RENDERING EQUATION AND BSDF

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1. Rendering Equation

$$L_o(\mathbf{p},\mathbf{o}) = L_e(\mathbf{p},\mathbf{o}) + \int_{\mathcal{S}^2} f_s(\mathbf{i},\mathbf{o},\mathbf{n})\, L_i(\mathbf{p},\mathbf{i})\, |\mathbf{i}\cdot\mathbf{n}|\, d\mathbf{i}$$

L_o**:** The light going out of a point in a given direction.
L_e**:** The light emmited from a point in a given direction.
Li**:** The light comming into a point from a given direction.
p: Location of point in space.
$\mathcal{S}^2$**:** Is the unit sphere, containing all possible **i**.
Ω**:** Is the unit hemisphere centered around **n**.
i: Is the direction of incident ray of light, for the most case this will be the negative of the direction along which we just ray traced/marched.
o: Is the negative direction of the outgoing ray oflight, this will be the direction along which we send new rays for tracing.
n: Is the surface normal at the point **p**.
f_s**:** Is the BSDF given from the objects material.

2. BSDF

$$f_s(\mathbf{i},\mathbf{o},\mathbf{n}) = f_r(\mathbf{i},\mathbf{o},\mathbf{n}) + f_t(\mathbf{i},\mathbf{o},\mathbf{n})$$

$$f_r(\mathbf{i},\mathbf{o},\mathbf{n}) = \frac{F(\mathbf{i},\mathbf{h_r})\, G(\mathbf{i},\mathbf{o},\mathbf{h_r})\, D(\mathbf{h_r})}{4\,|\mathbf{i}\cdot\mathbf{n}|\,|\mathbf{o}\cdot\mathbf{n}|}$$

$$f_t(\mathbf{i},\mathbf{o},\mathbf{n}) = \frac{|\mathbf{i}\cdot\mathbf{h_t}|\,|\mathbf{o}\cdot\mathbf{h_t}|}{|\mathbf{i}\cdot\mathbf{n}|\,|\mathbf{o}\cdot\mathbf{n}|} \frac{\eta_o^2\,(1-F(\mathbf{i},\mathbf{h_t}))\, G(\mathbf{i},\mathbf{o},\mathbf{h_t})\, D(\mathbf{h_t})}{(\eta_i(\mathbf{i}\cdot\mathbf{h_t}) + \eta_o(\mathbf{o}\cdot\mathbf{h_t}))^2}$$

$$\mathbf{h_r} = \frac{\mathbf{i}+\mathbf{o}}{\|\mathbf{i}+\mathbf{o}\|}$$

$$\mathbf{h_t} = -\frac{\mathbf{i}+\eta\mathbf{o}}{\|\mathbf{i}+\mathbf{o}\|}$$

η_o**:** Is the index of refraction medium of the outgoing light.
η_i**:** Is the index of refraction medium of the incoming light.
η**:** is the ratio of the IORs of the two media, $\eta = \frac{\eta_o}{\eta_i}$.