Discrete Mathematics

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1 Task 1

We are given:

$$A = \{1, 2, 3, 4, 5\}$$

And a relation matrix a where $R \subseteq A \times A$:

$$a = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

We need to find whether relation a is **reflextive**, **symmtetric** and **transitive**, if not then find their **closures** respectively.

1.1 Reflexivity

A relation is **reflexive** if every element is related to itself, e.g., $a_{ii} = 1$ for all i.

This means that the diagonal elements are all 1's, but it's not in our case therefore relation a is **not reflexive**.

1.1.1 Reflexive Closure

Reflexive closure for relation a would be a_{ref} where all the diagonal elements are 1's:

$$a_{ref} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

1.2 Symmetry

A relation is **symmetric** if $a_{ij} = a_{ji}$.
We can check this by transposing our relation matrix and if $T(a) = a$ then the relation is **symmetric**:

$$T(a) = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

We can clearly observe that the transposed matrix differs from the original relation matrix therefore:

$$T(a) \neq a$$

So the relation a is **not symmetric**.

1.2.1 Symmetric Closure

The **symmetric closure** for relation a would be a_{sym} where every $a_{ij} = a_{ji} = 1$:

$$a_{sym} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

1.3 Transitivity

A relation is **transitive** if whenever $(i, j) \in R$ and $(j, k) \in R$, then $(i, k) \in R$. We can easily check by taking the **boolean** square of the relation matrix a:

$$a^2 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

If all the 1's in a^2 exists in a then the relation is **transitive**, but this is not the case for us therefore the relation a is **not transitive**.

1.3.1 Transitive Closure

We can use Warshall's algorithm to find the transitive closure for relation a. After applying Warshall's algorithm to our initial relation matrix we get:

$$a_{trans} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

2 Task 2

We are asked to find how many integers below or equal to 1000 are divisible by 8, 22 or 32.

Let:

- A: Numbers divisible by 8
- B: Numbers divisible by 22
- C: Numbers divisible by 32

2.1 Calculate the numbers that 1000 is divisible by each number

We start by finding how many nymbers up to 1000 are divisible by:

8: Divide 1000 by 8:

$$\left\lfloor \frac{1000}{8} \right\rfloor = 125$$

22: Divide 1000 by 22:

$$\left\lfloor \frac{1000}{22} \right\rfloor = 45$$

32: Divide 1000 by 32:

$$\left\lfloor \frac{1000}{22} \right\rfloor = 31$$

2.2 Substract overlap

Then we will need to subtract the overlap numbers counted twice.
Some numbers are disible by both 8 and 22, or 8 and 32, etc.
We subtract them so we don't count them twice.

To do this, we find how many numbers are divisible by both pairs:

- 8 and 22: Find LCM (Least Common Multiple) of 8 and 22

$$LCM(8, 22) = 88$$

$$\left\lfloor \frac{1000}{88} \right\rfloor = 11$$

- 8 and 32: Find LCM of 8 and 32

$$LCM(8,32) = 32$$

$$\left\lfloor \frac{1000}{32} \right\rfloor = 31$$

- 22 and 32: Find LCM of 22 and 32

$$LCM(22,32) = 352$$

$$\left\lfloor \frac{1000}{352} \right\rfloor = 2$$

2.3 Add back numbers counted 3 times

Some numbers are divisible by all three so we add these back because we subtracted them earlier.

- $LCM(8,22,32) = 352$

$$\left\lfloor \frac{1000}{352} \right\rfloor = 2$$

2.4 Putting it all together using Inclusion-Exclusion Formula

The **Inclusion-Exclusion** Formula states the following:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Plug in our values:

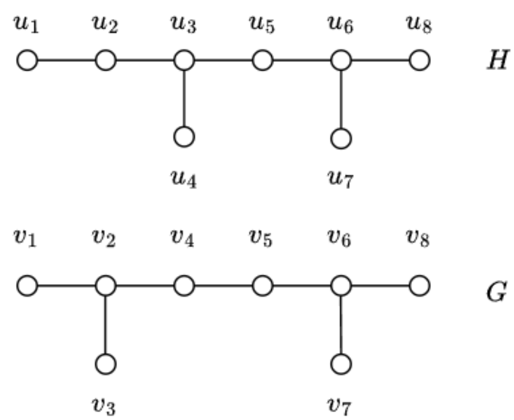
- $A = 125$
- $B = 45$
- $C = 31$
- $AB = 11$
- $AC = 31$
- $BC = 2$
- $ABC = 2$

Therefore:

$$125 + 45 + 31 - 11 - 31 - 2 + 2 = 159$$

3 Task 3

We are given graphs H and G:



We must determine if the given graphs are **isomorphic**. First we recall the definition of two isomorphic graphs:

Let $G_1 = (V_1, E_1)$ & $G_2 = (V_2, E_2)$ be two simple graphs where:

- V_1, V_2 are the sets of vertices
- E_1, E_2 are the sets of edges

Then G_1 & G_2 are **isomorphic** if there exists a bijective function $\varphi : V_1 \to V_2$ such that:

$$\{u, v\} \in E_1, \text{ if any only if } \{\varphi(u), \varphi(v)\} \in E_2$$

In simpiler terms G & H are **isomorphic** if there exists a bijection function φ that directly maps G's vertices to H's vertices (and vice-versa) and that each degree of both vertices match.

In our case it is visually apparent that these two graphs are not **isomorphic**. More specificaally, the degrees of u_3, v_3 do not match, u_3 has degree of two while v_3 has degree of just one.
Therefore these two graphis are not **isomorphic**.