(A) Basics of Image & Convolution

Image can be seen as a 2D function can be plotted in 3D.

$$\text{value} = I(x, y)$$

V_1 (1,1)	V_2 (1,2)	V_3 (1,3)	V_4 (1,4)
V_5 (2,1)	V_6 (2,2)	V_7 (2,3)	V_8 (2,4)
V_9 (3,1)	V_{10} (3,2)	V_{11} (3,3)	V_{12} (3,4)
V_{13} (4,1)	V_{14} (4,2)	V_{15} (4,3)	V_{16} (4,4)

$[I]$

$\therefore$ $V_6 = I(2,2)$

$V_{15} = I(4,3)$

(a) These (V_i)'s can be 0/1

$\Rightarrow$ Binary Image
off/ON

1	0	0
0	1	0

Often called as BLACK/WHITE image.

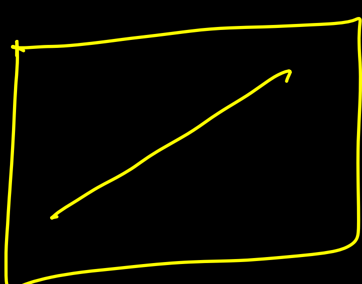
(b) These (V_i's) can be a value b/w (0 – 255)

Basically 8 bit-value per pixel

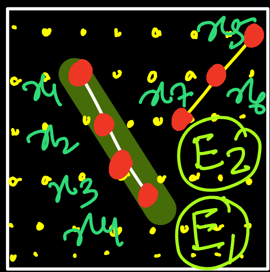
0 0 0 0 0 0 0 0 $\Rightarrow$ 0 (Black)
0 0 0 0 0 0 0 1
⋮
1 1 1 1 1 1 1 1 $\Rightarrow$ 255 (White)

acquiring the full gray level spectrum

$\Rightarrow$ Such images will be called as gray level images.



(*) What are things that are important in an image.



(*) do all pixels are useful/important

(*) Only few of them that have edges

Edges $\Rightarrow$ Changes $\Rightarrow$ gradient/ high derivative

(*) Do pixels on edges have same information?

x_1, x_2, x_3, x_4 all are equally important?

$\{x_1\ x_2\ x_3\ x_4\}$ share edge E_1 $\{x_5, x_6, x_7\}$ share edge E_2

Do they share information within their set/group?

Do they share information across the set/group?

$\Rightarrow$ Each of (x_i) have one thing common "They have high derivative"

But in which direction (x) or (y).

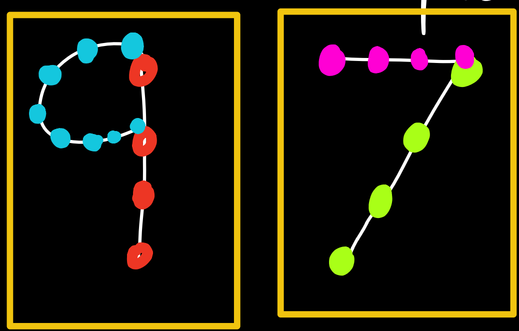
$\therefore$ Along with the magnitude of the gradient pixel will also have a direction.

$$I(x, y) \;;\; \nabla_x I(x, y),\; \nabla_y I(x, y)$$

$\longleftrightarrow$ $\updownarrow$

Similar to $\leftrightarrow$ & $\updownarrow$ Can we have directional derivatives, may be like $\nwarrow\!\searrow$ $\swarrow\!\nearrow$

(*) Can we design a filter/function that can compute these directional derivatives



$D(\leftrightarrow)$, $D(\updownarrow)$, $D(\nwarrow\!\searrow)$, $D(\swarrow\!\nearrow)$

$\Downarrow$ Yes

using Convolution.

One can observe that this directional information is important for learning the discriminative features

(Responce is saved in a seperate signal)

These filters are used to ask specific questions to each pixel.

$D(\leftrightarrow)[x, y] \Rightarrow R(x, y)$

(DO you belong to a horizontal line?)

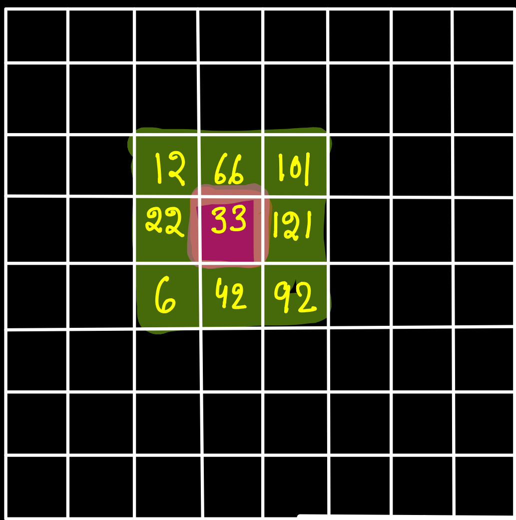
(B) CONVOLUTION

$$\underbrace{I(x, y)}_{\text{Signal Convoluted using } F(x,y)} \circledast F(x, y) \Longrightarrow \text{Responce}$$

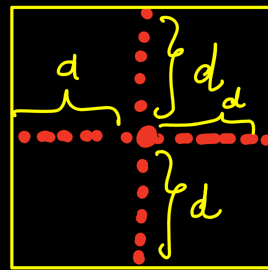
$F(x, y) \Rightarrow$ Pixel level operator, (Computed wrt a neighbourhood)

Let $I(x,y) = m \times n$ & $F(x,y) = (K \times K)$

where $(K = 2d+1)$

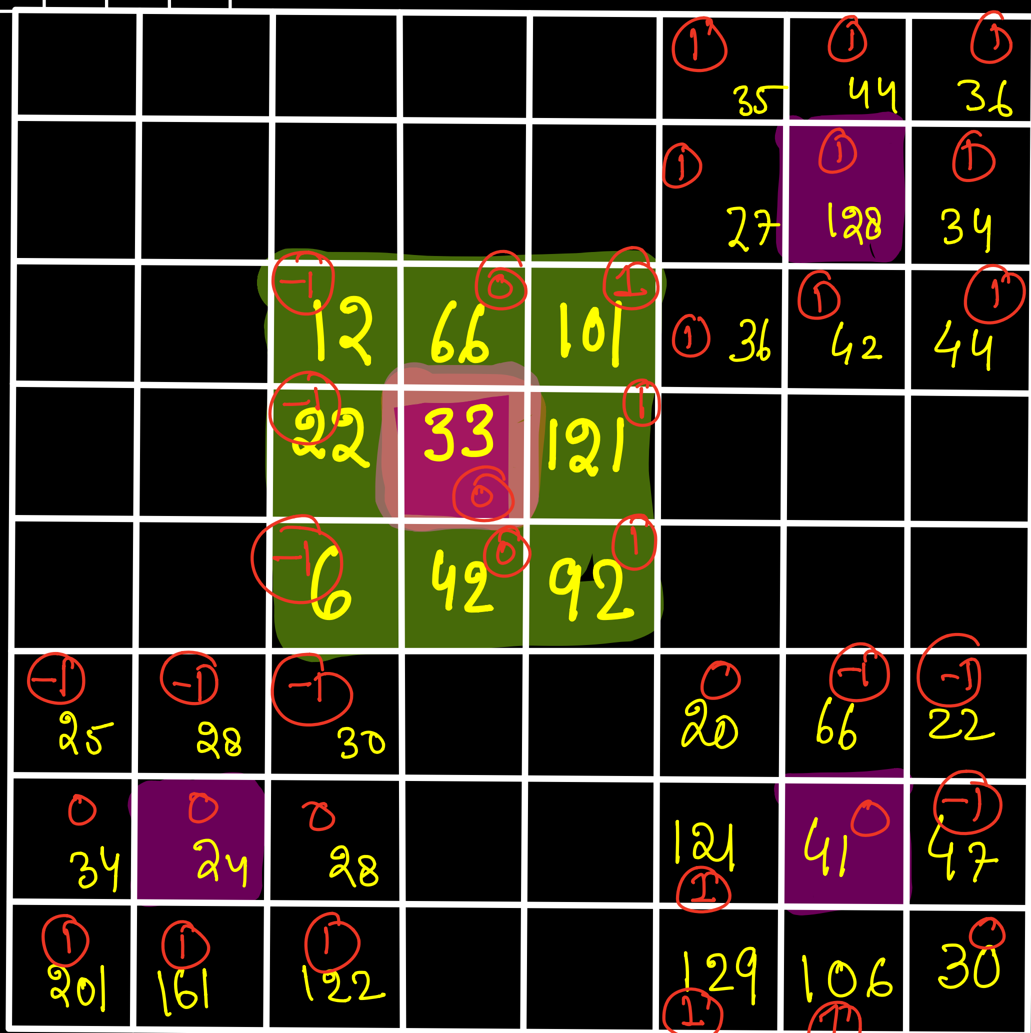


$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$



(*) What is your (x) gradient w.r.t (3x3) neighbourhood.

Do you have a pattern like this.



$-12 \; +101$

$-22 \; +121$

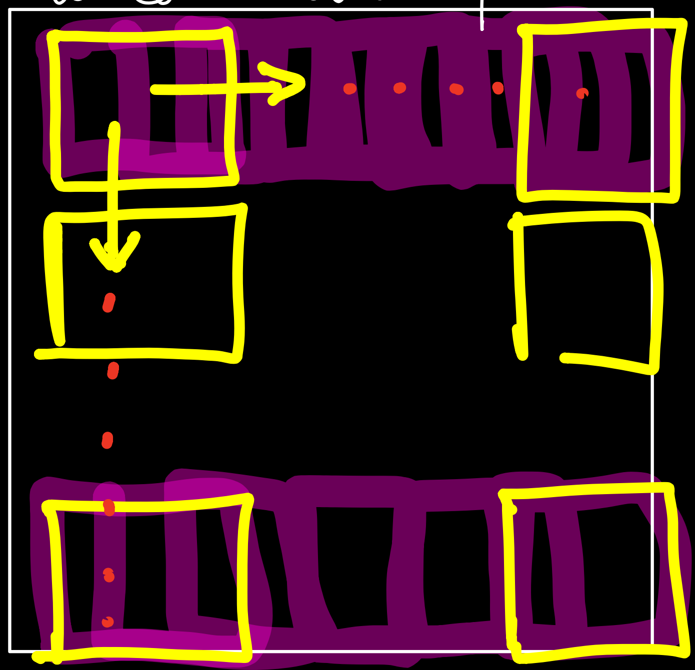
$-6 \; +92$

$\Rightarrow$ Some high value implies that Yes it has the pattern.

Hence one can see very useful questions can be embedded in these filters/Kernels.

$$\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

Full Convolution operation is like (Sliding Window)



if image is 128x128
& filter is (3x3)

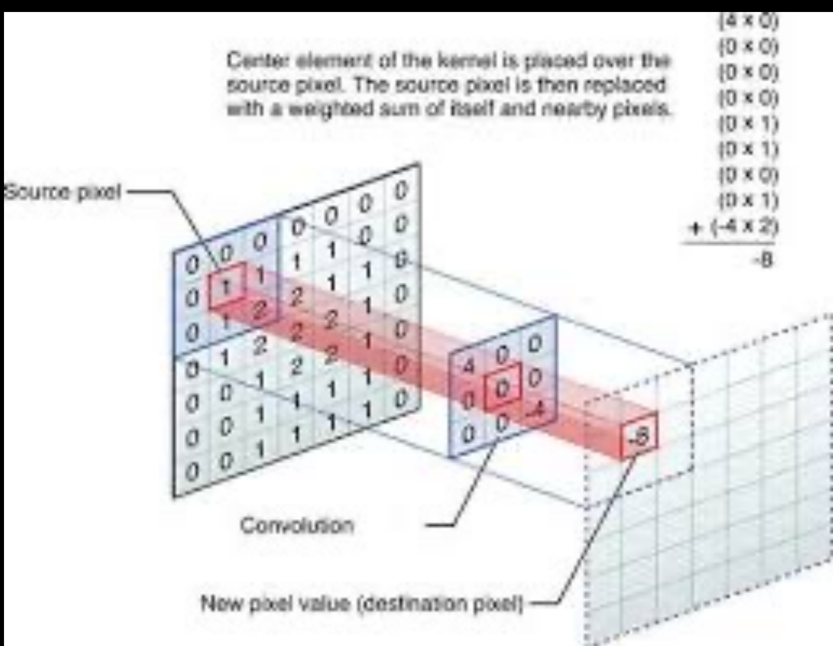
O/P $\Rightarrow$ (126 x 126)
will be the o/P

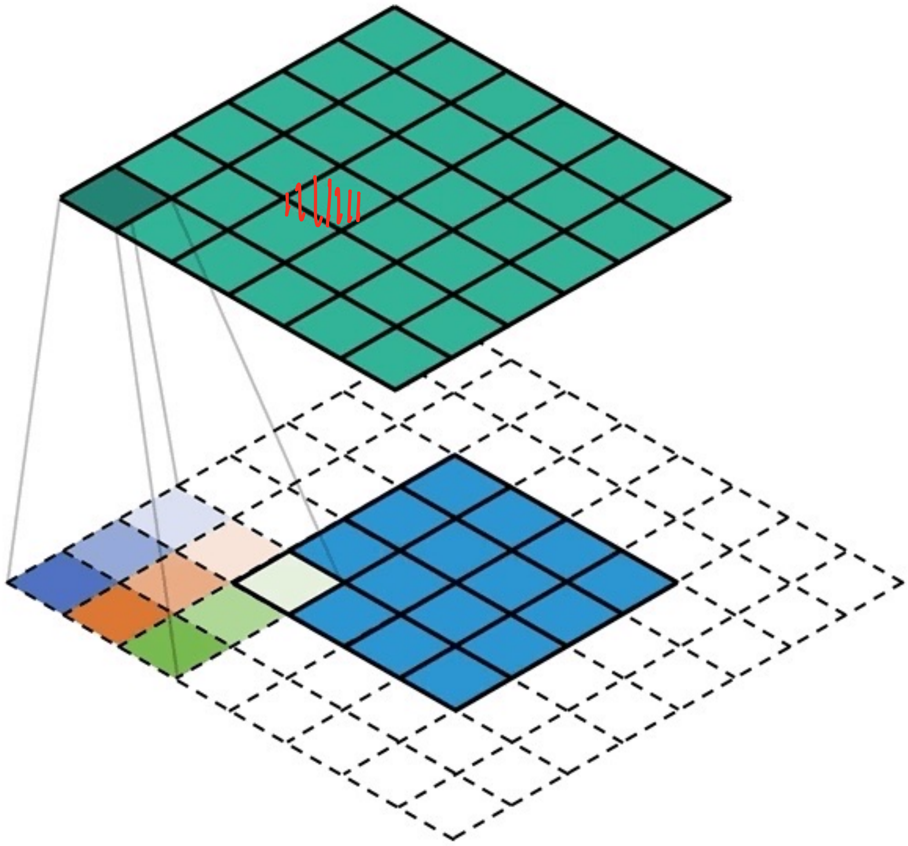
top & bottom Row &
left & right Column do
not have full neighbourhood.

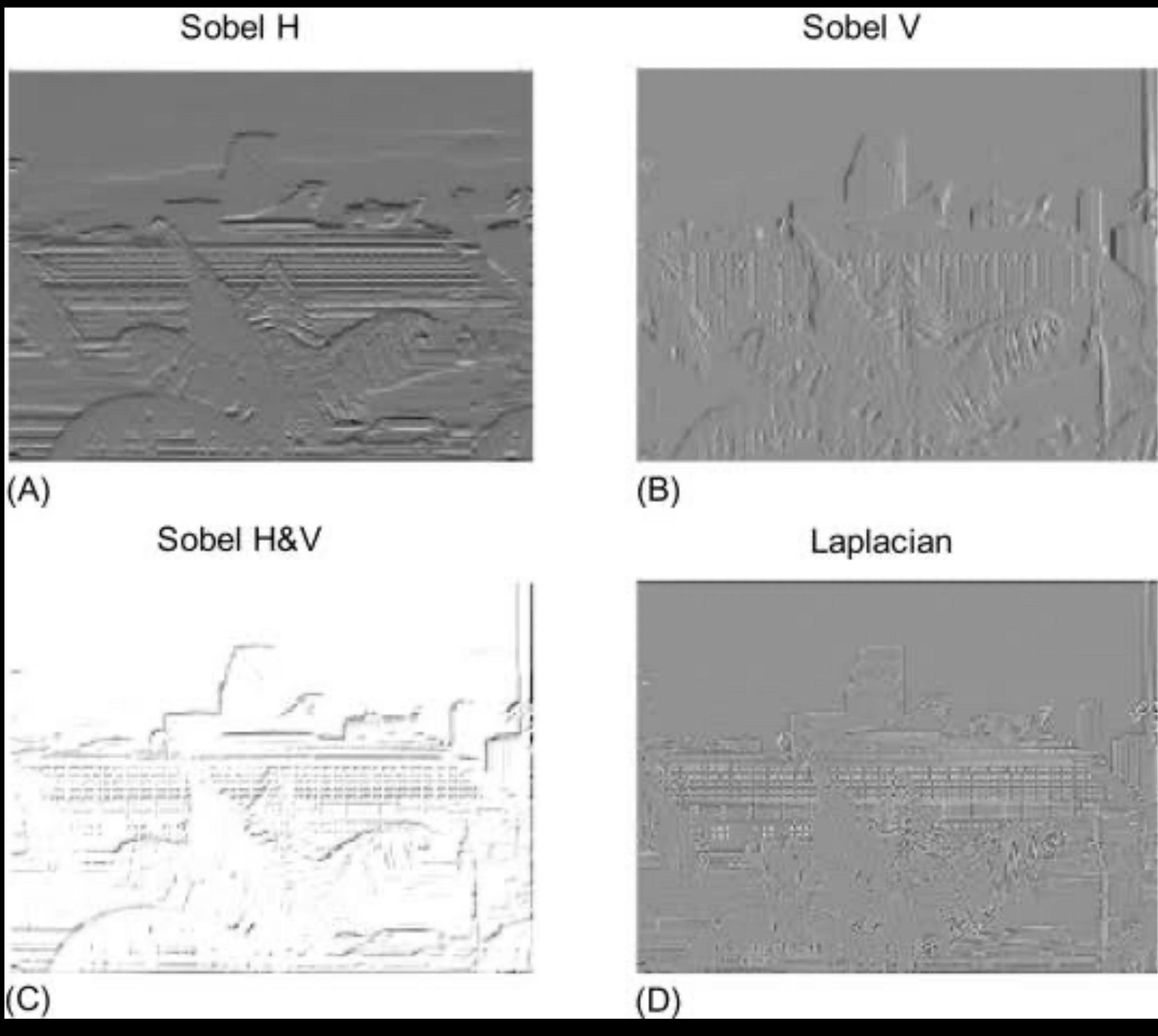
* There are several ways to manage Boundary Conditions
  - Ignore
  - Zero padding

Hence

$$\boxed{I(m,n) \circledast F(K \times K) \Rightarrow R(m,n)}$$






Sobel H
Sobel V
(A)
(B)
Sobel H&V
Laplacian
(C)
(D)

I → Features → Classifier

I

Feature extraction $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} \Rightarrow$ Classifier

Image proc

Computer vision

Manually

Handcrafted

$X \Rightarrow [x_1 - x_n]$

Bishop

Duda Hart

Conv NN = F.E + Class

Conv