FEM

ARDEN RASMUSSEN

1. Time dependent

$$
\begin{aligned}
\partial_t u &= Lu + f \\
L &= A^{ij}\partial_i\partial_j + B^i\partial_i + C \\
R(x,t) &= \partial_t u - Lu - f \\
\partial_t \langle u|\varphi_i\rangle &= \langle Lu|\varphi_i\rangle + \langle f|\varphi_i\rangle \\
\sum_j \langle \varphi_j|\varphi_i\rangle \, \partial_t u_j &= \sum_j \langle L\varphi_j|\varphi_i\rangle \, u_j + \langle f|\varphi_i\rangle \\
G\partial_t U &= MU + F \\
\widetilde{A} &= G + \frac{1}{2}\Delta t m \\
\widetilde{B} &= G - \frac{1}{2}\Delta t m \\
\widetilde{C}^n &= \Delta t \frac{F^{n+1} + F^n}{2} \\
Q^n &= \widetilde{B}U^n + \widetilde{C}^n \\
\widetilde{A}U^{n+1} &= Q^n
\end{aligned}
$$

1.1. Notes.

- G and M are only dependent on the mesh, and so they can be computed once and saved.
- F is only dependent on time t, so it can be computed separately from the main process.
- $\widetilde{A}$, $\widetilde{B}$ are independent of both time t and iteration n, so they can be computed once and then saved.
- $\widetilde{C}^n$ is only dependent on n and t, it is not dependent on the iteration, it is not necessary to know U^n in order to compute $\widetilde{C}^n$.

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2. Time independent

$$
\begin{aligned}
0 &= Lu + f \\
L &= A^{ij}\partial_i\partial_j + B^i\partial_i + C \\
R(x,t) &= Lu + f \\
0 &= \langle Lu|\varphi_i\rangle + \langle f|\varphi_i\rangle \\
\sum_j \langle L\varphi_j|\varphi_i\rangle\, u_j &= -\langle f|\varphi_i\rangle \\
MU &= -F
\end{aligned}
$$

2.1. **Notes.**

- M is dependent only on the mesh, and so can be computed once and saved.
- F is only dependent on the forcing function, so it can be computed once there and saved.