# **Beautiful Mind**

Abacus 2017 - Questions

1. The game of Nim is a fairly popular strategy game. For those of you who are not familiar with it, here are the rules:  
    a) Two players play the game.  
    b) There are some number of piles, each pile consisting a non-zero number of murukkus.  
    c) During each turn, a player picks a non-zero number of murukkus from a non-empty pile. The turn alternates between the players.  
    d) The player who picks the last murukku from the last non-empty pile wins.  
   Now, there are ten piles and these are the number of murukkus in each pile - {4567, 234, 678, 909, 345, 8888, 789, 1234, 987, 56789}. It is Geek Sundaram’s turn to play. He realises he has no chance of winning as he knows his opponent Nair plays optimally. So, Geek decides to cheat by adding murukkus to one or more piles when Nair is not looking, so as to make sure he never loses if he plays optimally. Help Geek find the minimum number of murukkus he must add to the piles.  
   **Answer : 39770 Marks : 5**
2. Geek Sundaram is playing with stones. He has 127 stones with each stone having a number from 1 to 127 painted on it. No two stones have the same number painted on them. Initially, the stones arranged on top of one another in tile A. Tile B and tile C are empty. Now, Geek can do the following things:  
    a) He can pick the stone currently at the top of the pile at tile A and place it either at the top of the pile at tile B or at the top of the pile at tile C.  
    b) He can pick the stone currently at the top of the pile at tile B and place it at the top of the pile at tile C.  
   Geek’s goal is to move all the stones in tile A to tile C in a such a way that the resulting pile at tile C will have numbers in an increasing order bottom to top. Geek wonders if he can achieve his goal irrespective of how the stones were arranged initially in tile A. Help Geek find the number of initial permutations of stones in tile A that makes his goal achievable.  
   **Answer : 11311095732253345760960290897769189975961199415637572612957718759342193629 Marks : 2**
3. Geek Sundaram buys a packet of Gems. There are 140 gems, 20 for each of the seven colors available. Geek Sundaram picks 15 gems from the packet at random. What is the number of distinct colors he can expect to be present among the 15 gems chosen? (Enter your answer with a precision of 5 after the decimal point)  
   **Answer : 6.39442 Marks : 2**
4. Geek Sundaram takes integers consisting only the digits ‘5’ and ‘8’ and arranges them in the ascending order to form a list. The first element of the list is 5, the second element 8, the third element 55, the fourth element 58 and so on. Geek asks you at what position in the list is 58885858858858585858858588558885855555558858558 found?  
   **Answer : 205448374861928 Marks : 2**
5. Geek Sundaram likes playing with numbers. He comes across an interesting set of numbers called magical numbers. A number is said to be a magical number if  
    a) The number should be divisible by the sum of its digits and the resulting quotient should be a prime.  
    b) All its prefixes should be divisible by their respective sum of digits.  
   Geek is interested in magical primes. A prime p is said to be a magical prime if p/10 is a magical number.  
   For example, take 8017. 8017 is a prime. 801/(8+0+1) = 89 which is a prime. Also 80 is divisible by 8 and so is 8. Therefore, 801 is a magical number and hence, 8017 is a magical prime.  
   Help Geek find the sum of all the magical primes less than 10^15.  
   **Answer : 3546157627889484 Marks : 3**
6. Lets keep it simple. Geek just wants you to find the number trailing 0s in (1903 ^ 17)!.  
   **Answer : 14073412249547760930585545662280791438492236744049737071 Marks : 1**
7. Geek Sundaram forms a set in such a way that each digit from 1 to 9 occurs exactly once in any one of the set member. Lets call these sets Geek sets. For example, {123, 456, 789}, {1, 54, 698, 73, 2}, {123456789}, {192837465} are all examples of Geek sets. A Geek set is said to be special if all of its members prime. An example of a special Geek set is {2, 7, 31, 84659}.  
   How many special Geek sets are there?  
   **Answer : 44680 Marks : 2**
8. A telecom company hires Geek Sundaram to build a telephone network. The network consists of telephone exchanges located in strategic positions and there may or may not be a connection between a pair of exchanges. Geek Sundaram submits the following report to the company:  
   *I have finished building the network. Each exchange has exactly five connections going out of/coming into it. The total number of connections in the network is 145.*  
   Help the manager of the company figure out the number of exchanges Geek built.  
   **Answer : 58 Marks : 1**
9. There are 100000 Sodexo coupons in Geek Sundaram’s laptop bag. Every time he opens his bag, he takes out a random number of coupons between 1 and the number coupons remaining in the bag (inclusive). For example, he first picks a random number of coupons between 1 and 48. Say he picks 21 coupons. In the next pick, there are 27 coupons left. So, he picks some number of coupons between 1 and 27.  
   On an average, how many picks are necessary to empty the bag of coupons?  
   (Enter your answer with a precision of 5 after the decimal point)  
   **Answer : 12.09015 Marks : 2**
10. Geek Sundaram is very interested in finding the index of a fibonacci number in the sequence. The number is:  
     13364717134737417754062593861828404531263435190428108114698768622357646897629468257016987943468611723750750644872353505803227785959782061868465300406943681221329363205566052252297  
      
    Help Geek find its index.  
    **Answer : 854 Marks : 1**
11. Find the sum of the first 1000 Primonacci numbers. Two consecutive numbers a and b of the fibonacci sequence belong to the primonacci sequence if  
     a) the two consecutive numbers are prime  
     b) their difference is non-prime  
    **Answer : 0 Marks : 2**
12. Scaling mountains fraught with unpredictable perils is a passion for most people. People usually start from the base of the mountains and reach the top.  
    But this was too mainstream for Geek Sundaram. He decided to start from the peak and reach the bottom. (Nobody knew how he reached the top, though rumours suggest that he stole Batman's cape :P)  
    There were so many routes from the top to the base, each costing some health.  
    Given the route and the damage each route causes, help Geek find his health when he reaches the base. Assume Geek's initial health as 500.  
    [The Mountain](http://cs.annauniv.edu/img/BM/mountain.txt)  
    Note that Geek can only move from one point to the two of its adjacent points in the bottom.  
    **Answer : 12 Marks : 2**
13. During early life, Geek Sundaram used to be a bus conductor before becoming the man he is now.  
    During his life as a conductor, Geek expected his passengers to provide exact amount of money for the ticket. However, some people boarded the bus without the exact amount just to annoy Geek. There were N people in the bus with Rs.10 and M people with Rs.5. The cost of the ticket was Rs.5.   
    Being a geek, he wondered if he can somehow arrange the passengers such that he can always provide change. Output the probability upto 6 decimal digits.  
    Assume M = N = 101.  
    **Answer : 0.009804 Marks : 2**
14. Geek Sundaram has been a great father. He has n children with whom he plays game every now and then. (Beware, the number n can be very large.)  
    One day during snacks time, he arranged his n children in a circle and numbered them in clockwise order. He had m momos. He stood in the middle of the circle and started giving momos from child 1 moving clockwise. The child i gets i momos. If Geek is not able to give any of his child the required number of momos, then he eats the remaining momos and completes the snacks session.  
    Determine the number of momos Geek eats. Assume n = 999999999 and m = 999999999999999999.  
    **Answer : 38439 Marks : 2**
15. After getting hit by a football on the head, Geek went mad and started walking in the path as given in [this image](http://cs.annauniv.edu/img/BM/spiral.png). Each second he takes a step in the direction as given by the image. (There must be no overlaps.)  
    Given a list of time, calculate the direction in which Geek will move in the next second.  
    For example,  
    At time 0, he will move North (N)  
    At time 1, he will move East (E)  
    At time 2, he will move South (S)  
    At time 3, he will move South (S)  
    and so on...  
    Find Geek's next direction at time   
    9  
    12  
    801803  
    12345678  
    Give the output as a single string containing the first letter of each direction (use uppercase characters).  
    **Answer : ESES Marks : 1**
16. Start with a positive integer ‘n’. Replace that number by the sum of squares of it’s digits and keep repeating that process until the result becomes ‘1’ (i,e, the loop will end here). Or else it will run endlessly in a loop and will never reach 1.   
    For e.g, 1st number in such a series is 1 itself. 2nd number is 7 and it goes on. So your task is to find the 1234567th number in this series.  
    **Answer : 8754253 Marks : 2**
17. Dr.Bash designs a robot that makes N turns.  
     On the first turn, the MVPD goes 1 unit to the right.  
     On the second turn, the MVPD goes 2 units up.  
     On the third turn, the MVPD goes 3 units to the left.  
     On the fourth turn, the MVPD goes 4 units down.  
     On the fifth turn, the MVPD goes 5 units to the right and this goes on for N turns.  
    Find the coordinates of his robot at 999999993th turn. Give the answer in the format   
    “<X-coord><single space> <Y-coord>”  
    **Answer : 499999997 -499999996 Marks : 1**
18. The following brainf\*\*k program prints “HQ” as it’s output when given the appropriate input. Refer [here](https://en.wikipedia.org/wiki/Brainfuck) to know about brainf\*\*k.  
    Program:  
      
    ,>++++++[-<-------->]<[>++++++++>+++++++++<<-]>.>.  
      
    As mentioned in the article, ‘,’ refers to the input. Your task is simple here. Find what input should be given to the above program so that it will print the mentioned string “HQ”.  
    **Answer : 9 Marks : 2**
19. Geek wants to know if there is a 200 digit number consisting only of the digits ‘6’ and ‘7’ which is a multiple of 2^200. If yes, enter the number. Otherwise, enter 0.  
    **Answer : 77766767767766666666767766666777767666676776667676666777776676766676666766676766777676667677766776677676776676776776776667666667776766677767667676666776766667777767666677766776777777777777666766667776 Marks : 3**
20. Find the last two digits of the sum of the following series

*1* ***601803788*** *+ 2* ***601803788*** *+ 3* ***601803788***  *+ … + 922368136* ***601803788***

**Answer : 64 Marks : 2**