$$J_{neg-sample}(o, \boldsymbol{v}_c, \boldsymbol{U}) = -\ln(\sigma(\boldsymbol{u}_o^\top \boldsymbol{v}_c)) - \sum_{k=1}^{K} \ln(\sigma(-\boldsymbol{u}_k^\top \boldsymbol{v}_c)) \tag{1}$$

$$\sigma(x) = \frac{1}{1+e^{-x}} \tag{2}$$

$$\begin{aligned}
\frac{\partial \sigma(x)}{\partial x} &= -\frac{1}{(1+e^{-x})^2} \cdot e^{-x} \cdot (-1) \\
&= \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}} \\
&= \frac{1}{1+e^{-x}} \cdot \left(1 - \frac{1}{1+e^{-x}}\right) \\
&= \sigma(x) \cdot (1-\sigma(x))
\end{aligned} \tag{3}$$

$$\begin{aligned}
\frac{\partial J}{\partial \boldsymbol{v}_c} &= -\frac{\partial \ln(\sigma(\boldsymbol{u}_o^\top \boldsymbol{v}_c))}{\partial \boldsymbol{v}_c} - \frac{\partial \sum_{k=1}^{K} \ln(\sigma(-\boldsymbol{u}_k^\top \boldsymbol{v}_c))}{\partial \boldsymbol{v}_c} \\
&= -\frac{1}{\sigma(\boldsymbol{u}_o^\top \boldsymbol{v}_c)} \sigma(\boldsymbol{u}_o^\top \boldsymbol{v}_c)(1-\sigma(\boldsymbol{u}_o^\top \boldsymbol{v}_c)\boldsymbol{u}_o \\
&\quad - \sum_{k=1}^{K} \frac{1}{\sigma(-\boldsymbol{u}_k^\top \boldsymbol{v}_c)} \sigma(-\boldsymbol{u}_k^\top \boldsymbol{v}_c)(1-\sigma(-\boldsymbol{u}_k^\top \boldsymbol{v}_c)(-\boldsymbol{u}_k) \\
&= (\sigma(\boldsymbol{u}_o^\top \boldsymbol{v}_c) - 1)\boldsymbol{u}_o + \sum_{k=1}^{K} (\sigma(-\boldsymbol{u}_k^\top \boldsymbol{v}_c) - 1)(-\boldsymbol{u}_k)
\end{aligned} \tag{4}$$

$$\frac{\partial J}{\partial \boldsymbol{u}_o} = (\sigma(\boldsymbol{u}_o^\top \boldsymbol{v}_c) - 1)\boldsymbol{v}_c \tag{5}$$

$$\frac{\partial J}{\partial \boldsymbol{u}_k} = \sum_{k=1}^{K} (\sigma(-\boldsymbol{u}_k^\top \boldsymbol{v}_c) - 1)(-\boldsymbol{v}_c) \tag{6}$$