

## Poisson Distribution Definition

The Poisson distribution is a discrete probability function that means the variable can only take specific values in a given list of numbers, probably infinite. A Poisson distribution measures how many times an event is likely to occur within “x” period of time. In other words, we can define it as the probability distribution that results from the Poisson experiment. A Poisson experiment is a statistical experiment that classifies the experiment into two categories, such as success or failure. Poisson distribution is a limiting process of the binomial distribution.

A Poisson random variable “x” defines the number of successes in the experiment. This distribution occurs when there are events that do not occur as the outcomes of a definite number of outcomes. Poisson distribution is used under certain conditions. They are:

- The number of trials “n” tends to infinity
- Probability of success “p” tends to zero
- $np = 1$  is finite

## Poisson Distribution Formula

The formula for the Poisson distribution function is given by:

$$f(x) = (e^{-\lambda} \lambda^x) / x!$$

Where,

e is the base of the logarithm

x is a Poisson random variable

$\lambda$  is an average rate of value

## Poisson Distribution Examples

An example to find the probability using the Poisson distribution is given below:

### Example 1:

A random variable X has a Poisson distribution with parameter  $\lambda$  such that  $P(X = 1) = (0.2) P(X = 2)$ . Find  $P(X = 0)$ .

### Solution:

For the Poisson distribution, the probability function is defined as:

$$P(X = x) = (e^{-\lambda} \lambda^x) / x!, \text{ where } \lambda \text{ is a parameter.}$$

$$\text{Given that, } P(x = 1) = (0.2) P(X = 2)$$

$$(e^{-\lambda} \lambda^1) / 1! = (0.2)(e^{-\lambda} \lambda^2) / 2!$$

$$\Rightarrow \lambda = \lambda^2 / 10$$

$$\Rightarrow \lambda = 10$$

Now, substitute  $\lambda = 10$ , in the formula, we get:

$$P(X = 0) = (e^{-\lambda} \lambda^0) / 0!$$

$$P(X=0) = e^{-10} = 0.0000454$$

Thus,  $P(X=0) = 0.0000454$

### Example 2:

Telephone calls arrive at an exchange according to the Poisson process at a rate  $\lambda = 2/\text{min}$ . Calculate the probability that exactly two calls will be received during each of the first 5 minutes of the hour.

### Solution:

Assume that “N” be the number of calls received during a 1 minute period.

Therefore,

$$P(N=2) = (e^{-2} \cdot 2^2)/2!$$

$$P(N=2) = 2e^{-2}.$$

Now, “M” be the number of minutes among 5 minutes considered, during which exactly 2 calls will be received. Thus “M” follows a binomial distribution with parameters  $n=5$  and  $p=2e^{-2}$ .

$$P(M=5) = 32 \times e^{-10}$$

$P(M=5) = 0.00145$ , where “e” is a constant, which is approximately equal to 2.718.

## What is a Poisson distribution?

A Poisson distribution is defined as a discrete frequency distribution that gives the probability of the number of independent events that occur in the fixed time.

## When do we use Poisson distribution?

Poisson distribution is used when the independent events occurring at a constant rate within the given interval of time are provided.

## What is the difference between the Poisson distribution and normal distribution?

The major difference between the Poisson distribution and the normal distribution is that the Poisson distribution is discrete whereas the normal distribution is continuous. If the mean of the Poisson distribution becomes larger, then the Poisson distribution is similar to the normal distribution.