Mechanics II Formula Sheet

Lagrangian Mechanics

Lagrangian:

$$L = T - V$$

where T is kinetic energy, V is potential energy.

Euler-Lagrange Equation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

where q_i are generalized coordinates, $\dot{q}_i = \frac{dq_i}{dt}$.

Generalized Momentum:

$$p_i = \frac{\partial L}{\partial \dot{q}_i}$$

Conserved Quantities:

$$\text{If } \frac{\partial L}{\partial q_i} = 0, \text{ then } p_i \text{ is conserved.}$$

Hamiltonian Mechanics

Hamiltonian:

$$H = \sum_i p_i \dot{q}_i - L$$

where H is typically $T + V$ for standard systems.

Hamilton's Equations:

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

Oscillations

Simple Harmonic Motion:

$$\ddot{x} + \omega^2 x = 0, \quad \omega = \sqrt{\frac{k}{m}}$$

where ω is angular frequency, k is spring constant, m is mass.

Solution:

$$x(t) = A\cos(\omega t + \phi), \quad v(t) = -A\omega \sin(\omega t + \phi)$$

Energy in SHM:

$$E = \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

Damped Harmonic Motion:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0, \quad \beta = \frac{b}{2m}, \quad \omega_0 = \sqrt{\frac{k}{m}}$$

where b is the damping constant.

Driven Oscillations:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = \frac{F_0}{m}\cos(\omega t)$$

Central Force Motion

Effective Potential:

$$V_{\text{eff}}(r) = V(r) + \frac{L^2}{2\mu r^2}$$

where μ is reduced mass, L is angular momentum.

Orbital Equation:

$$\frac{d^2u}{d\theta^2} + u = -\frac{\mu}{L^2}\frac{dV}{du}, \quad u = \frac{1}{r}$$

Kepler's Laws:

$$\text{1. Elliptical orbits, 2. Equal areas in equal times, 3. } T^2 = \frac{4\pi^2}{GM}a^3$$

where T is period, a is semi-major axis, $G = 6.674\,30 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$.

Rigid Body Dynamics

Moment of Inertia:

$$I = \sum m_i r_i^2 \quad \text{or} \quad I = \int r^2\,dm$$

Angular Momentum:

$$\mathbf{L} = I\omega$$

Torque:

$$\tau = \frac{d\mathbf{L}}{dt} = I\alpha$$

Rotational Kinetic Energy:

$$K = \frac{1}{2}I\omega^2$$

Parallel-Axis Theorem:

$$I = I_{\text{cm}} + Md^2$$

where I_{cm} is moment of inertia about center of mass, d is distance to parallel axis.

Non-Inertial Frames

Apparent Forces:

$$\mathbf{F}_{\text{eff}} = m\mathbf{a} - m\mathbf{a}_{\text{frame}} - 2m(\mathbf{v} \times \square) - m(\square \times (\mathbf{r} \times \square))$$

where $\mathbf{a}_{\text{frame}}$ is frame acceleration, □ is frame angular velocity, Coriolis and centrifugal terms included.

Special Relativity (Introductory)

Time Dilation:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where Δt_0 is proper time, $c = 3.00 \times 10^8\,\mathrm{m\,s^{-1}}$.

Length Contraction:

$$L = \frac{L_0}{\gamma}$$

where L_0 is proper length.

Relativistic Momentum:

$$\mathbf{p} = \gamma m\mathbf{v}$$

Relativistic Energy:

$$E = \gamma mc^2, \quad E_0 = mc^2, \quad E^2 = (pc)^2 + (mc^2)^2$$