dq}{dt}$ (basic def) $i=JA$ $i=\int_S\mathbf{J}\cdot d\mathbf{A}$ (using current density)

Current density $\mathbf{J}=\sigma\mathbf{E}=\frac{1}{\rho}\mathbf{E}$ (using conductivity σ or resistivity ρ)

Drift velocity $J=en_e v_d$ $\rho=m_e/e^2 n_e\tau$ (resistivity)

Resistance $V = iR \quad R = \frac{\rho L}{A} \quad R_{\text{eq}} = R_1 + R_2$ (series) $\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$ (parallel)

Temp dependence $\rho - \rho_0 = \rho_0 \alpha \left(T - T_0\right)$

RC circuits $\tau_{RC} = RC \quad q = q_{\max}\left(1 - e^{-t/\tau_{RC}}\right)$ (charging) $q = q_{\max} e^{-t/\tau_{RC}}$ (discharging)

Circuits (1) Current entering junction = current leaving junction (2) $\sum_i V_i = 0$ (over loop)

Power in circuit $P = iV$ (general power eqn) $P = i^2 R$ (power lost in resistor)

Chapter 28 – 29 (Magnetic fields)

Magnetic force $\mathbf{F} = q\mathbf{v} \times \mathbf{B} \quad F = qvB\sin\phi$ (charge) $\mathbf{F} = i\mathbf{L} \times \mathbf{B} \quad F = iLB\sin\phi$ (current)

Magnetic dipole $\mu = NiA$ (current loop)

$\vec{\tau} = \vec{\mu} \times \mathbf{B} \quad \tau = \mu B \sin\theta$ (torque) $U = -\vec{\mu} \cdot \mathbf{B} = -\mu B \cos\theta$ (potential energy)

Generating B field $d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{id\mathbf{s} \times \hat{\mathbf{r}}}{r^2} \qquad dB = \frac{\mu_0}{4\pi} \frac{ids\sin\theta}{r^2}$ (Biot-Savart law)

$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i_{\text{enc}}$ (Ampere's law) $B = \frac{\mu_0 i}{2\pi r}$ (long wire)

$B = \frac{\mu_0 i}{2R}$ (circular loop) $B = \frac{\mu_0 i\phi}{4\pi R}$ (partial loop) $B = \mu_0 ni$ (solenoid)

Force between currents $F_{ab} = \frac{\mu_0 i_a i_b L}{2\pi d}$

B field of dipole $\mathbf{B}\left(z\right) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3}$ (very far from dipole)

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Chapter 30 – 31 (Induction, RLC circuits)

Magnetic flux	$\Phi_B = \mathbf{B}\cdot\mathbf{A}$ (constant B, flat surface) $\Phi_B = \oint \mathbf{B}\cdot d\mathbf{A}$ (general)
Induction	$\mathcal{E} \equiv \oint \mathbf{E}\cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$ (Faraday's law) $\mathcal{E} = -N\frac{d\Phi_B}{dt}$ (multi-loop)
Inductance	$L = N\Phi_B / i$ (def) $L = \mu_0 n^2 Al$ (solenoid) $v_L = Ldi/dt$ $v_1 = M_{12}di_2/dt$
LR circuit	$\tau_{LR} = L/R$ $i = i_0\left(1-e^{-t/\tau_{LR}}\right)$ (current charging) $i = i_0 e^{-t/\tau_{LR}}$ (current decay)
Energy	$U_B = \frac{1}{2}Li^2$ (energy in inductor) $u_B = B^2/2\mu_0$ (energy density)
AC circuits	$\omega_0 = 1/\sqrt{LC}$ (resonant ω) $\mathcal{E} = \mathcal{E}_m \sin(\omega_d t)$ $i = I_m \sin(\omega_d t - \phi)$ (driven circuit)
	$X_L = \omega L$ $X_C = 1/\omega C$ (reactances) $\tan\phi = \frac{X_L - X_C}{R}$ (lag angle)
	$Z = \sqrt{R^2 + (X_L - X_C)^2}$ (impedance) $I_m = \frac{\mathcal{E}_m}{Z}$
	$I_{\text{rms}} = \frac{I_m}{\sqrt{2}}$ $\mathcal{E}_{\text{rms}} = \frac{\mathcal{E}_m}{\sqrt{2}}$ $P_{\text{ave}} = \mathcal{E}_{\text{rms}} I_{\text{rms}} \cos\phi = I_{\text{rms}}^2 R$
Inst./max voltages	$v_L = Ldi/dt$ $v_C = q/C$ $v_R = iR$ $V_L = I_m X_L$ $V_C = I_m X_C$ $V_R = I_m R$
Transformer	$\frac{V_s}{V_p} = \frac{N_s}{N_p}$ (voltage ratio) $V_p i_p = V_s i_s$ (power in ideal transformer)
	$R_{eq} = (N_p/N_s)^2 R$ (impedance matching)

Chapter 32 – 33 (Maxwell's equations, E&M waves)

Maxwell' equations	$\oint_S \mathbf{E}\cdot d\mathbf{A} = \frac{Q_{\text{encl}}}{\varepsilon_0}$ $\oint_S \mathbf{B}\cdot d\mathbf{A} = 0$ $\oint_S \mathbf{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ $\oint_S \mathbf{B}\cdot d\vec{s} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$ $i_d \equiv \varepsilon_0 \frac{d\Phi_E}{dt}$
Magnetic dipoles	$\vec{\tau} = \vec{\mu}\times\mathbf{B}$ $\tau = \mu B\sin\theta$ (torque) $U = -\vec{\mu}\cdot\mathbf{B} = -\mu B\cos\theta$ (potential energy)
Angular momentum	$L_{\text{orb z}} = m_l \frac{h}{2\pi}$ $S_z = \pm\frac{1}{2}\frac{h}{2\pi}$
e^- magnetic moments	$\mu_{\text{orb z}} = m_l\left(\frac{-e}{2m_e}\right)L_{\text{orb z}} = -m_l\frac{eh}{4\pi m_e}$ $\mu_{\text{spin z}} = \pm\frac{eh}{4\pi m_e}$ $\mu_B = \frac{eh}{4\pi m_e}$
Magnetization	$M = (\text{Mag. moment})/V$ $M_{\max} = N\mu/V$
Curie's law	$M = CB_{\text{ext}}/T$

Thermal energy $K = \frac{3}{2} k_B T$ per molecule

EM waves $E = E_m \sin(kx - \omega t)$ $B = B_m \sin(kx - \omega t)$ $k = 2\pi / \lambda$ $\omega = 2\pi f$ $c = \lambda f = \omega / k$

$E = Bc$ $c = 1/\sqrt{\mu_0 \varepsilon_0}$ $E_m = \frac{1}{\sqrt{2}} E_{\max}$ $B_{\text{rms}} = \frac{1}{\sqrt{2}} B_m$

Intensity and power $\mathbf{S} = \dfrac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$ $I = S_{\text{ave}} = \dfrac{1}{c\mu_0} E_{\text{rms}}^2 = \dfrac{c}{\mu_0} B_{\text{rms}}^2$

$I = \dfrac{P}{4\pi r^2}$ $p_r = \dfrac{I}{c}$ (abs), $\dfrac{2I}{c}$ (refl)

Polarization $I = I_0 \cos^2 \theta$ $I = \frac{1}{2} I_{\text{unpolarized}}$

Refraction $n_1 \sin \theta_1 = n_2 \sin \theta_2$ $\sin \theta_c = \dfrac{n_2}{n_1}$ $\tan \theta_B = \dfrac{n_2}{n_1}$

Chapter 34 (Images)

Lens equations $\dfrac{1}{p} + \dfrac{1}{i} = \dfrac{1}{f}$ $m = -\dfrac{i}{p}$ $f = \frac{1}{2} R$ (mirror) $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)$ (lensmaker's eqn)

Magnifier $m_\theta = -\dfrac{25\,\text{cm}}{f}$ (image at ∞), $-\left(\dfrac{25\,\text{cm}}{f} + 1\right)$ (maximum)

Telescope, microscope $m_\theta = -\dfrac{f_{\text{obj}}}{f_{\text{eye}}}$ (telescope) $M = \left(-\dfrac{16}{f_{\text{obj}}}\right)\left(\dfrac{25}{f_{\text{eye}}}\right)$ (microscope)

Spherical lens $\dfrac{n_1}{p} + \dfrac{n_2}{i} = \dfrac{n_2 - n_1}{r}$

Chapter 35 – 36 (Interference and diffraction)

Interference $\lambda_n = \lambda / n$ $\Delta\phi = 2\pi \Delta L / \lambda_n$ $\Delta L =$ path length

Thin film $2d = m\lambda_{n_1}$ (constructive) $= \left(m + \frac{1}{2}\right)\lambda_{n_1}$ (destructive) $(n_0 < n_1 < n_2)$

Diffraction $d \sin \theta = m\lambda$ (max, 2 slit or grating) $a \sin \theta = m\lambda$ (min, single slit)

$\Delta\theta = 1.22\lambda / D$ (Rayleigh criterion) $\Delta\theta \approx \lambda / d$ (interferometer resolution)

Intensity diffraction $I = I_m \left(\dfrac{\sin \alpha}{\alpha}\right)^2$ (1 slit) $I = I_m \left(\dfrac{\sin \alpha}{\alpha}\right)^2 \cos^2 \beta$ (2 slit) $\alpha = \dfrac{\pi a \sin \theta}{\lambda}$ $\beta = \dfrac{\pi d \sin \theta}{\lambda}$

Diffraction grating $I = I_m \left(\dfrac{\sin \alpha}{\alpha}\right)^2 \left(\dfrac{\sin N\beta}{N \sin \beta}\right)^2$ (N slits) $\Delta\theta_{hw} = \dfrac{\lambda}{Nd \cos \theta}$ (half-width of maxima)

$D = \dfrac{\Delta\theta}{\Delta\lambda} = \dfrac{m}{d \cos \theta}$ (dispersion) $R \equiv \dfrac{\lambda_{\text{avg}}}{\Delta\lambda} = Nm$ (resolving power)