2D FEM REPORT

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1. Discretization

I have successfully implement a fast construction of an unconstrained Delaunay triangulation, but I am currently running into issues with the construction of the constrained Delaunay triangulation. I understand the method, there is some issue in my first few attempts at the code, so I will review the algorithm more and try again to implement it.

I have implemented the construction of local basis function based upon the *barycentric* coordinate system. I need to determine a method to implement the construction of the global basis functions. The global basis functions should be a piecewise combination of N different local basis functions which all have their peak at a given vertex. The difficulty is that each of the parts of the piecewise is dependent on which triangular element the point of evaluation is within. Thus I need to develop an efficient method for determining the triangle in the mesh that a point is contained within. With such a method, the evaluation of the global basis functions will be trivial.

2. Numerical Approximation

I have a fully functional and surprisingly accurate numerical integration over a triangle, which uses the barycentric coordinates and a set of points and weights to evaluate the integral on the triangular domain. Its an extension of Gaussian quadrature integration, just expanded into 2d and for a triangle. Currently I have mine enabled to be accurate up to a degree 8 polynomial, which I believe to be sufficient for most cases that the FEM will need it for.

I have also looked further into the residual approximation of sparse matrices, and the krylov spaces. I haven't much opportunity to create an implementation of it yet, but I will likely do so this weekend.

3. Post processing

A very important thing to to be able to display the images of the generated results. So I have put some work into implementing a method for generating the two dimensional images of the domains. So far I have implemented a system to accept an arbitrary function from $\mathbb{R}^2 \to \mathbb{R}$ and it utilizes color maps to display the z value of the function. I have also implemented the use of mesh based domains, where only points within the mesh are evaluated and plotted. Again with the same issue as above, I can only get this to work nicely with strictly convex meshes.

I still need to implement text rendering so that I can add X, Y, and Z axis labels, and I want to add a legend as well, which should not be too difficult.

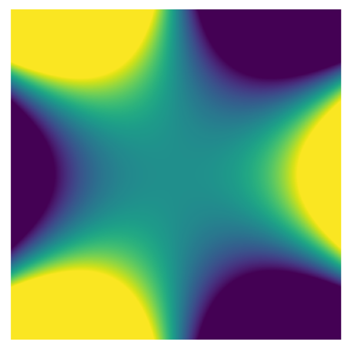

Figure 1. $z = x^3 - 3xy^2$ on $(-5,-5)$ to $(5,5)$

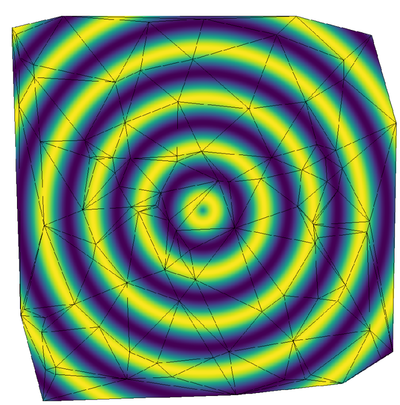

Figure 2. $z = \sin(0.05 * \sqrt{x^2+y^2})$ on a mesh

Date: November 22, 2018.