

1. GENERATION OF FINITE MESH

The first stage in the computational process of FEM, is to generate the finite mesh over the domain of the problem. There are several different methods that can be used to generate the mesh, the method that this paper utilizes is Delaunay triangulation. An alternative that is less utilized is the method of polygon clipping. The scope of this paper is restricted to two dimensional domains. However, the method for mesh generation is easily generalizable to higher dimensions.

The generation of a mesh using Delaunay triangulation has several stages.

- (1) Delaunay triangulation
- (2) Applying boundaries
- (3) Mesh refinement

1.1. Delaunay Triangulation. A Delaunay triangulation for a set of points P is a triangulation $DP(P)$ such that no point in P is in the circumcircle of any triangle in the triangulation $DP(P)$. This form of triangulation maximizes the minimum angle of all triangles in the triangulation, thus avoiding sliver triangles.

The generation of the Delaunay triangulation has several different algorithms, and is constantly advancing in efficiency. There are three main algorithms that are documented for the generation of the triangulation.

- (1) Edge flipping
- (2) Incremental
- (3) Divide and conquer

It is important to note that each algorithm will produce the same resulting triangulation. This is due to the fact that the Delaunay triangulation associated with a set of points is unique, with one exceptions of the points on a square.

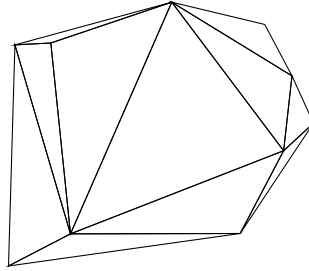


FIGURE 1. Example of Delaunay Triangulation

1.1.1. Edge Flipping. This method for generating the Delaunay triangulation, is a very simple algorithm, and not very efficient.

1.1.2. Incremental. This method for generating the Delaunay triangulation is better than the 1.1.1 algorithms, but can still be improved upon.

1.1.3. Divide and Conquer. This algorithm for constructing the Delaunay triangulation is currently shown to be the fastest, and allows for optimization in computational execution that the previous algorithms lacked.

1.2. Applying Boundaries. Now that the Delaunay triangulation has been generated, the next stage is to apply the constraints of the boundary conditions to the triangulation.

1.3. Mesh Refinement. The final stage is to refine the mesh, because the restriction of the boundary conditions are able to cause non-delaunay triangles. This stage also applies larger restrictions to the triangles, resulting is “nicer” triangles with restricted minimum angles.