3 Recursion

In this chapter

- You learn about recursion. Recursion is a coding technique used in many algorithms. It's a building block for understanding later chapters in this book.
- You learn what a base case and a recursive case is. The divide-and-conquer strategy (chapter 4) uses this simple concept to solve hard problems.

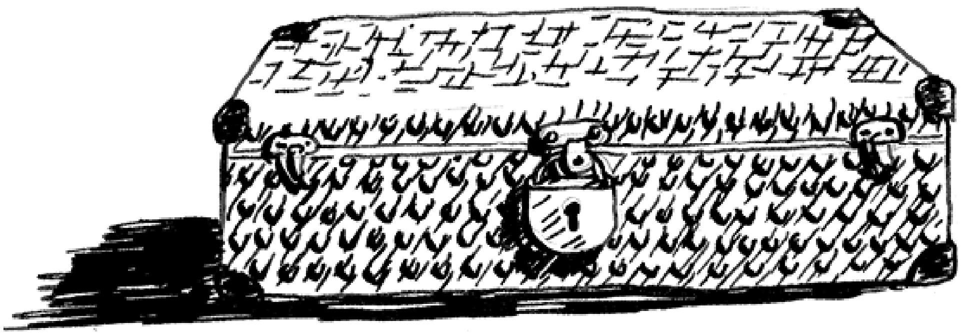
I'm excited about this chapter because it covers *recursion*, an elegant way to solve problems. Recursion is one of my favorite topics, but it's divisive. People either love it or hate it—or hate it until they learn to love it a few years later. I personally was in that third camp. To make things easier for you, I have some advice:

- This chapter has a lot of code examples. Run the code for yourself to see how it works.
- I'll talk about recursive functions. At least once, step through a recursive function with pen and paper: something like, "Let's see, I pass 5 into `factorial`, and then I return five times passing 4 into `factorial`, which is . . . ," and so on. Walking through a function like this will teach you how a recursive function works.

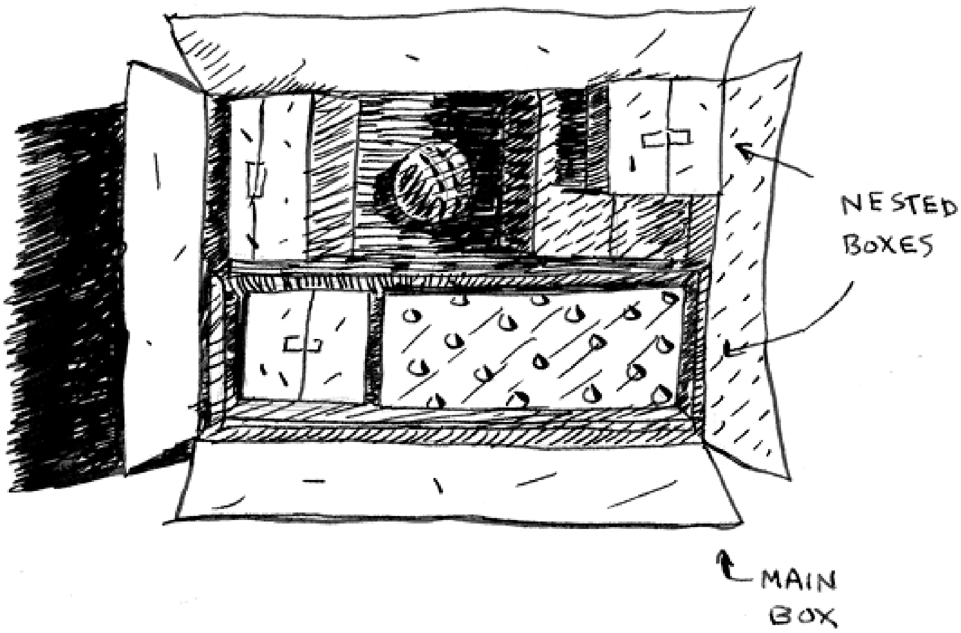
This chapter also includes a lot of pseudocode. *Pseudocode* is a high-level description in code of the problem you're trying to solve. It's written like code, but it's meant to be closer to human speech.

Recursion

Suppose you're digging through your grandma's attic and come across a mysterious locked suitcase.

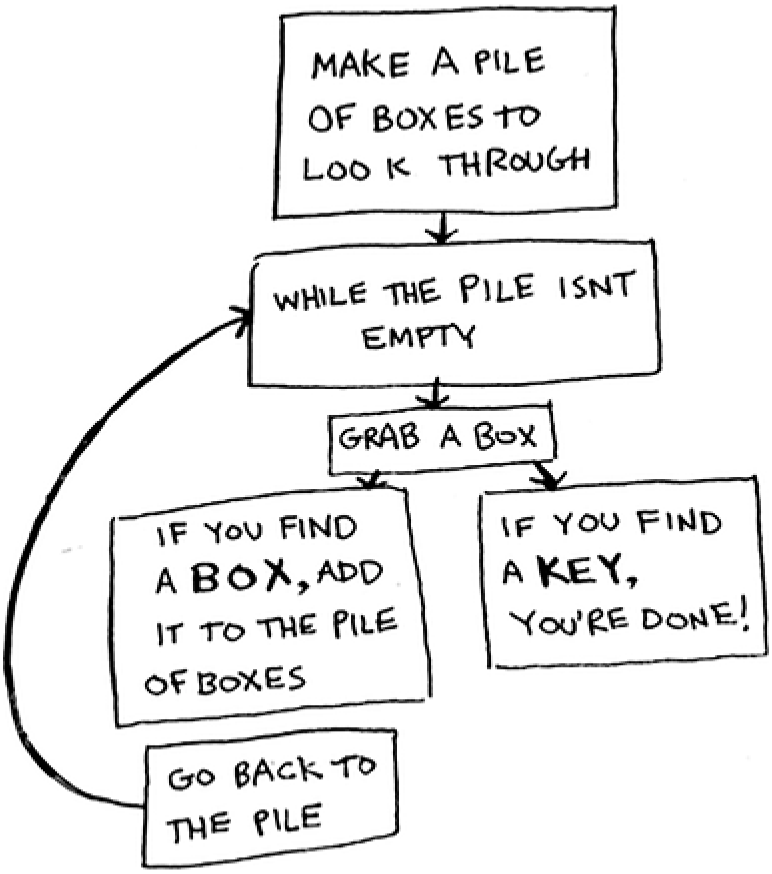


Grandma tells you that the key for the suitcase is probably in this other box.



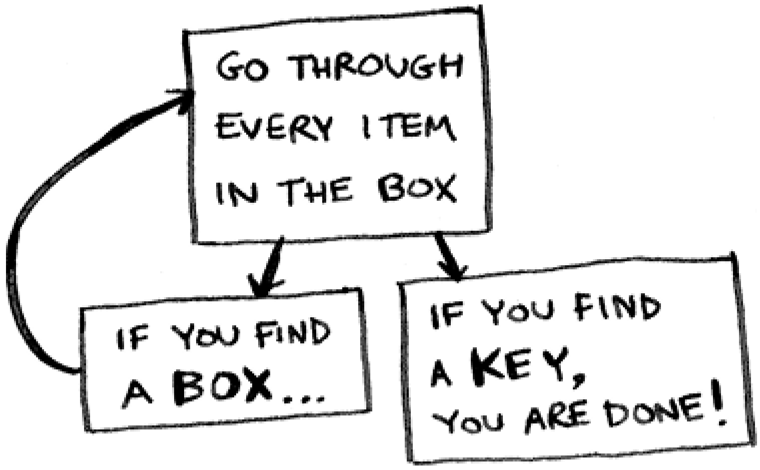
This box contains more boxes, with more boxes inside those boxes. The key is in a box somewhere. What's your algorithm to search for the key? Think of an algorithm before you read on.

Here's one approach:



1. Make a pile of boxes to look through.
2. Grab a box and look through it.
3. If you find a box, add it to the pile to look through later.
4. If you find a key, you’re done!
5. Repeat.

Here’s an alternate approach:



1. Look through the box.
2. If you find a box, go to step 1.
3. If you find a key, you’re done!

Which approach seems easier to you? The first approach uses a `while` loop. While the pile isn't empty, grab a box and look through it. Here's some pseudocode:

```
def look_for_key(main_box):
  pile = main_box.make_a_pile_to_look_through()
  while pile is not empty:
    box = pile.grab_a_box()
    for item in box:
      if item.is_a_box():
        pile.append(item)
      elif item.is_a_key():
        print("found the key!")
```

The second way uses recursion. *Recursion* is where a function calls itself. Here's the second way in pseudocode:

```
def look_for_key(box):
  for item in box:
    if item.is_a_box():
      look_for_key(item)      ①
    elif item.is_a_key():
      print("found the key!")
```

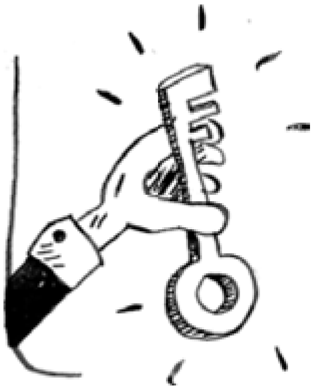
① Recursion!

Both approaches accomplish the same thing, but the second approach is clearer to me. Recursion is used when it makes the solution clearer. There's no performance benefit to using recursion; in fact, loops are sometimes better for performance. I like this quote by Leigh Caldwell on Stack Overflow: "Loops may achieve a performance gain for your program. Recursion may achieve a performance gain for your programmer. Choose which is more important in your situation!" (http://stackoverflow.com/a/72694/139117).

Many important algorithms use recursion, so it's important to understand the concept.

Base case and recursive case

Because a recursive function calls itself, it's easy to write a function incorrectly that ends up in an infinite loop.



For example, suppose you want to write a function that prints a countdown like this:

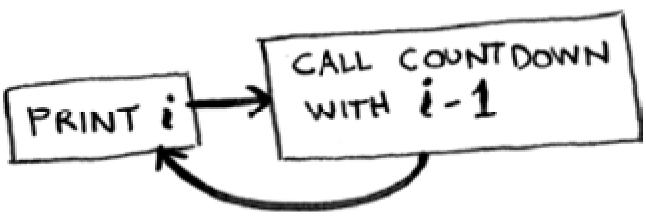
```
> 3...2...1
```

You can write it recursively like so:

```
def countdown(i):
  print(i)
  countdown(i-1)

countdown(3)
```

Write out this code and run it. You'll notice a problem: this function will run forever!



Infinite loop

```
> 3...2...1...0...-1...-2...
```

(Press Ctrl-C to kill your script.)

When you write a recursive function, you have to tell it when to stop recursing. That's why *every recursive function has two parts: the base case and the recursive case.* The recursive case is when the function calls itself. The base case is when the function doesn't call itself again, so it doesn't go into an infinite loop.

Let's add a base case to the countdown function:

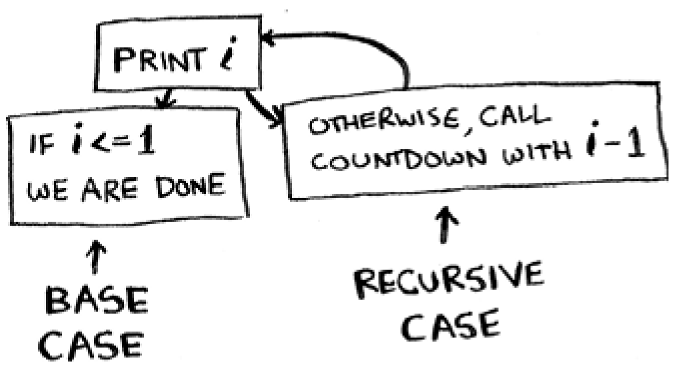
```
def countdown(i):
  print(i)
  if i <= 1:          ①
    return
  else:               ②
    countdown(i-1)

countdown(3)
```

① Base case

② Recursive case

Now the function works as expected. It goes something like this.



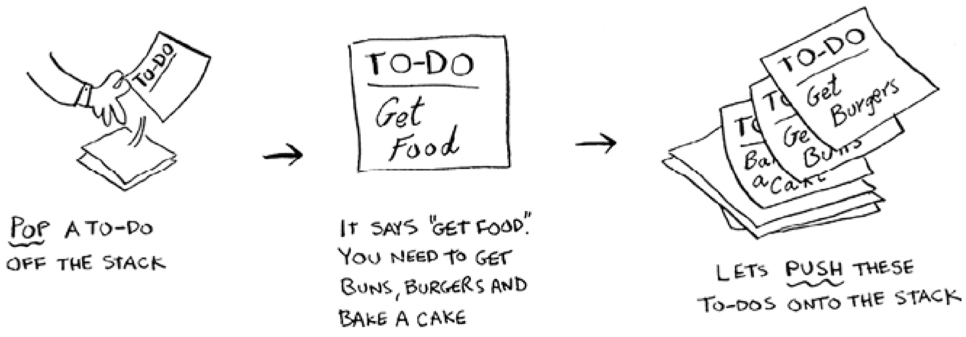
The stack

This section covers the *call stack*. The call stack is an important concept in general programming, and it's also important to understand when using recursion.

Suppose you’re throwing a barbecue. You keep a to-do list for the barbecue, in the form of a stack of sticky notes.

Remember back when we talked about arrays and lists, and you had a to-do list? You could add to-do items anywhere to the list or delete random items. The stack of sticky notes is much simpler. When you insert an item, it gets added to the top of the list. When you read an item, you only read the topmost item, and it’s taken off the list. So your to-do list has only two actions: *push* (insert) and *pop* (remove and read).

Let’s see the to-do list in action.



This data structure is called a *stack*. The stack is a simple data structure. You've been using a stack this whole time without realizing it!

The call stack

Your computer uses a stack internally called the *call stack.* Let's see it in action. Here's a simple function:

```
def greet(name):
    print("hello, " + name + "!")
    greet2(name)
    print("getting ready to say bye...")
    bye()
```

This function greets you and then calls two other functions. Here are those two functions:

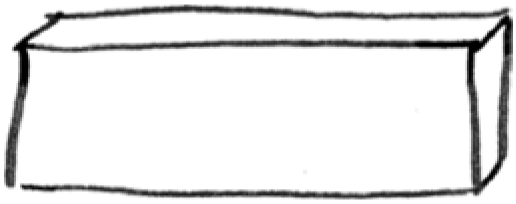
```
def greet2(name):
    print("how are you, " + name + "?")

def bye():
    print("ok bye! ")
```

Let's walk through what happens when you call a function.

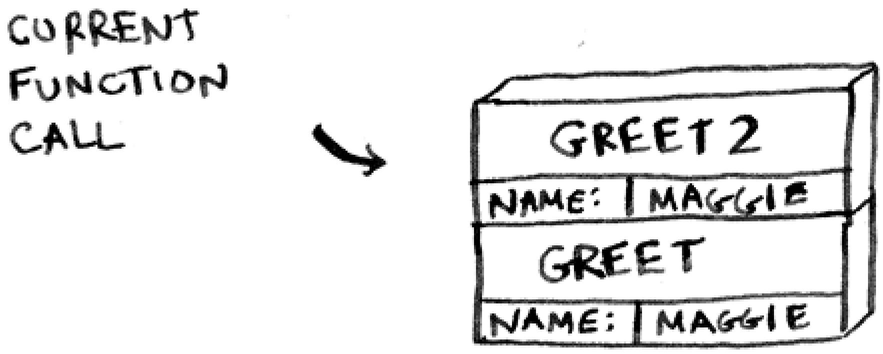
NOTE To keep things simple, I'm only showing the calls to `greet` , `greet2` , and `bye` . I'm not showing the calls to the `print` function.

Suppose you call `greet("maggie")` . First, your computer allocates a box of memory for that function call.

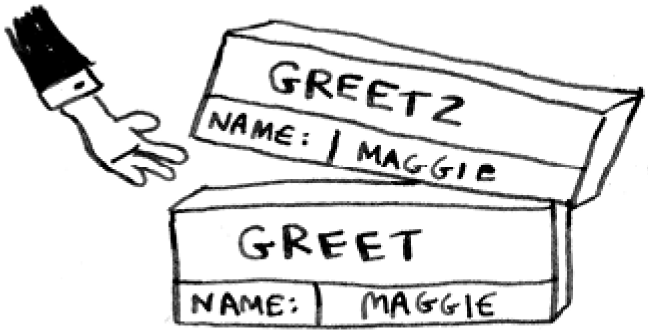


Now let's use the memory. The variable `name` is set to `"maggie"`. That needs to be saved in memory.

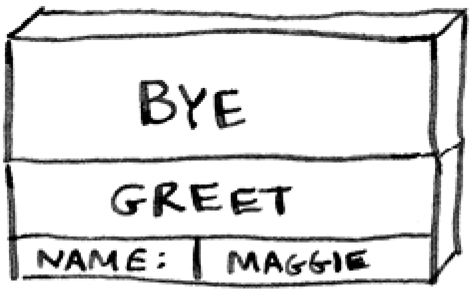
Every time you make a function call, your computer saves the values for all the variables for that call in memory like this. Next, you print `hello, maggie!` Then you call `greet2("maggie").` Again, your computer allocates a box of memory for this function call.



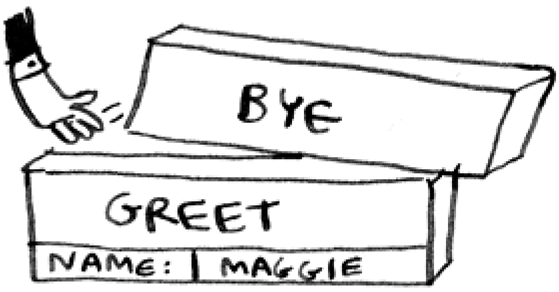
Your computer is using a stack for these boxes. The second box is added on top of the first one. You print `how are you, maggie?` Then you return from the function call. When this happens, the box on top of the stack gets popped off.



Now the topmost box on the stack is for the `greet` function, which means you returned to the `greet` function. When you called the `greet2` function, the `greet` function was *partially completed.* This is the big idea behind this section: *when you call a function from another function, the calling function is paused in a partially completed state.* All the values of the variables for that function are still stored on the call stack (i.e., in memory). Now that you're done with the `greet2` function, you're back to the `greet` function, and you pick up where you left off. First, you print `getting ready to say bye...` Then you call the `bye` function.

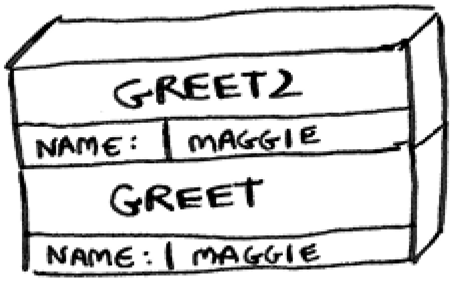


A box for that function is added to the top of the stack. Then you print `ok bye!` and return from the function call.



And you're back to the `greet` function. There's nothing else to be done, so you return from the `greet` function, too. This stack, used to save the variables for multiple functions, is called the *call stack*.

3.1 Suppose I show you a call stack like this.



What information can you give me, just based on this call stack?

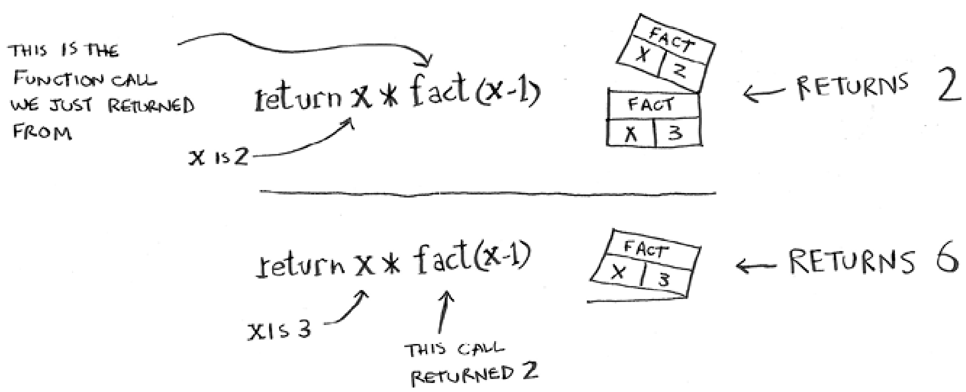
Now let's see the call stack in action with a recursive function.

The call stack with recursion

Recursive functions use the call stack, too! Let's look at this in action with the `factorial` function. `factorial(5)` is written as 5!, and it's defined like this: 5! = 5 * 4 * 3 * 2 * 1. Similarly, `factorial(3)` is 3 * 2 * 1. Here's a recursive function to calculate the factorial of a number:

```
def fact(x):
  if x == 1:
    return 1
  else:
    return x * fact(x-1)
```

Now you can call `fact(3)`. Let's step through this call line by line and see how the stack changes. Remember, the topmost box in the stack tells you what call to `fact` you're currently on.



Notice that each call to `fact` has its own copy of `x`. You can't access a different function's copy of `x`.

The stack plays a big part in recursion. In the opening example, there were two approaches to finding the key. Here's the first way again.

This way, you make a pile of boxes to search through, so you always know what boxes you still need to search.

But in the recursive approach, there's no pile.

If there’s no pile, how does your algorithm know what boxes you still have to look through? Here’s an example.

At this point, the call stack looks like this.

The “pile of boxes” is saved on the stack! This is a stack of half-completed function calls, each with its own half-complete list of boxes to look through. Using the stack is convenient because you don’t have to keep track of a pile of boxes yourself—the stack does it for you.

Using the stack is convenient, but there’s a cost: saving all that info can take up a lot of memory. Each of those function calls takes up some memory, and when your stack is too tall, that means your computer is saving information for many function calls. At that point, you have two options:

- You can rewrite your code to use a loop instead.
- You can use something called *tail recursion*. That’s an advanced recursion topic that is out of the scope of this book. It’s also only supported by some languages, not all.

Exercise

3.2 Suppose you accidentally write a recursive function that runs forever. As you saw, your computer allocates memory on the stack for each function call. What happens to the stack when your recursive function runs forever?

Recap

- Recursion is when a function calls itself.

- Every recursive function has two cases: the base case and the recursive case.
- A stack has two operations: push and pop.
- All function calls go onto the call stack.
- The call stack can get very large, which takes up a lot of memory.