Linear Algebra

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1 Task 1

Given the following basis vectors:

$$e_1 = (1,1,1)$$

$$e_2 = (1,1,2)$$

$$e_3 = (1,-1,1)$$

And vector x:

$$x = (6,9,14)$$

Check if the basis vectors are linearly independent by placing each vector as a column in matrix and calculating the **rank**:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$$

Calculate rank:

$$\Delta_1 = 1 \cdot 1 \cdot 1 + 1 \cdot (-1) \cdot 1 + 1 \cdot 1 \cdot 2 - 1 \cdot 1 \cdot 1 - 1 \cdot 1 \cdot 1 - 1 \cdot (-1) \cdot 2 = 2$$

$$rank(A) = 3$$

Because the **rank** is 3, the same amount as the length of the vectors, this means that the basis vectors are linearly independent.

x expressed as a vector inside our basis:

$$x = c_1 e_1 + c_2 e_2 + c_3 e_3$$

$$\begin{cases} c_1 + c_2 + c_3 = x_1 \\ c_1 + c_2 - c_3 = x_2 \\ c_1 + 2c_2 + c_3 = x_3 \end{cases}$$