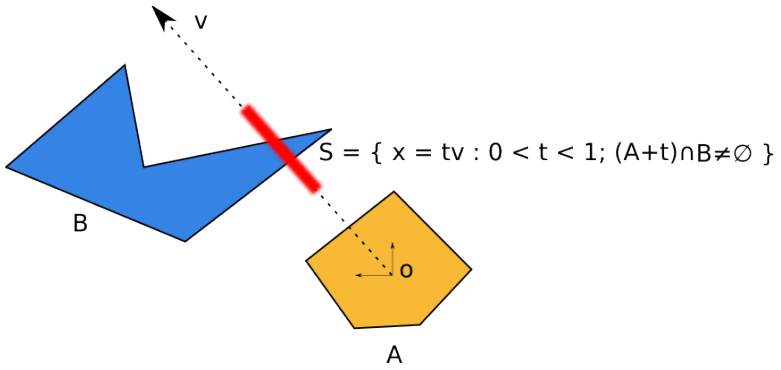
ME/CS558: Homework 5

1 Overview

For the final homework in this class, you will implement part of a basic 2D motion planning system. Your goal will be to find the amount of time that two bodies are in collision when traveling along a unit speed linear trajectory. One of these bodies will be specified by a convex polygon, A, while the other will be an arbitrary simple polygon, B. The trajectory will be given by a single translation vector, v, describing the movement of A in the plane relative to B. Visually, we can think of the situation as follows:



Your job will be to give find the amount of time that A intersects B assuming that A moves along v with unit speed until the end of the vector. The subset of the path that A spends in B is defined to be a set S such that,

$$S = \{x = tv : t \in [0, 1], (A + x) \cap B \neq \emptyset\}.$$

Since S is at most 1-dimensional, it has a well defined length measure given by the arc length. Your job will be to find the length of S.

2 Input

The input to your program will begin with a pair of floating point numbers representing $v = (v_x, v_y)$, where $-100 \leq v_x, v_y \leq 100$. This will then be followed by a pair of integers, n, m, with $3 \leq n \leq 20$ and $3 \leq m \leq 30$ which represent the number of vertices for A and B respectively. After this, there will be n pairs of floating point numbers, $-100 \leq a_{xi}, a_{yi} \leq 100$, with $0 \leq i < n$, representing the coordinates of the vertices of A given in counter clockwise order, starting with the vertex with lowest y-coordinate (which you may assume to be unique). Following this, there will be m pairs of floating point numbers, $-100 \leq b_{xj}, b_{yj} \leq 100$ representing the vertices of B, again given in counter-clockwise order.

2.1 Example Input

```
10 10
5 5
```

```
-1 -1
-2 1
-1 2
1 1
1 0
4 1
4 5
7 2
10 5
10 1
```

3 Output

Your output should be a single floating point number representing the length of S, and should be accurate to 8 significant digits.

3.1 Example Output

```
6.36396103
```

4 Written Assignment

1. Describe your proposed method and its time complexity as a function of m and n.
2. Suppose that you wanted to check if a piecewise linear translational trajectory for the body A was feasible, or in other words you want to determine if A collides with B at any point along the path. How would you modify your code to efficiently answer this question?
3. What if instead of being given a trajectory, you were instead given the start and end configuration of A and asked to find a trajectory of shortest length to move A to the desired point. How would you compute such a path in strongly polynomial time with respect to A, B?

5 Extra Credit

1. *+10 % (Programming)* Implement a program which works correctly if A is an arbitrary simple polygon.
2. *+10 % (Written)* Suppose that in addition to moving at a constant linear velocity, that A was also spinning with a constant angular velocity as well. How would you *detect* if A intersects B along such a trajectory in strongly polynomial time[1] with respect to n and m?

6 Grading

As usual, there will be 50% for the written assignment and 50% for the programming assignment. Good luck.

[1] This means the answer should be independent of the magnitude of any numerical quantities.